Trade Restrictiveness and Pollution

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Abstract: This paper proposes a trade restrictiveness indicator that explicitly incorporates environmental externalities. The index employs directional distance functions and use indicators (i.e. differences rather than ratios) modified to account for and evaluate efficiency changes in the face of simultaneous and multi-dimensional trade and environmental policy reforms. The index is made up of two components, one for production and one for consumption. Our overall trade restrictiveness indicator is accordingly the difference of the two. The properties of the indicator are developed and discussed together with its estimation.

JEL Classification: F 18.
1 Introduction

The emergence of trade liberalization as an environmental issue has given rise to heated policy debates and a body of research that addresses a range of concerns: What are the environmental quality consequences of trade reforms (Antweiler, Copeland and Taylor (2001), Dean (2002), Copeland and Taylor (2003))? What are the trade and welfare implications of environmental policy reforms in a setting where distortionary trade policies are also in the picture (Copeland and Taylor (1994), Copeland (1994), Basu, Chau and Grote (2006), Chau and Kanbur (2006))? What are the welfare implications of trade policy reforms in the context of a second-best world characterized by multiple environmental bads and a corresponding set of environmental policies (Baumol (1971), Copeland (1994))? Almost concurrently and independently, recent developments in theoretical and empirical research on trade restrictiveness measurement similarly focus on the welfare and efficiency impacts of trade policy reforms. These have introduced new approaches to trade restrictiveness measurement, which evaluate efficiency outcomes by collapsing typically multi-dimensional trade policy changes into a scalar indicator. Two types of approaches have been identified. The first involves price-based trade restrictiveness indicators (Anderson and Neary (1996, 1995, 1992) and Anderson and Neary (2006)), with origins that may be traced back to the Konüs (1924) cost of living index. In addition, quantity-based trade restrictiveness indicators (Chau, Färe and Grosskopf (2003), Bureau et. al (2003)) have also been developed, and are built upon the Mahler inequality, the coefficient of resource utilization of Debreu (1951), Malmquist (1953) and Shephard (1953).2

While the trade and environment literature and the trade restrictiveness measurement literature are both concerned with efficiency changes in open economies, as yet lacking is an approach that shows how existing trade restrictiveness measurement techniques can be modified to account for environmental bads and environmental policies. In this paper, we provide a set of tools which allows for an examination of one such modification. Of course, a first point of departure will involve the joint consideration of both goods and bads in the output set and in consumer preferences. Thus, unlike other forms of domestic distortions already dealt with in the trade restrictiveness measurement literature (Anderson, Bannister and Neary (1995), Chau,

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1A related theme concerns optimal environmental taxation in a second-best world with trade policies as one possible source of policy distortions. See for example Bovenberg and de Mooji (1995) and a sequence of subsequent work (e.g. Fullerton (1997), Bovenberg and de Mooji (1997)) that have origins that may be traced back to Atkinson and Stern (1974) on optimal public good provision with distortionary taxes.

2This index can additionally be decomposed into its consumer and a producer components, along with a domestic distortion component, thus accommodating production strictly inside the production frontier.
Färe and Grosskopf (2003)), our approach follows Färe, Grosskopf and Pasurka (1989) and formalizes an axiomatic model of the polluting technology. This formulation is also consistent with the general equilibrium specifications already adopted in the trade and the environmental literature, and addresses environmental externality of the producer to consumer variety (e.g. Copeland (1994)).

Our approach is rooted in trade restrictiveness quantity indices (Chau, Färe and Grosskopf (2003)). One of the advantages of this quantity-based approach is its formulation in terms of Shephard type distance functions, which are dual to the revenue function, allowing us to retrieve shadow prices. In place of the Shephard output and input distance functions, our modification adopts directional distance functions and benefit functions, which are respectively variants of the shortage and benefit functions of Luenberger (1995, 1992), modified in our context to incorporate the production and consumption of bads (Färe and Grosskopf (2004)). The Shephard distance functions seeks to expand all outputs proportionately, which is inappropriate when some outputs are undesirable as in the case of pollution. The directional distance function and benefit function both include the Shephard distance function as a special case; as a generalization they accommodate asymmetric treatment of components of the output vector, thus allowing for reductions in pollution while increasing desirable outputs.

Our overall trade restrictiveness indicator evaluates the overall efficiency effects of tariffs, taxes, subsidies—including those related to the environment. We show that the indicator reflects general equilibrium welfare consistent with equivalent variation. The indicator is also consistent with a number of desirable index number properties, and can be decomposed simply as the difference between a production efficiency and a consumption efficiency component. If environmental bads are taken out of our setting, these production and consumption efficiency components are in fact closely related to what Diewert and Woodland (2004) coined producer and consumer substitution functions except for a normalization factor. In addition, these components are also closely related to the output and consumption components of the trade restrictiveness quantity index in Chau, Färe and Grosskopf (2003), which are based on the Shephard distance functions and a Farrell-type decomposition using ratios rather than differences.

Finally, we will also illustrate the potential importance of such a decomposition particularly in the context of environmental externalities. Specifically, it is well known that trade policy distortions, in the absence of environmental externalities, generally give rise to

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3See also Färe and Grosskopf (2004).
4For a recent survey of the literature, see Copeland and Taylor (2004).
production and consumption distortions. As such, an appropriate trade policy reform can improve welfare by mitigating (i) overall production distortion through tax-cum-subsidies keeping consumption taxes fixed, (ii) overall consumption distortion again through tax-cum-subsidies keeping production subsidies fixed, or (iii) both, for example through a uniform reduction in production subsidies and consumption taxes if standard regularity conditions are met (Hatta (1977), Fukushima (1979), Dixit and Norman (1980), Woodland (1982)). With environmental externalities going from producers to consumers, overall consumption distortion is now directly linked to producer prices, in addition to the usual income effect that works through government to consumer transfers. As such, any production efficiency gains achieved may well necessitate a corresponding consumer efficiency tradeoff, and vice versa. With the possibility of such tradeoffs in mind, our decomposition of overall efficiency into its consumption and production efficiency components allows for two additional sets of issues to be addressed.\footnote{While beyond the scope of this paper, such a decomposition may also shed light on the political economy of trade and environmental policies. In particular, in addition to voting by specific factor owners (Mayer (1984)), or interest group lobbying among groups of specific factor owners (Grossman and Helpman 1994), an interesting question concerns whether the existing configurations of trade and environmental policies can endogenously impact the incentives for interest group formation among groups of producers and consumers.} In what ways can the existing configuration of trade and environmental policies affect the division of efficiency gains between producers and consumers subsequent to a welfare improving trade policy reform in general equilibrium? Conversely, in what ways can the same existing policy configurations affect the division of gains between producers and consumers subsequent to a welfare improving environmental policy reform?

The rest of the paper is organized as follows. Sections 2 - 4 respectively describe the production technology, consumer preferences and the trading equilibrium. The associated directional distance function and benefit function are defined. Our efficiency indicators are defined in Section 5, and their properties are shown in Section 6. In Section 7, we indicate an estimation approach and Section 8 concludes.

2 Technology

In this section, we introduce an environmental technology and a representative agent. Our approach is axiomatic, and we treat both the good outputs and the byproducts, the bads, as outputs. Desirable or good outputs and undesirable or bad outputs are denoted respectively as $y \in \mathbb{R}^M_+$ and $z \in \mathbb{R}^J_+$. Also let $v \in \mathbb{R}^N_+$ be the vector of input endowments. The output set

$$ P(v) \subseteq \mathbb{R}^M_+ \times \mathbb{R}^J_+, \quad v \in \mathbb{R}^N_+ $$

(1)
denotes the set of all good $y$ and bad $z$ outputs that the endowment vector $v$ can produce. For $P(v)$ to be an environmental technology, we require it to satisfy the following two axioms.\textsuperscript{6}

**Axiom 1** Null-jointness. If $(y, z) \in P(v)$ and $z = 0$, then $y = 0$.

**Axiom 2** Weak disposability of outputs. If $(y, z) \in P(v)$, $(\theta y, \theta z) \in P(v)$ for $\theta \in [0, 1]$.

Null-jointness requires that good outputs can only be produced if some bads are also produced as byproducts. Conversely, no bads imply that no good output can be produced. Weak disposability requires that proportional reduction of good and bad outputs is feasible. Intuitively, if bads are to be reduced, at the margin resources must be diverted away from the production of good outputs. This will be the case, for example, if there is an underlying abatement technology that uses resources to reduce environmental bads.

In addition to the above axioms, we assume that $P(v)$ satisfies standard assumptions: inputs are freely disposable, $P(v)$ is a compact and convex set with $P(0) = \{0\}$.\textsuperscript{7} Figure 1 illustrates an output set satisfying the above assumptions with two good outputs and a single bad. As shown, the output set satisfies null-jointness since if $z = 0$, then $y$ must also be equal to 0 for $(y, z) \in P(v)$. The figure also illustrates weak disposability. This is the case for any $(y, z)$ in $P(v)$, for proportional reduction of $(y, z)$ is always feasible, with $(\theta y, \theta z) \in P(v)$ for $0 \leq \theta \leq 1$.

Let $p^* \in \mathbb{R}_+^M$ be the vector of free trade prices of good outputs. In addition, let $p \in \mathbb{R}_+^M$ be the corresponding vector of producer prices. Departures from free trade as measured by the difference $p - p^*$ denote the vector of import tariffs / production subsidies. Also denote $t \in \mathbb{R}_+^J$ as the vector of environmental / emission taxes facing producers of bad outputs. Together, these imply that total producer revenue, evaluated at the vector of producer prices $p$ and environmental taxes $t$, is $py - tz$. Assuming competitive behavior among producers, economy-wide total revenue is given by the following revenue function:

$$G(p, t, v) = \max\{py - tz : (y, z) \in P(v)\}. \quad (2)$$

The supply functions of good and bad outputs can be respectively obtained by applying Shephard’s lemma:

$$y(p, t, v) = G_p(p, t, v), \quad z(p, t, v) = -G_t(p, t, v). \quad (3)$$

\textsuperscript{6}For a more detailed discussion, see Färe and Grosskopf (2004).

\textsuperscript{7}See Färe and Primont (1995) for details.
By standard arguments, $G(p, t, v)$ is homogeneous of degree one and convex in $(p, t)$. In addition, good and bad output supplies $y(p, t, v)$ and $z(p, t, v)$ are respectively homogeneous of degree zero in the same variables.

In order to derive shadow prices of the outputs, we need a functional representation of the environmental technology. This representation should credit the expansion of good output and the contraction of bads. To this end, we will use a directional distance function that embodies these properties. Let $g = (g_o, -g_z)$ be a directional vector such that expansion of goods and contraction of bads are credited. The directional distance function is defined as:

$$D^o(v, y, z; g) = \sup \{ \beta : (y + \beta g_o, z - \beta g_z) \in P(v) \}.$$ 

where the vector $(g_o, g_z) \geq 0$ has at least one strictly positive element. In terms of Figure 1, the directional vector in the third quadrant is added to the $(y, z)$ vector which is then scaled by $\beta$ projecting the $(y, z)$ vector onto the frontier of the output set $P(v)$ in the direction of $g = (g_o, -g_z)$.

This function satisfies, by definition, the translation property.

$$D^o(v, y + \alpha g_o, z - \alpha g_z; g) = D^o(v, y, z; g) - \alpha, \; \alpha \in \mathbb{R}.$$

$D^o$ also inherits a number of other properties related to the output set $P(v)$. In particular, $D^o(v, y, z; g)$ represents the technology in the sense that $D^o(v, y, z; g) \geq 0$ if and only if $(y, z) \in P(v)$.

It can also be shown that output derivatives of the distance function, if they exist, yield a vector of normalized shadow prices for good and bad outputs whenever $D^o(v, y, z; g) = 0$, respectively.

$$\nabla_y D^o(v, y(p, t, v), z(p, t, v); g) = \frac{-p}{py_o + tg_z},$$

$$\nabla_z D^o(v, y(p, t, v), z(p, t, v); g) = \frac{t}{py_o + tg_z}. \tag{4}$$

$$\nabla_z D^o(v, y(p, t, v), z(p, t, v); g) = \frac{t}{py_o + tg_z}. \tag{5}$$

3 Preferences

We consider an environmental externality of the producer to consumer variety and take the byproducts $z$ to be a vector of public bads. Also let $x \in \mathbb{R}^M_+$ be the vector of good outputs
consumed. Consumer preferences are represented by a utility function $u(x, z)$. We may call an output good if utility does not decrease with increases of those outputs. Similarly, we may term outputs bad if utility does not increase with increases of these outputs. More formally, outputs are good if

$$x' \geq x \Rightarrow u(x', z) \geq u(x, z),$$

and bad if

$$z' \geq z \Rightarrow u(x, z') \leq u(x, z).$$

Let $q \in \mathbb{R}^M_+$ be a consumer price vector of good outputs. Deviation of $q$ from free trade prices $p^*$, $q - p^*$, reflects the corresponding vector of import tariffs or consumption taxes. The price vector of bad outputs is given by the vector of environmental taxes $-t$ itself, and represents payment per unit of bad output consumed made possible through government to consumer transfers.

Of course, consumers do not choose a utility maximizing level of bads given prices, but rather take the level of public bad $z = z(p, t, v)$ as given. The corresponding restricted minimal expenditure function of the aggregate household $e(q, t, z, u)$ is given by:

$$e(q, t, z, u) = \min_x \{ qx - tz : u(x, z) \geq u \} = \min_x \{ qx : u(x, z) \geq u \} - tz = E(q, z, u) - tz. \tag{6}$$

Thus, $e(q, t, z, u)$ is made up of two parts. Respectively, these include the minimal expenditure spent on the consumption of goods, $E(q, z, u)$, and a payment received upon consuming $z$ units of bad outputs, $tz$. We note that the restricted expenditure function $e(q, t, z, u)$ is homogeneous of degree one and concave in $(q, t)$ at given $z$. Meanwhile, the first component of the expenditure function $E(q, z, u)$ is concave and homogeneous of degree 1 in $q$. Also, since $u(x, z)$ is non-increasing in $z$, $E(q, z, u)$ is non-decreasing in $z$, and as such $E_z(q, z, u) \geq 0$.

The Hicksian demand functions for good outputs are:

$$e_q(q, t, z, u) = E_q(q, z, u) = x(q, z, u). \tag{7}$$

Thus, $x(q, z, u)$ is independent of $t$ for given $z$, and is homogeneous of degree zero in $q$. Meanwhile, at given $z$, derivatives of the expenditure function with respect to environmental taxes gives the vector of public bads:

$$e_t(q, t, z, u) = -z. \tag{8}$$

Finally, from (6),

$$e_z(q, t, z, u) = E_z(q, z, u) - t.$$
Thus, the marginal harm inflicted on the consumer in expenditure terms $e_z(q, t, z, u)$ due to a small increase in $z$ can be positive or negative. As may be expected, this sign depends on whether the increase in expenditure required to sustain $u$, $E_z$, is sufficiently compensated by transfers through the environmental tax $t$.

In order to express the shadow price of consumption choices $(x, z)$, we model the representative consumer with a benefit function. This function is due to Luenberger (1992). In particular, choosing the directional vector $g = (g_o, -g_z)$, we have

$$b(x, z, u; g) = \sup \{ \beta : u(x - \beta g_o, z + \beta g_z) \geq u \}.$$

If the consumption vector $(x, z)$ is such that $b(x, z, u; g) = 0$, or if $u(x - b(x, z, u; g)g_o, z + b(x, z, u; g)g_z) = u$ along an indifference curve, the corresponding normalized shadow prices are

$$\nabla_x b(x(q, z, u), z, u; g) = \frac{\nabla_x u(x(q, z, u), z)}{\nabla_x u(x(q, z, u), z) g_o - \nabla u(x(q, z, u), z) g_z}$$

(9)

$$\nabla_z b(x(q, z, u), z, u; g) = \frac{\nabla_z u(x(q, z, u), z)}{\nabla_x u(x(q, z, u), z) g_o - \nabla u(x(q, z, u), z) g_z}.$$  

(10)

Thus, if the vector of environmental taxes $t$ is set pointwise to equal the marginal environmental harm in expenditure terms $E_z(q, z, u)$, normalized shadow prices are equal to observed prices, similarly normalized, and in familiar fashion:\footnote{To see this, note from the definition of $E(q, z, u)$ in (6) above that

$$E(q, z, u(q(z, u), z)) = qz(q, z, u).$$

Totally differentiating with respect to $q$ and $z$ gives: $\nabla_x u(x, z) = q/E_o(q, z, u)$, and $\nabla_z u(x, z) = -E_z(q, z, u)/E_o(q, z, u)$.

4 Trading Equilibrium

At a given world price vector $p^*$ and an environmental tax vector $t$, the trade balance function is given by:

$$B(p, q, t, p^*, v, u) \equiv c(q, t, z(p, t, v), u) - G(p, t, v) - (q - p^*)x(q, z, u) + (p - p^*)y(p, t, v).$$  

(15)
In the context of a small open economy, the general equilibrium utility of the aggregate household $u(p, q, t, p^*, v)$ is the solution of the following balance of trade relation:\footnote{We note that $B(q, p, t, p^*, v, u)$ is monotonically increasing in $u$ so long as the standard regularity condition $1 > (q - p^*)E_{qu}(q, z, u)/E_u(q, z, u)$ is satisfied. $E_{qu}(q, z, u)/E_u(q, z, u)$ denotes the vector of marginal propensities to consume. Thus, the solution to (16), if it exists, is uniquely determined.}

$$u(p, q, t, p^*, v) = \{u| B(p, q, t, p^*, v, u) = 0\}. \tag{16}$$

where consumer tax revenue net of producer subsidy expenditure, $(q - p^*)x(q, z, u) - (p - p^*)y(p, t, v)$, is assumed to be redistributed to the aggregate household in a lump sum fashion.

In what follows, we distinguish between two regimes. The first is a baseline in which the small open economy pursues free trade, at given world prices $p^* = p = q > 0$, and a given environmental tax vector $t^* \geq 0$. We do not put any further restrictions on the baseline environmental tax vector. This allows for the index to be applied given any base / observed vector of environmental taxes, rather than an endogenously determined “first-” or “second-best”, say. The equilibrium welfare of the aggregate household is $u^* = u(p^*, p^*, t^*, p^*, v)$, while production and consumption vectors of good and bad outputs are:

$$y^* = y(p^*, t^*, v), \quad z^* = z(p^*, t^*, v), \quad x^* = x(p^*, z^*, u).$$

A second regime refers more generally to any other set of circumstances under which the price of good outputs may be distorted away from free trade values, and any given set of environmental taxes $t$, which may or may not be equal to $t^*$,

$$u^d = \{u| B(p, q, t, p^*, v, u) = 0\}, \tag{17}$$

with

$$y^d = y(p, t, v), \quad z^d = z(p, t, v), \quad x^d = x(q, z^d, u).$$

The determinants of distorted welfare $u^d$ depend of course on the specifics of the underlying production technology and consumer preferences in question. $u^d$ also depends on the vectors of trade and environmental policies.\footnote{See Diewert, Turunen-red and Woodland (1991) for a generalized framework (including non-traded goods and multiple households) in which the impact of piecemeal tariff reforms with domestic distortion, though not environmental bads, is analyzed.} The general case of multiple trade and environmental distortions has been fully worked out in Copeland (1994) for example. As reference for our discussion in the sequel, we note two useful definitions set forth therein. In our notations:
**Definition 1** Output $i$ is intensive in pollutant $j$ if $-\partial y_i(p,t,v)/\partial t_j = \partial z_j(p,t,v)/\partial p_i > 0$.

**Definition 2** Let the vector $\delta z \equiv E_z(q,z,u) - (q - p^*)E_{qz}(q,z,u) - t$ be the net marginal damage to the consumer due to a small increase in bads.\footnote{These include $E_z(q,z,u) - t$ as already discussed above, in addition to $(q - p^*)E_{qz}(q,z,u)$ which additionally takes into account the effect of rising pollution on consumption distortion in the presence of a consumption tax $q - p^*$.} Output $i$ is said to be pollution damage intensive with respect to a set of pollutants $S$ if $\sum_{j \in S} \delta z_j \partial z_j(p,t,v)/\partial p_i > 0$.

Intuitively, output $i$ is intensive in pollutant $j$ if raising the price of output $i$ (tax on pollutant $j$) leads to an increase (a reduction) in the bad output $j$ (good output $i$), $-G_{t,p}(p,t,v) = -G_{p,t}(p,t,v) > 0$. Meanwhile, output $i$ is pollutant damage intensive with respect to an arbitrary set of pollutants $S$ if the weighted sum $\sum_{j \in S} \delta z_j \partial z_j(p,t,v)/\partial p_i = -\sum_{j \in S} \delta z_j G_{t,p}(p,t,v)$ is strictly positive, where the weights are given by the net marginal damage of each pollutant $j$ on the aggregate household $\delta z_j$.

Assuming that $q = p = p^* + \tau \geq p^*$, so only trade barriers in the form of import tariffs / export subsidies are in play, it can be verified that\footnote{For notational economy, the arguments of expenditure $E(\cdot)$, $(q,z,u)$, and the arguments of revenue $G(\cdot)$, $(p,t,v)$, are dropped whenever there is no risk of confusion.} $E_u(1 - (q - p^*)E_{qu}/E_u)du^d = -[\delta z G_{tp} - \tau(G_{pp} - E_{qq})]d\tau + [\delta z G_{tt} - \tau G_{pt}]dt$.\footnote{These include $E_z(q,z,u) - t$ as already discussed above, in addition to $(q - p^*)E_{qz}(q,z,u)$ which additionally takes into account the effect of rising pollution on consumption distortion in the presence of a consumption tax $q - p^*$.}

Thus, whenever the distortion multiplier, $1 - (q - p^*)E_{qu}/E_u$, is strictly positive

**Proposition 1** (Copeland 1994). A small equi-proportionate reduction in trade taxes and subsidies will not reduce welfare $u^d$ ($d\tau = \tau d\alpha$ where $\alpha$ is a positive scalar and $d\alpha < 0$), if all industries subject to trade protection are pollution damage intensive. A small increase in environmental taxes proportional to $\delta z$ will not reduce welfare ($dt = \delta z d\alpha$ and $d\alpha > 0$), if all industries subject to trade protection are pollution damage intensive.

## 5 The Trade Restrictiveness Indicator with Environmental Externalities

We are now in a position to introduce our trade restrictiveness indicator for the environmental producer and consumer model. We depart from Chau, Färe and Grosskopf (2003) by introducing environmental externalities, and propose efficiency measures that accommodate both goods and bads in the output set and consumption bundles. The indicators that we develop

in what follows will credit expansion of good outputs and contraction of bads, much like the
directional distance function and the benefit function.

In addition, we also depart from existing analysis of the welfare consequences of trade
and the environment by disentangling the trade policy responses of two distinct components of
welfare, namely, production efficiency and consumption efficiency in general equilibrium. The
indicators developed in what follows show precisely how this decomposition works. The de-
composition will in turn provide a set of tools that will allow us to address three sets of issues:
(i) differential production efficiency and consumption efficiency responses to welfare improving
and multi-dimensional trade reforms; (ii) differential production efficiency and consumption
efficiency responses to welfare improving and multi-dimensional environmental reforms; and
(iii) how proper accounting of environmental externalities and policies is important in cor-
rectly identifying the trade restrictiveness implications of policy reforms evaluated based on
production efficiency, consumption efficiency, or both.

We start with the production side of the economy, and introduce the directional efficiency
measure:

\[
OE^o (p, t, p^*, t^*, v; g) = \max \{\alpha : p^*(y^d + \alpha g_o) - t^*(z^d - \alpha g_z) \leq G(p^*, t^*, v)\} = \frac{G(p^*, t^*, v) - (p^*y^d - t^*z^d)}{p^*g_o + t^*g_z}.
\]  

(19)

\(OE^o\) ascertains the difference between maximum GDP and observed net supply, \(p^*y^d - t^*z^d\),
evaluated at free trade prices \(p^*\) and baseline environmental taxes \(t^*\). This difference is nor-
malized by the sum of the world price value of the directional (goods) vector \(g_o\), and the tax
revenue associated with the directional (bads) vector \(g_z\), \((p^*g_o + t^*g_z)\). In Figure 2, the produc-
tion side of the economy is illustrated with two goods and one bad output. The indicator \(OE^o\)
measures the normalized revenue difference between the baseline output vector \((y^*_1, y^*_2, z^*_1)\) and
the distorted vector \((y^d_1, y^d_2, z^d)\). The supporting hyperplanes \(p^*y^* - t^*z^*\) and \(py^d - tz^d\) evaluated
respectively at the baseline \((p^*, t^*)\) and the distorted \((p, t)\) price vectors are also illustrated.

Turning now to consumption efficiency evaluation, define the overall consumption direc-
tional efficiency measure,

\[
OE^c (q, p, t^*, w^d; g) = \min \{\beta : p^*(x^d + \beta g_o) - t^*(z^d - \beta g_z) \geq e(p^*, t^*, z^*, w^d)\} = \frac{e(p^*, t^*, z^*, w^d) - (p^*x^d - t^*z^d)}{p^*g_o + t^*g_z}.
\]  

(20)

\(OE^c\) measures consumption inefficiency as the normalized difference between two expenditures:
(i) minimal expenditure to achieve the distorted level of welfare \(w^d\) at baseline free trade prices
$p^*$ and baseline environmental taxes $t^*$, and (ii) observed expenditure, similarly evaluated at world prices of goods and the baseline vector of environmental taxes $t^*$. Without the normalization factor $p^*g_o + t^*g_z$ and without environmental bads, this difference in expenditure coincides with what Diewert and Woodland (2004) refer to as the consumption substitution function.\footnote{Diewert and Woodland (2004) also defines a corresponding production substitution function as a difference in profit levels, due to a change in netput and given netput prices, rather than outputs in our formulation in (19).}

To ascertain the level of overall trade distortion, we seek an equivalent variation directional efficiency measure, and do so by once again asymmetrically treating goods and bads, starting from the baseline equilibrium $(x^*, z^*)$:

$$
OE(p^*, t^*, u^d; g) = \max \{ \gamma : p^*(x^* - \gamma g_o) - t^*(z^* + \gamma g_z) \geq e(p^*, t^*, z^*, u^d) \}
$$

$$
= \frac{(p^*x^* - t^*z^*) - e(p^*, t^*, z^*, u^d)}{p^*g_o + t^*g_z}
$$

$$
= \frac{(p^*y^* - t^*z^*) - e(p^*, t^*, z^*, u^d)}{p^*g_o + t^*g_z}
$$

$$
= \frac{G(p^*, t^*, v) - e(p^*, t^*, z^*, u^d)}{p^*g_o + t^*g_z}.
$$

(21)

where the third equality follows from the balance of trade relation implied by (16), with $p^*x^* = p^*y^*$. Thus, $OE(p^*, t^*, u^d; g)$ measures inefficiency by evaluating the difference between the minimal expenditure needed to achieve distorted welfare $u^d$ and the maximal revenue attainable both evaluated at free trade prices and baseline environmental taxes $t^*$. Additionally, $OE(p^*, t^*, u^d; g)$ normalizes this difference by the world price value of the directional goods vector, and the value of the directional bads vector evaluated at the vector of baseline environmental taxes.

We note that $OE(p^*, t^*, u^d; g)$ accommodates any simultaneous trade and environmental reforms that depart from the baseline. As such, it also allows for purely trade or purely environmental reforms as special cases, which apply respectively when $t = t^*$, or when $p = q = p^*$.

6 Properties

The efficiency measures defined above have a number of desirable index number properties as well as intuitive interpretations. These properties will also help illustrate the potential usefulness of having separate indicators for production and consumption efficiency. Furthermore, the
important role of environmental policies on the efficiency implications of trade policy reforms, and conversely, the role of trade policy on the efficiency impact of environmental policy reforms, will also be illustrated.

**Decomposition:** The overall efficiency measure $OE (p^*, t^*, u^d; g)$ can be decomposed linearly into a producer efficiency component and a consumer efficiency component:

$$OE (p^*, t^*, u^d; g) = OE^p (p, t, p^*, t^*, v; g) - OE^c (p, t, p^*, t^*, u^d; g).$$

(22)

The equality in (22) follows from the endogenous link between the production and consumption sides of the small open economy through (16). Specifically,

$$OE^p - OE^c = G(p^*, t^*, v) - (p^* y^d - t^* z^d) \frac{p^* y_d + t^* g_z}{p^* g_o + t^* g_z} = G(p^*, t^*, v) - e(p^*, t^*, z^*, u^d) \frac{p^* y_d + t^* g_z}{p^* g_o + t^* g_z}$$

from (16), and the last equality follows by definition of the revenue function $G(p^*, t^*, v)$.\(^{17}\)

**Homogeneity:** The efficiency measure $OE^p$ is homogeneous of degree zero in $(q, p, t, p^*, t^*)$, whereas consumption efficiency $OE^c$ and overall efficiency $OE$ are both homogeneous of degree zero in $(q, p, t, p^*, t^*)$.

Note simply that for any scalar $\lambda > 0$,

$$OE^\lambda (\lambda p, \lambda t, \lambda p^*, \lambda t^*, v; g) = \frac{G(\lambda p^*, \lambda t^*, v) - \lambda p^* y(\lambda p, \lambda t, v) + \lambda t^* z(\lambda p, \lambda t, v)}{\lambda p^* g_o + \lambda t^* g_z} = \frac{G(p^*, t^*, v) - p^* y_d + t^* z^d}{p^* g_o + t^* g_z} = OE^\lambda (p, t, p^*, t^*, v; g).$$

Also, from (16) and (17), equilibrium welfare levels $u^d$ and $u^*$ are homogeneous of degree zero in $(q, p, t, p^*, t^*)$, since distorted Hicksian demand $(x(q, z(p, t, v), u))$ and supply functions $(y(p, t, v), z(p, t, v))$ are homogeneous of degree zero respectively in $(q, p, t)$ and $(p, t)$. Likewise, baseline Hicksian demand and supply functions are homogeneous of degree zero in $(p^*, t^*)$.

[17]We also note here the close relationship between the directional efficiency measure developed here and the trade restrictiveness quantity index (TRQI) of Chau, Färe and Grosskopf (2003). In particular, instead of the differences illustrated in (19) - (21), the consumption and production components of the TRQI are ratios: $G(p^*, v)/p^* y^d$ and $e(p^*, u^d)/p^* x^d$ in the absence of environmental bads. In addition, the TRQI itself is also a ratio of these production and consumption components.
Thus, we have

\[ O^\varepsilon (\lambda, \nu, \lambda^p, \lambda^t, u_1; g) = \frac{e(\lambda^p, \lambda^t, z, u_1) - \lambda p^d + \lambda^t z^d}{\lambda p^d g + \lambda^t g z} = O^\varepsilon (q, p^*, t^*, u^d; g). \]

Taken together, and by the decomposition property, it follows immediately that the overall efficiency measure \( O^e \) is homogeneous of degree zero in \( (q, p, p^*, t^*) \) as well.

**Welfare Change:** Since the expenditure function \( e(q, t, t^*, u) \) is increasing in utility \( u \), overall efficiency is thus a direct indicator of welfare change, as \( O^\varepsilon (p^*, t^*, u; g) \) is monotonically decreasing in \( u \) from (21).

Let superscripts 1 and 2 denote two distinct trade and environmental taxation regimes. It follows that \( O^\varepsilon (p^1, t^1, p^*, t^*, v; g) \) and \( O^\varepsilon (p^2, t^2, p^*, t^*, v; g) \) are both homogeneous of degree zero in \( (p, t) \). Thus, \( O^\varepsilon (p^1, t^1, p^*, t^*, v; g) \geq O^\varepsilon (p^2, t^2, p^*, t^*, v; g) \) if \( \alpha \) is a positive scalar. Otherwise, \( O^\varepsilon (p, t, p^*, t^*, v; g) \) is monotonically increasing in utility relative to the baseline.

Clearly, if \( \alpha p = p^* \), and \( \alpha t = t^* \), \( y^d = y^* \) and \( z^d = z^* \) since the supply functions of goods and bads are both homogeneous of degree zero in \( (p, t) \). Thus, \( O^\varepsilon (p, t, p^*, t^*, v; g) = 0 \). Conversely, if \( O^\varepsilon (p, t, p^*, t^*, v; g) = 0 \),

\[ G(p^*, t^*, v) = p^* y(p, t^*, v) - t^* z(p, t^*, v). \]

Differentiating both sides with respect to \( p^* \) and with respect to \( t^* \), we have, by applying Shephard’s lemma

\[ G_p(p^*, t^*, v) = y^* = y(p, t^*, v) = y^d \]
\[ -G_t(p^*, t^*, v) = z^* = z(p, t^*, v) = z^d \]

Note also from (4) and (5) that

\[ \nabla_y D^o (v, y^*, z^*; g) = \frac{-p^*}{p^* g + t^* g z} \quad \text{and} \quad \nabla_z D^o (v, y^*, z^*; g) = \frac{t^*}{p^* g + t^* g z}. \]
\[ \nabla_y D^o (v, y^d, z^d; g) = \frac{-p}{p g + t g z} \quad \text{and} \quad \nabla_z D^o (v, y^d, z^d; g) = \frac{t}{p g + t g z}. \]
Since \( y^* = y \) and \( z^* = z \), it follows that \( \alpha p = p^* \) and \( \alpha t = t^* \) where \( \alpha = (p^* g_o + t^* g_z)/(p g_o + t g_z) \) is a positive scalar.

Finally, note that since \( G(p^*, t^*, v) \geq p^* y - t^* z \) by definition of the revenue function for \((y, z) \in P(v)\), it follows that \( OE^e \geq 0 \).

**Consumption and Overall Efficiency Representations:** Consumption efficiency \(OE^c(q, t, p^*, t^*, u^d; g)\) is equal to zero if there is free trade, or \( q = p = p^* \), and if \( t = t^* \). Otherwise, \( OE^c \) can take on either positive or negative values, indicating respectively an improvement and a deterioration in efficiency relative to the free trade baseline.

Of course, there can be no change in efficiency if prices remain at their baseline levels, or \( p = q = p^* \) and \( t = t^* \). As such, \( u^d = u^* \), and \( OE^c(p^*, t^*, p^*, t^*, u^*; g) = OE(p^*, t^*, u^*; g) = 0 \). The converse, however, is generally not true, since the level of production generated public bad is determined endogenously by the vector of distorted producer prices \((p, t)\). In particular, suppose indeed that

\[
OE^c(q, t, p^*, t^*, u^d; g) = 0,
\]

\[
p^*(x(p^*, z(p^*, t^*, v), u^d) - x(q, z(p, t, v), u^d)) = t^*(z(p^*, t^*, v) - z(p, t, v))
\]

\[
\Leftrightarrow E(p^*, z(p^*, t^*, v), u^d) - p^* x(q, z(p, t, v), u^d) = t^*(z(p^*, t^*, v) - z(p, t, v))
\]

where the last term \( t^*(z(p^*, t^*, v) - z(p, t, v)) \) may take on positive or negative values. Thus, expenditure on the consumption of goods evaluated at free trade price may have changed \((E(p^*, z(p^*, t^*, v), u^d) - p^* x(q, z(p, t, v), u^d) \neq 0)\) even when overall consumption efficiency remains at zero, so long as the size of the transfer exactly compensates. Furthermore, whereas \((x(p^*, z(p^*, t^*), u^d), z(p^*, t^*, v))\) and \((x(q, z(p, t), u^d), z(p, t, v))\) are evidently points along the same indifference curve, the shadow prices of goods and bads, in general, need not be the same from (9) and (10) since consumers take the level of the public bad as given, rather than as choice variables.

Finally, with an arbitrary baseline vector of environmental taxes \( t^* \), welfare can indeed improve beyond the baseline via an appropriate choice of trade and environmental policy instruments. In particular, the first-best environmental tax vector \( \hat{t} \) is just the marginal damage in the absence of trade distortions \( E_z(p^*, z(p^*, \hat{t}, v), \hat{u}) \),\(^{18}\) where \( \hat{u} \) is the corresponding first best level of welfare which solves the balance of trade relation (16) at \( p = q = p^* \), and \( t = \hat{t} \). Thus, so long as \( t^* \neq t \), welfare can improve or deteriorate relative to the baseline benchmark \( u^* \). This corresponds to a level of consumption efficiency that may be either positive or negative. A

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\(^{18}\) To see this, note from (18) that utility is maximized and \( du^d = 0 \) if \( \tau = 0 \), or \( p = q = p^* \), and \( \delta_z = 0 \), or \( t = E_z(p^*, z(p^*, t, v), \hat{u}) \).
similar argument for overall efficiency also establishes that \( \overrightarrow{OE} \) can both take on either negative or positive values except in the case where \( t^{*} \) is set exactly at \( \hat{t} \).

**Piecemeal Trade Policy Reform:** We illustrate here the decomposed efficiency impacts of a trade policy reform that is known to be welfare improving in general equilibrium with environmental externality. Consider therefore a proportional reduction in all existing production subsidies as in Proposition 1 due to Copeland (1994). We begin with production efficiency. A production subsidy reform, \( dp \), improves production efficiency, if and only if

\[
\triangledown \rightarrow OEo = (p - p^{*})G_{pp}(p, t, v)dp + (t - t^{*})G_{tp}(p, t, v)dp < 0
\]

where the second equality follows since \( G_{p} \) is homogeneous of degree one in \((p, t)\). Also, \( G_{pp} \) is a positive semi-definite matrix of pure substitution effects. A proportionate reduction in production subsidies can be expressed as \( dp = (p - p^{*})d\alpha \), where \( \alpha \) is a scalar and \( d\alpha < 0 \). From (25), there are two distinct production efficiency effects, one with respect to the production of good outputs, and the other, bads. In particular, if (i) there is no environmental externality, or if (ii) the vector of environmental taxes never deviated from the baseline level \( t = t^{*} \),

\[
\triangledown \rightarrow OEo = (p - p^{*})G_{pp}(p, t, v)(p - p^{*})d\alpha < 0
\]

since \( G_{pp} \) is positive semi-definite and \( d\alpha < 0 \). As such, conditional on the satisfaction of (i) or (ii), a proportionate reduction in all existing production subsidies improves production efficiency based on our directional measure \( \overrightarrow{OE} \). Otherwise,

\[
\triangledown \rightarrow OEo = \frac{(p - p^{*})G_{pp}(p, t, v)(p - p^{*})d\alpha + (t - t^{*})G_{tp}(p, t, v)(p - p^{*})d\alpha}{p^{*}g_{o} + t^{*}g_{z}}.
\]

An additional set of sufficient conditions is now required to guarantee an improvement in production efficiency subsequent to the same trade policy reform. One such example would require that (iii) \( t - t^{*} < 0 \), and \( p > p^{*} \), and (iv) all protected sectors are pollution intensive with respect to all pollutants, so that each element of the matrix \( G_{tp}(p, t, v) \) is non-positive. These impose strong assumptions on the nature of the output set, but offers an intuitive interpretation as to the complications that arise upon introducing environmental policies and externalities: a reduction in subsidies on goods production will improve production efficiency if reducing goods output can also circumvent over-production of environmental bads. This applies whenever each element of the vector of environmental taxes \( t \) is lower than the baseline.
Turning now to consumption efficiency, recall that

\[ \text{OE}_c^c = e(p^*, t^*, z^*, u^d) - p^* x(q, z(p, t, v), u^d) + t^* z(p, t, v) \]

\[ p^* g_o + t^* g_z, \]

consumption efficiency is thus typically jointly determined by changes in welfare \( u^d \), and direct price effects through changes in \( p \) and \( q \) in this case of trade policy reforms. Consider now the case of a proportionate reduction in production subsidies \( (p^* - p) \) and consumption taxes \( (q - p^*) \). The effect of this trade reform on welfare \( u^d \) has already been shown in Proposition 1. The direct price effects on consumption efficiency are shown as follows:\(^{19}\)

\[ d \text{OE}_c^c|_{u^d \text{ const.}} = \frac{(q - p^*) E_{qq}(q, z(p, t, v), u^d) dq + (\delta_z + t - t^*) G_{tp}(p, t, v) dp}{p^* g_o + t^* g_z}. \]

In the absence of environmental externalities, a proportional reduction in all consumption taxes implies an improvement, since \( E_{qq} \) is a negative semi-definite matrix of pure substitution effects, and

\[ d \text{OE}_c^c|_{u^d \text{ const.}} = \frac{(q - p^*) E_{qq}(q, z(p, t, v), u^d)(q - p^*) d\alpha}{p^* g_o + t^* g_z} > 0. \]

Otherwise, the consumer efficiency impact of trade reform is once again no longer clear cut, and depends jointly on the signs of \( t - t^* \) and \( q - p^* \), the net marginal damage of bads on consumers \( \delta_z \) evaluated at the distorted price vector, \( (p, q, t) \), and whether protected sectors are pollution damage intensive, \( G_{tp} \) and \( \delta_z G_{tp} \). In particular, it can be easily verified that even when all of the sufficient conditions guarantee production efficiency and overall efficiency improvements are met ((iii) and (iv) above), the overall consumption efficiency impact of the same trade policy reform reform may still be negative.

In sum, even when conditions sufficient for a trade reform induced welfare improvement are assured, either production or consumption efficiency can nevertheless run in a direction opposite to the direction of the welfare change. In addition, suppose there is at hand a trade reform that leads to simultaneous welfare, consumption and production efficiency improvements in the absence of environmental externalities. Such uniform improvements can be negated as soon as bads are introduced, and even if such a trade reform in fact guarantees an overall improvement in efficiency.

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\(^{19}\)This follows from the homogeneity of Hicksian demand, with \( qE_q(q, z(p, t, v), u^d) = E(q, z(p, t, v), u^d) \). Totally differentiate with respect to \( q \) and \( p \) gives the expression below for \( d \text{OE}_c^c \).
Piecemeal Environmental Reform: We now examine the decomposed efficiency impacts of environmental reforms. Consider therefore an increase in environmental taxes proportional to $\delta z$ ($dt = \delta z \, d\alpha$ where $\alpha$ is a positive scalar and $d\alpha > 0$). From Proposition 1, $u^d$ rises if all industries subject to trade protection are pollution damage intensive, or, every element of the vector $\delta z G_{tp}$ is negative. The production efficiency impact of such an environmental reform can be expressed as follows:

$$d \rightarrow OE^o = \frac{(p - p^*) G_{pl}(p, t, v) \delta z \, d\alpha + (t - t^*) G_{tt}(p, t, v) \delta z \, d\alpha}{p^* g_o + t^* g_z}$$

which follows once again from the observation that $G(p, t, v)$ is homogeneous of degree one in $(p, t)$.

Evidently, the production efficiency impact of this welfare enhancing change in environmental taxes is in fact ambiguous. Note, however, that the introduction of trade distortion through production subsidies $(p - p^*) > 0$ in fact enhances the likelihood an improvement (a reduction in $\rightarrow OE^o$), if all protected sectors are pollution damage intensive, $\delta z G_{tp} < 0$. Intuitively, raising environmental taxes can indirectly enhance production efficiency by reducing the output distortion in sectors of the economy protected by a subsidy.

Finally, the consumption efficiency impact of a similar increase in environmental taxes can also be shown:

$$d \rightarrow OE^c |_{u^d \, const.} = \frac{[(t - t^*) G_{tt}(p, t, v) \delta z] \, d\alpha + [\delta z G_{tt}(p, t, v) \delta z] \, d\alpha}{p^* g_o + t^* g_z}$$

The second term in square bracket is positive for an increase in environmental taxes $d\alpha > 0$ for $G_{tt}$ is positive semi-definite. However, the first term in square bracket is once again of ambiguous sign. Like piecemeal trade policy reform, the production and consumption efficiency consequences of a welfare improving environmental policy reform can indeed run in opposite directions. In addition, note that the net marginal environmental damage term $\delta z = E_z - t - (q - p^*) E_{qz}$ depends on the size the consumption tax through $q$, and the size of the production subsidy through $p$. It follows therefore that deviations from free trade not only impacts the nature of a welfare improving environmental reform, but its decomposed impacts on production and consumption efficiency as well.

Finally, our indicators also show that the introduction of trade distortions need not always hamper the efficiency enhancing potential of environmental reforms. As shown, existing trade distortions in the form of a production subsidy in pollution damage intensive sectors in fact enhance the production efficiency impact of an environmental reform.
7 Estimation

The estimation of the trade restrictiveness indicator with environmental externalities would typically involve frontier estimation techniques. Given that we exploit duality to derive various shadow prices as derivatives of estimable functions, the most natural approach would be parametric, either stochastic or deterministic. The various components to be estimated include the revenue function and directional distance function for the producer side, the expenditure function and benefit function for the consumer side.

Since estimation of the revenue and expenditure functions is well-known, we focus on the directional distance function (the benefit function is the consumer side equivalent and is left to the reader). In parameterizing the directional distance function, the researcher should account for the fact that rather than homogeneity in outputs which characterizes the Shephard output distance function, this function satisfies the translation property,

$$D^o(v, y + \alpha g_o, z - \alpha g_z; g) = D^o(v, y, z; g) - \alpha, \quad \alpha \in \mathbb{R}.$$  

As shown in Färe, Martins-Filho and Vardanyan (2006), the translog functional form does not accommodate translation. However, the quadratic functional form does. For an illustration of estimation of a directional distance function with externalities using a quadratic specification, see Färe, Grosskopf, Noh and Weber (2005). For an illustration of estimation of trade restrictiveness (without externalities) using a deterministic, parametric frontier approach pioneered by Aigner and Chu (1967), see Bureau, Chau, Färe and Grosskopf (2003).

8 Conclusion

This paper extends our earlier work on trade restrictiveness by including the effects of environmental quality and regulation. In order to account for the effects of pollution and its regulation requires that we be able to account for the production and consumption of both desirable and undesirable outputs. Instead of the Shephard distance functions employed in our original trade restrictiveness index, we turn to directional distance functions which include the Shephard distance functions as a special case. The key advantage of this more general form is the ability to treat good and bad outputs asymmetrically, seeking increases in goods and decreases in bads. Like the Shephard distance function, it is dual to the revenue function which allows us to solve for shadow prices requiring only data on input and output quantities. These shadow prices are
Our overall trade restrictiveness indicator includes effects of tariffs, taxes, subsidies—including those related to the environment—on production and consumption. The overall indicator can be decomposed into a production efficiency and a consumption efficiency component. We show that these indicators are welfare indicators consistent with equivalent variation, as well as other desirable properties. And far from being just interesting theoretical constructs, they can be estimated with available data.

Reference


Figure 1. Environmental Output Set
Figure 2. Production Subsidies
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