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THE OPTIMAL MINIMUM WAGE FOR POVERTY MINIMIZATION

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Contents

Abstract

The effects of a minimum wage on employment and on poverty have been studied in the literature. This paper characterizes the poverty minimizing minimum wage, and shows how it depends on productivity, inequality and the degree of labor market competitiveness.

Keywords: inequality, labor productivity, market competitiveness, minimum wage, poverty.

JEL Classification: D6, I32, J38, J64.

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1. Introduction

There is a large literature on the impact of minimum wages on employment (for a recent survey, see Neumark and Wascher 2007). More recently, investigation has begun on the impact of a minimum wage on poverty (for example, Fields and Kanbur 2007 for competitive labor markets, and Basu, Chau, and Kanbur 2005 for non-competitive labor markets). The objective of this paper is to derive the poverty minimizing minimum wage, and to show how it depends on productivity, inequality and the degree of labor market competitiveness.

The plan of the paper is as follows. Section 2 presents the basic model. Section 3 derives the poverty minimizing minimum wage. Section 4 looks at the partial and the cross effects of competitiveness and productivity on the optimal minimum wage. Section 5 does a similar exercise for inequality and the minimum wage. Section 6 concludes.

2. The Model

We will develop a specialized, tractable model that allows us to address the questions posed. Let us suppose that individuals are distributed uniformly along a line segment on the x-axis, $[m-k, m+k]$, as depicted in Figure 1. Firms are located at $x=0$. So m is the average distance, or lack of access, to the firms and $k(\leq m)$ is a parameter describing the extent of inequality in terms of access to the labor market. Without loss of generality, population size is normalized to unity. Thus the density function of the distribution of individuals is given by $f(x)=1/2k$. If the firms offer some wage rate w, and if an individual at $x \in [m-k, m+k]$ works for a firm, her net income is given by $y(w,x)=w-tx$. The parameter $t(\geq 0)$ could be interpreted simply as the cost of mobility or, more generally, as transaction costs that are associated with finding and working for a firm. We assume that individuals have no earnings opportunity outside the economy. Hence given w, the individuals in $[m-k, w/t]$ work for the firms, while the individuals in $(w/t, m+k]$ do not.[3] Thus the labor supply function and the inverse labor supply function are respectively given by

[3]In this paper, we focus on the case in which labor productivity is not high enough to ensure full employment.

$$S(w)=\frac{1}{2k}[\frac{w}{t}-(m-k)] \quad (1)$$

and

$$w=2ktl+(m-k)t \quad (2)$$

For the demand side of the labor market, let us suppose that there exist n firms at $x=0$, where n is treated as a parameter to be varied. All firms have the same revenue function, $R(l_i,a)=al_i-bl_i^2/2$, where l_i denotes the number of workers employed, and $a>0$ and $b>0$ are technological parameters describing labor productivity and diminishing marginal product, respectively. In what follows, productivity growth is captured by increases in a. Given the revenue function and a wage rate w, the firm's profit function is given by $\pi(l_i)=R(l_i,a)-wl_i$. Each firm maximizes profit given the labor supply and the other firms' labor demand. Since the firms' technology is identical, we restrict ourselves to symmetric Nash equilibria in terms of employment.

The equilibrium employment and wage are calculated as follows. Given (2) and the other firms' labor demand, l_{-i}, firm i's profit function is of the form

$$\pi(l_i;l_{-i},a,m,k,t)=al_i-\frac{b}{2}l_i^2-[2kt(l_i+l_{-i})+(m-k)t]l_i$$

By differentiating π with respect to l_i, and then substituting $(n-1)l_i$ for l_{-i}, the equilibrium labor demand of each firm when there exist n firms in the market is given by

$$l_i^*=\frac{a-(m-k)t}{b+2kt(n+1)} \quad (3)$$

Thus the equilibrium (total) employment and wage are, respectively,

$$l^*=nl_i^*=\frac{n[a-(m-k)t]}{b+2kt(n+1)} \quad (4)$$

and

$$w^*=\frac{t[2akn+(b+2kt)(m-k)]}{b+2kt(n+1)} \quad (5)$$

So we suppose that $w/t<m+k$ always holds.

Note that letting $n \to \infty$ offers the competitive employment and wage:[4]

$$l_c = \frac{a-(m-k)t}{2kt}$$

$$w_c = a$$

Throughout our analysis, poverty is measured using the poverty measure which has been developed by Foster, Greer and Thorbecke (1984):

$$P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^\alpha g(y)dy,$$

where z is the (fixed) poverty line and g is the density function of income distribution. α is a parameter, increases in which make the measure more sensitive to the gaps between the poverty line and income levels below it. We consider $\alpha = 0$ and $\alpha \geq 1$.
By changing the variables, the poverty measure is also expressed as

$$P_\alpha = \int_{\frac{w-z}{t}}^{\frac{w}{t}} \left[\frac{z-(w-tx)}{z}\right]^\alpha f(x)dx + \int_{\frac{w}{t}}^{m+k} f(x)dx$$

if $y(w,m-k) > z$, and

$$P_\alpha = \int_{m-k}^{\frac{w}{t}} \left[\frac{z-(w-tx)}{z}\right]^\alpha f(x)dx + \int_{\frac{w}{t}}^{m+k} f(x)dx$$

if $y(w,m-k) \leq z$. In the case of a uniform distribution, $f(x) = 1/2k$ for all $x \in [m-k, m+k]$, so the above expressions are simplified as:

$$P_\alpha = \begin{cases} \dfrac{1}{2k}\left[\dfrac{z}{(1+\alpha)t} + (m+k) - \dfrac{w}{t}\right] & \text{if} \quad y(w,m-k) > z (6) \\ \dfrac{1}{2k}\left\{\dfrac{z}{(1+\alpha)t}\left[1-\left(\dfrac{z-[w-t(m-k)]}{z}\right)^{1+\alpha}\right] + (m+k) - \dfrac{w}{t}\right\} & \text{if} \quad 0 < y(w,m-k) \leq z (7) \end{cases} \qquad (8)$$

In this paper, we consider the case where the richest individuals in the economy are not poor: $y(w,m-k) > z$. In this case, by (2) and (8), the poverty measure is further simplified as follows:

[4]The numerator of l_c is the income of the richest individual when the market is competitive.

$$P_\alpha = \frac{z}{2kt(1+\alpha)} + (1-l^*) \qquad (9)$$

The above expression tells us that, given k and t, the poverty measure solely depends on the amount of employment.

3. The Optimal Minimum Wage

Let us restrict attention to minimum wages higher than the market wage. If a minimum wage $\overline{w}$ is set, in any symmetric Nash equilibria, the marginal cost of labor becomes $\overline{w}$ up to

$$\frac{S(\overline{w})}{n} = \frac{1}{2kn}[\frac{\overline{w}}{t} - (m-k)],$$

and $2kt(n+1)l_i + (m-k)t$ for $l_i \geq S(\overline{w})/n$:[5]

$$MC_i(l_i) = \begin{cases} \overline{w} & \text{for} \quad 0 \leq l_i \leq S(\overline{w})/n (10) \\ 2kt(n+1)l_i + (m-k)t & \text{for} \quad l_i \geq S(\overline{w})/n (11) \end{cases} \qquad (12)$$

The amount of labor at which the original marginal cost is $\overline{w}$ is always strictly less than $S(\overline{w})/n$. So given any $\overline{w}$, the amount of labor a firm could hire at the minimum wage always exceeds the amount of labor at which the original marginal cost is equal to the minimum wage. Thus the new marginal cost curve jumps at $l_i = S(\overline{w})/n$ (Figure 2).

So we can restrict our attention to the following three cases: (i) every firm hires the exact amount of labor available at the minimum wage, (ii) part of the labor supplied at the minimum wage is hired, and (iii) every firm hires more labor than available at the minimum wage (though this case never happens as is shown below).

First it can be easily seen that case (ii) occurs if and only if the marginal revenue at $S(\overline{w})/n$ is less than $\overline{w}$, in which case

$$\overline{w} > \frac{[2akn + b(m-k)]t}{b + 2knt} \equiv \tilde{w}$$

[5]Without government intervention, the marginal revenue and marginal cost functions in symmetric equilibria are, respectively, $MR_i(l_i) = a - bl_i$ and $MC_i(l_i) = 2kt(n+1)l_i + (m-k)t$ for all $l_i \geq 0$.

holds. Each firm hires $(a-\overline{w})/b$ so the total employment is $(a-\overline{w})n/b$.

Next, case (i) occurs if the marginal revenue at $l_i = S(\overline{w})/n$ is greater than $\overline{w}$ but the marginal cost at $S(\overline{w})/n$ is (weakly) greater than $\overline{w}$. After some calculations, we find that this case occurs iff

$$w^* < \overline{w} \le \tilde{w},$$

where w^* is the equilibrium wage without government intervention. Since we consider minimum wages beyond w^*, the first inequality always holds. Therefore case (iii) never occurs. The total employment in case (i) is $S(\overline{w})$. It should be noted that the marginal revenue at $l_i = S(\tilde{w})/n$ is $\tilde{w}$ (Figure 3).

(i) $w^* < \overline{w} \le \tilde{w}$

Let us start with case (i). In this case, if $\overline{w}$ is set, all the labor supplied at the wage rate is hired. By (8), the degree of poverty is given by

$$P_\alpha(\overline{w}) = \frac{1}{2k}\left[\frac{z}{(1+\alpha)t} + (m+k) - \frac{\overline{w}}{t}\right], \qquad (13)$$

which is clearly decreasing in $\overline{w}$. Thus the higher the minimum wage, the lower the degree of poverty. Therefore poverty is minimized at $\overline{w} = \tilde{w}$.

(ii) $\overline{w} > \tilde{w}$

Next, in case (ii), since each firm hires

$$l_i(\overline{w}) = \frac{a-\overline{w}}{b} < \frac{S(\overline{w})}{n}$$

total employment is $(a-\overline{w})n/b$. Let us assume that the individuals in $[m-k, \overline{x}]$ are hired, where

$$\overline{x} = \frac{2kn(a-\overline{w})}{b} + (m-k)$$

Then the value of the poverty measure is given by

$$P_\alpha(\overline{w}) = \frac{z}{2kt(1+\alpha)}\left[\frac{z-(\overline{w}-t\overline{x})}{z}\right]^{1+\alpha} + \frac{1}{2k}[(m+k)-\overline{x}] \qquad (14)$$

Differentiating (14) with respect to $\overline{w}$ gives

$$\frac{\partial P_\alpha(\overline{w})}{\partial \overline{w}} = \frac{n}{b} - \frac{b+2knt}{2bkt}\left[\frac{z-(\overline{w}-t\overline{x})}{z}\right]^{\alpha} \qquad (15)$$

Note that (13) and (14) take the same value at $\overline{w} = \tilde{w}$. So the poverty measure is continuous for all $\overline{w} > w^*$. Because the value of (15) evaluated at $\overline{w} = \tilde{w}$ is

$$\frac{\partial P_\alpha}{\partial \overline{w}} |_{\overline{w}=\tilde{w}} = -\frac{1}{2kt} < 0$$

and because the second derivative of (14) with respect to $\overline{w}$ is positive, the poverty minimizing minimum wage is higher than $\tilde{w}$. In fact, it is high enough for involuntary unemployment to occur. The intuition behind this is as follows. First, for minimum wages in the range of (i), both employment and wages increase as the minimum wage increases. After a critical value, namely $\tilde{w}$, as the minimum wage increases, the associated level of employment begins to fall. While the minimum wage is near $\tilde{w}$, however, the effect of higher minimum wages dominates that of lower employment.

The optimal minimum wage for poverty reduction can be obtained by equating (15) to zero:

$$\overline{w}^*(n,k) = \tilde{w} + \frac{bz}{b+2knt}[1 - (\frac{2knt}{b+2knt})^{\frac{1}{\alpha}}], \tag{16}$$

where the second term of the right side is positive. The associated poverty level is given by

$$P_\alpha(\overline{w}^*(n,k)) = \frac{b + n[(z-a)+(m+k)t]}{b+2knt} - \frac{\alpha z}{2kt(1+\alpha)}(\frac{2knt}{b+2knt})^{\frac{1+\alpha}{\alpha}} \tag{17}$$

In the case of a constant marginal value product of labor $(b = 0)$, the optimal minimum wage is equal to the competitive wage for any a, n, m and k. Otherwise, it depends on those variables.

Let $\Delta_{\overline{w}}$ denote the difference between the optimal minimum wage and the market wage:

$$\Delta_{\overline{w}} \equiv \overline{w}^*(n,k) - w(n,k) = \frac{4k^2nt^2[a-(m-k)t]}{(b+2knt)[b+2kt(n+1)]} + \frac{bz}{b+2knt}[1 - (\frac{2knt}{b+2knt})^{\frac{1}{\alpha}}]$$

First, note that

$$\lim_{n\to\infty} \Delta_{\overline{w}} = 0$$

holds, which means that the optimal minimum wage converges to the competitive wage as the number of firms approaches infinity. That in turn means that there is no need for government intervention in competitive labor markets.

Second, we have

$$\frac{\partial \Delta_{\bar{w}}}{\partial a} = \frac{4k^2nt^2}{(b+2knt)[b+2kt(n+1)]} > 0$$

So the difference between the poverty minimizing minimum wage and the prevailing market wage is greater the higher is labor productivity. Thus there is a greater need for government intervention the higher is labor productivity.

Third, as m increases, $\Delta_{\bar{w}}$ decreases:

$$\frac{\partial \Delta_{\bar{w}}}{\partial m} < 0$$

Therefore as individuals' access to the market decreases, the gap between the optimal minimum wage and the market wage becomes smaller.

4. The Effects of Competitiveness and Productivity

Let us begin with the effect of an increase in competitiveness on the optimal minimum wage. By differentiating (16) with respect to n, we have

$$\frac{\partial \bar{w}^*}{\partial n} = \frac{2bkt[a-(m-k)t]}{(b+2knt)^2} - \frac{2bktz}{(b+2knt)^2}[1-(\frac{2knt}{b+2knt})^{\frac{1}{\alpha}}] - \frac{2b^2ktz}{\alpha(b+2knt)^3}(\frac{2knt}{b+2knt})^{\frac{1-\alpha}{\alpha}}$$

The first term of the right side, which is the derivative of $\tilde{w}$ with respect to n, is positive. This is clearly seen by Figure 3. With $\tilde{w}$ held constant, $S(\tilde{w})/n$ decreases as n increases. So $\tilde{w}(n)$ must increase as n increases. Next, (the sum of) the second and third terms, which is the derivative of the second term in the right side of (16), are negative. It should be remembered that as long as the minimum wage is in the range of $[\tilde{w}, \bar{w}^*]$, the positive effect on poverty of increases in the minimum wage is greater than the negative effect of decreases in employment. Note that, as we have already seen, the total employment is given by $(a-\bar{w})n/b$, the derivative of which with respect to $\bar{w}$ decreases as n increases. In other words, for greater n, employment decreases more rapidly as $\bar{w}$ increases. So the optimum minimum wage becomes closer to $\tilde{w}$, which means that the derivative of the second term in the right side of (16) with respect to n is negative. Whether or not the optimal minimum wage increases as a result of an increase in n is determined by the tradeoff between those two effects. For example, in the case of $\alpha = 1$, the optimal minimum wage falls as the degree of competitiveness rises if

$$n < \frac{b\{2z - [a - (m-k)t]\}}{2kt[a - (m-k)t]}$$

In contrast, the effect of an increase in productivity on the optimal minimum wage is determinate:

$$\frac{\partial \overline{w}^*}{\partial a} > 0$$

The above inequality holds because $\tilde{w}$ always increases with increases in a, and because the marginal decrease in employment with an increase in the minimum wage (in the range of $\overline{w} \geq \tilde{w}$) is unchanged.

In addition, we have

$$\frac{\partial^2 \overline{w}^*}{\partial a \partial n} > 0$$

We have seen that the optimal minimum wage does not necessarily increase with an increase in n. However, the marginal increase in the optimal minimum wage with an increase in the number of firms always increases as productivity increases. This is because $\partial \tilde{w} / \partial n$ increases with an increase in productivity while $\partial(\overline{w}^* - \tilde{w}) / \partial n$ does not depend on labor productivity.

As for poverty, increases in productivity and competitiveness help decrease the minimized poverty:

$$\frac{\partial P_\alpha(\overline{w}^*)}{\partial a} = -\frac{n}{b + 2knt} < 0$$

$$\frac{\partial P_\alpha(\overline{w}^*)}{\partial n} = -\frac{b}{(b+2knt)^2}\{[a - (m-k)t] - z + z(\frac{2knt}{b+2knt})^{\frac{1}{\alpha}}\} < 0$$

The latter inequality holds because, by assumption, the richest in the economy (in the competitive market) is not poor: $a - (m-k)t > z$. The above results are intuitive because poverty decreases with increases in these variables without government intervention.

Besides,

$$\frac{\partial^2 P_\alpha(\overline{w}^*)}{\partial a \partial n} = -\frac{b}{(b+2knt)^2} < 0$$

also holds. Therefore the higher market competitiveness (labor productivity), the greater the marginal decrease in poverty with an increase in productivity (competitiveness). As we have already seen, $\partial\tilde{w}/\partial n$ is greater for greater a but $\partial(\overline{w}^* - \tilde{w})/\partial n$ does not depend on a. Since employment increases as $\tilde{w}$ increases, and decreases as the minimum wage increases in $\overline{w} \in [\tilde{w}, \overline{w}^*]$, we have the result.

5. The Effects of Inequality

Finally, let us explore how the optimal minimum wage and the minimized poverty change as the degree of inequality changes.

The derivative of the optimal minimum wage with respect to k is

$$\frac{\partial \overline{w}^*}{\partial k} = \frac{b\{2nt[(a-mt)-z]-bt\}}{(b+2knt)^2} - \frac{b^2 z}{\alpha k(b+2knt)^2}\left(\frac{2knt}{b+2knt}\right)^{\frac{1}{\alpha}}$$

For example, if the income of the individuals at the average distance from the firm in the competitive labor market, $a-mt$, is less than the poverty line, the optimal minimum wage is lower the greater is the degree of inequality.

Next, for the sake of simplicity, suppose that the revenue function is linear in terms of employment ($b=0$). Then the derivative of the poverty measure with respect to k is given by

$$\frac{\partial P_\alpha(\overline{w}^*)}{\partial k} = \frac{1}{2k^2 t}\left[(a-mt) - \frac{z}{1+\alpha}\right],$$

where, again, $a-mt$ is the competitive income of the individuals at the average distance from the firms. If m satisfies

$$m > \frac{a}{t} - \frac{z}{(1+\alpha)t} \quad (>0),$$

that is, if the extent of poverty is greater than certain degree at the beginning, the minimized poverty is lower the higher is inequality. Otherwise, the level of the minimized poverty is higher the greater is the degree of inequality.

6. Conclusion

Most of the theoretical literature has conducted comparative statics exercises on the impact of minimum wages on employment. This paper derives the poverty minimizing wage and conducts comparative statics on that optimal wage with respect to labor market competitiveness, productivity and inequality. It is shown that under certain conditions the optimal minimum wage falls with the degree of competitiveness, rises with productivity, and falls with inequality. Moreover, it is shown that the optimal minimum wage rises more with productivity the greater is the degree of competitiveness. These comparative static results can guide us in understanding the pattern of minimum wage across societies with different degrees of competitiveness, productivity and inequality.

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Figure 1. "Distance" to the Firms and Individual Incomes

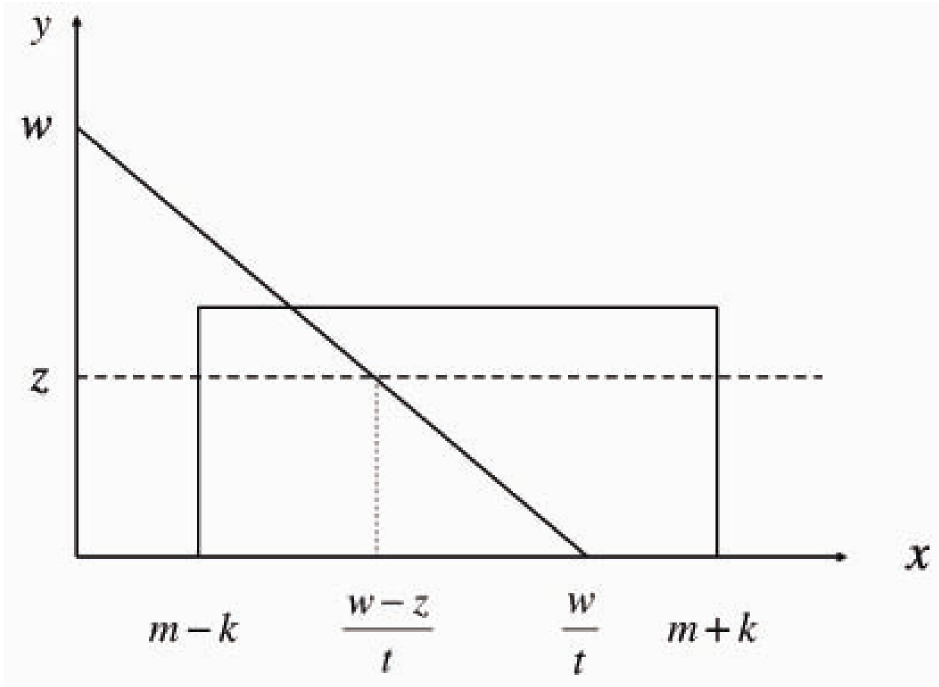


Figure 2. Minimum Wages and the Marginal Cost of Labor

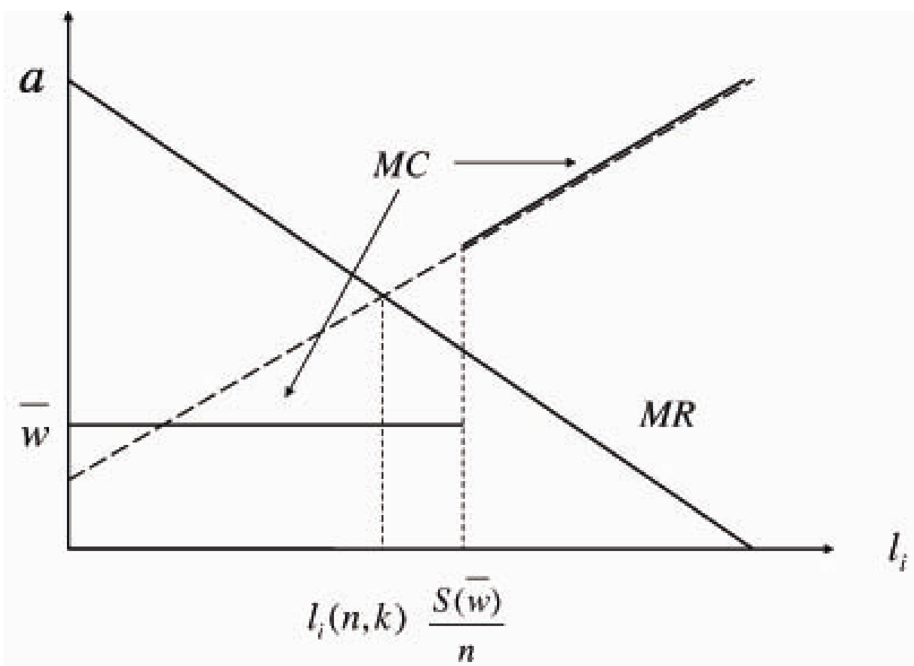
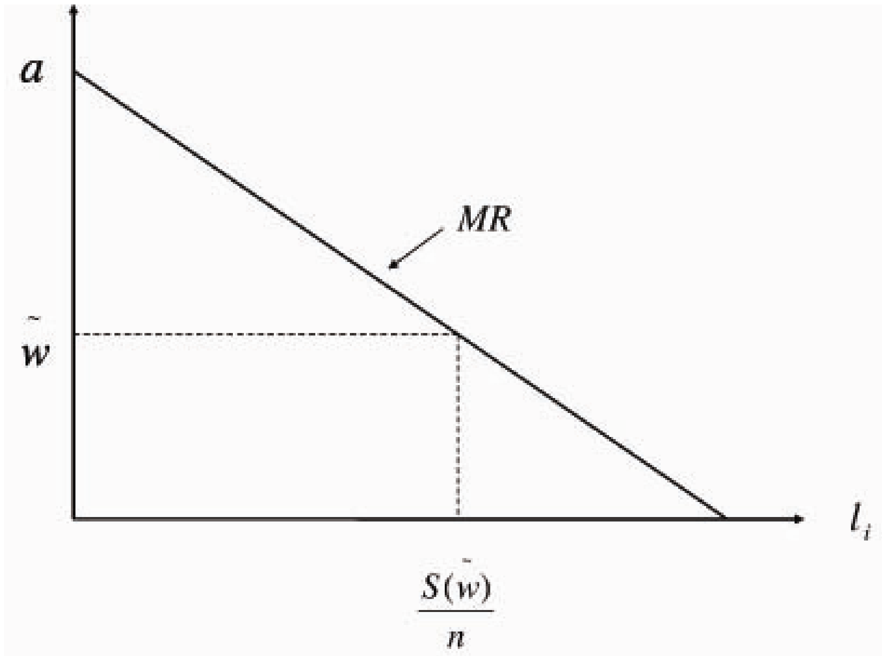


Figure 6.3. $\tilde{w}$



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