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OPTIMAL TAXATION AND PUBLIC PROVISION FOR POVERTY REDUCTION

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Abstract

The existing literature on optimal taxation typically assumes there exists a capacity to implement complex tax schemes, which is not necessarily the case for many developing countries. We examine the determinants of optimal redistributive policies in the context of a developing country that can only implement linear tax policies due to administrative reasons. Further, the reduction of poverty is typically the expressed goal of such countries, and this feature is also taken into account in our model. We derive the optimality conditions for linear income taxation, commodity taxation, and public provision of private and public goods for the poverty minimization case, and compare the results to those derived under a general welfarist objective function. We also study the implications of informality on optimal redistributive policies for such countries, and comment on the potential for minimum wage regulation. The exercise reveals non-trivial differences in optimal tax rules under the different assumptions. The derived formulae also capture the sufficient statistics that the governments need to pay attention to when designing poverty alleviation policies.

Key words: Redistribution, income taxation, commodity taxation, public good provision, poverty

JEL classification: H21, H40, O12.

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1 Introduction

High levels of within-country inequality in many otherwise successful developing countries have become a key policy concern in the global development debate. While some countries have very unequal inherent distributions (e.g. due to historical land ownership arrangements), in others the fruits of economic growth have been unequally shared. No matter what the underlying reason for the high inequality is, often the only direct way for governments to affect the distribution of income is via redistributive tax and transfer systems. Clearly, public spending on social services also has an impact on the distribution of well-being, although some of the effects (such as skill-enhancing impacts from educational investment) only materialize over a longer time horizon.

Reflecting the desire to reduce poverty and inequality, redistributive transfer systems have, indeed, proliferated in many developing countries. Starting from Latin America, they are now spreading to low-income countries, including those in Sub-Saharan Africa.[1] In low-income countries, in particular, redistributive arrangements via transfers are still at an early stage, and they often consist of isolated, donor-driven, programs. There is an urgent and well-recognized need to move away from scattered programs to more comprehensive tax-benefit systems.

This paper examines the optimal design of cash transfers, commodity taxes (or subsidies), the provision of public and private goods (such as education and housing), and financing them by a linear income tax. The paper also includes an analysis of optimal income taxation in the presence of an informal sector, and it also provides a brief discussion of the role of minimum wage as a poverty alleviation tool. The paper therefore provides an overview of many of the most relevant instruments for redistributive policies that are needed for a system-wide analysis of social protection. We build on the optimal income tax approach, which is extensively used in the developed country context,[2] but much less applied for the design of redistributive systems in developing country circumstances. This approach, initiated by Mirrlees (1971), allows for a rigorous treatment of efficiency concerns (e.g. the potentially harmful effect of distortionary taxation on employment) and redistributive objectives. Achieving the government's redistributive objectives is constrained by limited information: the social planner cannot directly observe individuals' income earning capacity, and therefore it needs to base its tax and transfer policies on observable variables, such as gross income. The most general formulation of optimal tax models apply non-linear tax schedules, but in a developing country context, using fully non-linear taxes is rarely feasible. In this paper we therefore limit the analysis to redistributive linear income taxes, which combine a lump-sum transfer with a proportional income tax, and which can be implemented by withholding at source if necessary.

Linear income taxes are not very common in practice: less than 30 countries had flat

[1] For a recent treatment and survey, see Barrientos (2013).

[2] See IFS and Mirrlees (2011) for an influential application of optimal tax theory to policy analysis for rich countries.

tax rates for personal income in 2012, with some concentration in ex-Soviet Eastern Europe (Peichl 2014). It is noteworthy that even though flat taxes are not particularly common in low-income countries, in many instances in such countries the progressive income tax reaches only a small share of the population. This would indicate that despite the existence of a progressive income tax, these countries do not yet possess enough tax capacity to implement well-functioning progressive income taxes. This is one motivation for our interest of modeling optimal linear taxes. Peichl (2014) suggests that simplification benefits can be especially relevant for developing countries.[3]

In conventional optimal taxation models, the government's objective function is modeled as a social welfare function, which depends directly on individual utilities. We depart from this welfarist approach by presenting general non-welfarist tax rules, as in Kanbur, Pirttilä, and Tuomala (2006), and, in particular, optimal tax, public good provision rules and minimum wage policies when the government is assumed to minimize poverty. We have chosen this approach as it resembles well the tone of much of the policy discussion in developing countries, including the Millennium Development Goals (MDGs), where the objective is explicitly to reduce poverty rather than maximize well-being. Similarly, the discussion regarding cash transfer systems is often couched especially in terms of poverty alleviation. Note also that the objective of poverty minimization is not at odds with the restriction of a linear tax scheme that we impose: a flat tax regime together with a lump-sum income transfer component can achieve similar amounts of redistribution towards the poor as a progressive tax system, if specified suitably (Keen, Kim, and Varsano 2008; Peichl 2014). In all our analyses, we first present welfarist tax rules (which are mostly already available in the literature) to provide a benchmark to examine how applying poverty minimization as an objective changes the optimal tax and public service provision rules.

We also deal with some extensions to existing models, which are motivated by the developing country context, such as the case where public provision affects the individuals' income-earning capacity, thus capturing (albeit at a very stylized way) possibilities to affect their capabilities. An important feature to take into account in tax analysis of developing countries is the presence of a large informal sector. We also examine the implications of the presence of an informal sector for optimal redistributive policies.

Our paper is related to various strands of earlier literature. First, Kanbur, Keen, and Tuomala (1994) and Pirttilä and Tuomala (2004) study optimal income tax and commodity tax rules, respectively, from the poverty alleviation point of view, but their papers build on the non-linear tax approach which is not well suited to developing countries. Kanbur and Keen (1989) do consider linear income taxation together with poverty minimization, but they do not produce optimal tax rules but focus on a tax reform perspective, and provide tax rate simulations. A second strand of literature considers taxation and development more generally,

[3] Note that it might be reasonable for some countries to move to a progressive income tax system as their tax capacity increases with development; the study of such dynamics is beyond the scope of this paper.

such as Gordon and Li (2009), Keen (2012) and Besley and Persson (2013).[4] This field, while clearly very relevant, has not concentrated much on the design of optimal redistributive systems.

Finally, we follow the approach in Piketty and Saez (2013), who use the 'sufficient statistics' approach 'whereby optimal tax formulas are derived and expressed in terms of estimable statistics including social marginal welfare weights capturing society's value for redistribution and labor supply elasticities capturing the efficiency costs of taxation' (Piketty and Saez 2013: 394). This sufficient statistic approach has proved very valuable for applied tax analysis, since it provides clear guidelines for the sort of empirical work that is needed to generate knowledge for implementing optimal tax rules. Piketty and Saez also emphasize how linear tax rules, while analytically more feasible, fit well with this approach as they lead to tax formulas that contain the same sufficient statistics than more complicated non-linear models. The linear tax rules, they argue, are robust to alternative specifications, and examining this forms part of our motivation: we study optimal linear tax policies, in our understanding for the first time, from the poverty minimization perspective.

The paper proceeds as follows. Section 2 examines optimal linear income taxation, while 3 turns to optimal provision rules for publicly provided private and public goods that are financed by such a linear income tax. Section 4 analyzes the combination of optimal linear income taxes and commodity taxation and asks under which conditions one should use differentiated commodity taxation if the government is interested in poverty minimization and also has optimal cash transfers at its disposal. The question on how optimal poverty-minimizing income tax policies are altered in the presence of an informal sector is examined in Section 5. The potential role of the minimum wage is briefly discussed in Section 6. Finally, conclusions are provided in Section 7.

2 Linear income taxation

2.1 Optimal linear income taxation under the welfarist objective

In this section we give an overview of some of the models and results for optimal linear income taxation as they have been presented in the literature. Many formulae for optimal taxation were developed in the 1970s and 1980s (see Dixit and Sandmo 1977; Tuomala 1985; and the survey by Tuomala 1990), and they are still being used, whereas Piketty and Saez (2013) offer fresh expressions of the tax rules. In what follows, we report both versions of the tax rules.

The government collects a linear income tax τ, which it uses to finance a lump-sum transfer b, along with other exogenous public spending R. The individuals differ in their income-earning capacity (w^i), and z^i denotes individual labor income ($w^i L^i$), where L^i represents

[4] Besley and Persson (2013) use a model with *groups* that can differ in their income-earning abilities. Their analysis focuses, however, on explaining how economic development and tax capacity are interrelated, and not on redistribution between *individuals*.

hours worked. Consumption equals $c^i = (1-\tau)z^i + b$, where the superscript-i refers to individuals.[5]

We start by following the analysis in Tuomala (1985). The government has redistributive objectives represented by a Bergson-Samuelson functional $W\left(V^1, ..., V^N\right)$ with $W' > 0$, $W'' < 0$. There is a discrete distribution of N individuals. The indirect individual utility function is denoted by $V^i(1-\tau, b)$, and we refer to the net-of tax rate as $1-\tau = a$. To simplify notation, subscript-a refers to the derivative with respect to the net-of-tax rate. The government's problem is to choose the tax rate τ and transfer b so as to maximize the social welfare function $\sum W\left(V^i(a,b)\right)$ under the budget constraint $(1-a)\sum z^i = Nb + R$.[6]

All the mathematical details are presented in Appendix A. There it is shown that the optimal tax rule is given by

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1 - \frac{z(\beta)}{\bar{z}}\right), \tag{2.1}$$

where $\varepsilon = \frac{d\bar{z}}{d(1-\tau)}\frac{(1-\tau)}{\bar{z}}$ is the elasticity of total income with respect to the net-of-tax rate, $\bar{z}$ is average income and $z(\beta)$ welfare-weighted average income. Define $\Omega = \frac{z(\beta)}{\bar{z}}$, so that $I = 1-\Omega$ is a normative measure of inequality or, equivalently, of the relative distortion arising from the second-best tax system. Clearly Ω should vary between zero and unity. One would expect it to be a decreasing function of τ (given the per capita revenue requirement $g = R/N$). There is a minimum feasible level of τ for any given positive g, and of course g must not be too large, or no equilibrium is possible. Hence any solution must also satisfy $\tau > \tau_{min}$ if the tax system is to be progressive. That is, if the tax does not raise sufficient revenue to finance the non-transfer expenditure, R, the shortfall must be made up by imposing a poll tax ($b < 0$) on each individual. One would also expect the elasticity of labor supply with respect to the net-of-tax rate to be an increasing function of τ (it need not be).

We can rewrite (2.1) as $\tau^* = \frac{1-\Omega}{1-\Omega+\varepsilon}$ to illustrate the basic properties of the optimal tax rate. Because $\varepsilon \geq 0$ and $0 \leq \Omega < 1$, both the numerator and denominator are non-negative. The optimal tax rate is thus between zero and one. The formula captures neatly the efficiency-equity trade off. τ decreases with ε and Ω and we have the following general results: (1) In the extreme case where $\Omega = 1$, i.e. the government does not value redistribution at all, $\tau = 0$ is optimal. We can call this case libertarian. According to the libertarian view the level of disposable income is irrelevant (ruling out both basic income b, and other public expenditures, g, funded by the government). (2) If there is no inequality, then again $\Omega = 1$ and $\tau = 0$. There is no intervention by the government. The inherent inequality will be fully reflected in the disposable income. Furthermore, lump-sum taxation is optimal; $b = -g$ or $T = -b$. (3) We can call the case where $\Omega = 0$ as "Rawlsian" or maxi-min preferences. The government

[5]We consider "income" here as the labor income of individuals, but, considering that our model is intended especially for the poorer countries, agricultural income could as well be included in the concept of income. In Section 5 we discuss the implications of untaxed home consumption in agricultural production.

[6]Summation is always over all individuals i, which is suppressed for simplification.

maximizes tax revenue (optimal $\tau = \frac{1}{\varepsilon}$) as it maximizes the basic income b (assuming the worst off individual has zero labor income). In fact, maximizing b can be regarded as a non-welfarist case, which is the focus in the next sub-section.

As mentioned above, Piketty and Saez (2013) offer another useful, alternative formulation of the optimal income tax rule in the welfarist case. They work with a social welfare function of the type $\int \omega^i W(u^i)\, d\nu(i)$, where ω is a Pareto weight and W is an increasing and concave transformation of utilities. They define $\beta^i = \frac{\omega^i W_u u_c^i}{\int \omega^i W_u u_c^i d\nu(i)}$ as a normalized social marginal welfare weight for individual i and $\bar{\beta} = \frac{\int \beta^i z^i d\nu(i)}{Z}$ as the average normalized social marginal welfare weight, weighted by labor incomes z^i (it can also be interpreted as the ratio of the average income weighted by individual welfare weights β^i to the average income Z). The elasticity of Z with respect to $1-\tau$ is denoted by ε. Using this notation, we arrive at a similar social welfare-maximizing tax rate as in (2.1)

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1-\bar{\beta}\right). \tag{2.2}$$

The welfare-maximizing tax rate is thus decreasing in both the average marginal welfare weight and the tax elasticity of aggregate earnings. A higher $\bar{\beta}$ reflects a lower taste for redistribution, and thus a lower desire to tax for redistributive reasons.[7]

2.2 Optimal linear income taxation under non-welfarist objectives

A non-welfarist government is one that follows a different set of preferences than those employed by individuals themselves (Kanbur, Pirttilä, and Tuomala 2006). Thus, instead of maximizing a function of individual utilities, the government has other, paternalistic objectives that go beyond utilities. A special case taken up in more detail below is the objective of minimizing poverty in the society. To be as general as possible, let us define a 'social evaluation function' (as in e.g. Kanbur, Pirttilä, and Tuomala 2006) as $S = \sum F(c^i, z^i)$, which the government maximizes instead of the social welfare function. $F(c^i, z^i)$ measures the social value of consumption c^i for a person with income z^i and can be related to $u(c^i, z^i)$ but is not restricted to it. Following Tuomala's model as above, given the instruments available, linear income tax τ, lump-sum grant b and other expenditure R the government thus maximizes $\sum F(az^i + b, z^i)$ subject to the budget constraint $(1-a)\sum z^i - Nb = R$. Define

$$\frac{\sum\left(F_c(z^i + az_a^i) + F_z z_a^i\right)}{\sum\left(F_c(1 + az_b^i) + F_z z_b^i\right)} \equiv \tilde{F},$$

which reflects the relative impact of taxes and transfers on the social evaluation function. Using this, and following the same steps as in the previous section, the optimal tax rate becomes

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1 - \frac{\tilde{F}}{\bar{z}}\right). \tag{2.3}$$

[7] A third version of the tax rule in the welfarist case, from Dixit and Sandmo (1977), is presented in Appendix A.

The result resembles the welfarist tax rules in (2.1) and (2.2). In addition to efficiency considerations via the term $\frac{1}{\varepsilon}$, they all entail a term that measures the relative benefits of taxes and transfers, in the welfarist case via welfare-weighted income, in the non-welfarist case via $\tilde{F}$, the relative impact on the social evaluation function. However, the non-welfarist optimal tax rate can differ from the welfarist rate. The signs and magnitudes of F_c and F_z (note that there is no equivalent to F_z in the welfarist tax rules) and thus $\tilde{F}$ depend on the specific objective of the government, i.e. the shape of F. Let us consider the specific case of poverty minimization below.

2.2.1 Special case: Poverty minimization

Now let us derive the optimal linear tax results for a government whose objective is to minimize poverty in society. The instruments available to the government are the same, τ and b, and other exogenous expenditure is R. Note first that the revenue-maximizing tax rate is in fact equivalent to the tax rate obtained from a maxi-min objective function, since when the government only cares about the poverty (consumption) of the poorest individual, its only goal is to maximize redistribution to this individual, i.e. maximize tax revenue.

Let us first define the objective function of the government explicitly. Poverty is defined as deprivation of individual consumption c^i relative to some desired level $\bar{c}$ and measured with a deprivation index $D\left(c^i, \bar{c}\right)$, such that $D > 0 \; \forall \, c \in [0, \bar{c})$ and $D = 0$ otherwise, and $D_c < 0$, $D_{cc} \geq 0 \; \forall \, c \in [0, \bar{c})$, as in Pirttilä and Tuomala (2004). A typical example of such an index would be the P_α family of Foster-Greer-Thorbecke (FGT) poverty indices. We discuss the application of FGT indices in our model in Appendix B. The social evaluation function $F(c^i, z^i)$ becomes $D\left(c^i, \bar{c}\right)$ and the objective function $min\, P = \sum D\left(c^i, \bar{c}\right)$. Now $F_c = D_c$ and $F_z = 0$, so

$$\tilde{F} = \tilde{D} = \frac{\sum D_c\left(z^i + a z_a^i\right)}{\sum D_c\left(1 + a z_b^i\right)},$$

and the optimal tax rule becomes

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1 - \frac{\tilde{D}}{\bar{z}}\right). \tag{2.4}$$

Since now $F_z = 0$, the result is closer to (2.1) than (2.3) was. Here $\tilde{D}$ describes the relative efficiency of taxes and transfers in reducing deprivation. Both the numerator and denominator of $\tilde{D}$ depend on D_c, so the difference in the relative efficiency of the two depends on z_a^i and z_b^i. The more people react to taxes (relative to transfers) by earning less, the higher is $\tilde{D}$ and the lower should the tax rate be. In (2.1), the higher is the social value of income, the higher is $z(\beta)$ and the lower should the tax rate be.

We can also rewrite $\tilde{D}$, using $a = 1-\tau$, as: $\frac{\sum D_c\left(z^i + (1-\tau)\frac{\partial z^i}{\partial(1-\tau)}\right)}{\sum D_c\left(1+(1-\tau)z_b^i\right)} = \frac{\sum D_c\left(1 + \frac{(1-\tau)}{z^i}\frac{\partial z^i}{\partial(1-\tau)}\right)z^i}{\sum D_c\left(1+(1-\tau)z_b^i\right)} = \frac{\sum D_c(1+\varepsilon)z^i}{\sum D_c\left(1+(1-\tau)z_b^i\right)}$. Thus the $\tilde{D}$ in the optimal tax result (2.4) entails a further efficiency consideration, lowering optimal tax rates to induce the poor to work more. Kanbur, Keen, and Tuomala (1994) find a similar result in their non-linear poverty-minimizing tax model. Here, however, we are restricted to lower the tax on everyone instead of only the poorest individuals.

Similarly as in the welfarist case, alternative ways of expressing the optimal tax rule can be derived. A rule that resembles the Piketty-Saez approach is

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1 - \bar{\beta} - \bar{\beta}^{\varepsilon}\right), \tag{2.5}$$

where analogously to Piketty-Saez, $\bar{\beta} = \frac{\int \beta^i z^i d\nu(i)}{Z} \left(= \frac{\int D_c z^i d\nu(i)}{Z \int D_c d\nu(j)}\right)$ is an average normalized deprivation weight, weighted by labor incomes (or, analogously, average labor income weighted by individual deprivation weights). In addition we have defined $\bar{\beta}^{\varepsilon} = \frac{\int \beta^i z^i \varepsilon_c^i d\nu(i)}{Z} \left(= \frac{\int D_c z^i \varepsilon_c^i d\nu(i)}{Z \int D_c d\nu(j)}\right)$, which describes average labor incomes weighted by their corresponding individual elasticities and deprivation weights. This can be interpreted as a combined deprivation and efficiency effect.

As in the welfarist setting, the more elastic average earnings are to taxation, the lower is the optimal tax rate (a regular efficiency effect). The optimal poverty-minimizing tax rate is decreasing in the average deprivation weight $\bar{\beta}$, as a higher taste for redistribution towards the materially deprived implies a lower $\bar{\beta}$ and thus higher taxation for redistributive purposes. The effect is analogous to the welfarist tax rate, of course with slightly different definitions for $\bar{\beta}$.

The new term $\bar{\beta}^{\varepsilon}$ can be interpreted as a combined deprivation weight and efficiency effect. The elasticity term implicit in $\bar{\beta}^{\varepsilon}$ takes into account the incentive effects of taxation on working and functions to reduce τ^*. To avoid discouraging the poor from working, their tax rates should be lower. But because the tax instrument is forced to be linear, tax rates are then lowered for everyone, as we found in the Tuomala model in Equation (2.4). The value of $\bar{\beta}^{\varepsilon}$ depends on the relationship of the individual earnings elasticities and income: if the elasticity is the same across income levels, there is just a level effect moving from $\bar{\beta}$ to $\bar{\beta}^{\varepsilon}$; however, if the elasticity were higher for more deprived individuals, for example, $\bar{\beta}^{\varepsilon}$ would most likely be higher than under a flat elasticity. This works towards a lower tax rate in order to avoid discouraging the poorest from working. However, whether $\bar{\beta}^{\varepsilon}$ is high or low does not depend only on the shape of the elasticity but also on the shape of the deprivation weights, which also affect $\bar{\beta}$.[8]

To summarize, the nonwelfarist tax rules diffes from the welfarist ones, depending on the definition of nonwelfarism in question (the F_c and F_z terms). However, when we take poverty minimization as the specific case of nonwelfarism, the tax rules are quite similar to welfarist ones. The basic difference is that equity is not considered in welfare terms but in terms of poverty reduction effectiveness. A more notable difference arises from efficiency considerations. With linear taxation, taking into account labor supply responses means that everybody's tax rate is affected, instead of just the target group's. If we want to induce the

[8] A version of the optimal tax rule in the poverty minimization case that is expressed in the spirit of Dixit and Sandmo (1977) is presented in Appendix A.

poor to work more to reduce their poverty, we need to lower everyone's tax rate. The welfarist linear tax rule does not take this into account. It is not however possible to state that under poverty-minimization tax rates are optimally lower than under welfare maximization, since we cannot directly compare the welfare and deprivation terms. However, there is an additional efficiency consideration involved under poverty minimization. Non-linear tax rules of course make it possible to target lower tax rates on the poorer individuals, but in a developing country context with lower administrative capacity this is not necessarily possible, and such considerations affect everyone's tax rate.

3 Public good provision with linear income taxes

3.1 Optimal public provision under the welfarist objective

Let us first extend the welfarist model of linear taxation to include the provision of pure public goods. The government offers a universal pure public good G, which enters individual utilities in addition to the consumption of private goods. The government's objective function is now $\sum W\left(V^i(a,b,G)\right)$, whereas the budget constraint becomes $(1-a)\sum z^i - Nb - \pi G = R$ where π is the producer price of the public good. The producer price of private consumption is normalized to 1. Let us now define the marginal willingness to pay for the public good by the expression $\sigma = \frac{V_G}{V_b}$ and $\sigma^* = \frac{\sum \beta^i \sigma^i}{\sum \beta^i}$ as the welfare weighted average marginal rate of substitution between public good and income for individual i. The rule for public provision can then be written as

$$\pi = \sigma^* - \tau\left(\sigma^* \bar{z}_b - \bar{z}_G\right). \tag{3.1}$$

This public good provision rule is a version of a modified Samuelson rule. It equates the relative cost of providing the public good to the welfare weighted sum of marginal rates of substitution (MRS). It also includes a revenue term, which takes into account the impacts of public good provision and income transfers on labor income tax. If, for example, labor supply is independent of the level of public good provision, $\bar{z}_G = 0$ and $\bar{z}_b < 0$, and if σ^* is positive, then the second term in (3.1) would make π higher than the welfare weighted aggregate marginal rate of substitution. The intuition in this case is the conventional idea that the public good needs to be financed using distortionary taxation, which increases is costs. However, if labor supply and public provision are positively related, the financing costs of the public good are reduced.

Extending the Piketty and Saez approach to include public provision leads to an alternative formulation of the public good provision rule

$$\frac{\int \omega^i W_u \left(u^i_G + u^i_x (1-\tau)\frac{\partial z^i}{\partial G}\right) \mathrm{d}\nu(i)}{\int \omega^i W_u u^i_x \,\mathrm{d}\nu(i)} = \pi - \tau \frac{dZ}{dG}. \tag{3.2}$$

The left-hand side relates the welfare gains of public good provision (a direct (u_G) and indirect effect $(u_x(1-\tau)\frac{\partial z^i}{\partial G}$ via labor supply reactions)) to the welfare gains of directly increasing consumption (cash transfers) and the right-hand side relates the costs of providing the public good (both its price and the effect it has on tax revenue) to the costs of directly increasing consumption (equal to 1 in this model).[9]

3.2 Optimal provision of public goods under poverty minimization

Now consider a non-welfarist government interested in minimizing poverty. The public good G which it offers, enters the deprivation index separately from other, private consumption x: $D\left(x, G, \bar{x}, \bar{G}\right)$. The government still offers a lump-sum cash transfer b as well, and finances its expenses with the linear income tax τ.

Again alternative formulations of the public good provision rule can be written. The first is

$$\pi = D^* - \tau\left(D^* \bar{z}_b - \bar{z}_G\right), \tag{3.3}$$

which can be compared with Equation (3.1). Here, $D^* = \frac{\sum D_G + \sum D_x a z_G^i}{\sum D_x\left(1 + a z_b^i\right)}$ captures the efficiency of the public good in reducing deprivation relative to the income transfer (because D_G, $D_x < 0$, $D^* > 0$). This rule differs considerably from standard modified Samuelson rules, reflecting instead of MRS the direct poverty reduction impact of the public good and its indirect impact via labor supply on consumption. As previously, the right-hand side includes a tax revenue term. Using the same example as in the context of (3.1), if $\bar{z}_G = 0$ and $\bar{z}_b < 0$, the price π of the public good would be higher than its relative efficiency in eliminating deprivation.

Suppose now that the consumers' welfare does not directly depend on the public good provision but the public good can have a productivity increasing impact. An example could be publicly provided education services that affect individuals' productivity via the wage rate. We therefore suppose that the direct impact of the public good on deprivation cancels out (i.e. $D_G = 0$), whereas the wage rate becomes an increasing function of G, i.e. $w'(G) > 0$ (denoting $z = w(G)L$). This means that the expression for D^* is rewritten as

$$D^* = \frac{\sum D_x a\left(w\frac{\partial L}{\partial G} + w' L\right)}{\sum D_x\left(1 + a w\frac{\partial L}{\partial b}\right)}. \tag{3.4}$$

This means that even if labor supply would not react to changes in public good provision, such provision would still be potentially desirable through its impact on the wage rate. In this way, public good provision can be interpreted as increasing the capability of the individuals to earn a living wage, which serves as a poverty reducing tool, and which can in some cases be a more effective way to reduce poverty rather than direct cash transfers. The optimality depends on the relative strength of $w'(G) > 0$ versus the direct impact of the transfers.

[9] In Equation (3.2), we could define a normalized marginal social welfare weight, similar as before, $\beta^i = \frac{\omega^i W_u u_x^i}{\int \omega^i W_u u_x^i d\nu(i)}$ to get $\frac{\int \omega^i W_u u_G^i \,\mathrm{d}\nu(i)}{\int \omega^i W_u u_x^i \,\mathrm{d}\nu(i)} + \int \beta^i (1-\tau)\frac{\partial z^i}{\partial G}\,\mathrm{d}\nu(i) = \pi + \tau\frac{dZ}{dG}$.

An alternative provision rule for the public good, using the Piketty-Saez approach, in the usual case where it also enters individuals' utility function is

$$\frac{\int \left(D_G + D_x(1-\tau)\frac{\partial z^i}{\partial G} \right) \, \mathrm{d}\nu(i)}{\int D_x \, \mathrm{d}\nu(i)} = \pi - \tau \frac{dZ}{dG}. \tag{3.5}$$

In the numerator of the left-hand side, the first term is the direct deprivation effect of G and the second term captures the indirect deprivation effect, operating via the labor supply impacts of the public good, which affect the level of private consumption x. These impacts are scaled by the poverty alleviation impact of private consumption itself (the impact of a cash transfer). The right-hand side reflects the costs of public good provision: besides the direct cost of the good there is an indirect tax revenue effect operating through labor supply. The condition is directly comparable to (3.2) because even though the welfarist case relies on utilities, in the FOC for G no envelope condition is evoked. The only difference between Equations (3.2) and (3.5) is that the utility and welfare weight terms are exchanged for deprivation terms.

Consider finally the provision of a quasi-private good, such that in addition to the publicly provided amount, individuals can purchase ("top-up") the good themselves as well. The good is denoted s and its total amount consists of private purchases h and public provision G: $s = G + h$. In addition to good s, individuals consume other private goods denoted x. The individual budget constraint is thus $c^i = x^i + ph^i = (1-\tau)z^i + \tau Z(1-\tau) - R - \pi G$. Deprivation is determined in terms of consumption of x and s, so the objective function is $min\, P = \int D\left(x^i, s^i, \bar{x}, \bar{s}\right) \mathrm{d}\nu(i)$. In this case, the provision rule is

$$\frac{\int \left[D_x \left((1-\tau)\frac{\partial z^i}{\partial s}\frac{\partial s}{\partial G} - p\frac{\partial h^i}{\partial G} \right) + D_s \frac{\partial s^i}{\partial G} \right] \mathrm{d}\nu(i)}{\int D_x \, \mathrm{d}\nu(i)} = \pi - \tau \frac{dZ}{dG}. \tag{3.6}$$

The result is analogous to the pure public good result in (3.5), with the difference that now the impact G has on poverty depends on whether public provision fully crowds out private purchases of the good (i.e. $\frac{dh}{dG} = -1 \Leftrightarrow \frac{ds}{dG} = 0$) or not (i.e. $\frac{dh}{dG} = 0 \Leftrightarrow \frac{ds}{dG} = 1$). If there is full crowding out, an increase in public provision of G that is fully funded via a corresponding increase in the tax rate has no impact on the consumption of s and consequently no impact on poverty. If there is no crowding out, however, the FOC becomes

$$\frac{\int \left[D_x \left((1-\tau)\frac{\partial z^i}{\partial s} \right) + D_s \right] \mathrm{d}\nu(i)}{\int D_x \, \mathrm{d}\nu(i)} = \pi - \tau \frac{dZ}{dG}, \tag{3.7}$$

which is the same as in the case of a pure public good in Equation (3.5).

To summarize, the welfarist public provision rule, when public goods are financed with linear income taxes and supplemented with lump-sum transfers, differs from the standard modified Samuelson rule. It equates a welfare-weighted sum of MRS to the marginal cost where tax revenue impacts are taken into account. Indirect effects of public provision (through labor supply decisions and thus private consumption) are incorporated. The poverty-minimizing public provision rule however replaces the welfare-weighted sum of MRS with the relative marginal returns to deprivation reduction. Here the "MRS" term measures how well public good is translated to reduced poverty (incorporating indirect effects as well), relative to private consumption. Finally, when the public good has positive effects on productivity, its provision can be desirable even if it would not have any direct impact on poverty.

4 Commodity taxation with linear income taxes

4.1 Optimal commodity taxation with linear income tax under the welfarist objective

This section considers the possibility that the government also uses commodity taxation (subsidies) to influence consumers' welfare. We follow the modeling of Diamond (1975). Unlike the analysis above, there are J consumer goods x_j instead of just two. Working with many goods is used to be able to more clearly describe the conditions under which uniform commodity taxation occurs at the optimum. The governments levies a tax t_j on the consumption of good x_j, so that its consumer price is $q_j = p_j + t_j$, where p_j represents the producer price (a commodity subsidy would be reflected by $t_j < 0$). Let q denote the vector of all consumer prices. In addition, the government can use a lump-sum transfer, b. Note that in this exposition, leisure is the untaxed numeraire commodity. Alternatively, one could also imply a linear tax on labor supply as above and treat one of the consumption goods as untaxed numeraire. However, choosing leisure as the numeraire makes the exposition easier. Thus, the consumer's budget constraint is $\sum_j q_j x_j^i = z^i + b$.

The government maximizes $\sum_i W\left(V^i(b,q)\right)$ subject to its budget constraint $\sum_i \sum_j t_j x_j^i - Nb = R$. It is useful to define, following Diamond (1975),

$$\gamma^i = \beta^i + \lambda \sum_j t_j \frac{\partial x_j^i}{\partial b} \tag{4.1}$$

as the net social marginal utility of income for person i. This notion takes into account the direct marginal social gain, β^i , and the tax revenue impact arising from commodity demand changes. The rule for optimal commodity taxation for good k is shown to be

$$\frac{1}{N} \sum_i \sum_j t_j \frac{\partial \tilde{x}_k^i}{\partial q_j} = \frac{1}{\lambda} cov(\gamma^i, x_k^i). \tag{4.2}$$

The left-hand side of the rule is the aggregate compensated change (weighted by commodity taxes) of good k when commodity prices are changed. The right-hand side refers to the covariance of the net marginal social welfare of income and consumption of the good in question. The rule says that the consumption of those goods whose demand is the greatest for people with low net social marginal value of income (presumably, the rich) should be discouraged by the tax system. Likewise the consumption of goods such as necessities should be encouraged by the tax system.

The key policy question is whether or when uniform commodity taxes are optimal, or, in other words, when would a linear income tax combined with an optimal demogrant be sufficient to reach the society's distributional goals at the smallest cost. Deaton (1979) shows that weakly separable consumption and leisure and linear Engel curves are sufficient conditions for the optimality of uniform commodity taxes. These requirements are quite stringent and unlikely to hold in practice; however, the economic importance they imply is unclear. If implementing differentiated commodity taxation entails significant administrative costs, they may easily outweigh the potential benefits of distributional goals, and that is why economists have typically been quite skeptical about non-uniform commodity taxation when applied to practical tax policy.

4.2 Optimal commodity taxation with linear income tax under poverty minimization

Poverty could be measured in many ways with multiple commodity goods: the government may care about overall consumption, the consumption of some of the goods (those that are in the basket used to measure poverty) or then it cares about both the overall consumption and the relative share of different kinds of consumption goods (such as merit goods). We discuss these measurement issues in Appendix B, but here we examine the simplest set-up where deprivation only depends on disposable income, $c^i = z^i + b$. Using the consumer's budget constraint, this is equal to the overall consumption level, $\sum_j q_j x^i_j$.

The government thus minimizes the sum of the poverty index $D\left(\sum_j q_j x^i_j, \bar{c}\right)$, and the budget constraint is the same as before. It is again useful to define

$$\gamma^i_P = D_c \sum_j q_j \frac{\partial x^i_j}{\partial b} + \lambda \sum_j t_j \frac{\partial x^i_j}{\partial b} \tag{4.3}$$

as the net poverty impact of additional income for person i. This notion takes into account the direct impact on poverty and the tax revenue impact arising from commodity demand changes.

As shown in Appendix A.3, this leads to an optimal tax rule as below:

$$\frac{1}{N} \sum_i \sum_j t_j \frac{\partial \tilde{x}^i_k}{\partial q_j} = -\frac{1}{\lambda} \left[\frac{1}{N} \sum_i D_c x^i_k + \frac{1}{N} \sum_i \sum_j D_c q_j \frac{\partial \tilde{x}^i_k}{\partial q_j} \right] + \frac{1}{\lambda} cov\left(\gamma^i_P, x^i_k\right). \tag{4.4}$$

In this formulation, the left-hand side is the same as in the welfarist case and it reflects the aggregate compensated change in the demand of good k. The first two terms in the squared brackets at the right-hand side capture the impacts of tax changes on poverty: the first term is the direct impact of the price change (keeping consumption unaffected) on measured poverty, whereas the second depends on the behavioral shift in consumption. Multiplied by the minus sign, the former term implies that the consumption of the good should be encouraged, whereas if demand decreases when the prices increase, the latter term actually serves to discourage consumption. The last term on the right reflects the same principles as the covariance rule in Equation (4.2), the correlation of the net poverty impact of income and the consumption of the good in question. That is, the covariance part of the tax rule moves the rule in the direction of favoring goods that have a high poverty reduction impact on the poor (i.e. those that the poor consume more).

The key lesson to note from the optimal commodity tax rule in the poverty minimization case is that the conventional conditions for uniform commodity tax to be optimal are not valid anymore. The reason is that even if demand was separable from labor supply, the first term at the right still remains in the rule, and its magnitude clearly varies depending on the quantity of good k consumed. Thus, income transfers are not sufficient to alleviate poverty when the government aims to minimize poverty that depends on disposable income. The intuition is very simple: commodity tax changes have a direct effect on the purchasing power of the consumer and these depend on the amount consumed. The extent of encouraging the consumption of the goods is the greater the larger their share of consumption among the consumption bundles of the poor is. A formal proof is provided in Appendix A.

In sum, the rule for optimal commodity taxation is changed when we shift from welfare maximization to poverty minimization. The welfarist rule reflects a fairly straightforward trade-off between efficiency (tax revenue) and equity (distributional impacts). The poverty-minimizing commodity tax rule brings new terms, the interrelations of which are not easy to entangle. It however also takes into account efficiency considerations (tax revenue through indirect labor supply effects) and equity (direct impact of the taxed good on poverty and indirect impact via labor supply effects). Most importantly, the conventional wisdom of when uniform commodity taxation is sufficient fails to hold in the poverty minimization case. Thus, observed commodity subsidies in developing countries, such as fuel or food subsidies, can be considered optimal given the preference for poverty minimization. In practice, it would be wise to limit the number of differentiated commodity tax rates to a few essential categories such as fuel and food, in order to keep the administrative complexity at a minimum.

5 Poverty minimization in the presence of an informal sector

An important issue for a developing country attempting to collect taxes is the issue of a large informal sector. If part of tax revenue is lost due to tax evasion in the informal sector, which is likely to be the case in the less developed economies, then the income transfer is reduced and redistributive targets may not be met. In this section we discuss the implications of informality for optimal redistributive policies for a government wishing to minimize poverty. The results can thus be contrasted to those obtained in previous sections.

Following Kanbur (2015) and Kanbur and Keen (2014), informal operators can be categorized as those who should comply with regulations but illegally choose not to, and those who legally remain outside regulation e.g. due to the smaller size of operations (either naturally or by adjusting size as a response to regulation). For our purposes however, it is enough to lump these categories into one 'informal sector', where it is possible to avoid taxes at least to some extent. It is also possible for workers to work in both sectors, such that part of total income is declared for taxation and part is evaded (consider e.g. supplementing official employment income with street vendoring). Note also that especially in the case of agriculture, evasion can also consist of home production. In this case, the reason for "informality" would be the small size of the producing entity, such that they are naturally not liable for taxes. Production for own consumption is however still relevant for the wellbeing and measured poverty of the family.

In our model, we follow the approach pioneered in Besley and Persson (2013). They work with a model that fits into the description above, where part of the tax base evades taxes. We thus take informality as given, and do not consider whether informality is 'natural', illegal or a response to taxation. Furthermore, this intensive margin model (what extent of income is earned at informal sector), they argue, yields essentially similar results as an extensive margin model (whether to participate in the formal job market).

Consider the case of income taxation. We can incorporate informality into the model by noting that people can shelter part e of their labor income from taxation. The extent of evasion is assumed to increase when the tax rate goes up, and thus $\frac{\partial e}{\partial a} < 0$. Income taxes are only paid from income $z^i - e^i$. It is noteworthy that for a government wishing to minimize income poverty, this is in fact beneficial: disposable incomes rise. The more this effect is concentrated among the poor who enter the deprivation index, the better. Individual consumption is now $z^i - \tau(z^i - e^i) + b = e^i + a(z^i - e^i) + b$. On the other hand, tax collections are reduced: the budget constraint becomes $(1-a)\sum(z^i - e^i) = Nb + R$. Our formulation follows that of Besley and Persson (2013) but we simplify it in order to explicitly consider the problem of optimal taxation, whereas they focus on the issue of investments in the state's fiscal capacity (we abstract from this issue here and take evasion as given).[10] The framework still nicely captures the essential tradeoffs a government faces when there is tax evasion.

The government now minimizes the Lagrangian $L = \sum D\left(e^i + a(z^i - e^i) + b, \bar{c}\right) + \lambda \left((1-a)\sum(z^i - e^i) - Nb - R\right)$. The first order condition with respect to the net-of-tax rate is

$$\sum D_c \left(\frac{\partial e^i}{\partial a} + z^i - e^i + a\left(\frac{\partial z^i}{\partial a} - \frac{\partial e^i}{\partial a}\right)\right) = \lambda\left(a\sum(z^i - e^i) - (1-a)\sum\left(\frac{\partial z^i}{\partial a} - \frac{\partial e^i}{\partial a}\right)\right), \tag{5.1}$$

whereas, under the assumption that there are no income effects in evasion, the first order

[10] Another difference is that in their original formulation, people face costs of evasion. When the tax rate goes up, the relative attractiveness of tax evasion increases, producing the same kind of effect $\left(\frac{\partial e}{\partial a} < 0\right)$ we assume directly here for brevity. (These costs could be related to e.g. Allingham-Sandmo-type risk of being caught and facing sanctions.) Also Slemrod's (1990) review suggests that higher tax rates tend to increase the supply of labor to the informal sector.

condition with respect to b stays the same. From here, we can derive a rule for the optimal tax following the same steps as in Section 2.2:

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon^e}\left(1 - \frac{\tilde{D}^e}{\bar{z}^e}\right), \tag{5.2}$$

where now ε^e is a tax elasticity of the net-of-evasion tax base $\bar{z}^e = \bar{z} - \bar{e}$ and $\tilde{D}^e$ represents the relative impact of taxes and transfers on the deprivation index (see Appendix for further detail). The rule represents a tradeoff between poverty reduction and efficiency, both of which are now altered by evasion. There is a pressure towards lower tax rates, as now distortions of taxation are increased by evasion behavior, so $\varepsilon^e > \varepsilon$. Contrary to this effect, $\tilde{D}^e$ is reduced compared to $\tilde{D}$ because reducing taxes (increasing a) is now a less useful instrument for poverty reduction, as part of the taxes have been evaded. As $\frac{\partial e}{\partial a} < 0$, people pay more taxes when tax rates are reduced, and therefore poverty in fact increases. $\tilde{D}^e$ thus works to increase tax rates.

Therefore, an interesting tradeoff arises: informality increases the cost of raising taxes, but it also means that higher taxes are less harmful as those in the informal sector do not need to pay them (and they are still entitled to the lump-sum transfer).[11] These countervailing forces have not been noted by the literature before. The presence of informality therefore seems to give rise to tax policy rules that are far from trivial. Future work could also look more deeply into the issue of the tax mix in the presence of informality. If income tax is more easily evaded than commodity taxation, as Boadway, Marchand, and Pestieau (1994) suggest, this could give rise to policies that focus taxation and redistribution on commodity taxes and subsidies, instead of income taxes and lump-sum transfers. This topic certainly merits more study.

6 A role for the minimum wage?

Our model can be used to analyse other instruments and situations relevant to developing economies as well. In Appendix C we illustrate here one particular application: examining whether it is desirable to complement the optimal income tax and transfer policy with a minimum wage on low-paying jobs. The approach builds on the recent important contribution by Lee and Saez (2012), who show that in a model with a discrete number of jobs and endogenously determined wage rates for different types of occupations, introducing minimum wages can be useful even in the presence of an optimal non-linear income tax under welfarist social objectives. This is in strong contrast with many earlier optimal tax models with minimum wages which have found very limited support for the minimum wage.[12]

[11] The idea that those in the informal sector can still receive transfers matches well with reality: many of the cash transfer systems reach those with little or no connection to the formal sector.

[12] Notice that in the conventional Mirrlees model with exogenously set wage rates, there would be no role for the minimum wage as it does not help to alleviate gross income inequality. That is why we also work with a model of endogenous wages, as other papers in the literature, such as Lee and Saez (2012), Marceau and Boadway (1994), and Allen (1987). Boadway and Cuff (2001), Allen (1987) and Guesnerie and Roberts (1987) consider linear income taxes whereas the majority of the literature focuses on minimum wage optimality together with non-linear taxation.

A key assumption for the result in Lee and Saez (2012) is that of efficient rationing. It implies that those who lose their job first when the minimum wage is raised above the market clearing level are those with the smallest surplus from work. This means that the social planner is indifferent regarding whether they work or not (implying little efficiency losses from the minimum wage) and what is left is the desirable redistributive effect in terms of higher wages for those earning the minimum wage. The purpose of our exercise is to see how the conclusions from the Lee-Saez analysis change if the government's objective function is poverty minimization. We employ this particular model as it helps to illustrate the problems of minimum wage legislation for the developing country context that we are focusing on.

The result reveals that the minimum wage is potentially a useful additional tool for the government if the weight of the working poor, whose income minimum wages increase, is sufficently high in the deprivation index. This is essentially the same result as in the welfarist case, but the importance of it could differ: it may well be the case that the working poor have a lower deprivation weight if the government is strongly concerned with the fate of those at the bottom of the distribution, i.e. those without work, and in such a case, the weight of the working poor would not necessarily be large enough.[13]

Notice that applying minimum wage regulation adds to the information needs of the government, which is at odds with our general set-up. We have motivated that linear tax systems are more easily implementable for a developing country government which cannot necessarily observe the salaries paid to individual employees but perhaps just the total sum of salaries paid by an employer (as motivated in Piketty and Saez 2013 as well), so that taxes can be withheld at source, for example. Regulating a minimum wage for a specific occupation obviously imposes a much higher informational requirement. We can however assume, as in Lee and Saez (2012), that the government can impose a minimum wage without having information on individual wage rates by relying on workers to whistle-blow on employers who do not comply with the regulation and pay wages below the regulated level. This would require the government to be able to accurately audit suspect firms, to reward whistle-blowers, and in addition that the workers cannot collude with employees to not report violations of minimum wage. Even under these assumptions, it is obvious that minimum wages impose higher requirements on the government's fiscal and legal capacity than the models in the previous sections. We can still assume that these new instruments can become available as the societies advance on their development path, even though they do not seem relevant for the developing country context we are considering in this paper.

7 Conclusion

This paper examined optimal linear income taxation, public provision of public and private goods and the optimal combination of linear income tax and commodity taxes when the

[13] Although also in the welfarist model, the mimimum wage is never desirable in the case of Rawlsian preferences.

government's aim is to minimize poverty. The linear tax environment was chosen because such taxes are more easily implementable in a developing country context and as they also depend on similar sufficient statistics than more complicated non-linear formulas, this giving rise to similar guidelines for empirical work in the area.

The results show that the linear income tax includes additional components that work towards lowering the marginal tax rate. This result arises from the goal to boost earnings to reduce income poverty. Unlike in the optimal non-linear income tax framework, this lower marginal tax affects all taxpayers in the society. Public good provision in the optimal tax framework under poverty minimization was shown to depend on the relative efficiency of public provision versus income transfers in generating poverty reductions. One particular avenue where public provision is useful is via its potentially beneficial impact on individuals' earnings capacity. Thus, public provision can be desirable even if its direct welfare effects were non-existent.

Perhaps more importantly, poverty minimization as an objective changes completely the conditions under which uniform commodity taxation is optimal. When the government objective is to minimize poverty that depends on disposable income, uniform commodity taxation is unlikely to be ever optimal: this is because the commodity tax changes have first-order effects on consumer's budget via the direct impact on the cost of living, and this direct effect depends on the relative importance of different goods in the overall consumption bundle. Separability in demand coupled with linear Engel curves is not sufficient to guarantee optimality of uniform commodity taxes. In reality, the administrative difficulties of implementing commodity taxation with many tax rates must, of course, be taken into account, as well.

We also examined the implications of the presence of an informal sector for optimal tax-and-transfer policies. The results revealed that when the government is concerned about income poverty, the presence of the informal sector is, on the one hand, useful, as it reduces the poverty-increasing effect of higher taxes but, on the other hand, it is also costly since it is likely to increase the elasticity of the tax base. Examining the implications of informality on the role of other instruments of government policies is an important avenue for future work.

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Appendices

A Mathematical appendix

A.1 Linear income taxation

A.1.1 Welfarism

Consider first the welfarist case. Using λ to denote the multiplier associated with the budget constraint, the government's Lagrangian is $L = \sum W\left(V^i(a,b)\right) + \lambda\left((1-a)\sum z^i - Nb - R\right)$.

Using Roy's theorem, $V_a^i = V_b^i z^i$, the first order conditions with respect to a and b, respectively, are

$$\sum \beta^i z^i = \lambda\left(\sum z^i - (1-a)\sum z_a^i\right) \tag{A.1}$$

$$\sum \beta^i = \lambda\left(N - (1-a)\sum z_b^i\right), \tag{A.2}$$

where $\beta^i = W_V V_b^i$ is the social marginal utility of income ($\Rightarrow W_V V_a^i = W_V V_b^i z^i = \beta^i z^i$).

Divide (A.1) by (A.2) to get

$$\frac{\sum \beta^i z^i}{\sum \beta^i} = \frac{\sum z^i - (1-a)\sum z_a^i}{N - (1-a)\sum z_b^i}. \tag{A.3}$$

Denote average income $\bar{z} = \frac{\sum z^i}{N}$ and welfare-weighted average income $z(\beta) = \frac{\sum \beta^i z^i}{\sum \beta^i}$ to get

$$z(\beta) = \frac{\bar{z} - (1-a)\bar{z}_a}{1-(1-a)\bar{z}_b}. \tag{A.4}$$

Multiply the government's revenue constraint by $\frac{1}{N}$ and define $g = \frac{R}{N}$ to get $(1-a)\bar{z} - b = g$, and totally differentiate, keeping g constant

$$\frac{db}{da}|_{gconst} = \frac{\bar{z} + (1-a)\bar{z}_a}{-\left[1-(1-a)\bar{z}_b\right]} = -z(\beta). \tag{A.5}$$

The fact that $z(\beta) = -\frac{db}{da}|_{gconst}$ tells us that welfare-weighted labor supply should be equal to the constant-revenue effect of tax rate changes in b.

By totally differentiating average labor income $\bar{z}$ and using (A.5), we have

$$\frac{d\bar{z}}{da}|_{gconst} = \bar{z}_a + \bar{z}_b\frac{db}{da}|_{gconst} = \bar{z}_a + \bar{z}_b z(\beta). \tag{A.6}$$

When we impose g as a constant we have to give up one of our degrees of freedom. Now the interpretation of $\frac{d\bar{z}}{da}|_{gconst}$ is then the effect on labor supply when a is changed, as is b, in order to keep tax revenue constant. Using (A.6) we can write (A.4)

$$z(\beta) - \bar{z} = (1-a)\frac{d\bar{z}}{da}|_{gconst} = -\tau\frac{d\bar{z}(1-\tau)\bar{z}}{d(1-\tau)(1-\tau)\bar{z}}, \tag{A.7}$$

from which we get the optimal tax rate of Equation (2.1).

We now derive the results in the form of the Piketty and Saez (2013) model. In their model, there is a continuum of individuals, whose distribution is $\nu(i)$ (population size is normalized to one). Individuals maximize their utility $u^i((1-\tau)z^i + b, z^i)$, and their FOC implicitly defines the Marshallian earnings function $z_u^i(1-\tau, b)$. Using this, aggregate earnings are $Z_u(1-\tau, b)$. The government's budget constraint $b + R = \tau Z_u(1-\tau, b)$ implicitly defines b as a function of τ, and consequently Z_u can also be defined solely as a function of τ: $Z(1-\tau) = Z_u(1-\tau, b(\tau))$. Z has elasticity $\varepsilon = \frac{1-\tau}{Z}\frac{dZ}{d(1-\tau)}$.

To start, note that if the government only cared about maximizing tax revenue $\tau Z(1-\tau)$, it would set τ such that $\frac{\partial(\tau Z(1-\tau))}{\partial \tau} = 0$: $Z(1-\tau) - \tau \frac{dZ}{d(1-\tau)} = 0$. Using $\frac{\tau}{Z}\frac{dZ}{d(1-\tau)} = \frac{\tau}{1-\tau}\varepsilon$, this gives

$$\begin{aligned} \frac{\tau^*}{1-\tau^*} &= \frac{1}{\varepsilon} \\ \Leftrightarrow \tau^* &= \frac{1}{1+\varepsilon}. \end{aligned} \tag{A.8}$$

When the government is concerned about social welfare, its problem is to $max\ SWF = \int \omega^i W(u^i((1-\tau)z^i + \tau Z(1-\tau) - R, z^i)\, d\nu(i)$, where use has been made of the individual consumption $c^i = (1-\tau)z^i + b = (1-\tau)z^i + \tau Z(1-\tau) - R$. Here ω is a Pareto weight and W is an increasing and concave transformation of utilities. The FOC $\frac{\partial SWF}{\partial \tau} = 0$ is

$$\int \omega^i W_u \left[u_c^i \left(-z^i + (1-\tau)\frac{\partial z^i}{\partial \tau} + Z + \tau \frac{dZ}{d\tau} \right) + u_z^i \frac{\partial z^i}{\partial \tau} \right] \mathrm{d}\nu(i) = 0,$$

which, using the individual's envelope condition, becomes

$$\int \omega^i W_u u_c^i \left(-z^i + Z - \tau \frac{dZ}{d(1-\tau)} \right) \mathrm{d}\nu(i) = 0.$$

Taking $Z - \tau\frac{dZ}{d(1-\tau)}$ out of the integrand and leaving it to the left-hand side we have on the right-hand side $\frac{\int \omega^i W_u u_c^i z^i \,\mathrm{d}\nu(i)}{\int \omega^j W_u u_c^j \,\mathrm{d}\nu(j)}$. Piketty and Saez define $\beta^i = \frac{\omega^i W_u u_c^i}{\int \omega^i W_u u_c^i d\nu(i)}$ as a normalized social marginal welfare weight for individual i, so that the term can be simplified to

$$Z - \tau \frac{dZ}{d(1-\tau)} = \int \beta^i z^i \,\mathrm{d}\nu(i).$$

Using the definition of aggregate elasticity of earnings and defining $\bar{\beta} = \frac{\int \beta^i z^i d\nu(i)}{Z}$ as the average normalized social marginal welfare weight, weighted by labor incomes z^i (it can also be interpreted as the ratio of the average income weighted by individual welfare weights β^i to the average income Z), we can rewrite this as

$$1 - \frac{\tau}{1-\tau}\varepsilon = \bar{\beta}.$$

According to Piketty and Saez, $\bar{\beta}$ "measures where social welfare weights are concentrated on average over the distribution of earnings". The social welfare maximizing tax rate thus gives the optimality rule of Equation (2.2) in the main text.

Piketty and Saez also note that (2.2) can be written in the form of $\tau^* = \frac{-cov\left(\beta^i, \frac{z^i}{Z}\right)}{-cov\left(\beta^i, \frac{z^i}{Z}\right)+\varepsilon}$. If higher incomes are valued less (lower β) then the covariances are negative and the tax rate is positive. This is a similar formulation as in Dixit and Sandmo (1977), Equation (20), where $\tau^* = -\frac{1}{\lambda}\frac{-cov\left(z^i,\mu^i\right)}{\frac{\partial \bar{z}}{\partial(1-\tau)}|_{comp.}}$ (here λ represents the government's budget constraint Lagrange multiplier and μ^i the individual's marginal utility of income, s.t. $u_c = \mu^i$). Here the numerator reflects the equity element and the denominator the efficiency component, similar as in (2.2).

A.1.2 Non-welfarism

In the non-welfarist case, the Lagrangean is $L = \sum F\left(az^i + b, z^i\right) + \lambda\left((1-a)\sum z^i - Nb - R\right)$. The first-order conditions with respect to a and b are

$$\sum\left(F_c(z^i + az_a^i) + F_z z_a^i\right) = \lambda\left(\sum z^i - (1-a)\sum z_a^i\right) \tag{A.9}$$

$$\sum\left(F_c(1 + az_b^i) + F_z z_b^i\right) = \lambda\left(N - (1-a)\sum z_b^i\right). \tag{A.10}$$

Dividing the first equation with the second and dividing through the right hand side with N we get

$$\frac{\sum\left(F_c(z^i + az_a^i) + F_z z_a^i\right)}{\sum\left(F_c(1 + az_b^i) + F_z z_b^i\right)} = \frac{\bar{z} - (1-a)\bar{z}_a}{1 - (1-a)\bar{z}_b}, \tag{A.11}$$

which gives Equation (2.3). Minimizing a deprivation index D is a special case of this, such that $F_c = D_c$ and $F_z = 0$. Otherwise the derivation of (2.4) is analogous to the above.

Let us next derive the poverty minimizing tax rule following the formulation of Piketty and Saez. Given the government's instruments, consumption is $c^i = (1-\tau)z^i + b = (1-\tau)z^i + \tau Z(1-\tau) - R$. The poverty-minimization objective in the continuous case thus reads as

$$\begin{aligned} min\, P &= \int D\left(c^i, \bar{c}\right)\, \mathrm{d}\nu(i) \\ &= \int D\left((1-\tau)z^i + \tau Z(1-\tau) - R, \bar{c}\right)\, \mathrm{d}\nu(i). \end{aligned} \tag{A.12}$$

The optimal tax rate is found from the government's FOC, $\frac{\partial P}{\partial \tau} = 0$:

$$\begin{aligned} &\int D_c\left(-z^i + (1-\tau)\frac{\partial z^i}{\partial \tau} + Z + \tau\frac{dZ}{d\tau}\right)\, \mathrm{d}\nu(i) = 0 \\ \Leftrightarrow &\int D_c\left(-z^i - (1-\tau)\frac{\partial z^i}{\partial(1-\tau)} + Z - \tau\frac{dZ}{d(1-\tau)}\right)\, \mathrm{d}\nu(i) = 0. \end{aligned} \tag{A.13}$$

Define a "normalized marginal deprivation weight" as $\beta^i = \frac{D_c}{\int D_c d\nu(j)}$. Using this definition, $\left(Z - \tau\frac{dZ}{d(1-\tau)}\right)\int D_c\, \mathrm{d}\nu(i) = \int D_c\left(z^i + (1-\tau)\frac{\partial z^i}{\partial(1-\tau)}\right)\, \mathrm{d}\nu(i)$ can be written as

$$Z - \tau\frac{dZ}{d(1-\tau)} = \int \beta^i\left(z^i + (1-\tau)\frac{\partial z^i}{\partial(1-\tau)}\right)\, \mathrm{d}\nu(i). \tag{A.14}$$

Using the definition of the elasticity of individual labor earnings $\varepsilon_c^i = \frac{1-\tau}{z^i}\frac{\partial z^i}{\partial(1-\tau)}$, we have $(1-\tau)\frac{\partial z^i}{\partial(1-\tau)} = z^i\varepsilon_c^i$ and using elasticity of aggregate earnings $\varepsilon = \frac{1-\tau}{Z}\frac{dZ}{d(1-\tau)}$ we have $Z - \tau\frac{dZ}{d(1-\tau)} = 1 - \frac{\tau}{1-\tau}\varepsilon$ and we can rewrite the above as

$$Z\left(1 - \frac{\tau}{1-\tau}\varepsilon\right) = \int \beta^i \left(z^i + z^i\varepsilon_c^i\right)\, \mathrm{d}\nu(i). \tag{A.15}$$

Which leads to the poverty minimizing rule of

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon}\left(1 - \bar{\beta} - \bar{\beta}^{\varepsilon}\right),$$

which leads to Equation (2.5).

Finally, the third way for expressing the optimal tax rule in the case of poverty minimization is one following the Dixit and Sandmo (1977) formulation and it can be written as

$$\tau^* = -\frac{1}{\lambda}\frac{cov\left(D_c, z^i\right) + \frac{1}{N}\sum D_c a\tilde{z}_a^i + cov\left(D_c a z_b^i, z^i\right)}{\frac{1}{N}\sum \tilde{z}_a^i}. \tag{A.16}$$

In this expression, the denominator is the same as in Equation (20) of Dixit and Sandmo (1977) presented in Section A.1, that is, the average derivative of compensated labor supply with respect to the net-of-tax rate. In the numerator, the first term measures the strength of the association between income and poverty impact: when the association between overall poverty and small income is strong (this would the case of squared poverty gap), the tax should be high so that it will finance a sizable lump-sum transfer. If the association is weaker (as in the headcount rate), the tax rate is optimally smaller. The second and the third terms in the numerator are new. They measure the indirect effects from changes in the tax rate on labor supply. Here $\tilde{z}$ is the compensated (Hicksian) labor supply. The greater is the reduction in the labor supply following an increase in the tax rate (it is the compensated change as the tax increase is linked with a simultaneous increase in the lump-sum transfer), the smaller should the tax rate be in order to avoid increases in deprivation arising from lower earned income. The last two terms in the numerator are closely linked with a formulation $D_c(1-\tau)\frac{\partial \bar{z}}{\partial q}|_{comp}$, where the idea is that the last covariance term serves as a corrective device for the mean impact of taxes on labor supply (similarly as in the denominator in the original Dixit-Sandmo formulation).

A.2 Public good provision

A.2.1 Welfarism

The Lagrangian is $L = \sum W\left(V^i(a, b, G)\right) + \lambda\left((1-a)\sum z^i - Nb - \pi G - R\right)$. Maximizing the Lagrangean with respect to b and G gives

$$\sum \beta^i = \lambda\left(N - (1-a)\sum z_b^i\right) \tag{A.17}$$

$$\sum W_V V_G^i = \lambda\left(\pi - (1-a)\sum z_G^i\right). \tag{A.18}$$

Dividing (A.18) by (A.17) we obtain

$$\frac{\sum \beta^i \sigma^i}{\sum \beta^i} = \frac{\pi - (1-a)\bar{z}_G}{1-(1-a)\bar{z}_b}, \tag{A.19}$$

where we define $\sigma^* = \frac{\sum \beta^i \sigma^i}{\sum \beta^i}$ to be the welfare weighted average marginal rate of substitution between public good and income for individual i. Rewriting this rule gives Equation (3.1) in the main text.

In the Piketty and Saez formulation, the government's goal function is

$$SWF = \int \omega^i W\left(u^i\left(\left(1-\tau\right)z^i + \tau Z(1-\tau) - R - \pi G, G, z^i\right)\right)\,\mathrm{d}\nu(i).$$

The FOC for τ is as before, and the FOC for public good provision G is

$$\int \omega^i W_u\left(u_G^i + u_x^i\left((1-\tau)\frac{\partial z^i}{\partial G} + \tau\frac{dZ}{dG} - \pi\right)\right)\mathrm{d}\nu(i) = 0,$$

which produces Equation (3.2) in the text.

A.2.2 Poverty minimization

Using Tuomala's model, and the deprivation index $D\left(x, G, \bar{x}, \bar{G}\right)$ defined over consumption of the public good G and other private consumption x, we can divide the government's first order condition for G (analogous to Equation (A.17)) with that of b (analogous to Equation (A.18)) to get the following relationship:

$$D^* = \frac{\pi - (1-a)\bar{z}_G}{1-(1-a)\bar{z}_b}, \tag{A.20}$$

where $D^* = \frac{\sum D_G + \sum D_x a z_G^i}{\sum D_x\left(1+a z_b^i\right)}$. This can be rewritten to get Equation (3.1).

In the Piketty-Saez type of model, individual private consumption is $x = (1-\tau)z^i + b = (1-\tau)z^i + \tau Z(1-\tau) - R - \pi G$. The government's problem is then

$$\begin{aligned} min\ P &= \int D\left(x^i, G, \bar{x}, \bar{G}\right)\,\mathrm{d}\nu(i) \\ &= \int D\left((1-\tau)z^i + \tau Z(1-\tau) - R - \pi G, G, \bar{x}, \bar{G}\right)\,\mathrm{d}\nu(i). \end{aligned} \tag{A.21}$$

The first-order condition for optimal tax τ is unchanged, and the FOC for public good provision is $\int\left[D_G + D_x\left((1-\tau)\frac{\partial z^i}{\partial G} + \tau\frac{dZ}{dG} - \pi\right)\right]\,\mathrm{d}\nu(i) = 0$, which gives the public provision rule of (3.2).

The poverty minimization problem in the case of provision of a quasi-private good is

$$
\begin{aligned}
min\ P &= \int D\left(x^i, s^i, \bar{x}, \bar{s}\right) \mathrm{d}\nu(i) \\
&= \int D\left((1-\tau)z^i + \tau Z(1-\tau) - R - \pi G - ph^i, s^i, \bar{x}, \bar{s}\right) \mathrm{d}\nu(i).
\end{aligned} \tag{A.22}
$$

The FOC for public good provision G is $\int \left[D_x \left((1-\tau)\frac{\partial z^i}{\partial s}\frac{\partial s}{\partial G} + \tau \frac{dZ}{dG} - \pi - \pi \frac{\partial h^i}{\partial G}\right) + D_s \frac{\partial s^i}{\partial G}\right] \mathrm{d}\nu(i) = 0$, which gives the public provision rule (3.6).

A.3 Commodity taxation

A.3.1 Welfarism

Lagrangean of the government's optimization problem is the following:

$$
L = \sum_i W\left(V^i(b,q)\right) + \lambda \left(\sum_i \sum_j t_j x_j^i - Nb - R\right) \tag{A.23}
$$

The first-order conditions with respect to b and q_k are

$$
\sum_i \beta^i + \lambda \sum_i \sum_j t_j \frac{\partial x_j^i}{\partial b} - \lambda N = 0 \tag{A.24}
$$

$$
-\sum_i \beta^i x_k^i + \lambda \sum_i \sum_j t_j \frac{\partial x_j^i}{\partial q_k} + \lambda \sum_i x_k^i = 0, \tag{A.25}
$$

where Roy's identity has been used in (A.25), i.e. $\frac{\partial V^i}{\partial q_k} = -\frac{\partial V^i}{\partial b} x_k^i$. Using the definition of γ^i, this means that (A.24) can we rewritten as

$$
\frac{\sum_i \gamma^i}{N} = \lambda, \tag{A.26}
$$

implying that the average net social marginal utility of income must equal the shadow price of budget revenues at the optimum. Next use the definition of γ and the Slutsky equation for the commodity demand

$$
\frac{\partial x_j^i}{\partial q_k} = \frac{\partial \tilde{x}_j^i}{\partial q_k} - x_k^i \frac{\partial x_j^i}{\partial b},
$$

where $\tilde{x}_j^i$ denotes the compensated (Hicksian) demand for good x_j^i, in (A.25), to get

$$
\sum_i \sum_j t_j \frac{\partial \tilde{x}_j^i}{\partial q_k} = \frac{1}{\lambda} \sum \left(\gamma^i - \lambda\right) x_k^i. \tag{A.27}
$$

The covariance between γ^i and the demand of the good x_k can be written as (using (A.26))

$$
cov\left(\gamma^i, x_k^i\right) = \frac{\sum_i \gamma^i x_k^i}{N} - \frac{\sum_i \gamma^i}{N}\frac{\sum_i x_k^i}{N} = \frac{\sum_i \gamma^i x_k^i}{N} - \lambda \frac{\sum_i x_k^i}{N}.
$$

Using Slutsky symmetry, Equation (A.29) can therefore be written as a covariance rule

(4.2).

A.3.2 Poverty minimization

The deprivation index to be minimized is $D\left(\sum_j q_j x_j^i, \bar{c}\right)$. The first-order conditions with respect to b and q_k are

$$\sum_i D_c \sum_j q_j \frac{\partial x_j^i}{\partial b} + \lambda \sum_i \sum_j t_j \frac{\partial x_j^i}{\partial b} - \lambda N = 0 \tag{A.28}$$

$$\sum_i D_c x_k^i + \sum_i D_c \sum_j q_j \frac{\partial x_j^i}{\partial q_k} + \lambda \sum_i \sum_j t_j \frac{\partial x_j^i}{\partial q_k} + \lambda \sum_i x_k^i = 0. \tag{A.29}$$

Using the Slutsky equation in Equation (A.29) and dividing by N leads to

$$\begin{aligned} &\frac{1}{N}\sum_i D_c x_k^i + \frac{1}{N}\sum_i D_c \sum_j q_j \left(\frac{\partial \tilde{x}_j^i}{\partial q_k} - x_k^i \frac{\partial x_j^i}{\partial b}\right) \\ &\quad + \frac{\lambda}{N}\sum_i \sum_j t_j \left(\frac{\partial \tilde{x}_j^i}{\partial q_k} - x_k^i \frac{\partial x_j^i}{\partial b}\right) + \frac{\lambda}{N}\sum_i x_k^i = 0. \end{aligned} \tag{A.30}$$

Multiplying Equation (A.28) by $\sum_i x_k^i / N^2$ and adding it with Equation (A.30) gives

$$\begin{aligned} &\frac{1}{N}\sum_i D_c x_k + \frac{1}{N}\sum_i D_c \sum_j q_j \frac{\partial \tilde{x}_j^i}{\partial q_k} - \frac{1}{N}\sum_i \sum_j D_c q_j x_k^i \frac{\partial x_j^i}{\partial b} \\ &\quad + \frac{1}{N}\frac{\sum_i D_c}{N}\sum_j q_j \frac{\partial x_j^i}{\partial b}\sum_i x_k^i + \frac{\lambda}{N}\sum_i \sum_j t_j \frac{\partial \tilde{x}_j^i}{\partial q_k} \\ &\quad - \frac{\lambda}{N}\sum_i \sum_j t_j x_k^i \frac{\partial x_j^i}{\partial b} + \frac{1}{N}\frac{\lambda}{N}\sum_i \sum_j t_j \frac{\partial x_j^i}{\partial b}\sum_i x_k^i = 0. \end{aligned} \tag{A.31}$$

Noticing that the covariance γ_P^i and x_j^i can be written as $\frac{1}{N}\sum_i \sum_j D_c q_j x_k^i \frac{\partial x_j^i}{\partial b} + \frac{\lambda}{N}\sum_i \sum_j t_j x_k^i \frac{\partial x_j^i}{\partial b} - \frac{1}{N}\frac{\sum_i D_c}{N}\sum_j q_j \frac{\partial x_j^i}{\partial b}\sum_i x_k^i - \frac{1}{N}\frac{\lambda}{N}\sum_i \sum_j t_j \frac{\partial x_j^i}{\partial b}\sum_i x_k^i$, the rule above can be written as Equation (4.4) in the main text.

A.3.3 Non-optimality of uniform commodity taxation

We demonstrate formally how uniform commodity taxation is not optimal in the case of poverty minimization. To see this, rewrite first the FOC with respect to b (A.28) as

$$\frac{1 - \frac{1}{N}\sum_i \sum_j t_j \frac{\partial x_j^i}{\partial b}}{\frac{1}{N}\sum_i D_c \sum_j q_j \frac{\partial x_j^i}{\partial b}} = \frac{1}{\lambda}. \tag{A.32}$$

Next, rewriting the FOC for q_k (Equation (A.30)) yields

$$\frac{1}{N}\sum_i\sum_j t_j\frac{\partial \tilde{x_k}^i}{\partial q_j} = -\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\frac{x_k^i}{N} - \frac{1}{\lambda}\frac{1}{N}\sum_i D_c\sum_j q_j\frac{\partial \tilde{x_k}^i}{\partial q_j}$$
$$+\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\sum_j q_j\frac{\partial x_j^i}{\partial b}x_k^i + \frac{1}{N}\left(\sum_j t_j\frac{\partial x_j^i}{\partial b} - 1\right)\sum_i x_k^i. \tag{A.33}$$

Substituting for $\frac{1}{\lambda}$ from (A.32) in the first term at the lower row of Equation (A.33) gives

$$\frac{1}{N}\sum_i\sum_j t_j\frac{\partial \tilde{x_k}^i}{\partial q_j} = -\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\frac{x_k^i}{N} - \frac{1}{\lambda}\frac{1}{N}\sum_i D_c\sum_j q_j\frac{\partial \tilde{x_k}^i}{\partial q_j}$$
$$+\frac{1-\frac{1}{N}\sum_i\sum_j t_j\frac{\partial x_j^i}{\partial b}}{\sum_i D_c\sum_j q_j\frac{\partial x_j^i}{\partial b}}\sum_i D_c x_k^i\sum_j q_j\frac{\partial x_j^i}{\partial b}$$
$$+\frac{1}{N}\sum_i x_k^i\left(\sum_j t_j\frac{\partial x_j^i}{\partial b} - 1\right). \tag{A.34}$$

Following Deaton (1979: 359-360), when preferences are separable and Engel curves are linear, demand is written as $x_j^i = \delta_j^i(q) + \theta_j(q)c^i$, hence the derivative of demand with respect to disposable income c or transfer b is $\theta_j(q)$, i.e. independent of the person i. By writing out explicitly the solution that the derivative of demand w.r.t b is independent of i and write $\frac{\partial x_j^i}{\partial b} = \theta_j(q)$ we have

$$\frac{1}{N}\sum_i\sum_j t_j\frac{\partial \tilde{x_k}^i}{\partial q_j} = -\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\frac{x_k^i}{N} - \frac{1}{\lambda}\frac{1}{N}\sum_i D_c\sum_j q_j\frac{\partial \tilde{x_k}^i}{\partial q_j}$$
$$+\frac{1-\frac{1}{N}\sum_i\sum_j t_j\theta_j(q)}{\sum_i D_c\left(\sum_j q_j\theta_j(q)\right)}\sum_i D_c x_k^i\left(\sum_j q_j\theta_j(q)\right)$$
$$+\frac{1}{N}\sum_i x_k^i\left(\sum_j t_j\theta_j(q) - 1\right), \tag{A.35}$$

where in the second row we can cancel out the $\sum_j q_j\theta_j(q)$ terms and rewrite $\sum_i\sum_j t_j\theta_j(q) = N\sum_j t_j\theta_j(q)$ in the numerator because the term is independent over i:

$$\frac{1}{N}\sum_i\sum_j t_j\frac{\partial \tilde{x_k}^i}{\partial q_j} = -\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\frac{x_k^i}{N} - \frac{1}{\lambda}\frac{1}{N}\sum_i D_c\sum_j q_j\frac{\partial \tilde{x_k}^i}{\partial q_j}$$
$$+\frac{1-\sum_j t_j\theta_j(q)}{\sum_i D_c}\sum_i D_c x_k^i + \frac{1}{N}\sum_i x_k^i\left(\sum_j t_j\theta_j(q) - 1\right). \tag{A.36}$$

Note next that due to homogeneity of degree 0 of compensated demand, $\sum_j q_j\frac{\partial \tilde{x_k}^i}{\partial q_j} + w_i\frac{\partial \tilde{x_k}^i}{\partial w_i} = 0$. This, together with the observation that if a uniform commodity tax t was a solution to a problem at hand, this would mean that the left-hand side of (A.33) could be written as $-\frac{t}{N}\sum_i w_i \frac{\partial \tilde{x_k}^i}{\partial w}$. Because of separability, the substitution response is linked to the full income derivative, so that $\frac{\partial \tilde{x_k}^i}{\partial w} = \phi^i \theta_j(q)$. Because of these arguments, (A.33) becomes

$$-\frac{t}{N}\theta_j(q)\sum_i w_i\phi^i = -\frac{1}{\lambda}\frac{1}{N}\sum_i D_c\frac{x_k^i}{N} - \frac{1}{\lambda}\frac{1}{N}\theta_j(q)\sum_i D_c w_i\phi^i + \frac{1-t\sum_j\theta_j(q)}{\sum_i D_c}\sum_i D_c x_k^i + \frac{1}{N}\sum_i x_k^i\left(t\sum_j\theta_j(q)-1\right). \tag{A.37}$$

Note that terms incorporation $\theta_j(q)$ cannot be canceled out from the equation so the result remains dependent on j. In addition, even if the terms were canceled, the term $\sum_i D_c\frac{x_k^i}{N}$ still depends on j. This shows that uniform commodity taxation is not optimal when the objective function of the government is to minimize poverty.

A.4 Optimal income taxation with an informal sector

A.4.1 Welfarism

The welfarist Lagrangian, in the presence of informality, is $L = \sum W\left(V^i(a,b,e)\right) + \lambda((1-a)\sum(z^i - e^i) - Nb - R)$. We can denote the effective tax base as $z^e = z - e$. The derivative of this tax base with respect to tax rate a is denoted $z_a^e = z_a - \frac{\partial e}{\partial a}$, where we assume $\frac{\partial e}{\partial a} < 0$ (whereas $\frac{\partial e}{\partial b} = 0$). The first-order conditions with respect to a and b are

$$\sum W_V V_a^e = \lambda\left(\sum z^e - (1-a)\sum z_a^e\right)$$
$$\sum W_V V_b = \lambda\left(N - (1-a)\sum z_b\right),$$

where V_a^e is a shorthand for the derivative of the indirect utility function that takes individual evasion behavior into account. Should there be no evasion, the individual would maximize her utility over income $az + b$ and $V_a = \lambda z$. Under evasion, consumption is $a(z-e) + e + b$ and, by envelope theorem, $V_a^e = \lambda(z-e) = \lambda z^e$. Roy's theorem adapts in this case to: $V_a^e = V_b z^e$, and welfare-weighted average income can be denoted as $z^e(\beta) = \frac{\sum \beta^i z^{e,i}}{\sum \beta}$. The ratio of the first-order conditions is

$$z^e(\beta) = \frac{\bar{z}^e - (1-a)\bar{z}_a^e}{1-(1-a)\bar{z}_b},$$

and we can derive the optimal tax rate by following the same steps as in Section A.1, by considering the evasion-modified tax base z^e instead of z

$$\frac{\tau^*}{1-\tau^*} = \frac{1}{\varepsilon^e}\left(1 - \frac{z^e(\beta)}{\bar{z}^e}\right).$$

The intuition behind the derivation and the tax rule is the same as before, but we must consider the relevant tax base in the context of evasion. Both the elasticity of labour income with respect to the tax rate and the relevant welfare concepts change when part of the income base evades taxation.

A.4.2 Poverty minimization

The derivation of Equation (5.2) follows the same steps as presented above and in part A.1.2. The first-order conditions with respect to a and b are

$$\sum D_c \left(\frac{\partial e^i}{\partial a} + z^e + a z^e_a \right) = \lambda \left(a \sum z^e - (1-a) \sum z^e_a \right)$$
$$\sum D_c (1 + a z^i_b) = \lambda \left(N - (1-a) \sum z^i_b \right).$$

From the ratio of the two conditions we get the measure of relative deprivation impact under tax evasion, $\tilde{D}^e$:

$$\tilde{D}^e \equiv \frac{\sum D_c \left(z^e + a z^e_a + \frac{\partial e}{\partial a} \right)}{D_c (1 + a z^i_b)} = \frac{\bar{z}^e - (1-a)\bar{z}^e_a}{1 - (1-a)\bar{z}_b},$$

which gives us Equation (5.2) in the text. $\tilde{D}^e$ measures the relative efficiency of taxes and transfers. The latter impact (the denominator) is the same as before, but the impact of taxation (numerator) is different in the presence of tax evasion.

B Measuring multidimensional poverty

B.1 Linear income tax

One of the most popular poverty measures is the P_α category developed by Foster, Greer and Thorbecke. It is usually written in the form of $P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^\alpha f(y)\,\mathrm{d}(y)$ where z is the poverty line and y is income. Following Kanbur and Keen (1989), who define the poverty index in terms of disposable income, and using our notation, this becomes: $P_\alpha = \int_0^{\bar{c}} \left(\frac{\bar{c}-c^i}{\bar{c}}\right)^\alpha \mathrm{d}\nu(i)$, where c^i is disposable income, in the linear tax case $c^i = (1-\tau)z^i + b = (1-\tau)z^i + \tau Z(1-\tau) - R$. In the Piketty and Saez model, we can use this specification of the functional form to define the derivative $D_c = -\frac{\alpha}{\bar{c}} \left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1}$ (note that $D_c < 0$ as long as $c^i < \bar{c}$). We can follow the same steps to arrive at the optimal tax rate $\tau^* = \frac{1-\bar{\beta}-\bar{\beta}^\varepsilon}{1-\bar{\beta}-\bar{\beta}^\varepsilon+\varepsilon}$ where now

$$\beta^i = \frac{D_c}{\int_0^{\bar{c}} D_c \,\mathrm{d}\nu(i)} = \frac{-\frac{\alpha}{\bar{c}}\left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1}}{\int_0^{\bar{c}} -\frac{\alpha}{\bar{c}}\left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1} \mathrm{d}\nu(i)} = \frac{\left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1}}{\int_0^{\bar{c}} \left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1} \mathrm{d}\nu(i)}$$

and consequently $\bar{\beta} = \frac{\int_0^{\bar{c}} \beta^i z^i \,\mathrm{d}\nu(i)}{Z}$ and $\bar{\beta}^\varepsilon = \frac{\int_0^{\bar{c}} \beta^i z^i \varepsilon^i_c \,\mathrm{d}\nu(i)}{Z}$ as before. Everything else stays exactly the same as in the calculations of Section 2.2.1. Also in the case of Tuomala's and Dixit and Sandmo's models, the results stay the same, and we can plug in the explicit definition for D_c, the derivative of the poverty measure with respect to disposable income, into the results.

B.2 Public good provision

Employing the FGT poverty measure in the context of public good provision for poverty reduction is a more complicated than in the linear tax case. In Section 3.2 the government's objective function was defined as $min\ P = \int D\left(x^i, G, \bar{x}, \bar{G}\right) \mathrm{d}\nu(i)$, that is, deprivation was measured both as deprivation in private consumption (i.e. disposable income) as well as with respect to the public good. But the FGT index is a uni-dimensional measure, measuring deprivation with respect to one dimension only (e.g. disposable income). If one wants to consider publicly offered goods such as education as separate from private consumption, a multidimensional FGT measure is needed. Multidimensionality however entails a difficult question of determining when a person should be determined as deprived.

There are several approaches to multidimensionality of FGT-type poverty measures.[14] For example, Besley and Kanbur (1988), who consider the poverty impacts of food subsidies, employ the uni-dimensional FGT measure but define deprivation in terms of equivalent income: $P_\alpha = \int_0^z \left(\frac{z_E - y_E}{z_E}\right)^\alpha f(y)\,\mathrm{d}(y)$, where y_E is equivalent income, defined implicitly from $V(p, y_E) = V(q, y)$, and z_E is the poverty line corresponding to equivalent income. But given our aim of defining optimal policy in terms of poverty reduction, irrespective of individual welfare, the use of equivalent income is problematic as it forces the solution to be such that, by definition, individuals are kept as well off as before. Pirttilä and Tuomala (2004) employ shadow prices in a poverty-minimizing context to allow for several goods in the poverty measure. For them, deprivation is measured as $D\left(z, y\left(q, w\right)\right)$ where $z^h = s_x x^* - s_L^h L^*$and $y^h(q, w^h) = s_x x(q, w^h) - s_L^h L(q, w^h)$. This approach requires determining shadow prices s_x, s_L for consumption and leisure in order to construct a reference bundle respective to which deprivation can be measured, but there is no clear guideline to the choice of the shadow prices.

The approach in Bourguignon and Chakravarty (2003) is more suitable for our purposes. They provide a multidimensional extension of the FGT measure, according to which a person is poor if she is deprived in at least one dimension. A simple example of such an extension of the FGT is

$$P_\theta = \frac{1}{n} \sum_{j=1}^{m} \sum_{i \in S_j} a_j \left(\frac{z_j - x_{ij}}{z_j}\right)^{\theta_j},$$

where θ_j and a_j are weights given to dimension j, and S_j is the group of people who are poor in dimension j. Alkire and Foster (2011) for their part provide a similar measure which uses a weighted count of dimensions in which the person is deprived to determine whether she is poor. An aspect of this is also whether the goods under consideration are complements or substitutes. Following the Bourguignon-Chakravarty approach and defining $x_{i1} = x_i$ as private consumption, $z_1 = \bar{x}$, $x_{i2} = G$ as the amount of public good, and $z_2 = \bar{G}$ would give us $P_\theta = \frac{1}{n} \sum_{i \in S_j} \left(a_1 \left(\frac{\bar{x} - x^i}{\bar{x}}\right)^{\theta_1} + a_2 \left(\frac{\bar{G} - G}{\bar{G}}\right)^{\theta_2}\right)$. Using this measure, $D_x = -\frac{\theta_1 a_1}{\bar{x}} \left(\frac{\bar{x} - x^i}{\bar{x}}\right)^{\theta_1 - 1}$

[14] See Foster, Greer, and Thorbecke (2010, p.504-5) for a brief overview of multidimensional FGT extensions that allow the inclusions of dimensions such as health, education and nutrition in addition to other consumption.

and $D_G = -\frac{\theta_2 a_2}{\bar{G}}\left(\frac{\bar{G}-G}{\bar{G}}\right)^{\theta_2-1}$. These can then be inserted to the public provision rules. For example, (3.5) becomes

$$\frac{\int\left(\frac{\theta_2 a_2}{\bar{G}}\left(\frac{\bar{G}-G}{\bar{G}}\right)^{\theta_2-1}+\frac{\theta_1 a_1}{\bar{x}}\left(\frac{\bar{x}-x^i}{\bar{x}}\right)^{\theta_1-1}(1-\tau)\frac{\partial z^i}{\partial G}\right)\mathrm{d}\nu(i)}{\int\frac{\theta_1 a_1}{\bar{x}}\left(\frac{\bar{x}-x^i}{\bar{x}}\right)^{\theta_1-1}\mathrm{d}\nu(i)} = p-\tau\frac{dZ}{dG}$$

and (A.20) becomes

$$\frac{\sum\frac{\theta_2 a_2}{\bar{G}}\left(\frac{\bar{G}-G}{\bar{G}}\right)^{\theta_2-1}+\sum\frac{\theta_1 a_1}{\bar{x}}\left(\frac{\bar{x}-x^i}{\bar{x}}\right)^{\theta_1-1}az_G^i}{\sum\frac{\theta_1 a_1}{\bar{x}}\left(\frac{\bar{x}-x^i}{\bar{x}}\right)^{\theta_1-1}(1+az_b^i)} = \frac{p-(1-a)\sum z_G^i}{1-(1-a)\sum z_b^i},$$

from where it can be seen that the relative efficiency of the public good versus cash transfers on reducing poverty can be directly traced back to the magnitudes of θ_1 and θ_2.

B.3 Commodity taxation

In the case of commodity taxes, we run into the same issues regarding deprivation measurement as with public goods. However, in Section 4.2 deprivation was measured only in terms of disposable income, c. We thus escape the multidimensionality issue and employing the FGT poverty measure is thus as simple as in the linear income tax case: we simply need to define $D = P^\alpha$ and thus $D_c = -\frac{\alpha}{\bar{c}}\left(\frac{\bar{c}-c^i}{\bar{c}}\right)^{\alpha-1}$ in Equation (4.4). Potentially the government might also consider weighting different goods according to their importance to measured poverty.

C The role of the minimum wage

In the model of Lee and Saez (2012), there are two different ability levels for workers, w_1 (low ability) and w_2. There are h_i number of workers at ability level w_i. The production function is $F(h_1, h_2)$, and different types of labor are imperfect substitutes (which also implies that the equilibrium wage of type 2 is a function of w_1). The workers face different kinds of cost of effort, θ_i. The government can observe the wage rates (=income) of different ability types, but not the individuals' cost of effort. Individual utility is assumed to be $u_i = c_i - \theta_i$, where $c_i = w_i - T_i$, i.e. consumption equals earnings minus taxes. The tax function can also be of the linear variety, $T_i = \tau w_i - b$. Notice that Lee and Saez assume away income effects.

The government minimizes poverty, and here we assume that those without a job (whose number is denoted by h_0) and those working in the low-skilled occupation are below the poverty line. We therefore consider the case with working poor. Notice that without any taxes and transfers, if all those who work would be above the poverty line, the minimum wage would never be optimal: all it would do would be to increase the number of the poor (Fields and Kanbur 2007). When the overall number of workers is normalized to unity, the government's objective function is $(1-h_1-h_2)D(c_0)+\int_{\theta_1}D(c_1)dH(\theta)$. Its budget constraint is $h_0c_0 + h_1c_1 + h_2c_2 = h_1w_1 + h_2w_2$. It is useful to write the average social marginal value of poverty reduction as $g_0 = D'(c_0)/\lambda$ and $g_1 = \int_{\theta_1} D'(c_1)dH(\theta)/h_1\lambda$ for those without a work and for the low-skilled workers, respectively.

The government considers introducing a minimum wage $\bar{w}_1 = w_1$, that is, it is set at the market-clearing level. The poverty measure in the presence of optimal tax policies is $(1-h_1-h_2)D(c_0)+\int_{\theta_1} D(c_0+\Delta c_1)dH(\theta)$, where $\Delta c_1 = c_1-c_0$. Writing down the Lagrangean, and denoting the Lagrange multiplier of the government budget constraint by λ, consider a variation dc_1 with a binding minimum wage. Taking a derivative of the Lagrange function leads to

$$\frac{dL}{dc_1} = \int_{\theta_1} D'(c_0 + \Delta c_1)dH(\theta) - \lambda h_1 = \lambda(g_1 - 1)h_1, \tag{C.1}$$

which follows as the minimum wage implies that there is no change in w_1 by definition, and thus, in the absence of general equilibrium effects, no change also in w_2. The result means that the usefulness of the minimum wage depends on the deprivation function the government uses: the derivative above is positive only if $g_1 > 1$. Since the mean value of the social marginal weight at the given income is one, this is likely to be the case if the government's deprivation function is only mildly dependent on the income of the poor. If deprivation heavily depends only on the income of the very poor (in this model, those without a job), introducing a minimum wage is not optimal, since this would mean that g_1 would be below one. In comparison to the welfarist case analyzed by Lee and Saez, the case for the minimum wage appears to be somewhat weaker when the government's objective is poverty minimization.

One problematic assumption is the ability to increase the consumption of type 1 workers, that is, setting $dc_1 > 0$ while holding c_0 and c_2 constant. This could mean e.g. an in-work subsidy for type 1 jobs (as Lee and Saez 2012 assume), as taxes and benefits are universal and therefore changing them would alter also the consumption of the other types besides type 1 workers. However, this violates the assumption that the government is able to implement only linear policies. Due to these issues, the Lee-Saez result of minimum wage optimality does not easily carry over to our set-up.

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