Heterogeneous firms and informality: the effects of trade liberalization on labor markets

Dennis Becker
It is the policy of Cornell University actively to support equality of educational and employment opportunity. No person shall be denied admission to any educational program or activity or be denied employment on the basis of any legally prohibited discrimination involving, but not limited to, such factors as race, color, creed, religion, national or ethnic origin, sex, age or handicap. The University is committed to the maintenance of affirmative action programs which will assure the continuation of such equality of opportunity.
Heterogeneous firms and informality: 
the effects of trade liberalization on labor markets∗

Dennis Becker†
Cornell University
March 20, 2014

Abstract
The informal sector is often seen as a coping mechanism for firms that choose to evade registration fees or pay low wages. In this paper, I investigate the role of the informal sector in the impact of trade liberalization on welfare, employment and wage inequality in a model of trade with heterogeneous firms. The findings suggest that trade liberalization reduces informal employment unambiguously. Contrary to the extant literature, however, its impact on welfare, total employment and wage inequality is country-specific.

JEL-Classification: F12, F16, O17

Keywords: heterogeneous firms; informality; trade liberalization; fair wages

∗I am grateful to Nancy Chau, Ravi Kanbur, Joel Landry, Ankita Patnaik, the 2013 NEUDC participants and 2013 ISI conference participants for helpful comments and suggestions. I gratefully acknowledge financial support from the Cornell Population Center.

1 Introduction

Valued at about 17% of the world’s GDP, the informal sector represents an important component of developing countries in particular (Schneider et al., 2010). In addition to its size, the ability to transition in and out of the informal sector has been shown to be a crucial coping mechanism allowing firms to weather adverse market conditions and thus could be important with regard to trade liberalization (Mullainathan and Schnabl, 2010; Ousman and Hallward-Driemeier, 2012). That leaves an important question, what is the role of the informal sector on the impact of trade liberalization in general equilibrium?

In the seminal heterogeneous firm trade model by Melitz (2003), trade liberalization induces firm selection on productivity into exporting and forces the least-productive firms to exit the market. The result is improved welfare and higher average productivity. Yet, empirical studies provide suggestive evidence that informality causes the dispersion of marginal products between firms and acts as a channel through which the firm-level resource allocation process can be distorted (La Porta and Shleifer, 2008; Hsieh and Klenow, 2009; McCaig and Pavcnik, 2013). For instance, Nataraj (2011) shows that the productivity improvement as a result of the Indian trade liberalization in 1991 predominantly stems from the exit of the least-productive, informal firms. Additionally, Perry et al. (2007) find that the majority of informal salaried workers would prefer to work in the formal sector due to a formal sector wage premium. This raises two important questions: First, how does trade liberalization affect labor markets in the presence of an informal sector? Second, how does the presence of an informal sector mediate the effect of trade liberalization on welfare?

To answer these questions, this paper develops a model that incorporates informality into a trade model with heterogeneous firms and can replicate the salient features of the empirical literature on trade and informality. The model consists of four crucial pieces. First, I define informality as firm-level non-compliance with registration (de Soto, 1989). Second, firms are heterogeneous in productivity. Third, firms opt in or out of the informal sector according to profitability considerations. Informal sector participation is therefore an

---

1 On a firm-level, informality is commonly defined as either registration non-compliance, tax evasion or both of these (e.g. Fajnzylber et al., 2011). My choice of definition is motivated by several empirical findings. de Paula and Scheinkman (2011) find that registration non-compliance and tax evasion are highly correlated. Exploring the impact of policies on the size of the informal sector, Ulyssea (2010) finds that the reduction of entry costs, rather than payroll taxes are effective in reducing the incidence of informality. Moreover, several studies find that high firm registration costs do not just function as barriers to formality (Djankov et al., 2002; Auriol and Warlters, 2005; Antunes and de V. Cavalcanti, 2007), but also lead to few and larger active firms in an industry (Klapper et al., 2006; Fisman and Allende, 2010). Importantly, in the most cases the registration costs are not an efficient transfer to the government, but rather red tape caused by bureaucratic hurdles and complex registration procedures (Djankov et al., 2002).
active entrepreneurial choice to maximize firm profits given the costs and benefits of informal and formal sector participation.\footnote{Another common view sees informality as a last resort of business-owners waiting for formal employment. Bruhn (2013) provides evidence for the coexistence of both types of informal businesses. Given the firm-level focus, the model covers the entrepreneurial segment corresponding to the findings of e.g. Maloney (2004), Ousman and Hallward-Driemeier (2012) and de Mel et al. (2013).} Fourth, labor markets are imperfect. Labor market frictions are caused by workers’ fair wage expectations,\footnote{Labor market frictions arising from fair wage preferences can empirically be shown through the existence of wage curves (Blanchflower and Oswald, 1995). Several empirical studies find wage curves for informal and formal salaried workers (Bucheli and González, 2007; Ramos et al., 2010; Baltagi et al., 2013), which indicates fair wage expectations of workers in both sectors. Moreover, as surveyed by Zenou (2008), informal workers are commonly employed by friends and relatives and therefore search frictions are often negligible in the informal sector. Hence, the fair wage specification, opposed to the search-and-matching framework, is suitable to model wage formation in both the informal and formal sector.} and informal workers are excluded from the formal labor market.\footnote{Alternatively, informal employment is seen as voluntary choice. Current empirical evidence indicates a dual structure in the labor market, where a share of workers is excluded from formal employment and another share voluntarily chooses informal employment (Günther and Launov, 2012). Given that workers are salaried workers, the model corresponds to the exclusion view.} The pieces come together by introducing informality into the model of Egger and Kreickemeier (2009), which is a heterogeneous firm trade model à la Melitz (2003) with a fair wage specification along the lines of Akerlof and Yellen (1990).

The model delivers several predictions on the impact of trade liberalization in the presence of informality that correspond closely to the findings from empirical studies. First, higher-productivity firms are more likely to be formal, large, pay high wages and participate in international trade (Bernard and Jensen, 1999; Tybout, 2000; Perry et al., 2007), whereas lower-productivity firms are more likely to be informal, smaller, compensate workers with lower wages and earn less profits than their formal counterparts (La Porta and Shleifer, 2008; Dabla-Norris et al., 2008; McKenzie and Sakho, 2010; de Paula and Scheinkman, 2011).

Second, trade liberalization ambiguously affects formal sector employment and leads to a decrease of informal sector employment, but the effect on total employment is ambiguous. This result derives not just from export selection and firm exit, both well known from Melitz (2003), but a new adjustment mechanism - the informalization of firms. Intuitively, trade liberalization induces the selection of high-productivity formal firms into exporting and hiring more workers. With trade liberalization, competitive pressure in the market rises and the demand for each firm’s product falls. Lower-productivity formal firms switch to informal production to remain profitable, and accordingly shed labor. Depending on the characteristics of the economy, such as firm registration costs, the additional hires by exporting firms may or may not compensate for the labor shedding in the formal sector. The lowest-productivity informal firms are forced to exit the market, resulting in a reduction in informal sector
employment. The net effect on total employment is ambiguous. Therefore, the model provides an explanation for a decrease in informal sector employment amidst conflicting empirical evidence (Goldberg and Pavcnik, 2003; Fiess and Fugazza, 2012; McCaig and Pavcnik, 2012). Additionally, the result corresponds to the ambiguous findings on the impact of trade on unemployment (Davidson and Matusz, 2009; Dutt et al., 2009; Felbermayr et al., 2011b; Menezes-Filho and Muendler, 2011).

Third, the effect of trade liberalization on aggregate output, and hence welfare, is ambiguous. Similar to the employment effect, the increase in aggregate output by shifting resources towards the highest-productivity exporting firms may or may not compensate for the loss in aggregate output due to the exit of the least-productive informal firms and the informalization of the least-productive formal firms. Therefore, the existence of an informal sector distorts the resource allocation process, found for example by Bruhn (2013), and trade liberalization can either alleviate (McCaig and Pavcnik, 2013) or aggravate the distortion by affecting informal sector participation. This result is particularly interesting, as it stands in contrast to the clear increase in welfare predicted by Egger and Kreickemeier (2009) and commonly established in the theoretical literature on trade and heterogeneous firms.

Lastly, wage inequality is caused by a wage gap between informal and formal workers, a frequent result in the empirical literature (de Paula and Scheinkman, 2011; Bargain and Kwenda, 2011; Günther and Launov, 2012), and is employment-based. Openness to trade increases wage inequality between informal and formal workers. Because of the reduction in informal sector employment and sector switching, the increase of the informal sector average wage is lower than the increase in formal sector average wage. Hence, the average wages of both sectors diverge and between-group wage inequality increases. Wage inequality among all employed workers is ambiguously affected through the above employment effect and depends on the proportion of formal sector firms prior to trade liberalization.

This paper contributes to the literature on trade models featuring heterogeneous firms and labor market frictions, and models of the informal economy. There exist two groups of trade models with labor market frictions: In the first set of papers the labor market imperfections arise from a search-and-matching setup (Helpman and Itskhoki, 2010; Felbermayr et al., 2011a). The second group of models builds on fair wage specifications (Egger and Kreickemeier, 2009; Davis and Harrigan, 2011; Amiti and Davis, 2012). These models readily address and explain the labor market concerns arising from trade liberalization and correspond to a wide range of empirical facts with regard to the formal economy. However, they do not consider the possibility of an informal sector and they therefore overlook a potentially crucial determinant of resource allocation and wage dispersion among firms.
Models of the informal economy have a long history. Fields (1975) uses a Harris-Todaro-type model to analyze unemployment, whereas Rauch (1991) provides a rationale for size-dualism with large formal and small informal firms. Loayza (1996) examines the interaction of tax evasion and public good congestion. Wage dualism between formal and informal workers is explained by Basu et al. (2011). While these models extend the understanding of informality in various dimensions, their assumptions of an open economy preclude an explicit analysis of how informality may affect the transition of an economy from autarky to trade liberalization.

The major contribution of this paper therefore is to combine both informality and trade in a heterogeneous firm setting, which few papers have sought to do. To my knowledge, only two studies to date, Aleman-Castilla (2006) and Paz (2014), have considered this. Both feature a labor market perspective by defining informality as payroll tax evasion and assume that informal firms are prevented from exporting. In Aleman-Castilla (2006) there are no labor market frictions and therefore the model features full employment with one equilibrium wage for all workers. Alternatively, Paz (2014) features labor market imperfections using an efficiency wage framework in a model where firms draw both productivity and ability to monitor their workers. The model presented in this paper extends previous research in several ways. First, by defining informality as firm registration non-compliance, the model complements the two previous works that focus on labor market regulations as causes of informality. Second, my model rests on the parsimonious assumption that the only source of heterogeneity stems from firm productivity differences, yet captures a wide range of empirical findings. Third, this model provides a more comprehensive picture than previous research by featuring indicators for employment, welfare and wage inequality in one framework and gives a rationale for unemployment as well as size and wage dualism. Fourth, the exclusion of informal firms from international trade is a result of the model rather than an assumption, which makes the model more realistic than its counterparts. Therefore, this model provides a new perspective on the impact of trade in the presence of informality and, given its wide scope, has important policy implications.

I proceed as follows. Section 2 characterizes the closed economy specification of the model. Section 3 extends the model to an open economy and discusses the impact of trade liberalization. Section 4 concludes the paper.
2 The closed economy model

The economy consists of L units of labor, the only factor of production. There are two types of goods: a final output and intermediate goods. The final output is homogeneous and its market is in perfect competition. The intermediate goods are differentiated and produced under monopolistic competition by a mass of heterogeneous firms (Melitz, 2003; Melitz and Redding, 2013).

2.1 The final output

The final output \( Y \) is an aggregate of all intermediate goods and is characterized by the following CES-production function (Blanchard and Giavazzi, 2003):

\[
Y = \left[ M^{-(1-\rho)} \int_{v \in V} q(v)^\rho dv \right]^{\frac{1}{\rho}}, 0 < \rho < 1. \tag{1}
\]

Intermediate good varieties are indexed with \( v \) and \( V \) is the set representing the mass of available intermediate goods \( M \). \( q(v) \) and \( p(v) \) are the quantity and the price of variety \( v \). \( \sigma \equiv \frac{1}{1-\rho} \) is the elasticity of substitution between the varieties of intermediate goods. The final output market is under perfect competition and the final output acts as numeraire. The resulting CES-price index \( P \) is thus normalized to 1. The price index is described by

\[
P = \left[ M^{-1} \int_{v \in V} p(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}. \tag{2}
\]

Through profit maximization of the producer of the final good, the demand for variety \( v \) of the intermediate good is

\[
q(v) = \frac{Y}{M} p(v)^{-\sigma}. \tag{3}
\]

2.2 The intermediate goods: informal and formal sector

The intermediate goods are produced under monopolistic competition by firms in two sectors: an informal and a formal sector. Henceforth all informal sector variables feature subscript \( i \).
and formal sector variables subscript $f$. The firms are heterogeneous in their productivity and every firm produces one unique variety of input for the final output production. Hence, $M$ can interchangeably be used to account for the number of firms or total number of input varieties $v$. $M = M_i + M_f$ is the number of domestically active firms, which consists of all informal sector firms, $M_i$, and formal sector firms, $M_f$. Firms face fixed costs of production in terms of final output $Y$ and variable costs that are directly related to the firm’s productivity $\varphi$. Firm output is linear in labor input $l$ and productivity $\varphi$, that is $q = \varphi l$. Firms with the same productivity behave in the same manner. Therefore, I henceforth index firms solely in terms of their productivity $\varphi$.

Firms voluntarily choose to become either informal or formal producers. Informal producers face fixed cost $f_i$ and formal producers face $f_f$ to start production. I assume $f_f > f_i$, as the formal fixed cost reflects complex firm registration and red tape (Djankov et al., 2002; Auriol and Warlters, 2005; Antunes and de V. Cavalcanti, 2007). As unregistered firms, informal firms are deprived of access to public goods and face a probability of detection $\delta$ by the authorities, which leads to the total loss of firm revenues. The probability of audit and detection is a reflection of the institutional quality of the economy (Loayza, 1996; Cebula, 1997; Dabla-Norris et al., 2008). As a result of formal sector participation, formal firms experience a productivity bonus $\lambda \in [0,1)$, which can be imagined as the result of access to public goods and rule of law (de Soto, 1989; La Porta and Shleifer, 2008; Dabla-Norris et al., 2008). Formal sector productivity is then described by $\varphi - \lambda$ and $\lambda$ is assumed to be exogenously given.

The profit maximizing price of both types of firms is a constant markup $\frac{1}{\rho}$ over marginal cost, where $w(\varphi)$ denotes the wage paid by a firm with productivity $\varphi$ and $\epsilon$ is the effort level of workers.

$$p_i(\varphi) = \frac{w_i(\varphi)}{(1 - \delta)\rho \varphi \epsilon} \quad \text{and} \quad p_f(\varphi) = \frac{w_f(\varphi)}{\rho \varphi \epsilon^{1-\lambda}}. \quad (4)$$

In combination, demand for the individual input variety (3) and the profit-maximizing price (4) lead to the informal and formal revenues

$$r_i(\varphi) = \frac{Y}{M} \left( \frac{w_i(\varphi)}{(1 - \delta)\rho \varphi \epsilon} \right)^{1-\sigma} \quad \text{and} \quad r_f(\varphi) = \frac{Y}{M} \left( \frac{w_f(\varphi)}{\rho \varphi \epsilon^{1-\lambda}} \right)^{1-\sigma}, \quad (5)$$

6While occurring at the same time in the model, intuitively a firm has to first pay the fixed cost to start production and only then earns revenue. Government enforcement only comes into play once the entry costs are sunk and the firm is meanwhile generating revenue. Hence, enforcement leads to a loss of revenue independent of the fixed cost.
and profits

\[
\pi_i(\varphi) = (1 - \delta)^\sigma \frac{Y}{\sigma M} \left( \frac{w_i(\varphi)}{\rho \sigma \epsilon} \right)^{1-\sigma} - f_i \quad \text{and} \quad \pi_f(\varphi) = \frac{Y}{\sigma M} \left( \frac{w_f(\varphi)}{\rho \sigma \epsilon} \right)^{1-\sigma} - f_f,
\]

which can be concave or convex in productivity depending on whether \(\xi - 1 < 0\) or \(\xi - 1 > 0\) holds. \(\xi\) is defined as \(\xi \equiv (1 - \sigma)(\theta - 1)\). Besides firm productivity and fair wages, firm revenue depends on two factors. First, the number of firms operating in the domestic market \(M\) (and hence the number of varieties sold in the domestic market) influence the demand for each individual variety. A strong domestic competition (thus, a high \(M\)) reduces the demand for each variety and therefore the profitability of each firm’s operation. Second, the demand for each firm’s variety rises with the aggregate revenue in the economy (here \(Y\), as \(P\) is normalized to 1).

2.3 The labor market and wage determination with informality

There are three groups of workers: informal workers \(L_i\), formal workers \(L_f\) and unemployed workers \(L_u\). Together they make up the total labor force in the economy \(L \equiv L_i + L_f + L_u\). The informal sector employment share, formal sector employment share and share of unemployed workers are described by \(E_i = (L_i/L)\), \(E_f = (L_f/L)\) and \(U = (L_u/L)\). \(E \equiv E_i + E_f\) is the fraction of employed workers.

Workers are identical and have a preference for fair wages along the lines of Akerlof and Yellen (1990). As workers are identical, they are randomly selected by employers in the hiring process and can become either informal or formal salaried workers or remain unemployed. Therefore, I distinguish the informal sector average wage \(\bar{w}_i\) from the formal sector average wage \(\bar{w}_f\). The average wages are calculated as follows. The wage bill of the informal sector \(W_i\), i.e. the sum of all wages \(w_i(\varphi)\) paid in the informal sector, is divided by the amount of informal employment \(L_i\). This provides the informal sector average wage \(\bar{w}_i = (W_i/L_i)\). The calculation of the formal sector average wage follows the same procedure. To take the potential unemployment into account, the average wage income per worker in the economy \((E_i\bar{w}_i + E_f\bar{w}_f)\) consists of the two average wages weighted by the respective employment shares, that is:

\[
E_i\bar{w}_i + E_f\bar{w}_f = \frac{W_i + W_f}{L}.
\]

The reference wage of workers consists of two parts: a firm-internal and a firm-external
factor (Howitt, 2002; Bewley, 2005; Danthine and Kurmann, 2007). The firm-internal component captures the economic success of the firm as measured by the firm’s productivity $\varphi$. When hired by a formal firm, workers are aware of the formal sector productivity bonus and take the firm’s effective productivity $\frac{\varphi}{1-\lambda}$ into account. In line with Kreickemeier and Nelson (2006), the firm-external component relates to the labor market and is the outside option of workers. The external reference point is therefore described by the average wage income per worker $(E_i\bar{w}_i + E_f\bar{w}_f)$.

The workers’ fairness consideration between firm-internal and -external reference points is a geometric average weighted by the fairness parameter $\theta \in (0,1)$.

Through the firm-internal component that relates the wage to firm productivity, the fair wage setup induces heterogeneous wages among workers that are identical before the hiring process. The informal and formal reference wages take the following functional form:

$$\hat{w}_i(\varphi) = \varphi^\theta (E_i\bar{w}_i + E_f\bar{w}_f)^{1-\theta}$$
$$\hat{w}_f(\varphi) = (\varphi - \lambda)^\theta (E_i\bar{w}_i + E_f\bar{w}_f)^{1-\theta}. \quad (8)$$

Workers exert effort level $\epsilon$ relative to what they perceive to be a fair wage $\hat{w}$. For $w(\varphi) \geq \hat{w}$ workers effort their maximum effort level $\epsilon = 1$, that is $\epsilon = \min\{w(\varphi)/\hat{w}, 1\}$. Additionally, I assume that worker effort level $\epsilon$ cannot be contracted and the only mechanism to induce effort is a wage offer. Therefore, outside workers are not able to underbid the wage of currently employed workers, all firms pay the firm-specific reference wage $\hat{w}$ and all employed workers exert effort $\epsilon = 1$.

This is supported by the experimental evidence of Fehr and Falk (1999), who show that worker effort level is positively related to their wage and that employers are not interested in wage underbidding by outside workers in a world of incomplete contracts. With all employed workers exerting full effort $\epsilon = 1$, I henceforth ignore $\epsilon$.

The presence of two sectors leads to two crucial differences of this model’s wages to the wage specification of Egger and Kreickemeier (2009). First, the firm-external reference point depends on both informal and formal sector wages. Due to the random selection of workers, both employment types are viable options for workers. Second, workers are aware of the productivity bonus that formal sector firms experience and adjust their firm-internal reference point accordingly. This is supported by de Paula and Scheinkman (2011), who find that formal firm status is correlated with higher wages even after controlling for firm

---

7 For $\theta = 0$, all firms would pay the average wage income per worker in the economy and wage adjustment would lead to full employment similar to Melitz (2003) and the special case of $\theta = 0$ in Egger and Kreickemeier (2009).

8 If an outsider were to be hired for a lower wage, she would adjust her reference wage $\hat{w}$ and, given the wage below reference level, decrease her effort level accordingly. Therefore, in equilibrium wage underbidding is not attractive to firms.
2.4 The firm’s decision to enter the informal or formal sector

A firm’s decision to become informal or formal is established as follows. Before the start of operation, the firm pays a fixed cost \( f \) and draws a productivity \( \varphi \), a mechanism explained in detail in subsection 2.6. A firm’s productivity influences its variable costs but not the fixed cost it faces for entering either the informal or formal sector. Having full information over entry costs and productivity bonus accruing to formal production, firms choose the sector in which they can produce the most profitable or do not produce at all, that is \( \max \{ \pi_i(\varphi), \pi_f(\varphi), 0 \} \). Without further specification there are three ways informal and formal firms may be distributed over the productivity spectrum. First, informal firms could be the lowest-productivity firms in an economy and formal firms could be characterized by high-productivity levels. Second, formal firms could be low-productivity firms and informal firms could be found in the high-productivity spectrum. Lastly, informal and formal firms could be distributed all over the spectrum without any distinct pattern. Given empirical evidence on the productivity of informal sector firms, I assume the first case of low-productivity informal firms (La Porta and Shleifer, 2008; Dabla-Norris et al., 2008; de Paula and Scheinkman, 2011). That is \( \varphi^*_f > \varphi^*_i \), where \( \varphi^*_i \) and \( \varphi^*_f \) are the cutoff productivity levels at which informal and formal firms start profitably operating. The relationship between the benefits and costs to achieve this sorting are summarized in Proposition 1:

**Proposition 1.** If \( \left( 1 - \frac{\delta}{1 - \lambda} \right)^{\frac{1}{2}} < \frac{1}{\sqrt{\frac{\lambda}{\lambda - 1}}} \left( \frac{\delta}{\lambda} \right)^{\frac{1}{2}} \), then formal sector firms are higher-productivity firms than informal sector firms; that is \( \varphi^*_f > \varphi^*_i \).

The sorting of sectors along the productivity spectrum according to Proposition 1 can be seen in Figure 1. The drivers of this sorting are the formal sector productivity bonus \( \lambda \), the sector entry costs \( f_i \) and \( f_f \) as well as the enforcement parameter \( \delta \) that steer the profitability of production in both sectors. Intuitively, the formal productivity bonus has to outweigh the cost of informal enforcement to make formal sector participation economically viable. However, the bonus cannot be bigger than the relative formal sector entry cost, which induces only high-productivity firms to enter formal production. Accordingly, informal firms break even at a lower productivity level than formal firms do.\(^{10}\) Additionally, the marginal profitability in terms of productivity of the informal sector is lower than the marginal

---

\(^9\)This is a reflection of the entrepreneurial view on informal firms (Maloney, 2004; Ousman and Hallward-Driemeier, 2012; de Mel et al., 2013).

\(^{10}\)Breaking even at a lower productivity requires that \( \varphi^*_i < \varphi^*_f \) for \( \varphi^*_i \) from \( \pi_i(\varphi^*_i) = 0 \) & \( \varphi^*_f \) from \( \pi_f(\varphi^*_f) = 0 \).
profitability of the formal sector,\(^{11}\) that is, for high-productivity levels the profitability of the formal sector is above the profitability of the informal sector.\(^{12}\) As both profit functions are monotonically increasing in productivity, a single-crossing of the two functions is guaranteed and only formal sector firms can be found at higher productivity levels. This is also consistent with the empirical evidence that formal firm profits are higher than informal firm profits (McKenzie and Sakho, 2010; de Paula and Scheinkman, 2011; Fajnzylber et al., 2011; de Mel et al., 2013). As a result, \(\phi^*_i\) and \(\phi^*_f\) are determined by the following two conditions:

\[
\pi_i(\phi^*_i) = 0 \quad (9)
\]

and

\[
\pi_f(\phi^*_f) = \pi_i(\phi^*_f) \quad (10)
\]

From equation (6): \(\phi^*_i = \left[ f_i \frac{\sigma M}{\psi Y} \right]^{\frac{1}{\xi}} (E_i \bar{w}_i + E_f \bar{w}_f) (1 - \delta)\rho \frac{1}{1-\sigma} \) and \(\phi^*_f = \left[ f_f \frac{\sigma M}{\psi Y} \right]^{\frac{1}{\xi}} (E_i \bar{w}_i + E_f \bar{w}_f) (1 - \lambda)\rho \frac{1}{1-\sigma} \) and

\[
\frac{\partial \pi_i}{\partial \phi} < \frac{\partial \pi_f}{\partial \phi}. \quad \text{From equation (6)}:
\]

\[
\frac{\partial \pi_i}{\partial \phi} = (1 - \delta)^\sigma \xi \frac{1}{\sigma M} \{ (E_i \bar{w}_i + E_f \bar{w}_f)^{1-\theta} \rho^{-1} \}^{1-\sigma} \phi^{\xi-1} \text{ and } \frac{\partial \pi_f}{\partial \phi} = \xi \frac{1}{\sigma M} \{ (1 - \lambda)^{1-\theta} (E_i \bar{w}_i + E_f \bar{w}_f)^{1-\theta} \rho^{-1} \}^{1-\sigma} \phi^{\xi-1} \text{.}
\]

Hence, \((1 - \delta)^\sigma \xi < \frac{1}{\sigma M} \) and in combination ensure \(\phi^*_f > \phi^*_i\) as illustrated in Figure 1.

\[\text{Figure 1: Sector sorting along the productivity spectrum according to Proposition 1.}\]
2.5 Firm-specific variables

The relative difference between any two firms can entirely be described by their productivity $\varphi$, government enforcement $\delta$, the formal sector productivity bonus $\lambda$ and their resulting formality status. Given that the result of Proposition 1 holds, the model parsimoniously captures the findings of the empirical literature. To begin with, using (8) and (4) results in

$$\frac{w_i(\varphi_i)}{w_f(\varphi_f)} = \left(\frac{\varphi_i}{\varphi_f}\right)^\theta (1 - \lambda)\theta < 1 \text{ and } \frac{p_i(\varphi_i)}{p_f(\varphi_f)} = \left(\frac{\varphi_i}{\varphi_f}\right)^{\theta - 1} (1 - \lambda)\theta - 1(1 - \delta)^{-1} > 1. \quad (11)$$

This highlights the firm-specificity of wages and the difference between wages in the two sectors. Consistent with the empirical literature, informal wages are lower than formal wages for two reasons. First, wages are a function of firm productivity and formal sector firms are high-productivity firms. Second, the formal sector productivity bonus acts as a wage premium for formal sector employment. This results in an informal wage gap, as commonly found in the empirical literature (de Paula and Scheinkman, 2011; Bargain and Kwenda, 2011; Günther and Launov, 2012). Similarly, (11) allows for a comparison of the relative prices $p_i(\varphi_i)/p_f(\varphi_f)$, and shows that formal firms charge lower prices than informal firms. Intuitively, due to their productivity bonus and higher productivity level, formal firms are able to translate lower marginal costs into lower prices for their products. This is in line with the findings of Foster et al. (2008), who show a negative correlation between physical firm productivity and output prices. Moreover, higher enforcement, as a driver of informal sector production costs, translates into higher prices charged by informal producers. In addition, using (3) and (5), I can compare the quantities and revenues between firms of the two sectors.

$$\frac{q_i(\varphi_i)}{q_f(\varphi_f)} = \left(\frac{\varphi_i}{\varphi_f}\right)^{\sigma(1 - \theta)} (1 - \lambda)\sigma(1 - \theta)(1 - \delta)^\sigma < 1 \text{ and } \frac{r_i(\varphi_i)}{r_f(\varphi_f)} = \left(\frac{\varphi_i}{\varphi_f}\right)^{\xi} (1 - \lambda)\xi(1 - \delta)^{\sigma - 1} < 1. \quad (12)$$

Formal firms are not just able to translate higher productivity into lower prices, but they also produce larger quantities than their informal counterparts, as found by La Porta and Shleifer (2008). In combination, the revenue of firms in the formal sector is higher than in

---

La Porta and Shleifer (2011) also suggest that informal firms produce lower-quality products than their formal counterparts. In the model, firms produce symmetric varieties at different costs. As mentioned in Melitz (2003), an alternative interpretation of productivity in Melitz (2003)-type models is that firms produce...
the informal sector. This corresponds to the empirical findings of Fajnzylber et al. (2011). Furthermore, (13) illustrates the labor demand of an informal sector firm \( l_i(\varphi) \) relative to the labor demand of a formal sector firm \( l_f(\varphi) \).

\[
\frac{l_i(\varphi_i)}{l_f(\varphi_f)} = \left( \frac{\varphi_i}{\varphi_f} \right)^{\sigma(1-\theta)-1} (1-\delta)^\sigma (1-\lambda)^{\sigma(1-\theta)-1} < 1. \quad (13)
\]

To capture the positive correlation between productivity level and employment, that is \( l_i(\varphi) < l_f(\varphi) \), I assume \( \sigma(1-\theta)-1 > 0 \), analogous to Egger and Kreickemeier (2009). In this setup the assumption is relevant for another reason. Given this assumption, informal firms hire less labor than formal firms due to the productivity bonus of formal sector firms found by La Porta and Shleifer (2008), de Paula and Scheinkman (2011) and Fajnzylber et al. (2011). Moreover, high-productivity firms pay not just higher wages, but also hire more workers, in line with the literature on firm size wage premiums (Brown and Medoff, 1989). Using the zero profit conditions for both sectors (9) and (10), I can specify the distance of informal to formal sector cutoff productivity level relative to the formal sector cutoff productivity level \( \varphi^*_f - \varphi^*_i \).

\[
\frac{\varphi^*_f - \varphi^*_i}{\varphi^*_f} = 1 - \left( \frac{f_i}{f_f - f_i} \right)^{1/\xi} \left( (1-\lambda)^{-\xi} - (1-\delta)^\sigma \right)^{1/\xi} \xi (1-\delta)^{-1}. \quad (14)
\]

This allows the analysis of inter-firm productivity differences between the marginal informal and the marginal formal firm with regard to policy changes and is summarized in Proposition 2.

**Proposition 2.** The relative productivity distance of formal to informal sector firms \( \frac{\varphi^*_f - \varphi^*_i}{\varphi^*_f} \) is decreasing in \( f_i, \delta \) and \( \lambda \) and increasing in \( f_f \).

**Proof.** See appendix A.

The intuition for Proposition 2 is as follows. An increase in factors that make informal sector participation more costly (\( f_i, \delta \)) or factors that make formal sector production more profitable (\( \lambda \)) raises the informal sector productivity cutoff level \( \varphi^*_i \) relative to the formal sector productivity cutoff level \( \varphi^*_f \) and thereby diminishes the distance in relative productivity levels. The distance in relative productivity levels increases for an increase in the formal sector entry cost \( f_f \) through a reduction in the profitability of formal sector participation.

varieties of different quality at the same cost. Given this interpretation, the model implicitly reflects the lower quality of informal sector products. Verhoogen (2008) models product quality explicitly, albeit does so using heterogeneous workers.
2.6 Firm productivity distribution and free entry

Firms are indexed by their productivity $\varphi$, hence firm size distribution, aggregate employment and aggregate output of the economy hinge on the productivity distribution. A commonly used distribution in Melitz (2003)-type models is the Pareto distribution. It is both tractable and fits empirical findings on firm size and the productivity distribution well (Axtell, 2001; Helpman et al., 2004). The distribution is given by $G(\varphi)$ with density $g(\varphi)$ and shape parameter $k > \xi$.

The lower bound of productivities is normalized to 1.

$$G(\varphi) = 1 - \varphi^{-k} \quad \text{and} \quad g(\varphi) = k\varphi^{-(k+1)}.$$  

(15)

In a manner well known from Melitz (2003), I assume an unbounded mass of prospective entrants, all identical ex-ante, to the intermediate sector. Entering entails a fixed cost of $f_e > 0$ and allows firms to draw a productivity $\varphi$ from the distribution $G(\varphi)$. Firms only start producing if the expected profit of production is non-negative. In equilibrium the average profit of active firms, conditional on successful market entry, is equal to the sunk cost $f_e$. This is described by the free entry condition:

$$\int_{\varphi^*_f}^{\varphi^*} \frac{\pi_i(\varphi)g(\varphi)d\varphi}{G(\varphi^*_f) - G(\varphi^*_f)} + \int_{\varphi^*_f}^{\infty} \frac{\pi_f(\varphi)g(\varphi)d\varphi}{1 - G(\varphi^*_f)} = f_e. \quad (16)$$

Using the cutoff productivity levels and wage equations for both sectors, I can describe (7), the average wage income per worker in the economy, as

$$E_i\bar{w}_i + E_f\bar{w}_f = L^{-1} \left[ \int_{\varphi^*_i}^{\varphi^*_f} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*_f}^{\infty} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi \right]. \quad (17)$$

As firms price their goods with a constant markup $1/\rho$ over marginal costs, the wage income of employed workers is equal to a constant share of output $(E_i\bar{w}_i + E_f\bar{w}_f)L = \rho Y$. Through (16), the expected profits of all firms equal their initial investment $f_e$. Therefore,

14A Pareto distribution has a finite mean only if its shape parameter $k > 1$. Productivity is Pareto distributed and labor as well as revenue are power functions of productivity. Therefore, also firm size and revenue are Pareto distributed. For the distributions of firm size and revenue to have a finite mean, $\frac{k}{\xi - 1} > 1$ and $\frac{k}{\sigma - 1} > 1$, respectively, have to hold. Thus, as in Egger and Kreickemeier (2009), $k > \xi$ is assumed.

15The aggregate revenue in the economy is described by $Y$, because $P = 1$. It consists of both the firms’ and the workers’ share of the aggregate revenue. Due to the monopolistic competition assumption and resulting prices characterized by a constant markup $1/\rho$ over marginal costs, the shares are constant proportions. A constant fraction $1/\sigma$ of the firm revenue accrues to the firm and $(\sigma - 1)/\sigma = \rho$ accrues to the workers of the firm. Hence, the wage income of all employed workers $(E_i\bar{w}_i + E_f\bar{w}_f)L$ has to equal their constant share of the aggregate revenue $\rho Y$. 

14
the workers’ income is the only disposable income for consumption and a natural utilitarian measure for welfare, as in Egger and Kreickemeier (2009), and can be written as:

\[ Y = \frac{E_i \bar{w}_i + E_f \bar{w}_f}{\rho}. \] (18)

2.7 Employment

The share of employment in the economy consists of two parts: informal sector employment share \( E_i \) and formal sector employment share \( E_f \). Building on the condition of Proposition 1, the employment share can be written as

\[ E = E_i + E_f = L^{-1} \left[ \int_{\varphi_i^*}^{\varphi_f^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi_f^*}^{\infty} l_f(\varphi)g(\varphi)d\varphi \right]. \] (19)

Next, I examine the relative employment share \( \frac{E_i}{E_f} \) to determine the effect of policy changes.

\[ \frac{E_i}{E_f} = (1 - \delta)^\sigma (1 - \lambda)^{\chi + k} \left[ \left( \frac{\varphi_i^*}{\varphi_f^*} \right) - 1 \right], \] (20)

where \( \chi \equiv \sigma(1 - \theta) - k - 1 < 0. \)\(^{16}\)

**Proposition 3.** The ratio of informal employment share to formal employment share \( \frac{E_i}{E_f} \) is increasing in \( f_f \) and decreasing in \( f_i, \delta, \lambda \).

**Proof.** See appendix B.

Proposition 3 and Proposition 2 jointly shed light on the mechanics of the economy. A reduction in informal sector profitability or increase in formal sector profitability (i.e. increase in \( f_i, \delta \) and \( \lambda \)) leads to a decrease in relative productivity distance and a decrease in informal sector employment relative to formal sector employment. Similarly, a decrease in formal sector profitability (i.e. increase in \( f_f \)) increases the relative productivity distance and increases relative informal sector employment. Intuitively, reducing informal sector profitability or increasing formal sector profitability drives the least-productive informal sector firms out of the market. As a result, the informal sector sheds labor and informal sector average productivity increases. This extends Egger and Kreickemeier (2009) to a new adjustment margin. Informal and formal sector employment are affected differently by changes in the

\(^{16}\)\( k > \xi \) is assumed. This implies \( \sigma(1 - \theta) - k - 1 + \theta < 0 \) and thus \( \sigma(1 - \theta) - k - 1 < 0 \).
economy. The employment effect of one sector can buffer the employment effect of the other one.

The setup of this model in combination with the aforementioned propositions capture the major stylized facts emerging from the empirical literature on the informal sector. First, informal sector firms hire less workers, pay a lower wage and are described by lower productivity. (Perry et al., 2007; La Porta and Shleifer, 2008). By contrast, higher-productivity firms hire more workers, pay a higher compensation for the work and are more likely to be part of the formal sector (Bernard and Jensen, 1999; Perry et al., 2007). The empirical literature classically states low government enforcement and high firm registration costs as main drivers of informal sector participation (Schneider and Enste, 2000; Djankov et al., 2002; Auriol and Warlters, 2005; Dabla-Norris and Inchauste, 2008). The model replicates this finding, as lower enforcement $\delta$ and higher fixed cost $f_f$ lead to higher informal sector employment relative to formal sector employment.

2.8 Wage inequality

Prior to the hiring process workers are identical and subsequently can be employed in either the informal or formal sector. Additionally, wages in this model are firm-specific. Therefore, two types of wage inequality can be disentangled. First, given the productivity difference between the two sectors and the informal sector wage gap, I consider the wage inequality between informal and formal workers. Second, as all workers are identical in skill level, I analyze wage inequality among all employed workers similar in spirit to Egger and Kreickemeier (2012).

The measure of between-group wage inequality is the ratio of the formal sector average wage relative to the informal sector average wage:

$$\frac{\bar{w}_f}{\bar{w}_i} = (1 - \lambda)^{-\theta} \left[ 1 - \left( \frac{\varphi_i^*}{\varphi_f^*} \right) \right] \left[ 1 - \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{\xi-k} \right]^{-1} > 1.$$ (21)

As formal production entails a productivity bonus and is characterized by high-productivity firms, the average wage in the formal sector is higher than in the informal sector and the ratio is strictly greater than one.

Second, I measure the wage inequality among all employed workers using the Gini-coefficient. Calculating the Gini-coefficient for the two-sector economy requires two steps. First, I calculate the Lorenz curve $Q(\gamma)$ by relating the share of employment to the share of wage bill for firms with productivity below $\bar{\varphi} \in [\varphi_i^*, \infty]$. Because employment and wages
in the informal sector differ from the formal sector, the Lorenz curve consists of two segments and requires lengthy calculations that can be found in appendix C. Second, the Gini-coefficient $G_{(a)}$, where subscript $(a)$ stands for autarky, follows from the Lorenz curve through $G_{(a)} = 1 - 2 \int_0^1 Q(\gamma) d\gamma$. $G_{(a)}$ is then described by

$$G_{(a)} = G_f \left[ 1 + \frac{2 \left( \frac{\varphi^*_i}{\varphi^*_f} \right)^{k-\xi}}{\frac{\theta \Gamma \Delta}{\theta \Gamma \Delta}} \left\{ \chi \left[ \Upsilon - \Xi \left( \frac{\varphi^*_i}{\varphi^*_f} \right)^\theta + [\Xi - \Upsilon] \left( \frac{\varphi^*_i}{\varphi^*_f} \right)^{-\chi} \right] \right\} \right] + \frac{\theta \Upsilon}{1 - (\varphi^*_i/\varphi^*_f)^{\xi}} \right\},$$

where $G_f \equiv \frac{(1 - \delta)^{\sigma} - \left[ (1 - \delta)^{\sigma} - (1 - \lambda)^{-(\xi + \theta)} \right] \left( \frac{\varphi^*_i}{\varphi^*_f} \right)^{\xi}}{(1 - \delta)^{\sigma} - (1 - \lambda)^{-(\xi + \theta)}}$, $\Gamma \equiv (1 - \delta)^{\sigma} - (1 - \lambda)^{-(\xi + \theta)}$, $\Delta \equiv (1 - \delta)^{\sigma} - (1 - \lambda)^{-(\xi + \theta)}$, $\Upsilon \equiv (1 - \delta)^{2\sigma} - (1 - \delta)^{\sigma}(1 - \lambda)^{-(\xi + \theta)}$, $\Xi \equiv (1 - \delta)^{2\sigma} - (1 - \delta)^{\sigma}(1 - \lambda)^{-(\xi + \theta)}$. The Gini-coefficient in the two-sector economy $G_{(a)}$ depends on the ratio of the two cutoff productivity levels $(\varphi^*_i/\varphi^*_f)$, which is a proxy for the relative sector size. For the extreme cases of $(\varphi^*_i/\varphi^*_f) = 1$, i.e. all firms are formal, and $(\varphi^*_i/\varphi^*_f) = 0$, i.e. all firms are informal, the specification collapses to the single-sector economy Gini-coefficient $G_f$. In the two-sector economy, that is $(\varphi^*_i/\varphi^*_f) \in (0, 1)$, $G_{(a)} > G_f$ holds and the wage distribution is more unequal than in the single-sector economy. Moreover, for $\theta \in (0, 1)$, i.e. workers value firm-specific wages, the Gini-coefficient is strictly greater than 0.\footnote{For the extreme case of $\theta = 0$, i.e. all workers receive the same wage, $G_f = 0$ and the economy would be perfectly equal.}

### 3 The open economy

To explore how the presence of an informal sector may mediate the impact of trade liberalization, I extend the closed economy specification by adding international trade with $n$ symmetric countries. The symmetry assumption allows me to focus on firm-level effects and renders country indices obsolete. Moreover, a world in which every country is characterized by sector dualism is sensible, since informality is a global phenomenon (Schneider et al., 2010). Two types of costs are distinguished for firms participating in international trade. As has been empirically shown by Roberts and Tybout (1997), sunk costs of exporting critically
determine export participation. Firms have to cover a fixed exporting cost $f_x > f_f$, in addition to the domestic entry cost, to participate in trade. The fixed cost $f_x$ can be interpreted as a one-time expense for knowledge or infrastructure needed to engage in international trade and allows firms to access all $n$ markets. Subscript $x$ is used henceforth to describe variables related to export activities. In addition, firms face a variable trade cost that is modeled in the form of an iceberg trade cost $\tau > 1$, i.e. for one unit to arrive at the destination market, $\tau$ units have to be shipped (Anderson and van Wincoop, 2004).

### 3.1 The firm’s decision to export

Given the previous constraint on informal firms being characterized by lower productivities than formal firms, it is never profitable for informal firms to export.\(^{19}\) Intuitively, informal firms decide against formality out of profitability considerations arising from the formal sector fixed cost $f_f$. Exporting induces an even higher fixed cost $f_x$ than the formal sector participation already does. Hence, the same profitability considerations will lead informal sector firms to not be able to profitably export. The complete exclusion of informal sector firms from exporting is stylized. Yet, this model result is supported by the empirical literature that finds that informal firms rarely export (Batra et al., 2003; Bigsten et al., 2004; La Porta and Shleifer, 2008). The result is summarized in Proposition 4:

**Proposition 4.** If Proposition 1 holds, informal sector firms will never find it profitable to export.

In regards to the formal sector, empirical studies find a clear correlation between export participation and firm productivity, i.e. the highest-productivity firms in an economy self-select into exporting (Bernard and Jensen, 1995; Roberts and Tybout, 1997; Delgado et al., 2002; Wagner, 2007). Hence, the results focus on parameters that satisfy $\phi^*_x > \phi^*_f$, where $\phi^*_x$ stands for the cutoff productivity level at which exporting becomes profitable, and assumes that exporters are characterized by a higher productivity level than non-exporters. Therefore upon drawing a productivity $\phi$, a firm in the open economy decides on its formality status and export participation according to $\max \{\pi_i(\phi), \pi_f(\phi), \pi_f(\phi) + \pi_x(\phi), 0\}$. The number of firms operating in the domestic market then consists of informal sector firms and formal

\(^{19}\)What is required for informal exporting to be profitable at a lower productivity level than formal exporting, i.e. $\phi_i < \phi_f$ from $\pi_i(\phi_i) = (1 - \delta)^{n_x} n \frac{\gamma}{\sigma_M} \left( \frac{\phi_i}{\phi_f} \right)^{\delta - 1} (E_i \bar{w}_i + E_f \bar{w}_f)^{1 - \theta} \rho^{-\theta - 1} \tau^{1 - \sigma} - f_x = 0$ &

$\pi_f(\phi) = n \frac{\gamma}{\sigma_M} \left( \frac{\phi_f}{\phi_f} \right)^{\delta - 1} (E_i \bar{w}_i + E_f \bar{w}_f)^{1 - \theta} \rho^{-\theta - 1} \tau^{1 - \sigma} - f_x = 0$. The resulting requirement is $(1 - \delta) \bar{\pi} > 1 - \frac{1}{1 - \lambda}$, which contradicts $(1 - \delta) \bar{\pi} < 1 - \frac{1}{1 - \lambda}$ of Proposition 1.
sector firms, which are domestic, as well as foreign exporters, i.e. $M = M_i + M_f + (1 + n)M_x$. The sufficient condition for the productivity sorting is summarized in Proposition 5:

**Proposition 5.** If \( \frac{f_x \tau f_f}{n(1-\lambda)^{-\xi}} > \frac{f_f - f_i}{(1-\lambda)^{-\xi} - (1-\delta)^{\sigma}} \), then exporting firms are higher-productivity firms than non-exporting formal sector firms; that is $\varphi_x^* > \varphi_f^*$.

The sorting depends on the entry costs to the informal and formal sector ($f_i$ and $f_f$), informal sector enforcement ($\delta$), formal sector productivity bonus ($\lambda$) and the variables determining the costs and benefits of trade ($f_x$, $\tau$ and $n$). Intuitively, the inequality compares two cost-benefit ratios. If the cost-benefit ratio of exporting \( \frac{f_x \tau f_f}{n(1-\lambda)^{-\xi}} \) is higher than the cost-benefit ratio of domestic production \( \frac{f_f - f_i}{(1-\lambda)^{-\xi} - (1-\delta)^{\sigma}} \), then a higher productivity is required to be able to profit from exporting.

Given the variable and fixed cost, the formal sector firm revenue function is

\[
r(\varphi) = \begin{cases} 
  r_f(\varphi) & \text{if the firm sells domestically,} \\
  r_f(\varphi) + n\tau^{1-\sigma}r_f(\varphi) & \text{if the firm exports.} 
\end{cases}
\]  

A firm’s profit from exporting is described by

\[
\pi_x(\varphi) = \frac{r_f(\varphi)n\tau^{1-\sigma}}{\sigma} - f_x.
\]  

In addition to (9) and (10), there is a new condition to determine the export participation productivity cutoff level $\varphi_x^*$

\[
\pi_x(\varphi_x^*) = 0.
\]

In summary, to achieve the productivity sorting of firms in the open economy according to the empirical literature, the model builds on the results of Proposition 1, 4 and 5. That is, informal sector firms are assumed to be the lowest-productivity firms followed by domestic formal firms. Lastly, formal firms that export are the highest-productivity firms. Given this, there are no informal sector exporters.

\[19\]

---

\[20\] Ensuring $\varphi_i^* > \varphi_f^*$ for $\pi_i(\varphi_f^*) = \pi_f(\varphi_f^*)$ and $\varphi_x^*$ from $\pi_x(\varphi_x^*) = 0$ is sufficient to sort domestic productivity levels below export productivity levels. This results in

\[
\varphi_f^* = \left[ \frac{(f_f - f_i)M \sigma}{(1-\lambda)^{-\xi} - (1-\delta)^{\sigma}} \right] \tau (E_i \bar{w}_i + E_f \bar{w}_f) \rho^{\frac{1}{\frac{\sigma}{\tau}}} \]  

and $\varphi_x^* = \left[ \frac{f_x \tau f_f}{n(1-\lambda)^{-\xi}} \right] \tau (E_i \bar{w}_i + E_f \bar{w}_f) (\varphi_f^*)^{\frac{1}{\frac{\sigma}{\tau}}} (1-\lambda)$. In combination, \( \frac{f_x \tau f_f}{n(1-\lambda)^{-\xi}} > \frac{f_f - f_i}{(1-\lambda)^{-\xi} - (1-\delta)^{\sigma}} \).
3.2 Firm-specific variables

I can express the relationship between formal firms and formal exporting firms as ratios solely in terms of their productivity levels, variable trade costs $\tau$ and the number of countries $n$, independent of assumptions on the distribution of firm productivity. The price and quantities refer solely to the export markets. However, the profits of exporting firms stem from both domestic and foreign sales. Hence, the total labor demand and revenues of exporting firms consist of both the ones for the domestic market and the export market. Exporters pay the same wage as formal sector producers, i.e. $w_f(\varphi)$, and the model does not feature an explicit exporter wage premium. Since exporters are assumed to be more productive than non-exporting formal firms, $w_f(\varphi^*_x) > w_f(\varphi^*_f)$ holds. With the fair wages being power functions of productivity, the productivity differential between exporters and non-exporters entails a wage differential. As a result, the model captures the empirical observation that exporting firms pay higher wages than non-exporting firms (Bernard and Jensen, 1995; Schank et al., 2007).

\[
\frac{p_f(\varphi_f)}{p_x(\varphi_x)} = \left( \frac{\varphi_f}{\varphi_x} \right)^{\theta-1} \tau^{-1} < 1 \quad \text{and} \quad \frac{q_f(\varphi_f)}{q_x(\varphi_x)} = \left( \frac{\varphi_f}{\varphi_x} \right)^{\sigma(1-\theta)} \tau^\sigma > 1. \quad (26)
\]

Given the same productivity, in foreign markets the price is higher and the quantity sold is lower than in the domestic market.

\[
\frac{r_f(\varphi_f)}{r_x(\varphi_x)} = \left( \frac{\varphi_f}{\varphi_x} \right)^{\xi} \frac{1}{1 + n\tau^{1-\sigma}} < 1 \quad \text{and} \quad \frac{l_f(\varphi_f)}{l_x(\varphi_x)} = \left( \frac{\varphi_f}{\varphi_x} \right)^{\sigma(1-\theta)-1} \frac{1}{1 + n\tau^{1-\sigma}} < 1. \quad (27)
\]

With regard to revenue and labor demand, both are increasing in the number of countries $n$ and decreasing in the variable trade cost $\tau$ for exporters relative to formal non-exporters. These model results are in line with the commonly stated firm-level evidence on exporters being characterized by higher employment and higher revenues than their non-exporting counterparts (Bernard and Jensen, 1995; Bernard et al., 2012).

Equation (28) allows me to analyze the distance between formal sector productivity cutoff $\varphi^*_f$ and exporting cutoff productivity level $\varphi^*_x$ relative to the exporting cutoff productivity level $\varphi^*_x$:

\[
\frac{\varphi^*_x - \varphi^*_f}{\varphi^*_x} = 1 - \left( \frac{(f_f - f_x)n\tau^{1-\sigma}}{f_x} \right)^{\frac{1}{\xi}} \left( (1 - \lambda)^{-\xi} - (1 - \delta)^\sigma \right)^{\frac{-1}{\xi}} (1 - \lambda)^{-1}. \quad (28)
\]

The results are summarized in Proposition 6 and can be separated into two groups. First,
the effects that make international trade more attractive to firms (increase in \( n \) or decrease in \( f_x \) or \( \tau \)) close the relative distance between the productivities. A more open economy allows for increased sales of exporting firms, directly benefiting their profitability, and decreases the required productivity level for participation. For non-exporting formal sector firms, an increased number of foreign competitors in the domestic market drives down profitability and increases productivity requirements. The second group are factors that also influence domestic firms directly. Factors increasing the profitability of formal sector participation relative to informal sector participation (increase in \( f_i \), \( \delta \) and \( \lambda \)) lower the productivity threshold of becoming formal, but do not affect export participation as much. As a result, the relative distance between exporting and domestic formal firms increases. The opposite holds true for the formal sector entry cost \( f_f \). Formal sector participation is more affected than export participation, as exporters are high-productivity firms, and the productivity distance decreases.

**Proposition 6.** The relative distance in cutoff productivities \( \frac{\phi_x^* - \phi_f^*}{\phi_x^*} \) is decreasing in \( f_f \) and \( n \). It is increasing in \( f_i \), \( f_x \), \( \delta \) and \( \lambda \).  

*Proof.* See appendix D. \( \square \)

Lastly, comparing \( \phi_x^* = (f_x/f_i)^{\frac{1}{\xi}}(n\tau^{1-\sigma})^{-\frac{1}{\xi}}(1-\lambda)(1-\delta)\xi \phi_f^* \), i.e. the cutoff productivity level of the marginal informal sector and marginal exporting firms highlights what drives their difference. The ratio of sector entry costs, trade variables and the productivity bonus lead to a higher productivity requirement for exporting firms relative to informal sector producers. Similar to the cutoff productivity levels, the difference between these key variables is driven by potential government enforcement for informal sector firms, the productivity bonus of formal sector firms and the trade parameters.

To derive the new free entry condition in the open economy, I extend (16) to include potential exporting profit:

\[
\int_{\phi_f^*}^{\phi_x^*} \pi_i(\varphi) g(\varphi) d\varphi + \int_{\phi_f^*}^{\phi_x^*} \pi_f(\varphi) g(\varphi) d\varphi + \int_{\phi_x^*}^{\infty} \left[ \pi_f(\varphi) + \pi_x(\varphi) \right] g(\varphi) d\varphi = f_e. 
\]  

(29)

### 3.3 Employment

I can rewrite the equilibrium employment in terms of the cutoff productivity levels. Employment in the economy consists of three segments: informal, formal and formal exporter employment. The employment share in the economy then is
$E = L^{-1} \left[ \int_{\varphi_1^*}^{\varphi_2^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi_1^*}^{\varphi_2^*} l_f(\varphi)g(\varphi)d\varphi + \int_{\varphi_2^*}^{\infty} l_x(\varphi)g(\varphi)d\varphi \right].$  \hspace{1cm} (30)

To analyze the impact of trade liberalization on employment, I first delineate the effect into the employment of the informal and formal sectors. Formal sector employment adjusts along two margins. The high-productivity formal firms become exporters and hire additional workers to be able to serve the foreign demand. After trade liberalization, foreign competitors enter the domestic market and more varieties of the intermediate good are sold domestically (increase in $M$). As competitive pressure rises, the demand for each variety decreases and the profitability of all firms is reduced. Low-productivity formal firms informalize to remain profitable. As a result, formal sector employment is affected negatively, dampening total employment. Informal sector firms do not experience the productivity bonus and thus hire fewer workers than formal sector firms at the same productivity level. This is obvious when comparing the labor demand ratio $l_i(\varphi_1)/l_f(\varphi_2) = (\varphi_1/\varphi_2)^{\sigma(1-\theta)-1} (1-\delta)^\sigma (1-\lambda)^{\sigma(1-\theta)-1}$. Whether the hiring effect of exporting firms or the labor shedding of informalized firms dominates depends on the key characteristics of the economy. Thus, the effect of trade liberalization on formal sector employment is ambiguous.

Informal sector employment is also affected along two margins. With falling demand for each input variety, the lowest-productivity informal producers are forced out of the market and release labor. Along the other margin, the least-productive formal sector firms become informal and thereby increase informal sector employment. Given the Pareto productivity distribution, the labor releasing effect is stronger than the labor hiring and therefore informal sector employment unambiguously decreases upon trade liberalization.\textsuperscript{21} The empirical evidence on the adjustment of informal sector employment through trade liberalization is ambiguous with a wide range of definitions of informality and data sets in use.\textsuperscript{22} My research provides theoretical support for the finding of decreasing informal sector employment with

\textsuperscript{21} $E_{(t)} \over E_{(a)} = \left[ 1 + n \left( \frac{l_x}{f_{i,n+1}} \right) \frac{\lambda^{-k}}{1-\lambda} \left( 1-\delta \right)^{-\frac{\lambda}{1-\lambda}} \right] \frac{\theta}{\xi} \left[ 1 + \frac{(1-\lambda)^{-k}(n^{1-\sigma})^\frac{\lambda}{1-\lambda} \frac{x-k}{x} (1-\delta)^{\frac{\sigma(x-k)}{1-\delta}}}{(1-\lambda)^{-\xi - (1-\delta)\alpha} \frac{\lambda}{\xi} \left( \frac{l_x}{f_{i,n+1}} \right) \frac{\lambda^{-k}}{1-\lambda} \left( 1-\delta \right)^{\frac{\sigma(x-k)}{1-\delta} + (1-\delta)\alpha} } \right]^{\frac{\theta}{\xi} - 1}$,

where the subscript (a) and (t) stand for autarky and trade. \( E_{(t)} \over E_{(a)} < 1 \) because $-\frac{\theta}{\xi} < 0$ and $\frac{\theta}{\xi} - 1 < 0$. Hence, informal sector employment unambiguously decreases upon trade liberalization.

\textsuperscript{22} My model focuses on firm-level informality as registration non-compliance. Thus, all workers employed by a firm are either formal or informal workers. Defining informality from a labor market perspective, i.e. as the evasion of labor market regulations, allows firms to hire both informal and formal workers by, for instance, evading social security contributions for only some of their workers. As a consequence of trade liberalization and increased competitive pressure, firms might substitute formal with informal workers, potentially leading to an increase or no change in informal employment found by Goldberg and Pavcnik (2003).
Lastly, I analyze the total employment in the economy as a combination of the employment adjustments of both sectors. As illustrated in Figure 2, liberalizing trade changes the position of both sectors along the productivity distribution. Three forces determine the change in total employment: an employment gain as a result of exporter hiring, an employment loss caused by the informalization of the least-productive formal sector firms and an employment loss that occurs as the least-productive informal sector firms exit the market. While the informal sector unambiguously reduces its size, the effect of trade liberalization on formal sector employment is ambiguous, thus rendering the total employment effect ambiguous. The magnitude of each sectoral adjustment and accordingly the direction of total employment adjustment is determined by the economy’s characteristics, such as entry costs to both sectors. The move from autarky to full integration is stylized. The aforementioned results hold also for gradual trade integration, as measured by a reduction in $\tau$ or $f_x$.\footnote{Proof for this is available from the author upon request.}

Figure 2: Sector sorting along the productivity distribution in autarky and the open economy.

This highlights a key contribution of this model. In Egger and Kreickemeier (2009) there are only two forces at work. The highest-productivity firms become exporters and hire additional workers; the lowest-productivity firms exit the market and shed labor. In sum, Egger and Kreickemeier (2009) find an unambiguous employment decrease. The existence of an informal sector gives rise to a third force, i.e. the informalization of low-productivity formal firms, which dampens the reallocation of labor towards more productive firms. The economy’s characteristics affect the three forces and accordingly the magnitude of each. This result, as summarized in Proposition 7, bridges the gap between the original model of Egger
and Kreickemeier (2009) and the mixed empirical evidence on the relationship between trade openness and unemployment (Davidson and Matusz, 2009; Dutt et al., 2009; Felbermayr et al., 2011b; Menezes-Filho and Muendler, 2011).

**Proposition 7.** Trade liberalization reduces informal sector employment unambiguously and can either reduce or increase formal sector employment. In combination, the effect of trade liberalization on total employment in the economy is ambiguous in the presence of informality.

*Proof.* See appendix F.

To gain further insight into the mechanics of the model, analogously to the closed economy case, I can describe the informal sector employment share relative to the formal sector employment share:

\[
\left( \frac{E_i}{E_f} \right)_{(t)} = \eta \left( \frac{E_i}{E_f} \right)_{(a)},
\]

where \( \eta \equiv \left[ 1 + n \tau^{1-\sigma} \left( \frac{x^*_i}{x^*_f} \right)^{-\chi} \right]^{-1} < 1 \). Subscript \((a)\) and \((t)\) stand for autarky and trade. The intuition follows from the earlier result. Informal sector employment unambiguously decreases, while formal sector employment may either increase or decrease. In combination, trade liberalization unambiguously reduces the informal sector employment share relative to the formal sector share. This leads to Proposition 8:

**Proposition 8.** The ratio of informal employment share to formal employment share is lower in the open economy than under autarky.

### 3.4 Welfare

The average wage income per worker \((E_i\bar{w}_i + E_f\bar{w}_f)\) in the open economy is described by all three cutoff productivities and, in combination with (18), determines the aggregate output in the open economy.

\[
(E_i\bar{w}_i + E_f\bar{w}_f) = L^{-1} \left[ \int_{\varphi^*_i}^{\varphi_f^*} l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_f^*}^{\varphi^*_x} l_f(\varphi) w_f(\varphi) g(\varphi) d\varphi + \int_{\varphi^*_x}^{\infty} l_x(\varphi) w_f(\varphi) g(\varphi) d\varphi \right].
\]

(32)
The effect of trade liberalization on welfare, as measured by the aggregate output of the economy per capita, is similar to the employment effect. The intuition is as follows. Trade liberalization allows the highest-productivity firms to become exporters and shifts resources towards the most-productive firms in the economy. Thereby aggregate formal sector output is increased. The lowest-productivity formal firms, however, switch to informal sector production. The result is a reduction in aggregate formal sector output through the loss of the formal sector productivity bonus. In sum, the effect of trade on the aggregate formal sector output is ambiguous. The informal sector is affected along two margins, as well. The lowest-productivity informal sector firms cease production and decrease aggregate informal sector output, while the informalization of the lowest-productivity formal firms increases the aggregate output of the informal sector. Depending on the economy’s characteristics the former may or may not compensate for the latter, rendering the effect on aggregate informal sector output ambiguous. As before, depending on the key parameters of the economy, the net effect of trade liberalization on the aggregate output of the whole economy can be positive or negative. This result also holds for gradual trade liberalization (decrease in $\tau$ or $f_x$).\(^{24}\) The effect of trade on aggregate output in this model is more nuanced than in Egger and Kreickemeier (2009), who find an unambiguous increase in aggregate output through trade liberalization. The ambiguous result highlights the distortive effect of the informal sector on resource allocation, as suggested by, for example, Hsieh and Klenow (2009) and Bruhn (2013), that can either be alleviated (McCaig and Pavcnik, 2013) or aggravated through trade. Proposition 9 summarizes this result.

**Proposition 9.** *Trade liberalization in the presence of informality has an ambiguous effect on the aggregate output of the informal sector, the formal sector and in sum on the welfare of the economy.*

*Proof. *See appendix F.

### 3.5 Wage inequality

Trade liberalization affects wage inequality indirectly by adjusting the number of workers employed in the informal and formal sector.\(^{25}\) As before, I first analyze between-group wage inequality.

\(^{24}\)Proof for this is available from the author upon request.

\(^{25}\)My model focuses on the interaction between the informal and formal sector and accordingly ignores an exporter wage premium. The model can be extended to include a fair wage constraint that uses firm revenue and not firm productivity as firm-internal reference point. This would lead to the exporter wage premium and could provide another source for wage inequality, even among formal workers.
\[
\left( \frac{w_f^i}{w_i^i} \right)_{(t)} = \omega \left( \frac{w_f^i}{w_i^i} \right)_{(a)}, \tag{33}
\]

where \( \omega \equiv \left[ 1 + n \tau^{1-\sigma} \left( \frac{\varphi^i}{\varphi_f^i} \right)^{\xi-k} \right] \left[ 1 + n \tau^{1-\sigma} \left( \frac{\varphi^i}{\varphi_f^i} \right)^{\chi} \right]^{-1} > 1 \). Between-group wage inequality in the open economy is higher than under autarky. Intuitively, trade liberalization raises the competitive pressure in the economy and forces the least-productive informal firms to exit and the lowest-productivity formal firms to informalize. The informal firms paying the lowest wages exit and higher-wage formal firms start informal production. This raises the informal sector average wage. With regard to the formal sector, the highest-wage exporters hire more workers and the lowest-wage formal firms informalize. The average wage of the formal sector increases and does so at a greater magnitude than the informal sector average wage. Hence, the average wages diverge and between-group inequality increases.

Similarly, wage inequality among all employed workers measured by the Gini-coefficient hinges on the share of employment in both sectors. The derivation of the Gini-coefficient is analogous to the closed economy, albeit more complicated.\(^{26}\) The Lorenz curve consists of not just informal and formal workers, but also workers employed by exporting firms.\(^{27}\) Trade liberalization affects wage inequality indirectly through the employment shares and can either increase or decrease wage inequality. The intuition for this result derives from Proposition 8. Both a purely-formal and a purely-informal economy feature the same Gini-coefficient, which is strictly lower than that of an economy featuring both sectors.\(^{28}\) Trade liberalization increases the formal sector employment share relative to informal sector employment share. If initially formal sector employment is large relative to informal sector employment, a relative increase in the formal labor share pushes the economy closer to a purely-formal economy. Hence, wage inequality decreases. The opposite holds true if the formal sector is relatively small before trade liberalization. Due to the relative formalization of labor through trade, the economy diverges from a purely-informal economy. Trade then increases wage inequality.\(^{29}\)

\(^{26}\)Given its complicated nature, the Gini-coefficient for the open economy is derived in Appendix E.

\(^{27}\)For the extreme case of \((\varphi^i/\varphi_f^i) = 0\), i.e. no firm exports, the open economy Gini-coefficient collapses to the autarky specification. If in addition to that, \((\varphi^i/\varphi_f^i) = 0\) or \((\varphi^i/\varphi_f^i) = 1\) are imposed, the coefficient further collapses to the formal-sector-only specification.

\(^{28}\)For a derivation of this result, see Appendix C. As shown by Helpman et al. (2010), the Gini-coefficient depends only on the shape parameter of the wage distribution, but not its lower limit. Both informal and formal sector wage distribution feature the same shape parameter and thus the same inequality.

\(^{29}\)The intuition here is similar to the effect of a conditional exporter wage premium on wage inequality, as shown empirically by Helpman et al. (2012), Akerman et al. (2013), Baumgarten (2013) and theoretically by Helpman et al. (2010). The findings suggest that a major share of overall wage inequality arises from the wage differences between firms in the same industry paid to workers with similar characteristics, i.e. within-industry
Proposition 10. Trade liberalization increases between-group wage inequality and has an ambiguous effect on wage inequality among all employed workers.

Proof. See appendix F.

4 Conclusion

Previous trade models did not reconcile heterogeneous firms, labor market frictions and informality in the form of registration non-compliance. In this paper, I developed a simple general equilibrium trade model with one production factor, namely labor. Firms in the model are heterogeneous in productivity and pay a fair wage depending on the firm’s productivity and the average wage of employed workers in the economy. Depending on their productivity, firms select into informal sector production, formal sector production or exporting. By introducing informality into heterogeneous firm trade models with labor market frictions, the model shows analytically how informality distorts resource allocation in an economy. Trade liberalization leads to a decrease in informal sector employment and affects formal sector employment ambiguously. Depending on the characteristics of the economy, total employment and welfare can either decrease or increase. Opening the economy to trade affects wage inequality among employed workers ambiguously and ultimately depends on the number of formal sector firms relative to informal sector firms in the economy. Wage inequality between informal and formal workers increases as the formal sector average wage rises faster than the average wage earned by informal sector workers.

The implication of this framework for policy-makers is clear. While trade liberalization achieves the often targeted reduction in informal employment, the economic conditions in a country ultimately determine whether trade is beneficial or detrimental in regard to employment, welfare and wage inequality in the presence of informality. Hence, this setup emphasizes the need to consider the existence of an informal sector and the economic environment jointly in policy decisions on trade.

Several extensions of this work would provide for interesting future research endeavors. First, replacing the productivity sharing motif of the fair wage specification with revenue sharing would shed light on an additional source of wage inequality, i.e. an exporter wage premium, and can possibly entail different distributional consequences than the present work. Second, including heterogeneous workers and allowing firms to hire both informal and formal wage inequality. Moreover, wage inequality is driven by the employment adjustments of these firms upon trade liberalization.
workers is a useful extension to capture the empirical findings of works with labor market-specific definitions of informality. Lastly, introducing informality with a broader definition as tax evasion and registration non-compliance and in a public finance framework would inform optimal taxation and enforcement decisions in the presence of an informal sector.
Appendix

A  Comparative statics on the relative productivity difference in autarky

I analyze (14) in a comparative statics exercise to analyze the effect of the parameters of interest on the relative productivity distance:

\[
\frac{\varphi^*_f - \varphi^*_i}{\varphi^*_f} = 1 - \varphi^*_i \left( \frac{f_i}{f_f - f_i} \right)^{\frac{1}{\xi}} \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi}} \frac{1}{\tau}. \\
\frac{\partial \varphi^*_i}{\partial f_i} = \frac{1}{\xi} \left( \frac{f_i}{f_f - f_i} \right)^{\frac{1}{\xi} - 1} \left( \frac{f_f}{(f_f - f_i)^2} \right) \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi}} \frac{1}{\tau} > 0. \\
\frac{\partial \varphi^*_i}{\partial f_f} = \frac{1}{\xi} \left( \frac{f_i}{f_f - f_i} \right)^{\frac{1}{\xi} - 1} \left( -\frac{f_i}{(f_f - f_i)^2} \right) \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi}} \frac{1}{\tau} < 0. \\
\frac{\partial \varphi^*_i}{\partial \delta} = \left( \frac{f_i}{f_f - f_i} \right)^{\frac{1}{\xi}} \left[ \frac{\sigma}{\xi} (1 - \delta)^{-\sigma} \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi}} \frac{1}{\tau} \right] + (1 - \delta)^{-\sigma} \frac{\sigma}{\xi} (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi} - 1} > 0. \\
\frac{\partial \varphi^*_i}{\partial \lambda} = \left( \frac{f_i}{f_f - f_i} \right)^{\frac{1}{\xi}} \left( 1 - \delta \right)^{-\sigma} \left( (1 - \lambda)^{-\xi} - (1 - \delta)^{\sigma} \right)^{\frac{1}{\xi} - 1} (1 - \lambda)^{-\xi - 1} > 0.
\]
B Comparative statics on informal relative to formal employment share in autarky

I analyze (20) in a comparative statics exercise to analyze the effect of the parameters of interest on the relative employment in the two sectors:

\[
\frac{E_i}{E_f} = (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 1} \left[ \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^\chi - 1 \right].
\]

\[
\frac{\partial E_i}{\partial f_i} = (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 1} \chi \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{\chi - 1} \frac{\partial \varphi_i^*}{\partial f_i} < 0.
\]

\[
\frac{\partial E_i}{\partial f_f} = (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 1} \chi \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{\chi - 1} \frac{\partial \varphi_i^*}{\partial f_f} > 0.
\]

\[
\frac{\partial E_i}{\partial \delta} = - \sigma (1 - \delta)^{\sigma - 1} (1 - \lambda)^{\sigma(1 - \theta) - 1} \left[ \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^\chi - 1 \right] + (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 1} \chi \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{\chi - 1} \frac{\partial \varphi_i^*}{\partial \delta} < 0.
\]

\[
\frac{\partial E_i}{\partial \lambda} = - (\sigma (1 - \theta) - 1) (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 2} \left[ \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^\chi - 1 \right]
\]

\[
+ (1 - \delta)^\sigma (1 - \lambda)^{\sigma(1 - \theta) - 1} \chi \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{\chi - 1} \frac{\partial \varphi_i^*}{\partial \lambda} < 0.
\]

C Wage inequality in autarky

First, I compute the Lorenz curve by relating the share of employment to the share of the wage bill. Using the Lorenz curve, I derive the Gini-coefficient.

Purely-formal economy
Share in employment of firms with productivity below \( \bar{\varphi} \):

\[
\gamma_f(\varphi) = \frac{\int_{\varphi}^{\bar{\varphi}} l_f(\varphi) \gamma(\varphi) d\varphi}{\int_{\varphi}^{\bar{\varphi}} l_f(\varphi) \gamma(\varphi) d\varphi} = 1 - \left( \frac{\varphi}{\bar{\varphi}} \right)^\chi
\]

Share in wage bill of firms with productivity below \( \bar{\varphi} \):
\[ Q_f(\varphi) = \int_{\varphi_i}^{\varphi} l_f(\varphi) w_i(\varphi) g(\varphi) d\varphi = 1 - \left( \frac{\varphi}{\varphi_i} \right)^{\xi-k} \]

The Lorenz curve is \( Q_f(\gamma_f) = 1 - (1 - \gamma_f)^{\xi-k} \) and the Gini-coefficient follows from
\[ G_f = 1 - 2 \int_0^1 Q_f(\gamma_f) d\gamma_f = \frac{\theta}{\theta-2(\xi-k)}. \]

**Purely-informal economy**

Share in employment of firms with productivity below \( \bar{\varphi} \):
\[ \gamma_i(\varphi) = \int_{\varphi_i}^{\bar{\varphi}} l_i(\varphi) g(\varphi) d\varphi = 1 - \left( \frac{\varphi}{\varphi_i} \right)^{\xi-k} \]

Share in wage bill of firms with productivity below \( \bar{\varphi} \):
\[ Q_i(\varphi) = \int_{\varphi_i}^{\bar{\varphi}} l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi = 1 - \left( \frac{\varphi}{\varphi_i} \right)^{\xi-k} \]

The Lorenz curve is \( Q_i(\gamma_i) = 1 - (1 - \gamma_i)^{\xi-k} \) and the Gini-coefficient follows from
\[ G_i = 1 - 2 \int_0^1 Q_i(\gamma_i) d\gamma_i = \frac{\theta}{\theta-2(\xi-k)}. \]

As a purely-formal and purely-informal economy have the same Gini-coefficient, they feature the same wage inequality. Because the wage distribution directly depends on the Pareto distribution, productivity dispersion among firms in the economy determines wage inequality, that is \( \frac{\partial G_f}{\partial k} = \frac{-2\theta}{(\theta-2(\xi-k))^2} < 0 \). Hence, wage inequality measured by the Gini coefficient, is increasing in firm productivity dispersion (lower \( k \)). As supported by the literature, higher dispersion of firm productivity creates higher wage inequality by spreading out the range of wages paid in the economy. This is empirically supported by Davis and Haltiwanger (1991) and Faggio et al. (2010) and has theoretically been shown in Helpman et al. (2010) and Egger and Kreickemeier (2012).

**Economy with informal and formal sector**

I first estimate the Lorenz curve and then derive the Gini-coefficient. However, I have to distinguish between informal and formal workers. The Lorenz curve then consists of two segments. The first share of workers is employed in the informal sector:

Share in employment of firms with productivity below \( \bar{\varphi} \):
\[ \gamma = \frac{\int_{\varphi}^{\bar{\varphi}} l_1(\varphi) g(\varphi) d\varphi}{\int_{\varphi_i}^{\bar{\varphi}} l_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i}^{\infty} l_f(\varphi) g(\varphi) d\varphi} = \frac{(1-\delta)\left[ 1 - \left( \frac{\varphi}{\varphi_i} \right)^{\xi-k} \right]}{1 - \left( \frac{\varphi}{\varphi_i} \right)^{\xi-k}}, \]
where $\Gamma \equiv (1 - \delta)^{\sigma} - [(1 - \delta)^{\sigma} - (1 - \lambda)^{-\xi + \theta}] \left( \frac{\phi_f^*}{\phi_f^*} \right)^{\xi - k}$. 

Share in wage bill of firms with productivity below $\varphi^*$:

$$Q_i = \frac{\int_{\varphi_f^*}^{\varphi_i^*} l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi}{\int_{\varphi_f^*}^\infty l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi} = \frac{(1 - \delta)^{\sigma} \left[ 1 - \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{(1 - \lambda)} \right]}{\Delta},$$

where $\Delta \equiv (1 - \delta)^{\sigma} - [(1 - \delta)^{\sigma} - (1 - \lambda)^{-\xi}] \left( \frac{\phi_f^*}{\phi_f^*} \right)^{\xi - k}$. 

The first segment of the Lorenz curve is $Q_i(\gamma) = \frac{(1 - \delta)^{\sigma}}{\Delta} \left[ 1 - \left( \frac{\phi_f^*}{\phi_f^*} \right)^{(1 - \lambda) - \xi + \theta} \right]$. 

Second, including workers employed in the formal sector:

Share in employment of firms with productivity below $\varphi^*$:

$$\gamma = \frac{\int_{\varphi_f^*}^{\varphi_i^*} l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi}{\int_{\varphi_f^*}^\infty l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi} = 1 - \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{(1 - \lambda)}.$$ 

Share in wage bill of firms with productivity below $\varphi^*$:

$$Q_f = \frac{\int_{\varphi_f^*}^{\varphi_i^*} l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi}{\int_{\varphi_f^*}^\infty l_i(\varphi) w_i(\varphi) g(\varphi) d\varphi + \int_{\varphi_i^*}^\infty l_i(\varphi) w_f(\varphi) g(\varphi) d\varphi} = \frac{1 - \left( \frac{\varphi_i^*}{\varphi_f^*} \right)^{(1 - \lambda)}}{\Delta}. $$

The second segment of the Lorenz curve is $Q_f(\gamma) = 1 - \frac{(1 - \delta)^{\xi}}{\Delta} \left[ (1 - \lambda) \xi - \gamma \right]^{\xi - k}$. 

Jointly, the Lorenz curve is described by $Q(\gamma) = \begin{cases} Q_i(\gamma) & \text{if } \gamma \in [0, b_i] \\ Q_f(\gamma) & \text{if } \gamma \in [b_i, 1] \end{cases}$, where $b_i = \frac{(1 - \delta)^{\sigma} \left[ 1 - \left( \frac{\phi_i^*}{\varphi_f^*} \right)^{(1 - \lambda)} \right]}{\Delta}$ is the share of workers employed in the informal sector.

The Lorenz curve $Q(\gamma)$ has the desired properties $Q_i(b_i) = Q_f(b_i)$, $Q(0) = 0$, $Q(1) = 1$ and $\frac{\partial Q(\gamma)}{\partial \gamma} > 0$.

The Gini-coefficient follows from $G = 1 - 2 \int_0^1 Q(\gamma) d\gamma$:

$$G = G_f \left[ 1 + \frac{2\phi^{k - \xi}}{\theta^\lambda} \left( \chi \left[ \Xi - \Xi \phi^{\theta} + [\Xi - \Xi] \phi^{-\lambda} \right] + \theta \Xi [1 - \phi^{-\lambda}] \right) \right], \text{ where } \phi \equiv \frac{\varphi_f^*}{\varphi_i^*} \in (0, 1).$$

$$G > G_f \text{ if } \frac{\chi}{\theta} \left[ 1 + \frac{\Xi [\phi^{-\lambda} - \phi^{\theta}]}{[1 - \phi^{-\lambda}]} \right] > 1.$$
Hence, I analyze $f(\phi) = \frac{1}{\phi} \left[ 1 + \frac{\phi^{\chi - \phi^\theta}}{1 - \phi^\theta} \right] - 1$ for $\phi \in (0, 1)$

$$\lim_{\phi \to 0} f(\phi) = \frac{k - \xi}{\theta} > 0$$

Using L’hôpital’s rule, it is clear that $\lim_{x \to c} \frac{g(\phi)}{h(\phi)} = \lim_{x \to c} \frac{g'(\phi)}{h'(\phi)}$. Hence,

$$\lim_{\phi \to 1} f(\phi) = \frac{k - \xi}{\theta} - \frac{\xi}{\theta} \lim_{\phi \to 1} \left[ -1 - \frac{\theta}{\chi} \phi^{\chi + \theta} \right] = \frac{k - \xi}{\theta} \left[ \frac{T - \Xi}{T} \right] > 0$$

Lastly, I show that $f(\phi)$ is strictly monotone in $\phi \in (0, 1)$. Using the L’hôpital’s monotonicity rule,\(^{30}\) it is clear that $\frac{g(\phi)}{h(\phi)}$ is strictly monotone in $\phi$ if $\frac{g'(\phi)}{h'(\phi)}$ is strictly monotone in $\phi$ on $(a, b)$ with $f(a) = g(a) = 0$ or $f(b) = g(b) = 0$. Accordingly, I find $\frac{\partial (f(\phi))}{\partial \phi} = \frac{\Xi}{T} (\xi - k) \phi^{\chi + \theta - 1} < 0$.

As $f(\phi)$ is strictly monotone and $> 0$ in $\phi \in (0, 1)$, the Gini-coefficient of the economy with informality is strictly larger than the one of a purely-formal economy. That means independent of its size, the existence of an informal sector increases wage inequality among ex-ante identical workers. Intuitively, the two-sector economy wage distribution is characterized by a discrete jump in the form of an informal sector wage gap. As part of the population receives a lower wage, not just because of the firm productivity dispersion, but also because of the wage gap, wage inequality has to be higher.

D Comparative statics on the relative productivity difference with trade

I analyze (28) in a comparative statics exercise to analyze the effect of the parameters of interest on the relative productivity distance:

\(^{30}\)See Lemma 2.2 in Anderson et al. (1993) for a detailed explanation and proof of the L’hôpital’s monotonicity rule.
\[
\frac{\varphi_x^* - \varphi_f^*}{\varphi_x^*} = 1 - \frac{f_f}{f_x} = 1 - \left( \frac{(f_f - f_i)n^{1-\sigma}}{f_x} \right)^{1/\xi} \left( (1-\lambda)^{-\xi} - (1-\delta)^{\sigma} \right)^{1/\xi} (1-\lambda)^{-1}.
\]

\[
\frac{\partial^2 \varphi_x^*}{\partial f_f^2} = - \frac{(f_f - f_i)^{1/\xi - 1} \left( n^{1-\sigma} \right)^{1/\xi}}{f_x} \left( (1-\lambda)^{-\xi} - (1-\delta)^{\sigma} \right)^{1/\xi} (1-\lambda)^{-1} < 0.
\]

\[
\frac{\partial^2 \varphi_x^*}{\partial f_f \partial f_x} = - \frac{(f_f - f_i)^{1/\xi - 1} \left( n^{1-\sigma} \right)^{1/\xi}}{f_x^2} \left( (1-\lambda)^{-\xi} - (1-\delta)^{\sigma} \right)^{1/\xi} (1-\lambda)^{-1} < 0.
\]

\[
\frac{\partial^2 \varphi_x^*}{\partial f_f \partial n} = \frac{(f_f - f_i)^{1/\xi - 1} \left( n^{1-\sigma} \right)^{1/\xi}}{f_x} \left( (1-\lambda)^{-\xi} - (1-\delta)^{\sigma} \right)^{1/\xi} (1-\lambda)^{-1} > 0.
\]

\[
\frac{\partial^2 \varphi_x^*}{\partial f_f \partial \delta} = \left( \frac{(f_f - f_i)n^{1-\sigma}}{f_x} \right)^{1/\xi} (1-\lambda)^{-1} \left( \frac{\sigma}{\xi} \right) \left( 1-\delta \right)^{\sigma-1} \left( (1-\lambda)^{-\xi} - (1-\delta) \right)^{1/\xi} < 0.
\]

\[
\frac{\partial^2 \varphi_x^*}{\partial f_f \partial \lambda} = \left( \frac{\varphi_x^*}{\varphi_f^*} \right) (1-\lambda)^{-1} \frac{1}{\left( (1-\delta)^{\sigma}/(1-\lambda)^{-\xi} \right) - 1} < 0.
\]

### E  Wage inequality with trade

I first estimate the Lorenz curve and then derive the Gini-coefficient. For this Gini-coefficient I have to distinguish three segments: informal workers, formal workers and export firm workers. Accordingly, the Lorenz curve consists of three segments. The first share of workers is employed in the informal sector:

Share in employment of firms with productivity below \(\bar{\varphi}^*\):

\[
\gamma = \frac{\int_\varphi^* f_1^\sigma_1 \, d\varphi \, g_1(\varphi) \, d\varphi}{\int_\varphi^* f_1^\sigma_1 \, d\varphi + \int_{\bar{\varphi}_1^*}^{\varphi_1^*} f_1^\sigma_1 \, d\varphi + \int_{\varphi_1^*}^{\infty} f_1^\sigma_1 \, d\varphi + \int_{\varphi_1^*}^{\infty} f_1^\sigma_1 \, d\varphi + \int_{\varphi_1^*}^{\infty} f_1^\sigma_1 \, d\varphi} = \frac{(1-\delta)^{\sigma} \left( 1 - \left( \frac{\varphi}{\varphi_1^*} \right)^\chi \right)}{\Gamma + \Psi},
\]

where \(\Gamma \equiv (1-\delta)^{\sigma} - [(1-\delta)^{\sigma} - (1-\lambda)^{-\xi + \theta}] \left( \frac{\varphi_1^*}{\varphi_1^*} \right)^\chi \) and \(\Psi = (1-\lambda)^{-\xi + \theta} n^{1-\sigma} \left( \frac{\varphi_1^*}{\varphi_1^*} \right)^\chi\).

Share in wage bill of firms with productivity below \(\bar{\varphi}^*\):
\[ Q_1 = \frac{\int_{\bar{\varphi}^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi}{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi} = \frac{(1-\delta)^\sigma \left[ 1 - \left( \frac{\varphi}{\varphi^*} \right)^{\xi-k} \right]}{\Delta + \Omega}, \]

where \( \Delta \equiv (1-\delta)^\sigma - [(1-\delta)^\sigma - (1-\lambda)^{-\xi}] \left( \frac{\varphi^*}{\varphi} \right)^{\xi-k} \) and \( \Omega = (1-\lambda)^{-\xi}n\tau^{-\sigma} \left( \frac{\varphi^*}{\varphi} \right)^{\xi-k}. \)

The first segment of the Lorenz curve is \( Q_1(\gamma) = \frac{(1-\delta)^\sigma}{\Delta + \Omega} \left( 1 - \frac{1}{\gamma} \frac{(1-\lambda)^{-\xi} \left( \frac{\varphi^*}{\varphi} \right)^{\xi-k}}{\Delta + \Omega} \right). \)

Second, including formal workers:

Share in employment of firms with productivity below \( \bar{\varphi}: \)
\[ \gamma = \frac{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi}{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)g(\varphi)d\varphi} = \frac{\Gamma - (1-\lambda)^{-\xi+\theta}\left( \frac{\varphi}{\varphi^*} \right)^{\xi-k}}{\Gamma + \Psi}. \]

Share in wage bill of firms with productivity below \( \bar{\varphi}: \)
\[ Q_f = \frac{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi}{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)g(\varphi)d\varphi} + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi = \frac{\Delta - (1-\lambda)^{-\xi}\left( \frac{\varphi^*}{\varphi} \right)^{\xi-k}}{\Delta + \Omega}. \]

The second segment of the Lorenz curve is \( Q_2(\gamma) = \frac{\Delta - (1-\lambda)^{-\xi}}{\Delta + \Omega} \left( 1 - \frac{1}{\gamma} \frac{(1-\lambda)^{-\xi+\theta}}{\Delta + \Omega} \right)^{\xi-k}. \)

Third, including workers employed in exporting firms:

Share in employment of firms with productivity below \( \bar{\varphi}: \)
\[ \gamma = \frac{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi}{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)g(\varphi)d\varphi} = 1 - \frac{1}{\Gamma + \Psi} \left( \frac{\varphi}{\varphi^*} \right)^{\xi-k}. \]

Share in wage bill of firms with productivity below \( \bar{\varphi}: \)
\[ Q_3 = \frac{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi}{\int_{\varphi^*}^{\varphi^*} l_i(\varphi)w_i(\varphi)g(\varphi)d\varphi + \int_{\varphi^*}^{\varphi^*} l_f(\varphi)w_f(\varphi)g(\varphi)d\varphi} + \int_{\varphi^*}^{\varphi^*} l_x(\varphi)w_f(\varphi)g(\varphi)d\varphi = \frac{1 - \frac{(1-\lambda)^{-\xi}}{\Delta + \Omega}}{\Delta + \Omega} \left( \frac{\varphi^*}{\varphi} \right)^{\xi-k}. \]

The last segment of the Lorenz curve is \( Q_3(\gamma) = 1 - \frac{(1-\lambda)^{-\xi}}{\Delta + \Omega} \left( \frac{1-\lambda}{\Delta + \Omega} \right)^{\xi-k}. \)

Jointly, the Lorenz curve is described by \( Q(\gamma) = \begin{cases} Q_1(\gamma) & \text{if } \gamma \in [0, b_i] \\ Q_2(\gamma) & \text{if } \gamma \in [b_i, b_f] \\ Q_3(\gamma) & \text{if } \gamma \in [b_f, 1] \end{cases}. \)
where $b_i = \frac{(1-\delta)^\gamma \left[1 - \left(\frac{\varphi i}{\chi} \right)^\chi\right]}{\Gamma + \Psi}$ is the share of workers employed in the informal sector and $b_f = \frac{\Gamma - (1-\lambda)^{-\xi+\alpha} \left(\frac{\varphi i}{\chi} \right)}{\Gamma + \Psi}$ is the share of workers employed in non-exporting firms.

The Lorenz curve $Q(\gamma)$ has the desired properties $Q_1(b_1) = Q_2(b_1)$, $Q_2(b_2) = Q_3(b_2)$, $Q(0) = 0$, $Q(1) = 1$ and $\frac{\partial Q(\gamma)}{\partial \gamma} > 0$.

The Gini-coefficient follows from $G = 1 - 2 \int_0^1 Q(\gamma) d\gamma$:

$$G = G_f \left[ 1 + \frac{2\phi^{k-\xi}}{\theta (\Gamma + \Psi) (\Delta + \Omega)} \left\{ \chi \left[ \Upsilon - \Xi \phi^\theta + [\Xi - \Upsilon] \phi^{-\chi} + (1-\lambda)^{-2\xi+\theta} n \tau^{1-\sigma} \left[ \mu^{-\chi} \phi^{k-\xi} - \mu^k \phi^{-\chi} \right] + \mu^k \phi^{-\chi} (1-\lambda)^{-\xi} n \tau^{1-\sigma} \left[ \phi^{-\chi} - 1 \right] + \mu^{-\chi} (1-\lambda)^{-\xi+\theta} n \tau^{1-\sigma} \left[ 1 - \phi^{k-\xi} \right] \right] + \theta \left[ \Upsilon \left[ 1 - \phi^{-\chi} \right] + (1-\lambda)^{-2\xi+\theta} n \tau^{1-\sigma} \mu^{k-\xi} \phi^{k-\xi} \right] \right\} \right],$$

where $\mu \equiv \frac{\varphi}{\varphi_x}$.

F Numerical simulation for the key variables

Because of the complexity of the equations, I show that the aggregate output of the informal sector, formal sector and entire economy, and the formal and total employment can increase or decrease upon trade liberalization. To do this I compute the numerical value of the key variables in autarky and upon trade liberalization for three scenarios with different economy parameters within the assumptions of the model, as summarized in table 1. I then calculate the ratio of the trade variable relative to the respective autarky variable and show that the ratio can be greater or less than 1.
Table 1: Economy parameters for two scenarios.

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total workforce</td>
<td>$L$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\sigma$</td>
<td>3.8</td>
<td>2.3</td>
</tr>
<tr>
<td>Pareto distribution parameter</td>
<td>$k$</td>
<td>3.4</td>
<td>3.5</td>
</tr>
<tr>
<td>Formal fixed cost</td>
<td>$f_f$</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Informal fixed cost</td>
<td>$f_i$</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Exporting fixed cost</td>
<td>$f_x$</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Fairness parameter</td>
<td>$\theta$</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Productivity bonus</td>
<td>$\lambda$</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Iceberg transportation cost</td>
<td>$\tau$</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Number of foreign countries</td>
<td>$n$</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Probability of detection</td>
<td>$\delta$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The parameters specifications in Table 1 are within the following constraints of the various Propositions and assumptions of the model:

- $[1 - \delta]^\sigma < \frac{1}{1 - \lambda} < \left[\frac{f_f}{f_i}(1 - \delta)^\sigma\right]^\frac{1}{\xi}$
- $\frac{k + 1 - \theta}{1 - \theta} > \sigma > \frac{1}{1 - \theta}$
- $\frac{f_x r^{\frac{1 - \sigma}{\xi}}}{n(1 - \lambda)^{-\xi}} > \frac{f_f - f_i}{(1 - \lambda)^{-\xi}(1 - \delta)^\sigma}$

The equations for the key variables are as follows:
\[
\frac{E(t)}{E(a)} = \left[ 1 + n \left( \frac{f_x}{f_i n \tau^{-1} - \sigma} \right)^{\frac{\theta}{\xi}} (1 - \lambda)^{-k} (1 - \delta)^{\frac{-k \sigma}{\xi}} \right]^{-\frac{\theta}{\xi}} \\
1 + \frac{(1 - \lambda)^{-k} (n \tau^{-1} - \sigma) \frac{k}{\xi} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} (1 - \delta)^{\frac{\sigma(\xi - k)}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\alpha} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma}}^\frac{\theta}{\xi}. \\
\]

\[
\frac{E_f(t)}{E_f(a)} = \left[ 1 + n \left( \frac{f_x}{f_i n \tau^{-1} - \sigma} \right)^{\frac{\theta}{\xi}} (1 - \lambda)^{-k} (1 - \delta)^{\frac{-k \sigma}{\xi}} \right]^{-\frac{\theta}{\xi}} \\
1 + \frac{(1 - \lambda)^{-k} (n \tau^{-1} - \sigma) \frac{k}{\xi} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} (1 - \delta)^{\frac{\sigma(\xi - k)}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\alpha} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma}}^\frac{\theta}{\xi}. \\
\]

\[
\frac{E_i(t)}{E_i(a)} = \left[ 1 + n \left( \frac{f_x}{f_i n \tau^{-1} - \sigma} \right)^{\frac{\theta}{\xi}} (1 - \lambda)^{-k} (1 - \delta)^{\frac{-k \sigma}{\xi}} \right]^{-\frac{\theta}{\xi}} \\
1 + \frac{(1 - \lambda)^{-k} (n \tau^{-1} - \sigma) \frac{k}{\xi} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} (1 - \delta)^{\frac{\sigma(\xi - k)}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\alpha} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma} \left( \frac{f_x}{f_i} \right)^{\frac{k - \xi}{\tau}} + (1 - \delta)^{\sigma}}^\frac{\theta}{\xi}. \\
\]
\[
\frac{(E_i\bar{w}_i + E_f\bar{w}_f)(t)}{(E_i\bar{w}_i + E_f\bar{w}_f)(o)} = \left[1 + n \left(\frac{f_x}{f_i n \tau^{1-\sigma}}\right)^{\frac{\xi}{\xi - \sigma}} (1 - \lambda)^{-k} (1 - \delta)^{-\frac{k \sigma}{\xi}}\right]^{-\frac{k}{\xi (k+1)}} \left[1 + \frac{(1 - \lambda)^{-k} (n \tau^{1-\sigma})^{\frac{k}{\xi}} \left(\frac{f_x}{f_i}\right)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}} + (1 - \delta)^{\sigma}}\right]^{\frac{k \xi}{\xi (k+1)}},
\]

where \(\frac{Y_{(t)}}{Y_{(o)}} = \frac{(E_i\bar{w}_i + E_f\bar{w}_f)(t)}{(E_i\bar{w}_i + E_f\bar{w}_f)(o)}\).

\[
\frac{(E_f\bar{w}_f)(t)}{(E_f\bar{w}_f)(o)} = \left[1 + n \left(\frac{f_x}{f_i n \tau^{1-\sigma}}\right)^{\frac{\xi}{\xi - \sigma}} (1 - \lambda)^{-k} (1 - \delta)^{-\frac{k \sigma}{\xi}}\right]^{-\frac{k}{\xi (k+1)}} \left[1 + \frac{(1 - \lambda)^{-k} (n \tau^{1-\sigma})^{\frac{k}{\xi}} \left(\frac{f_x}{f_i}\right)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}} + (1 - \delta)^{\sigma}}\right]^{\frac{k \xi}{\xi (k+1)}},
\]

where \(Y_f = \frac{E_f\bar{w}_f L}{\rho}\) and \(\frac{Y_{(t)}}{Y_{(o)}} = \frac{(E_f\bar{w}_f)(t)}{(E_f\bar{w}_f)(o)}\).

\[
\frac{(E_i\bar{w}_i)(t)}{(E_i\bar{w}_i)(o)} = \left[1 + n \left(\frac{f_x}{f_i n \tau^{1-\sigma}}\right)^{\frac{\xi}{\xi - \sigma}} (1 - \lambda)^{-k} (1 - \delta)^{-\frac{k \sigma}{\xi}}\right]^{-\frac{k}{\xi (k+1)}} \left[1 + \frac{(1 - \lambda)^{-k} (n \tau^{1-\sigma})^{\frac{k}{\xi}} \left(\frac{f_x}{f_i}\right)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}}}{(1 - \lambda)^{-\xi} (1 - \delta)^{\frac{k \xi}{\xi - \sigma}} (1 - \delta)^{-\frac{k \sigma}{\xi}} + (1 - \delta)^{\sigma}}\right]^{\frac{k \xi}{\xi (k+1)}},
\]

where \(Y_i = \frac{E_i\bar{w}_i L}{\rho}\) and \(\frac{Y_{(t)}}{Y_{(o)}} = \frac{(E_i\bar{w}_i)(t)}{(E_i\bar{w}_i)(o)}\).
\[ G_{(a)} = G_f \left[ 1 + 2 \frac{\phi_i^{\xi}}{\phi^f_a} \right] \left\{ \chi \left[ \Theta - \Xi \left( \frac{\phi_i}{\phi^f} \right)^\theta + \Theta - \Theta \right] \left( \frac{\phi_i}{\phi^f} \right)^{-\chi} \right\} + \theta \Theta \left[ 1 - \left( \frac{\phi_i}{\phi^f} \right)^{-\chi} \right]. \]

\[
G_{(t)} = G_f \left[ 1 + \frac{2\phi^{k-\xi}}{\Theta(\Gamma + \Psi)(\Delta + \Omega)} \left\{ \chi \left[ \Theta - \Xi \phi^\theta + \Theta - \Theta \right] \phi^{-\chi} + (1 - \lambda)^{-\xi + \theta n \tau - \sigma} \mu^k \phi^{k-\xi} - \mu^{k-\xi} \phi^{-\chi} \right\} + \mu^k \phi^k (1 - \delta)^\sigma (1 - \lambda)^{-\xi + \theta n \tau - \sigma} \left[ \phi^{-\chi} - 1 \right] + \mu^\chi (1 - \delta)\sigma (1 - \lambda)^{-\xi + \theta n \tau - \sigma} \left[ 1 - \phi^{k-\xi} \right] \right\} + \Theta \left[ 1 - \phi^{-\chi} \right] + (1 - \lambda)^{-\xi + \theta n \tau - \sigma} \mu^k \phi^{k-\xi} \left[ 1 - \phi^{k-\xi} \right]. \]

The results are summarized in table 2. The three scenarios highlight the different impact that trade liberalization has, given the various economic parameters. In scenario 1, total employment, formal employment and aggregate formal output increase, while informal employment, aggregate informal output, total aggregate output and the Gini-coefficient decrease. In scenario 2, formal employment, aggregate informal output, aggregate formal output, total aggregate output and the Gini-coefficient increase, while informal and total employment decrease. Lastly, scenario 3 leads to yet another outcome upon trade liberalization. Employment and aggregate output of all sectors and the entire economy decrease, but the Gini-coefficient increases. Hence, the effect of trade liberalization on all variables, except for informal employment, is ambiguous and depends on the parameters of the economy.

**Table 2: Numerical results of the key variables with trade liberalization relative to autarky.**

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{(t)}/E_{(a)})</td>
<td>1.11241</td>
<td>0.99977</td>
<td>0.99346</td>
</tr>
<tr>
<td>(E_{f(t)}/E_{f(a)})</td>
<td>1.25219</td>
<td>1.09506</td>
<td>0.98681</td>
</tr>
<tr>
<td>(E_{i(t)}/E_{i(a)})</td>
<td>0.67790</td>
<td>0.99947</td>
<td>0.89984</td>
</tr>
<tr>
<td>(Y_{(t)}/Y_{(a)})</td>
<td>0.66809</td>
<td>1.00097</td>
<td>0.99051</td>
</tr>
<tr>
<td>(Y_{f(t)}/Y_{f(a)})</td>
<td>1.45032</td>
<td>4.41278</td>
<td>0.99576</td>
</tr>
<tr>
<td>(Y_{i(t)}/Y_{i(a)})</td>
<td>0.59766</td>
<td>1.00005</td>
<td>0.89783</td>
</tr>
<tr>
<td>(G_{(t)}/G_{(a)})</td>
<td>0.91485</td>
<td>1.00796</td>
<td>1.00033</td>
</tr>
</tbody>
</table>
References


<table>
<thead>
<tr>
<th>WP No</th>
<th>Title</th>
<th>Author(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014-12</td>
<td>Minimum Wage Effects at Different Enforcement Levels: Evidence from Employment Surveys in India</td>
<td>Soundararajan.V.</td>
</tr>
<tr>
<td>2014-10</td>
<td>On the Political Economy of Guest Workers Programs in Agriculture</td>
<td>Richard, R</td>
</tr>
<tr>
<td>2014-09</td>
<td>Groupings and the Gains from Tagging</td>
<td>Kanbur, R. and M. Tuomala</td>
</tr>
<tr>
<td>2014-07</td>
<td>The Economics of China: Successes and Challenges</td>
<td>Fan, S., Kanbur, R., Wei, S. and X. Zhang</td>
</tr>
<tr>
<td>2014-06</td>
<td>Mindsets, Trends and the Informal Economy</td>
<td>Kanbur, R.</td>
</tr>
<tr>
<td>2014-05</td>
<td>Regulation and Non-Compliance: Magnitudes and Patterns for India’s Factories Act</td>
<td>Chatterjee, U. and R. Kanbur</td>
</tr>
<tr>
<td>2014-04</td>
<td>Urbanization and Agglomeration Benefits: Gender Differentiated Impacts on Enterprise Creation In India’s Information Sector</td>
<td>Ghani, E., Kanbur, R. and S. O’Connell</td>
</tr>
<tr>
<td>2014-03</td>
<td>Globalization and Inequality</td>
<td>Kanbur, R.</td>
</tr>
<tr>
<td>2014-02</td>
<td>Should Mineral Revenues be Used for Countercyclical Macroeconomic Policy in Kazakhstan?</td>
<td>Kyle, S.</td>
</tr>
<tr>
<td>2014-01</td>
<td>Performance of Thailand Banks after the 1997 East Asian Financial Crisis</td>
<td>Mahathanaseth, I. and L. Tauer</td>
</tr>
<tr>
<td>2013-20</td>
<td>What is a &quot;Meal&quot;? Comparing Methods to Determine Cooking Events</td>
<td>Harrell, S., Beltramo, T., Levine, D., Dlalock, G. and A. Simons</td>
</tr>
<tr>
<td>2013-19</td>
<td>University Licensing of Patents for Varietal Innovations in Agriculture</td>
<td>Rickard, B., Richards, T. and J. Yan</td>
</tr>
</tbody>
</table>