WP 2016-09
April 2016

Working Paper

The Charles H. Dyson School of Applied Economics and Management
Cornell University, Ithaca, New York 14853-7801 USA

Capital Flows, Beliefs, and Capital Controls*

Olena Rarytska[†] Viktor Tsyrennikov[‡]

Capital Flows, Beliefs, and Capital Controls*

Olena Rarytska† Viktor Tsyrennikov‡

April 28, 2016

Abstract

Belief heterogeneity generates speculative cross-border capital flows that are much larger than flows generated by the hedging/insurance motives. We show theoretically that limiting financial trades may generate welfare gains despite inhibiting insurance possibilities. Financial constraints tame speculation forces, limit movements of the net foreign wealth positions, and thus reduce consumption volatility. This provides a novel justification for capital controls.

Simulations indicate that welfare gains from imposing capital controls can be substantial, equivalent to a permanent consumption increase of up to 4%, or 80 times the cost of business cycles. Controls that activate only during substantial inflows or outflows are preferred to those constantly active, e.g. a transaction tax used by some emerging market economies. Yet, despite improving macroeconomic stability capital controls may unintentionally lead to increased volatility in the domestic financial markets.

Keywords: international portfolios, capital controls, foreign exchange intervention

JEL codes: F32

*We would like to thank Olivier Blanchard, Olivier Jeanne, Enrique Mendoza, seminar participants at the IMF research department, Georgetown University, NYU.

†email: elena-rarytska-at-gmail.com. Dyson School of Management, Cornell University

‡email: viktor.tsyrennikov-at-gmail.com. Promontory

1 Introduction

Recent episodes of rapid capital inflows and exchange rate appreciation had led governments in emerging market economies to impose capital controls or contemplate other forms of intervention. Government officials argued that these flows were speculative in nature and, hence, inefficient. Yet, none of the current models feature speculation. Most of the existing explanations are built upon models with some form of an externality that motivates intervention. Our model features no externality and instead we focus on the informational friction, namely heterogeneous beliefs in the spirit of Harrison and Kreps (1978), that enables me to model speculative flows. We start with the complete financial markets benchmark and highlight the issues that may arise when beliefs are heterogeneous. One could think of our model as of the U.S. financial sector betting on an emerging economy like Brasil or Mexico. Regardless of investors' beliefs they would assume large gross portfolio positions allowing them to profit substantially in case their predictions turn out to be correct. Investors are searching for high yield and they are willing to accept volatile consumption in anticipation of future financial reward. The excessive risk-taking, among other things, leads to volatile capital flows, exchange rates, and asset prices. Persistent capital flows can suddenly reverse and a steadily appreciating currency can plummet when a country's luck runs out. In turn, large exchange rate swings could impoverish the economy that receives capital inflows. All of these phenomena are driven by movements of NFW that would be constant had beliefs were homogeneous.

Capital flows in our model are speculative in nature because they are based on conflicting predictions of future events. Yet, these trades occur alongside traditional investment and hedging and often it is impossible to distinguish one from another. Despite this fact, we show that it is still desirable to impose capital controls even if it could harm insurance and/or investment opportunities.

The cost of speculation comes from a particular economic mechanism: it creates a volatile wealth distribution with a substantial tail "stickiness." That is when wealth is distributed evenly speculation is prevalent and there are large and frequent wealth transfers. If some economy comes to command a large wealth share, speculation slows and wealth transfers fade. This occurs because the wealthier economy has a disproportionate effect on asset prices pushing the latter closer to what is country's subjective valuation. For this reason the wealthier economy has little incentives to speculate in financial markets. Those who lost wealth believe that there is room for arbitrage as prices move away from their subjective valuations. But having little wealth poor economies cannot take large positions and, moreover, they must exercise caution not to lose more wealth. That is as financial luck favors some market participants the world economy looks more as a homogeneous beliefs economy. But in a homogeneous beliefs economy relative wealth positions persist over time. That is, if someone suddenly revealed the true process and speculation vanished then wealth positions would be instilled perpetually. Some economies would be permanently stuck in poverty. Friedman (1953) foresaw in his "survival hypothesis" that agents that are better able to predict the economic environment will prosper and the rest will be driven out of markets. What Friedman did not foresee is that transitional dynamics of wealth may be very erratic and often those with less-accurate beliefs may dominate the market, even if temporarily. Moreover, if disagreement persists uneven wealth distribution can persist for long time. This is the ultimate cost of speculation. Different capital management measures could limit the tail behavior of the wealth distribution and bring about substantial welfare gains equivalent to permanently increasing consumption by up to 4%, 80 times larger than the cost of business cycles in the same setting. Our estimates show that only a third is due to increased volatility caused by speculation.

We would like to emphasize the fact the aim of regulation is to restrict tails of the world wealth distribution. This can be done by ruling out large wealth losses by imposing capital flow or net foreign wealth limits. A transaction tax would increase the time that is needed to pass before any country could lose a substantial amount of wealth, thus postponing periods of low consumption and increasing welfare. The two measures achieve similarly high welfare levels, but this statement is conditional on countries starting out with equal financial wealth at the time of capital controls implementation. If the decision is made after a large capital outflow it is best to impose a capital outflow limit rather than tax financial trades. The reason is that the first measure would prevent the financial position of the damaged economy from worsening. The second measure would not, and, moreover, it would reduce speed at which lost wealth could be regained.

The rest of the paper starts with a description of the key results on survival in financial markets with heterogeneous beliefs and a brief overview of other work on capital controls. In section we describe the model and the key economic forces. Section 3 presents a motivating example and section 4 states the key theoretical result. Section 5 analyzes several alternative forms of controls. We conclude by proposing avenues for future work.

1.1 Key results from survival literature

It is instructive to introduce several key results from the literature on heterogeneous beliefs that this work builds on. All of these results were developed in the context of bounded endowment economies with complete, that is unregulated, financial markets. It is also crucial to point out that all information is common. That is agents understand that others have different beliefs which constitutes a departure from the rational expectations paradigm, an issue that We discuss later. In this setting, with complete financial markets and heterogeneous beliefs, any agent who has less-accurate information, as measured by relative entropy, is going to be driven out of the market. That is his consumption converges to zero on all paths of events except for a small set that has zero probability. This result was shown by Blume and Easley (2006) and, in a different form, in Sandroni (2001). Blume and Easley (2009) show that if all agents have equally accurate information then each agent will infinitely often have consumption that is arbitrarily close to zero. In either case, if the period utilities are unbounded below the above statements imply that agents will certainly experience a dismal flow of utility. In section 4 we show that the same analysis applies in the multi-good environments. Regulated financial markets are desirable because they do not allow for the full effect of survival forces. But belief heterogeneity *per se* does not mandate regulation of financial markets as the competitive equilibrium allocation is *subjective*-Pareto optimal after all. But as argued in Blume et al. (2014) the subjective Pareto criterion may lead to unreasonable social choices in environments with heterogeneous beliefs. Similar arguments are made by Brunnermeier et al. (2013) and Gilboa et al. (2012).

That is where the welfare criterion becomes important. Following Blume et al. (2014) we assume that neither the planner-regulator nor any agent knows the true evolution of the world. Every agent believes that he has the most accurate information. Everyone's belief is publicly known. But it is impossible to determine with certainty whose beliefs are more accurate for it is impossible to separate the effect of luck. If the planner-regulator knew the true process he should share this information with all the agents. The decision about the set of restrictions on financial markets has to be made without knowledge of the true evolution of the world. All that is known is that the true process comes from the same pre-specified set as the agents' beliefs. That is the set of financial restrictions is chosen behind the veil of ignorance: never knowing the truth. This argument introduces a new degree of freedom: the set of admissible beliefs. The larger it is the stronger disagreement between agents can be and, therefore, the faster agents can loose their wealth. So, more disagreement provides more incentives to regulate financial markets. We discuss ways to choose this set in section 5.

We assume that there is no learning. Learning from exogenous events is prohibited to separate the relevant forces in an already rich environment. Several defenses can be offered. The most plausible, in our view, is that the true process is more complicated than what agents believe in.[1] In this case, beliefs will likely never settle and disagreement will persist. Learning from prices is also unrealistic in large anonymous markets. Finally, prices would be uninformative if agents had private investment opportunities as in Albuquerque et al. (2007).

The survival forces borne in the environments with disagreement are consistent with many realistic phenomena. The most important are capital flow and exchange rate volatility. It is well-known that despite strong home bias in countries' equity portfolios observed cross-border capital flows are substantial and volatile. This makes financial trading costs an unlikely friction behind the observed lack of diversification for then capital flows would be small and relatively stable. In our model the home equity bias co-exists with large and volatile capital flows. At the same time observed exchange rates are very volatile despite a relatively stable supply of goods. In contrast, with homogeneous beliefs and complete financial markets the real exchange rates are determined by the relative supply of goods and are generally smooth. In our model, exchange rates are affected by belief heterogeneity directly via their influence on the equilibrium pricing kernel and indirectly via their effect on the world's wealth distribution.

Belief disagreement is observable to some degree. One possibility is to compare published forecasts of various institutions. For example, on December 7, 2014 Agence France-Presse reported that Troika considered the 2.9% forecast growth too optimistic and predicted a 3% budget deficit against the Greeks estimate of 0.2%. On September 22, 2014 Brazils central bank released its GDP forecast of 0.7% and two days later Morgan Stanley based in the U.S. released their own forecast of 1.0%. However, we are not aware of any systematic data analysis. Another possibility is to study professional

[1] We would like to thank David Easley for this suggestion.

forecast surveys. Unfortunately, no survey known to me asks about performance of different economies. Disagreement can be gauged indirectly from trades between market participants. Consider an environment with income realizations that are independent across time. In this case, if the beliefs were homogeneous, the maximum trade that one could observe in equilibrium would be bounded above by the size of the income support. Any trade that is larger in size must be speculative. We take this path in a different work. Here we only demonstrate that a failure to recognize belief heterogeneity may lead to substantial welfare losses. For our calculations we bound belief differences by the statistical uncertainty that is present in a typical macroeconomic series.

1.2 Related work

From the modelling point of view the closest work are Albuquerque et al. (2007) and Dumas et al. (2014). They too analyze endowment economies with heterogeneous beliefs. The model in Albuquerque et al. (2007) explains several important facts about the U.S. capital flows: two-way flows, momentum in equity positions, and return chasing. The key element of this model is heterogeneous financial sophistication of investors in each country. The more-sophisticated agents take a different position in foreign assets than the less-sophisticated ones and this generates two-way capital flows. Momentum and return chasing are driven by an endogenous level of investment in local equity markets by the more-sophisticated agents. Dumas et al. (2014) address a different set of facts: co-movement of returns and international capital flows, home equity bias, dependence of firm returns on home and foreign factors; and abnormal returns around foreign firm cross-listing in the home market. Both papers analyze one-good settings, ignoring fluctuations in the exchange rate that is a major factor determining capital flows in our view. Neither of these papers considers regulation of capital flows, which is the main objective if this paper.

Other justifications of capital controls in the literature are based either on pecuniary, as in e.g. Jeanne and Korinek (2010) and Bianchi and Bengui (2011), or demand externalities, as in Farhi and Werning (2013). Like Farhi and Werning (2013) we study complete financial markets. But the setting here could be viewed through the lens of missing markets, very much like in the overlapping generations (OG). While in an OG setting unborn agents are excluded from the financial markets, in our work agents cannot insure against a possibility of being born with inaccurate beliefs. This market is missing because neither planner nor any other agent in the economy has an ability to testify to a belief's accuracy. So, it is a model of capital controls under imperfect information. Our model features a novel for this type of models mechanism – survival. As long as there is disagreement, all or some agents, by poor luck or by inability to process financial information, will loose substantial amounts of wealth in financial markets. The survival mechanism may lead to unexpected consequences if there are other frictions in the model as in Cogley et al. (forthcoming). In this way this work is complementary to the vast literature on macro-prudential policy.

2 The Model

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. The exogenous state of the economy z_t is a first-order Markov process with finitely many states, $\mathcal{Z} = \{1, ..., S\}$, and a probability transition matrix Π^0. Initial state z_0 is given. A partial history of the state realizations $(z_0, ..., z_t)$ is denoted by z^t and its probability by $\pi(z^t|z_0)$.

There are two countries, each is populated by a representative consumer-investor.

There are two perishable goods traded every period. Country i produces good i that is traded at price $p_i(z^t)$.

Financial markets are dynamically complete. In each date and history financial markets trade S Arrow securities. An Arrow security j that is purchased in period t pays one unit of account in period $t+1$ if state $z_{t+1} = j$ realizes. The price of Arrow security j is denoted by $Q_j(z^t)$.

The portfolio of Arrow securities purchased by country i is denoted by $a^i(z^t) \equiv (a^i_1(z^t), ..., a^i_S(z^t))$. The initial distribution of financial wealth $(a^1(z_0), a^2(z_0))$ is given.

Household in country i trades in financial and goods markets to maximize the expected life-time utility given by:

$$E^i\left[\sum_{t=0}^{\infty}\beta^t u(g^i(c_1(z^t), c_2(z^t)))\bigg| z_0\right], \ \beta \in [0, 1). \quad (1)$$

Function g^i is a constant return to scale (CRS) consumption aggregator. We assume that households' display *consumption home bias*. Hence, consumption spending in country i is biased towards the domestically produced good i. In the case of the constant elasticity of substitution (CES) aggregator this assumption is isomorphic to assuming that trading is subject to a proportional cost that is rebated back to consumers.

Assumption 1.
$u(c)' > 0, u(c)'' < 0, \quad \forall c > 0.$

Household in country i receives $e_i(z_t)$ units of domestic good i.

Assumption 2.
There exists $\underline{e} > 0$ *such that* $e_i(z) > \underline{e}, \quad \forall z \in \mathcal{Z}, i \in \{1, 2\}$.

The above assumption guarantees that the utility in financial autarky, defined later, is always bounded below.

Budget constraint of a household living in country i after history z^t is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + \sum_j Q_j(z^t)a_j^i(z^t) = I^i(z^t), \tag{2}$$

where $I^i(z^t)$ is "cash-in-hand" that consists of his non-financial income e_i and the market value of his financial possessions:

$$I^i(z^t) \equiv p_i(z^t)e_i(z^t) + a_{z_t}^i(z^{t-1}). \tag{3}$$

A *competitive equilibrium* is a price system $\mathcal{P} = \{p_1(z^t), p_2(z^t), (Q_j(z^t))_{j=1}^S) : \forall z^t\}$, an allocation $\mathcal{C} = \{(c_1^i(z^t), c_2^i(z^t))_{i=1}^2 : \forall z^t\}$, and a security trading plan $\mathcal{A} = (a^1(z^t), a_2^i(z^t))_{i=1}^2 : \forall z^t\}$ such that:

1. given the price system $\mathcal{P}$, the allocation $\mathcal{C}$ and the security trading plan $\mathcal{A}$ solve each household's optimization problem;

2. financial and goods markets clear: $\forall z^t, j = 1, 2$,

$$c_j^1(z^t) + c_j^2(z^t) = e_j(z^t) \tag{4a}$$
$$a_j^1(z^t) + a_j^2(z^t) = 0. \tag{4b}$$

3 An Example Unregulated Economy

This section sets up a motivating numerical example of an unregulated economy. We study three belief specifications to illustrate the forces shaping the dynamics of wealth of the two countries. In the first setting both countries hold correct beliefs. It is a useful benchmark. In the second setting both countries hold equally-incorrect beliefs. This setting highlights speculative forces leading to large and volatile capital flows. In the third setting only one country holds correct beliefs. This setting demonstrates the survival force that drives the country with inaccurate beliefs towards financial extinction. To make the analysis simpler we assume that the two economies are symmetric under the true data generating process in all cases.

In this section we concentrate attention on the dynamics of net foreign wealth (NFW), real exchange rate (RER), and consumption. In D we describe how welfare changes over a large set of possible beliefs.

3.1 Functional forms and parameter assumptions

For the analysis in this section and other numerical examples in other sections we assume a CRRA utility function and a consumption CES aggregator:

$$u(c) = c^{1-\gamma}/(1-\gamma), \quad \gamma > 0 \tag{5a}$$

$$g^1(c_1, c_2) = (sc_1^\rho + (1-s)c_2^\rho)^{1/\rho}, \quad s \in [0.5, 1], \rho \leqslant 1 \tag{5b}$$

$$g^2(c_1, c_2) = ((1-s)c_1^\rho + sc_2^\rho)^{1/\rho} \tag{5c}$$

where ε denotes the elasticity of substitution (ES) between the two goods. Unless stated otherwise we assume the following preference parameters in our numerical examples:

$$\beta = 0.96, \quad \gamma = 2, \quad \rho = 0.8. \tag{6}$$

The above imply the following aggregate price indices:

$$P^1 = (s^\varepsilon p_1^{1-\varepsilon} + (1-s)^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}, \tag{7a}$$

$$P^2 = ((1-s)^\varepsilon p_1^{1-\varepsilon} + s^\varepsilon p_2^{1-\varepsilon})^{1/(1-\varepsilon)}. \tag{7b}$$

Let $q \equiv p_1/p_2$ and $Q \equiv P^1/P^2$ denote terms of trade and real exchange rate, respectively.

3.2 Both countries hold correct beliefs

In this case the equilibrium allocation and the price system are:

$$(c_1^1(z^t), c_2^1(z^t)) = (fe_1(z_t), (1-f)e_2(z_t)) \tag{8a}$$
$$(c_1^2(z^t), c_2^2(z^t)) = ((1-f)e_1(z_t), fe_2(z_t)) \tag{8b}$$

where

$$f \equiv \frac{s^\varepsilon}{s^\varepsilon + (1-s)^\varepsilon}.$$

Several important observations can be made. Because markets are complete the CE allocation is strongly stationary, namely it is a function of the exogenous state only. Moreover, there is full risk-sharing and the countries consume constant fractions of the world supply of each good. But the countries' aggregate consumption levels are less volatile than their outputs. Net foreign wealth of each country changes, and so capital flows are non-zero, only if the exogenous state changes. Capital flow, when non-zero, equals 0.1% of country 1's GDP and NFW of country 1 fluctuates between -0.06% and +0.06% of GDP. Real exchange rate oscillates between 0.917 and 1.091, and its average value remains constant at 1.

Welfare of each country equals -1.5936. To set a comparison benchmark, we compute cost of business-cycle fluctuations as in Lucas (1978). It is defined as a welfare effect of removing output volatility. If output were fixed at its expected level in each country then welfare of each economy would be -1.5908. This constitutes a gain that is equivalent to permanently increasing consumption by 0.2% and it is of a similar magnitude as found by Lucas (1978). We will use this magnitude as a benchmark for potential welfare losses or gains that are typically obtained in endowment economies like the one considered here.

3.3 Both countries hold incorrect beliefs

We assume that beliefs are symmetric with respect to the truth. So, both countries have equally incorrect beliefs. Namely, we assume that $p^1 = (0.525, 0.475)$, $p^2 = (0.475, 0.525)$. The truth remains the same: $p^0 = (0.5, 0.5)$. So, both economies hold optimistic beliefs about domestic output, although this fact is inconsequential. The mean consumption, NFW, RER remain constant over time. But their variability steadily increases. Figure 7 in E plots key macroeconomic variables. The world economy reaches "stationary state" around period 100, and table 1 column B reports country

	A. correct,correct	B. wrong,wrong	C. wrong,correct
NFW	0.000	-0.306	-0.813
	[0.000,-0.000,0.000]	[1.301,-5.496, 1.974]	[1.331,-5.753, 1.775]
RER	1.000	1.000	0.913
	[0.087, 0.917,1.091]	[0.259, 0.483, 2.070]	[0.214, 0.474, 1.835]
Consumption	0.800	0.781	0.744
	[0.036, 0.764,0.836]	[0.102, 0.418, 1.006]	[0.112, 0.363, 0.992]

Table 1: Selected statistics for the numerical examples described in section 3. In square brackets we report standard deviation, minimum and maximum.

1's key economic statistics in that period. Volatility of each variable is several times larger than in the economy with homogeneous beliefs. Country 1's NFW varies between -550% and 197% of its GDP. RER varies between 0.483 and 2.070. The extreme variation of RER and the strong negative correlation with NFW is the reason for NFW/GDP to be a significant negative value despite NFW being zero on average.

The increase in economic volatility takes its toll on welfare. Country 1's true welfare is -1.6117 which should be compared to -1.5936 in the homogeneous beliefs case. Welfare in country 2 is the same. The loss of utility in the case with heterogeneous beliefs is equivalent to a permanent reduction in consumption of 1.2%, more than 6 times larger than the cost of business cycle fluctuations. It is caused by speculative motives of agents who are willing to accept volatile consumption expecting to profit from financial trade with other market participants. All agents think that they have more accurate beliefs than the rest and along some paths it may look like their strategies pay off. But fewer paths lead to wealth accumulation than agents expect.

Large portion of the welfare loss stems from the fact that NFW becomes "sticky" when it deviates far from 0. When country 1's NFW is zero, that is financial wealth is distributed evenly, speculative trading is at its highest level. When country 1 accumulates wealth it commands a stronger influence on asset prices and valuations shift towards what is implied by its beliefs. For this reason country 1's incentives to engage in speculative trading decline. Country 2's speculative motives strengthen as asset prices deviate from what it considers to be a fair evaluation. However, country 2's wealth declines and so does its ability to speculate. The overall result is a decline in speculative trading and, hence, a decline in wealth fluctuations. That is agents protect themselves from exposure to large financial losses by taking smaller positions. But this also limits the upside potential and reclaiming lost wealth requires more time.

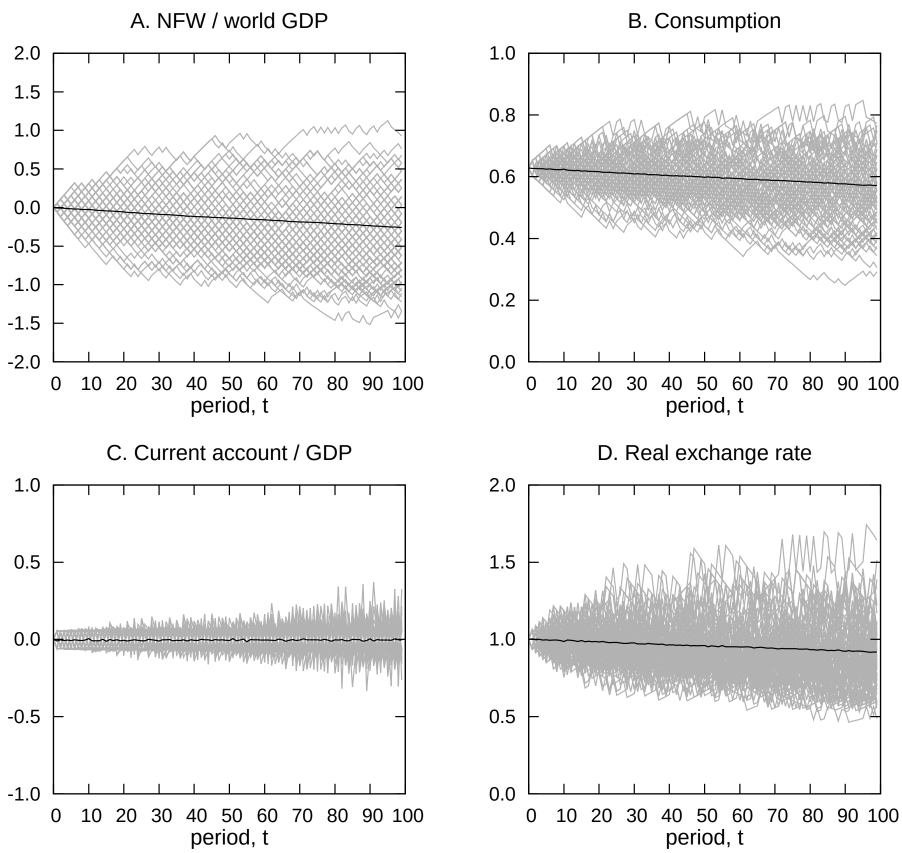

Figure 1: Dynamics of country 1 endowed with less-accurate beliefs. Grey lines represent 100 random paths, black lines are averages across paths.

3.4 Only one country holds correct beliefs

In the final configuration we assume that only country 2 has correct beliefs: $p^1 = (0.545, 0.455)$, $p^2 = (0.5, 0.5)$. These beliefs were chosen so that consumption volatility remains unchanged. The truth remains the same: $p^0 = (0.5, 0.5)$. So, economy 1 holds optimistic beliefs about domestic output, but again this fact is inconsequential. Table 1 column C reports country 1's key economic statistics in that period. Because country 1 has less accurate beliefs its financial wealth trends down. So, the mean consumption, NFW, RER are lower than in column B. Volatility of NFW remains essentially unchanged and that of RER declines 17% relative to column B. Volatility of consumption remains the same by design. Country 1's NFW worst and best case scenarios worsen. Real exchange rate reacts similarly. Country 1's true welfare is -1.6383. The loss relative to the homogeneous beliefs case is equivalent to permanently decreasing consumption by 2.8%. This welfare effect is 16 times larger than the cost of business cycles in the same setting.

Because consumption volatility is the same in this and the previous setting this allows me to compute the effect of survival forces on welfare: it is equivalent to a 1.6% permanent loss of consumption. It is larger than, but comparable to, the 1.2% loss due to speculative volatility.

Figure 1 plots 100 sample paths of the key macroeconomic indicators for this economy. Panel A plots the key statistic: net foreign wealth position of country 1. Country 2's wealth is the negative of country 1's value. If both countries held the same beliefs wealth position of each country would remain zero along any path. Because country 1 has less accurate beliefs its wealth is drifting down on average. In period 100 the average path of NFW reaches -25.7% of the world GDP. More strikingly, wealth of country 1 is extremely variable. By period 20 it can be anywhere between -95% and 80% of the *world* GDP. The dynamics of wealth translates directly into that of consumption and real exchange rate, see panel B and D. As country 1 loses financial wealth its consumption drifts down and the real exchange rate depreciates. Current account, shown in panel C, grows more volatile over time and the absolute value of a period balance can reach 25% of domestic GDP. This extreme volatility has profound implications for welfare as the numbers presented above attest. The survival forces present in this example are at the heart of the arguments in the next section.

4 Theoretical results

First, define welfare level of country i evaluated using belief π:

$$W_{\pi}^{i}(\pi^{1},...,\pi^{I},M) = (1-\beta)\sum_{t=0}^{\infty}\sum_{z^{t}}\beta^{t}\pi(z^{t})u(c^{i}(z^{t})). \tag{9}$$

It depends on the probability measure used to compute the expected utility and on the allocation assigned to agent i. The allocation, being part of a competitive equilibrium, depends on beliefs of all agents $(\pi^1, ..., \pi^I)$ and on the financial market structure M. we use the utilitarian social welfare function:

$$W_{\pi^0} = \sum_i W^i_{\pi^0}.$$

Observe that individual welfare levels are evaluated using the objective probability distribution. As stated in the introduction this paternalistic welfare criterion is motivated in Blume et al. (2014).

When the financial markets are complete the CE allocation is subjective-Pareto optimal, where the adjective "subjective" emphasizes the fact that the criterion uses subjective beliefs to evaluate individual welfare. For this reason the marginal utilities must be equalized across countries. That is, using $\lambda^i > 0$ to denote Pareto weight of country i:

$$\frac{\lambda^1 \beta \pi^1(z^t) u'(c^1(z^t)) g_1(c^1_1(z^t), c^1_2(z^t))}{\lambda^2 \beta \pi^2(z^t) u'(c^2(z^t)) g_1(c^2_1(z^t), c^2_2(z^t))} = 1.$$

As trade in goods is frictionless we can use relation (13) derived in the appendix to get:

$$\frac{u'(c^1(z^t))}{u'(c^2(z^t))} \cdot \frac{1}{Q(z^t)} = const \times \frac{\pi^2(z^t)}{\pi^1(z^t)}, \tag{10}$$

where $Q(z^t) = p^1(z^t)/p^2(z^t)$ is the real exchange rate.

It is useful now to relate to the setting in which only one good is produced and consumed. In this case $Q(z^t) \equiv 1$ and behavior of the ratio of marginal utilities is determined solely by the likelihood ratio $\pi^2(z^t)/\pi^1(z^t)$. The analysis of Blume and Easley (2006) is based on this relation. The same analysis continues to apply as long as one can show that $Q(z^t)$ is bounded. This is indeed the case and we first present an intuitive explanation. The key is to understand what happens if one of the countries vanished, that is $\limsup_{t\to\infty} c^1(z^t) = 0$. With one country present in the market the relative price of goods is $g_1(e_1(z^t), e_2(z^t))/g_2(e_1(z^t), e_2(z^t))$, a value that is bounded above and away from zero because endowments are. Because the real exchange rate is a continuous function of the relative good price it must be also bounded above and away from zero. Despite the total spending of country 2 approaching zero, the relative consumption of individual goods remains bounded. That is a possibility that one of the countries could vanish does not imply unbounded dynamics of the real exchange rate. The formal statement is given below and the proof is confined to B.

Proposition 1. *The real exchange rate is bounded:*

$$\exists m, M > 0 : Q(z^t) \in [m, M] \quad \forall t, z^t.$$

Because the real exchange rate is bounded it must converge to a constant as one of the countries is being driven to poverty. So, one can apply the same arguments as in Blume and Easley (2006) to show that the country with less accurate beliefs will be driven out of the market. We state this result without proof below for the case with $I = 2$.

Proposition 2. *The country with less-accurate beliefs is driven out of the market:*

$$E(\pi^1, \pi^0) > E(\pi^2, \pi^0) \rightarrow \lim \sup_{t\to\infty} c^1(z^t) = 0, \quad \pi^0 - a.s.,$$
$$E(\pi^1, \pi^0) = E(\pi^2, \pi^0) \rightarrow \lim \inf_{t\to\infty} c^i(z^t) = 0, \quad \pi^0 - a.s., \quad i = 1, 2.$$

Proposition 2 implies that when beliefs are heterogeneous countries opt for a more volatile consumption wrongfully expecting speculative financial gains. Leaving the speculative forces unrestricted has dismal implications for welfare. But, restricting financial markets means closing some insurance venues. The aim of any financial regulation is thus to balance speculation and insurance. To understand this consider the following case. Suppose that country 1 has less accurate beliefs and $\lim\sup_{t\to\infty} c^1(z^t) = 0, \pi^0 - a.s.$ Then for any $m > 0, p \in (0, 1)$ there exists $T > 0$ such that $prob(c^1(z^t) < m) \geqslant p, \forall t > T$. The maximum consumption of country 1 is also uniformly bounded above by some $M > 0$ because endowments are. So, the life-time utility is bounded above by $\beta^T u(M) + (1-\beta^T)u(m)$ that converges to $u(m)$ as β increases. Crucially, an increase in β does not affect T. Because m and p are arbitrary and the utility function is unbounded below the true welfare of country 1 can be arbitrarily low. In C we show that the utility under the financial autarky is bounded below. So, when agents are sufficiently patient even the financial autarky can dominate the unrestricted financial markets. This proves the main result of this paper that is stated formally below.[2]

[2]We thank Larry Blume for suggesting to study the effect of discounting.

Proposition 3. *Suppose that the period utility function is unbounded below and $\pi^1 \neq \pi^2$. If the agents are sufficiently patient then the true welfare under complete markets is lower than under financial autarky:*

$$\exists \bar{\beta} : \forall \beta > \bar{\beta} \quad W_{\pi^0}(\pi^1, \pi^2, CM) < W_{\pi^0}(Aut)$$

4.1 Strength of survival forces with multiple goods

In this section we ask the question of whether the survival forces are stronger or weaker in a multi-good environment. In a single-good environment their strength depends solely on the level of disagreement as measured by the entropy. With multiple goods, an endogenous response of the real exchange rate interferes with the survival process. This can be seen from equation (10). Suppose that country 1 has less accurate beliefs and that country 2. So, country 1 must be loosing wealth on average and with it leaves the strength of its currency. Keeping the relative likelihood π^1/π^2 fixed, as $Q(z^t)$ declines the relative consumption $c^1(z^t)/c^2(z^t)$ must increase according to (10). That is the endogenous response of the real exchange rate counteracts the survival forces.

There is another consideration. As the real exchange rate decreases country 1 must rely more heavily on consumption of the domestic good. The price incentives is one reason, but there is another – the natural borrowing limit of country 1 shrinks. So, it takes fewer negative shocks to exhaust the borrowing capacity in the multi-good setting than in the single-good setting. In the example of 3 countries can borrow up to approximately 2.5 times their GDP, while in the equivalent one good setting they would be able to borrow up to 20 times their GDP.

5 Capital controls

In this section we analyze several forms of capital controls and their effect on welfare. The first form of controls is a simple lower limit on the net foreign wealth (NFW) position. The second form of controls is a tax on foreign transactions. There is a profound difference between these two forms of controls. A limit on NFW position activates infrequently, but its impact on the allocation is significant when it does activate. A transaction tax while being always active has only a small effect on the allocation.

5.1 Lower bound on net foreign wealth

Consider the following restriction on financial trade:

$$a_j^i(z^t) \geqslant -B, \quad \forall j, z^t. \tag{11}$$

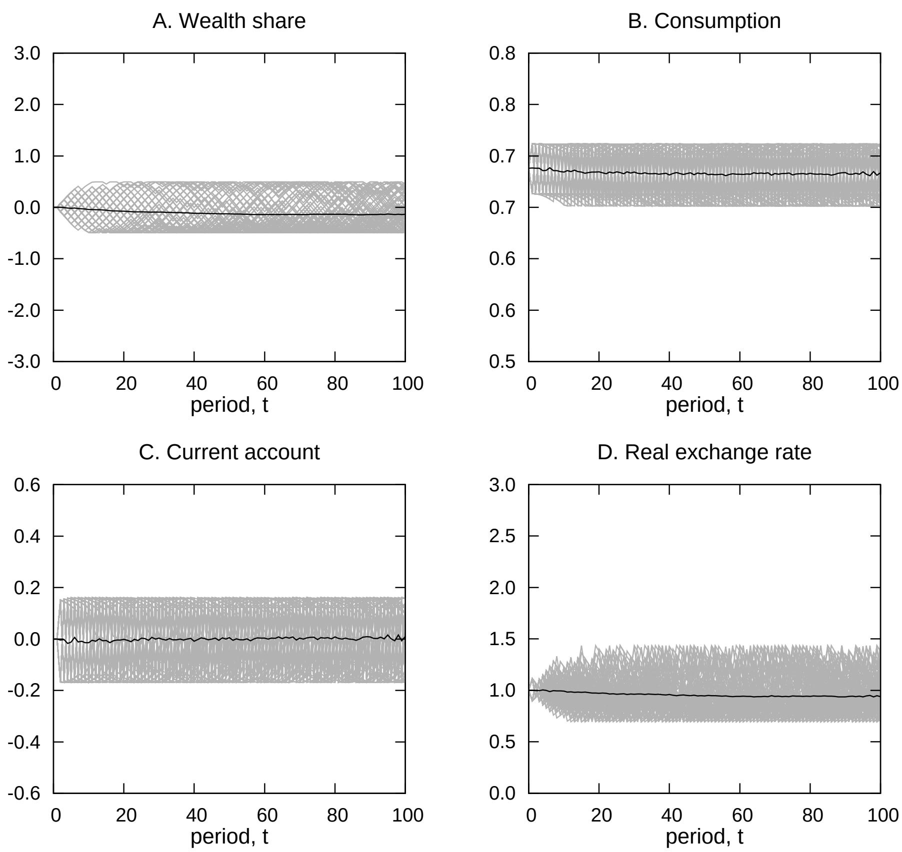

Figure 2: Dynamics of country 1 endowed with less-accurate beliefs. Grey lines represent 100 random paths, black lines are averages across paths. Financial trade is subject to an exogenous limit: B=0.5.

Figure 2 plots the key macroeconomic variables for economy 1 when both countries are subject to an exogenous lower bound on NFW position $B = 0.5$. Compared to figure 1 variability of all series is reduced significantly. Wealth, shown in panel A, is still trending down but now it never decreases below 50% of the world GDP. The average path of wealth ends at -13.5% of the world GDP as opposed to -42.3% when the financial markets are unrestricted. Similarly, the average path of consumption ends at 0.684 instead of 0.661, a 3.2% improvement. The average real exchange in period 100 is 6.4% stronger, settling at 0.939 instead of 0.883. As before consumption and the real exchange rate trend down but consumption does not decrease below 0.652 while before it could be as low as 0.436. The real exchange rate does not depreciate below 0.694 while before it could reach 0.321. It is the significant improvement in the worst case outcome that improves countries' welfare. The true welfare of country 1 and 2 is, respectively, -1.4812 and -1.4460. Compare these to -1.4991 and -1.4436 under the unrestricted financial markets. At the same time the limit is binding only 4.4 percent of time. The other time the financial constraint is inactive and neither risk-sharing nor speculation between consumers in different countries is restricted.

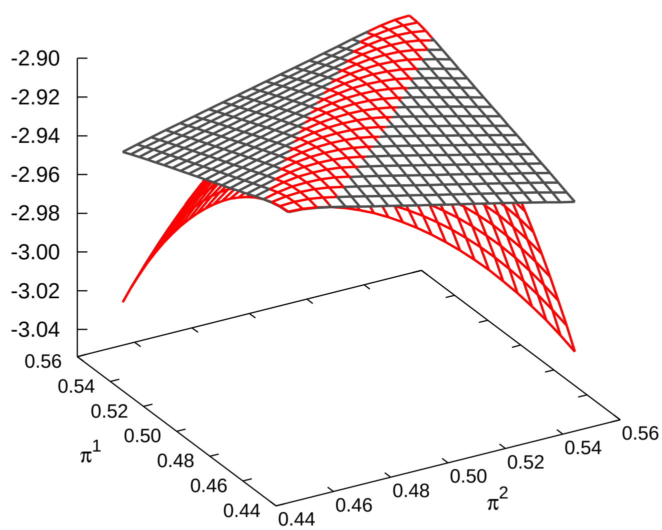

Figure 3: Welfare surfaces for the unregulated economy (red) and the economy with a net foreign wealth limit (black). Each point corresponds to a different admissible assignment of beliefs. Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

Controls?	Mean	Minimal
Unregulated	-2.932	-3.023
NFW limit	-2.922	-2.945
	(0.4%)	(2.6%)
Tobin tax	-2.923	-2.948
	(0.3%)	(2.5%)

Table 2: True welfare levels in different capital controls regimes. Values in brackets indicate the corresponding consumption equivalent variations. The NFW limit is $B = 0.5$; the transaction tax level is $\tau = 0.67\%$. ADD OTHER STATS: AVG and MIN AT T=100

Figure 3 plots the average true welfare for a range of admissible beliefs: $(\pi_1^1, \pi_1^2) \in [0.45, 0.55]^2$. Consider the world without capital controls (red surface). Along and close to the diagonal where $\pi_1^1 = \pi_1^2$ welfare in the world is high. The reason is that motives for speculation are limited and consumption of each economy is relatively stable. The lowest welfare corresponds to the case when the countries hold maximally different beliefs, that is along the diagonal $\pi_1^1 = 1 - \pi_1^2$. The maximal loss of utility, relative to the homogeneous beliefs benchmark, is large and it is equivalent to a 3.8% permanent reduction of consumption. Turn to the economy with a limit on the NFW (black surface). We set the limit at $B = 0.5$ which is equal to an average value of a country's output. If beliefs were homogeneous the welfare would be nearly as high as in the unregulated economy. The reason is that when disagreement is small controls are inactive. So, if one believes that there is little disagreement imposing capital controls would have a negligible impact (loss) on utility. In fact, at the homogeneous-beliefs diagonal welfare loss is identically zero. At the same time there is a substantial gain to be made: the maximal loss of utility is now equivalent to losing 1.1% of consumption. Importantly, this limit was chosen arbitrarily, benefits would be more substantial if this limit were chosen optimally.

There are two ways to measure welfare in the two economies. First, we could average across all possible belief realizations in $[0.45, 0.55]^2$. Second, we could choose the minimal (worst-case) welfare across all possible beliefs. Blume et al (2015) discusses different choices and argues in favor of the second. We report both in table 2. According to the first measure (mean) imposing capital controls is equivalent to permanently increasing every country's consumption by 0.4%. This is a substantial effect given that endowments are stationary and that standard deviation of endowments is only 5%. According to the second measure (worst-case) welfare improvement is equivalent to a 2.6% permanent increase in consumption. To put it in perspective, the present discounted value of this windfall is valued at more than 50% of a country's GDP.

How does the NFW limit affect the policy functions? Obviously, the limit impacts the saving decisions. So, consumption must be affected also. But the real exchange rate is affected only indirectly. That is the real exchange rate as a function of country 1's wealth is the same in the two settings, but the dynamics of wealth itself is different. Figure 4 plots the dynamics of the NFW of country 1. First, discrepancy between the two paths of the NFW appears immediately. This difference is attributable to variable strength of precautionary motives. For the fear of being constrained consumers take smaller bets in the financial markets in the regulated economy. Yet, as shown in panel A, initially the NFW follows a very similar path with $B = 0.5$ or without the controls. A significant difference builds in period 185 when outflows from country 1 hit the limit, and stay close to it afterwards, in the regulated economy. In the unregulated economy capital outflows keep increasing, reaching above 0.90 already in period 190. At the same time the effect on the real exchange rate is largely negligible until period 185. In the consequent periods, in the unregulated economy country 1 continues to accumulate wealth, its NFW position increases, and the real exchange rate appreciates more than in the regulated economy. Panel C shows that the impact of the regulation on the price of equity is substantial. The difference between the two paths becomes more substantial when country 1's NFW turns negative. This is so because country 1 is the major buyer of the domestic equity due to built-in preference for domestic goods.

We also want to point out the asymmetric effect of the terms of trade. For example, in period 190 the NFW/GDP of country 1 is 144.9% and for country 2 this indicator is -234.3%, a significantly larger magnitude. Most of this difference is explained by an exceptionally high terms of trade standing at 1.463. The effect is asymmetric because it dampens positive positions and amplifies negative positions.

Finally, observe that only the tail behavior of economic variables is distorted. As long as the NFW position does not deviate far form even distribution, namely zero, evolution of most macroeconomic indicators in the regulated and unregulated economies is nearly the same.

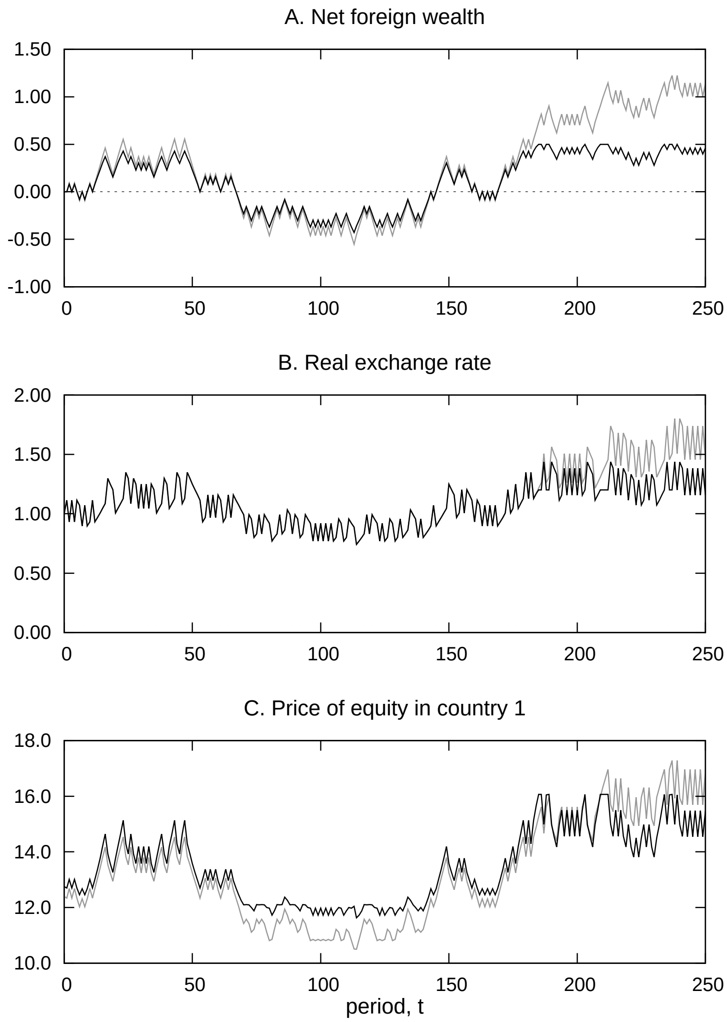

Figure 4: Selected simulated paths of the key macroeconomic variables for the unregulated economy (gray) and the economy with a net foreign wealth limit (black). Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

5.2 Transaction cost

Consider now taxing financial transactions at a fixed rate τ. The budget constraint of country i is:

$$p_1(z^t)c_1^i(z^t) + p_2(z^t)c_2^i(z^t) + \sum_j Q_j(z^t)[a_j^i(z^t) + \tau|a_j^i(z^t)|] = I^i(z^t) + T^i(z^t),$$

where $T^i(z^t)$ is the tax rebate received by country i. By the financial market clearing condition $\tau|a_j^1(z^t)| = \tau|a_j^2(z^t)|$; so, each country generates the same tax revenue. A country's budget constraint is unaffected because the tax collection is offset by the rebate; the good markets are not affected either. The effect on the economy comes about via distorted allocation and asset prices. Price of an Arrow security paying in state $z_{t+1} = j$ is determined from the following Euler equation:[3]

$$Q_j(z^t)(1 + \tau \cdot sign(a^i(z^t, j))) = \beta\pi^i(z^t, j|z^t)\frac{u'(c^i(z^{t+1}))}{u'(c^i(z^t))}. \qquad (12)$$

So, if a country purchases Arrow security j it pushes the price up by a factor $1 + \tau$. The other country must sell the security, and, if wealth is evenly distributed, this restores the original security price. But for the security price to remain unchanged consumption in both countries must adjust. If wealth is not evenly distributed then we expect a non-trivial effect both on the security price and the consumption allocation.

Figure 5 shows the dynamics of the NFW, the real exchange rate, and the price of country 1's equity for the transaction tax level $\tau = 0.005$. For this magnitude of the tax the maximal value of country 1's wealth share is close to 0.5, just like under the NFW limit discussed above. The effect of the tax is that countries scale down their security purchases. As a result the path of the NFW of country 1 is a scaled down version of that under the unregulated markets as panel A demonstrates. More stable wealth position translates into a less volatile consumption process and, hence, a less volatile real exchange rate shown in panel B. The transaction tax also stabilizes the price of country 1's equity as can be seen from panel C.

The welfare surface for the economy with a transaction tax is very similar to the one with a NFW limit. So, it is not reported here. The main difference lies in the fact that transaction tax negatively affect welfare even if there is no disagreement. That is the measure's main disadvantage. A skeptic who believes that large disagreement is unlikely could point to this potential loss. At the same time, potential gains remain large but they are achieved at the cost of losing welfare even when there is no disagreement/speculation. Imposing a limit on NFW does not have this trade-off and hence if preferable in our view.

To conclude we would like to point out that the transaction cost should less favorable measure for the following reason. At the time of announcement

[3]Function $sign(x)$ is defined as $x/|x|$ if $x \neq 0$ and zero otherwise.

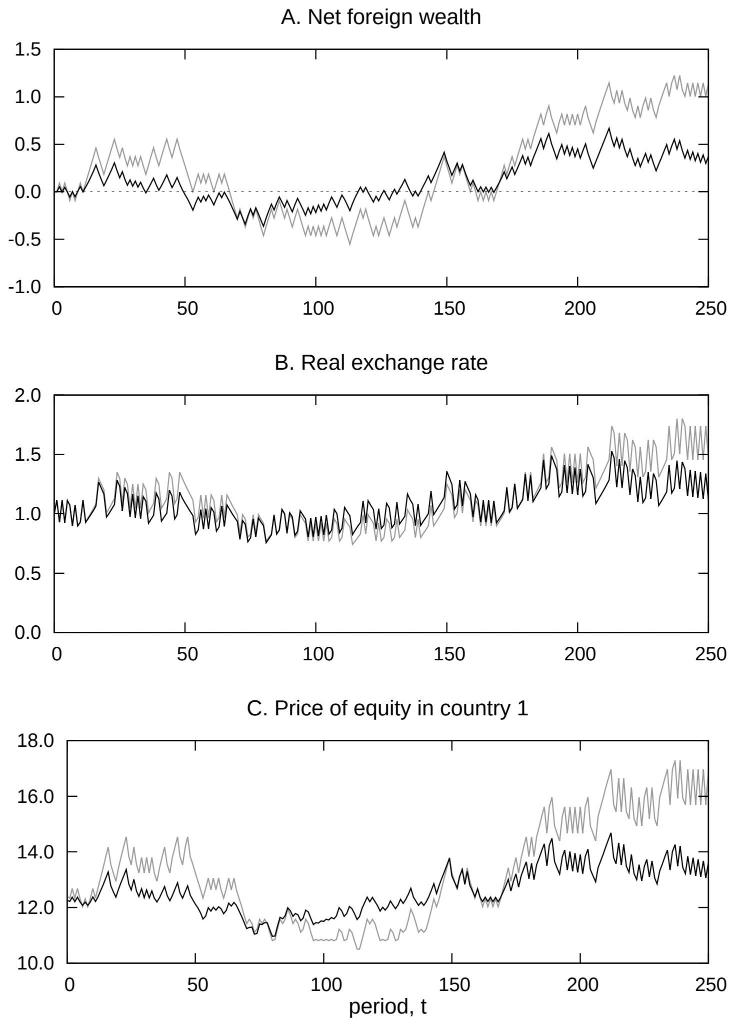

Figure 5: Selected simulated paths of the key macroeconomic variables for the unregulated economy (gray) and the economy with a transaction tax (black). Parameters: $\beta = 0.96, e_l = 0.95, e_h = 1.05, \pi^0 = (0.5, 0.5)$.

the transaction tax is going to have an immediate level effect on the asset markets: prices will be depressed because trading securities will become costlier. Announcement of a limit on current account will only have such effect if the country is close to exhausting this limit.

5.3 The effect of capital controls on asset markets

The proposed financial market restrictions impact the dynamics of the financial markets in unexpected way. To illustrate consider an example with symmetric beliefs as described in 3 and three regimes: an unregulated economy, an economy with a NFW limit, and an economy with a transaction tax. We choose the transaction tax to make the last two setups have the same volatility of a country's NFW. In the case with low and high ES we set $\tau = 0.67\%$ and $\tau = 0.44\%$, respectively. We report the key financial market statistics in table 3. With either form of capital controls macroeconomic stability, as measured by volatility of consumption, CA and NFW, is improved significantly. Volatility of consumption decreases by a factor of 10, while that of NFW decreases at least by a factor of 3. In the case of low ES the capital controls also increases financial stability as measured by volatility of bond and equity prices. Asset price volatility decreases by at least a factor of 2. However, the same is not true if we assume a high value of ES. The case with high ES is closer to a one-good environment in which tighter regulation may be in conflict with financial stability, as observed in Blume et al (2015). To explain this phenomenon consider the case with the NFW limit. There are two driving forces of asset price volatility in the model: exogenous changes of countries' output and endogenous fluctuations in wealth distribution. Changes in wealth have a larger impact on asset prices in the case with low ES as the relative price of goods is more elastic. So, after a low output shock equity value in country 1 declines. Country 1's wealth and the price of good 1 decrease also, extending the decline of the equity value. So, capital controls, by restricting movements in wealth, stabilize equity prices. The effect of the NFW limit, a binding financial constraint that typically increases asset price volatility, is relatively unimportant. In the case with high ES the importance of the wealth effect and of the NFW limit are reversed. So, when the NFW limit is imposed asset price volatility increases.

The above analysis highlights that stabilization of capital flows may amplify fluctuations in domestic asset markets. When gross asset positions are substantial imposing capital controls may initially lead to increased volatility of capital flows. This should pose no problem if the financial system is sufficiently developed to withstand large asset price swings.

Finally, we would like to remark that the transaction tax is less effective at controlling CA fluctuations. In this case financial trade is costly but not impossible unlike under the NFW limit that halts trading when financial positions reach a given level. The transaction tax also depresses asset prices while the NFW limit increases them. The first is good for the case when the domestic equity market is "overvalued" because of capital inflows. The NFW limit is good when the domestic market declines due to capital outflows.

	$\sigma(\text{NFA}^i)$	$\sigma(\text{CA}^i)$	$\sigma(C^i)$	$\sigma(Q)$	$\sigma(q_b^i)$	$\sigma(q_e^i)$
A. Low elasticity: $\varepsilon = 0.8$						
Unregulated	1.275	0.130	0.049	0.593	0.146	0.130
NFW limit	0.331	0.066	0.004	0.192	0.044	0.065
Tobin's tax	0.337	0.064	0.012	0.211	0.045	0.056
B. High elasticity: $\varepsilon = 2.0$						
Unregulated	0.940	0.110	0.080	1.182	0.076	0.073
NFW limit	0.314	0.043	0.008	0.613	0.039	0.093
Tobin's tax	0.313	0.047	0.022	0.587	0.032	0.086

Table 3: Selected moments for the economies with and without capital controls. Because countries are symmetric we report moments only for one country. NFA and CA are normalized by a country's GDP, all other variables were transformed using the natural logarithm function. Asset prices are denominated in units of a domestic consumption basket.

6 Conclusions

we study an international portfolio choice model with heterogeneous beliefs. Belief diversity generates speculative cross-border capital flows. The latter are much larger than flows generated by the hedging/insurance motives. Capital controls improve welfare because they limit movement of the relative wealth positions and, hence, consumption. Using numerical simulations we find that the controls that limit tail behavior of capital flows and/or exchange rates are most desirable. That is the controls that activate only during substantial inflows or outflows are preferred to those that are less distortive yet constantly active.

This work opens up a possibility to study quantitatively various recent episodes of speculative capital flows. Are the capital flow controls expected to work as intended if a policy is enacted after massive flows taken place? How effective are unilateral measures and can they fire back? The setting with heterogeneous beliefs also offers numerous advantages – from resolving the long-standing puzzles in the international finance like consumption-exchange rate disconnect to offering a plausible explanation of financial phenomena such as excess volatility and return predictability. Building upon this model allows matching the volume of observed financial activity that economists, and this paper in particular, strive to regulate. Further, heterogeneous beliefs can be easily included into existing models of macroprudential regulation.

Bibliography

References

Albuquerque, R., Bauer, G. H., Schneider, M., 2007. International equity flows and returns: A quantitative equilibrium approach. Review of Economic Studies 74, 130.

Bianchi, J., Bengui, J., 2011. Overborrowing and systemic externalities in the business cycle. American Economic Review 101 (2), 3400–3426.

Blume, L., Cogley, T., Easley, D., Sargent, T., Tsyrennikov, V., 2014. A case for market incompleteness. Cornell University manuscript.

Blume, L., Easley, D., 2009. Heterogeneity, selection and wealth dynamics. Cornell University manuscript.

Blume, L. E., Easley, D., 2006. If you're so smart, why aren't you rich? belief selection in complete and incomplete markets. Econometrica 74 (4), 929–966.

Brunnermeier, M., Simsek, A., Xiong, W., 2013. A welfare criterion for models with distorted beliefs.

Cogley, T., Sargent, T., Tsyrennikov, V., forthcoming. Wealth dynamics in a bond economy with heterogeneous beliefs. Economic Journal.

Dumas, B., Lewis, K. K., Osambela, E., 2014. Differences of opinion and international equity markets. unpublished.

Farhi, E., Werning, I., 2013. A theory of macroprudential policies in the presence of nominal rigidities. Harvard and MIT manuscript.

Friedman, M., 1953. Choice, chance, and the personal distribution of income. Journal of Political Economy 61 (4), 277–290.

Gilboa, I., Samuelson, L., Schmeidler, D., 2012. No-betting pareto dominance.

Harrison, M., Kreps, D., 1978. Speculative investor behavior in a stock market with heterogeneous expectations. Quarterly Journal of Economics 92 (2), 323–336.

Jeanne, O., Korinek, A., 2010. Excessive volatility in capital flows: a pigouvian taxation approach. American Economic Review, Papers and Proceeedings 100 (2), 403–407.

Lucas, R. E., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

[INCOMPLETE]

A Efficient allocations

Here we derive a condition that must be satisfied by any CE allocations as long as trade in goods is frictionless. The derivations below assume that the consumption aggregator is CRS. The first order conditions for (c_1^i, c_2^i) are:

$$\beta\pi^i(z^t)u'(c^i(z^t))g_1(c_1^i(z^t), c_2^i(z^t)) = \lambda p_1(z^t),$$
$$\beta\pi^i(z^t)u'(c^i(z^t))g_2(c_1^i(z^t), c_2^i(z^t)) = \lambda p_2(z^t).$$

Multiply the two equations by c_1^i and c_2^i respectively and add to get:

$$u'(c^1)c^1 = \lambda(p_1c_1^1 + p_2c_2^1) = \lambda p^1c^1.$$

The above implies that $g_1(c_1^i, c_2^i) = p_1/p^i$ and:

$$\frac{g_1(c_1^2, c_2^2)}{g_1(c_1^1, c_2^1)} = \frac{p^1}{p^2} = Q. \tag{13}$$

B RER is bounded

In this appendix we establish that the real exchange rate RER is bounded in any complete markets competitive equilibrium.

Let $h^i(x, y) = g_1^i(x, y)/g_2^i(x, y)$. Because the trade in goods is frictionless the following relation must hold:

$$h^1(c_1^1, c_2^1) = h^2(e_1 - c_1^1, e_2 - c_2^1) = p_1/p_2. \tag{14}$$

The question is if having any country's consumption converge to zero destabilizes the relative price. The answer is "no." To see this suppose that country 1's consumption aggregate is zero. This is possible only if consumption of one of the goods is zero. let it be consumption of good 1: $c_1^1 = 0$. Then the relative price must also be zero: $p_1/p_2 = 0$. But this is possible only if $e_1 - c_1^1 = 0$ that is impossible as e_1 is bounded away from zero.

It is easy to illustrate the above argument in the case of the CES good aggregator. In this case $h^1(x,y) = \frac{s}{1-s}(x/y)^{\rho-1}, h^2(x,y) = \frac{1-s}{s}(x/y)^{\rho-1}$. The equation (14) can be solved for c_2^1 in terms of c_1^1:

$$c_2^1 = e_2 \frac{s^{-2\varepsilon} c_1^1}{s^{-2\varepsilon} c_1^1 + (1-s)^{-2\varepsilon}(e_1 - c_1^1)},$$

and the relative price:

$$p_1/p_2 = s^2 e_2^{\rho-1}[s^{-2\varepsilon} c_1^1 + (1-s)^{-2\varepsilon}(e_1 - c_1^1)]^{1-\rho} \geqslant (s/1-s)^2 (e_1/e_2)^{\rho-1}. \quad (15)$$

C Welfare in financial autarky

Consider the financial autarky. For the optimization problem of each country is static we drop the time subscript in this subsection. Optimal trade in goods between countries solves the following system of equilibrium equations for any pair $(e_1, e_2) \gg 0$:

$$g_1^i(c_1^i, c_2^i)/g_2^i(c_1^i, c_2^i) = q, \quad i = 1,2, \quad (16a)$$

$$c_j^1 + c_j^2 = e_j, \quad j = 1,2, \quad (16b)$$

$$q(c_1^1 - e_1) + c_2^1 = 0. \quad (16c)$$

Given that g^1, g^2 satisfy the Inada condition it is easy to show that c_1, c_2 is an interior solution. The optimal allocation is a continuous function of (e_1, e_2). Because the latter is bounded the allocation must also be bounded, most importantly, below.

D Welfare in unregulated economy

This section complements the discussion in section 3 by describing how welfare changes with countries' beliefs. Because endowments are symmetric under the true dgp we report only welfare of country 1 in figure 6. First, welfare of country 1 is high when its beliefs are more accurate. In particular, it is also higher than in the homogeneous beliefs case (point C), although concavity of the period utility function limits the gain at 1.6%. But belief accuracy is not the only force. For example, at $(p^1, p^2) = (0.50, 0.45)$ country 1's information advantage is maximized, but its welfare is not. Under belief assignment $(p^1, p^2) = (0.50, 0.55)$ country 1's welfare is higher despite the informational advantage being the same. The reason is that under the latter belief assignment the world is biased towards thinking that country 1's expected output is higher. This affects the price system in favor of country 1 and allows it to achieve higher consumption. Country 1's welfare is still higher at $(p^1, p^2) = (0.525, 0.55)$ denoted by point A_1 where some of the informational advantage is given up and expectation are biased more in the country's favor. Country 1's welfare is also very high at $(p^1, p^2) = (0.48, 0.45)$ denoted by point A_2, but not as high as at A_1 because these beliefs are biased against country 1.

Country 1 welfare can lose as much as 4.5% relative to the benchmark with correct beliefs. The lowest welfare is achieved at $(p^1, p^2) = (0.45, 0.535)$ denoted by point B_1 where country 1 has less accurate beliefs, disagreement is nearly maximized, and expectations are stacked against it. Its welfare is very also at $(p^1, p^2) = (0.55, 0.46)$ denoted by point B_2, but not as low as at B_1 because these beliefs are slightly biased in favor of country 1.

Despite the saddle-shaped welfare function of each individual country, the total welfare in the world is concave surface with a maximum at point C where both countries hold correct beliefs. Welfare does not decline more than 0.3% if countries hold the same beliefs. But the loss increases to 4.3% at the points where disagreement is maximal: $(0.45, 0.55)$ or $(0.55, 0.45)$.

E Macro-dynamics with equally incorrect beliefs

Figure 1 plots key macroeconomic indicators for the economy in which both countries are optimistic about their own prospects. Endowments are specified as in section 3 and beliefs are:

$$\pi^1 = \begin{bmatrix} 0.525 \\ 0.475 \end{bmatrix}, \pi^2 = \begin{bmatrix} 0.475 \\ 0.525 \end{bmatrix}, \quad \pi^0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

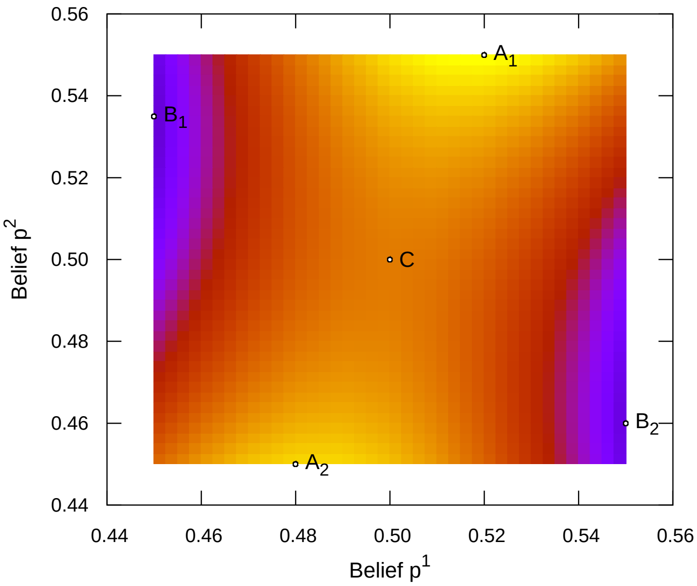

Figure 6: Welfare in the unregulated economy of section 3 for a range of possible belief assignments.

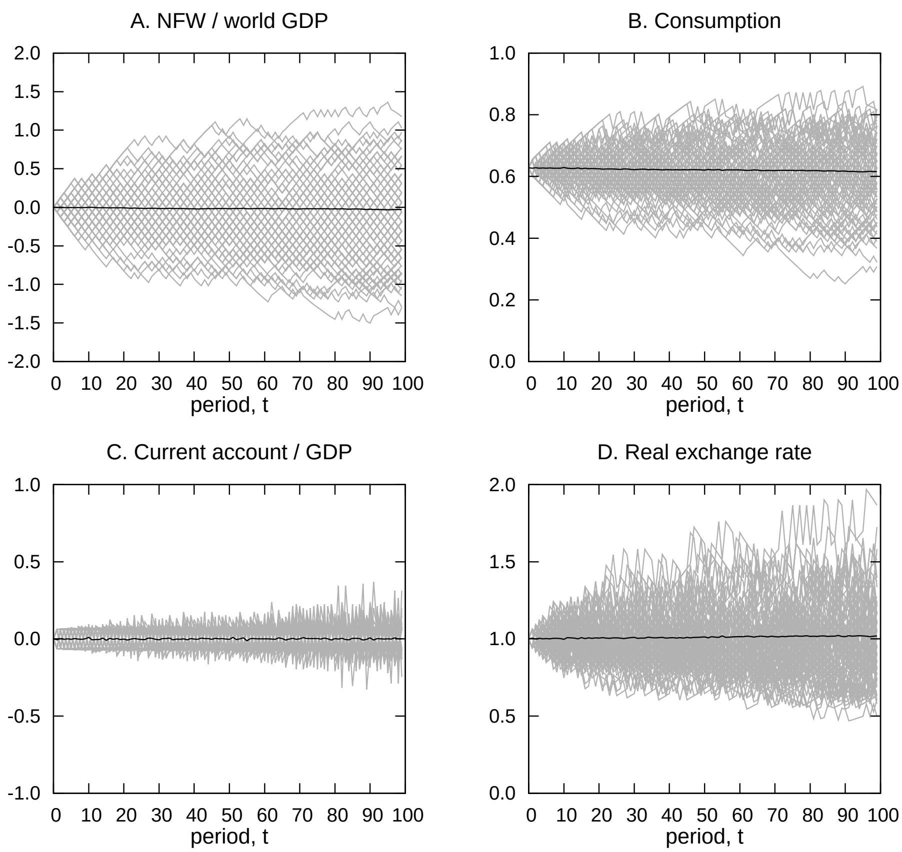

Figure 7: Dynamics of country 1. Countries have wrong but equally-inaccurate beliefs. Grey lines represent 100 random paths, black lines are averages across paths.

OTHER A.E.M. WORKING PAPERS

WP No	Title	Fee (if applicable)	Author(s)
2016-08	Using Unobtrusive Sensors to Measure and Minimize Hawthorne Effects: Evidence from Cookstoves		Simons, A., Beltramo, T., Blalock, G. and Levine, D.
2016-07	Economics and Economic Policy		R. Kanbur
2016-06	W. Arthur Lewis and the Roots of Ghanaian Economic Policy		R. Kanbur
2016-05	Capability, Opportunity, Outcome - -and Equality		R. Kanbur
2016-04	Assessment of New York's Pollution Discharge Elimination Permits for CAFO's: A Regional Analysis		Enahoro, D., Schmit T. and R. Boisvert
2016-03	Enforcement Matters: The Effective Regulation of Labor		Ronconi, L. and R. Kanbur
2016-02	Of Consequentialism, its Critics, and the Critics of its Critics		Kanbur. R
2016-01	Specification of spatial-dynamic externalities and implications for strategic behavior in disease control		Atallah, S., Gomez, M. and J. Conrad
2015-13	Networked Leaders in the Shadow of the Market - A Chinese Experiment in Allocating Land Conversion Rights		Chau, N., Qin, Y. and W. Zhang
2015-12	The Impact of Irrigation Restrictions on Cropland Values in Nebraska		Savage, J. and J. Ifft
2015-11	The Distinct Economic Effects of the Ethanol Blend Wall, RIN Prices and Ethanol Price Premium due to the RFS		de Gorter, H. and D. Drabik
2015-10	Minimum Wages in Sub-Saharan Africa: A Primer		Bhorat, H., Kanbur, R. and B. Stanwix
2015-09	Optimal Taxation and Public Provision for Poverty Reduction		Kanbur, R., Pirttilä, J., Tuomala, M. and T. Ylinen
2015-08	Management Areas and Fixed Costs in the Economics of Water Quality Trading		Zhao, T., Poe, G. and R. Boisvert

Paper copies are being replaced by electronic Portable Document Files (PDFs). To request PDFs of AEM publications, write to (be sure to include your e-mail address): Publications, Department of Applied Economics and Management, Warren Hall, Cornell University, Ithaca, NY 14853-7801. If a fee is indicated, please include a check or money order made payable to Cornell University for the amount of your purchase. Visit our Web site (*http://dyson.cornell.edu/research/wp.php)* for a more complete list of recent bulletins.