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## "Behavior, Production and Competition"

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# Behavior, Production and Competition

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**Abstract**

Previous studies have found underestimation of risk, or overconfidence, to be a key factor in entrepreneurship. We use a simple model of competitive equilibrium to show that an irrational under-estimation of risk provides a competitive advantage leading to a greater chance of survival under competitive pressures. Overconfidence leads to greater investment, production levels, average profit and greater variance of profits. Despite the greater variance of profits, if enough producers under-estimate their risk, they should collectively drive more rational decision-makers form the market. We illustrate a local equivalency between Kahneman and Tversky's prospect theory model, and a subjective expected utility model with decision-makers display overconfidence. This model allows us to characterize risk attitudes through two primary effects: diminishing marginal utility of wealth (rational), and diminishing distance perception (behavioral). Diminishing distance perception is a simple measure of misperception of risk. Results from economic simulations suggest that diminishing distance perception may be a more important determinant of market behavior, and entrepreneurial success, than diminishing marginal utility of wealth.

## 1. Introduction

While all would agree that starting a new business is an extremely risky venture, there is little evidence that entrepreneurs are more risk tolerant than other individuals. In fact, Low and MacMillan (1988) find specifically that propensity to take on risk does not differentiate entrepreneurs from non-entrepreneurs. Rather, many have discovered that entrepreneurs differ in the process by which they evaluate opportunities and asses the risks involved (Das and Teng, 1997). For example, Baron (2000) finds that entrepreneurs are less likely to engage in counterfactual thinking, not recognizing the possibility for alternative outcomes of their venture.

Many have found a link empirically between the under-estimation of risk and entrepreneurship activity (see for example, Simon, Houghton and Aquino, 2000). Camerer and Lovallo (1999) use economic theory to argue that such overconfidence should lead to excess entrepreneurial activity. Despite an increasingly evident link between overconfidence and entrepreneurship, little is known of the effects of overconfidence on business performance under competition. We employ basic concepts of competitive equilibrium, and comparative static results similar to Sandmo (1971) to illustrate that overconfidence may not only lead to entry, but give entrepreneurs a competitive edge not achieved by more rational decision-makers.

Behavioral theorists beginning with Kahneman and Tversky (1979) have noted that individuals appear to be loss averse. Loss aversion supposes that while individuals may be risk averse over gains, they are risk loving over losses. In other words, individuals will take great risks to avoid a loss.

Since the late 1970s, models of behavior under risk have become much more complicated and a little less clear. While expected utility supposes all risk response is due to diminishing marginal utility of wealth, more current models suppose that risk response may also be affected by probability communication and perception, and heuristics, like the use of a reference point. We find that loss aversion, and many other models describing experimental behavior can be equivalently represented by employing a subjective expected utility model, with the perceived probability displaying a shrinking of the distribution around the mean, similar to the overconfidence displayed by entrepreneurs. If these behavioral anomalies apply, they confound the effects of the canonical expected utility based risk aversion. Firms that display these anomalies may actually face an advantage over other firms, increasing their production, risk, and average profit, despite their preferences. Thus, behavioral anomalies, unlike diminishing marginal utility of wealth, may be self sustaining in markets, driving more rational firms from the market.

These results are radically different from the standard literature on production risk. First, we find that, while risk may reduce firm welfare, it may be a source of competitive power and something advantageous on a market scale. Secondly, we suppose that there are two opposing responses to risk, diminishing marginal utility of wealth (DMUW), and *diminishing distance perception* (DDP). The market response to risk will depend primarily on the strength of these forces. While there is little reason to believe that policy can affect the rate of DMUW, it may be possible to alter DDP by offering improved information, training, or other resources to the decision-makers. In fact much of the experimental literature that uncovered the behavioral anomalies behind DDP focuses on the possibility of eliminating anomalies through educational experiences. Thus, risk response may be targeted by policy.

In the following section we discuss the current literature on rationality in markets, outline a simple model of overconfidence using a Bayesian prior. We show that this model can be used to represent prospect theory like loss aversion, and other similar models. Using the techniques employed by Sandmo, we show that our brand of overconfidence (and thus loss aversion) yields a competitive advantage in production. Thus, in competitive markets rational behavior should be driven out by loss averse behavior. This provides a strong and neoclassical rationale for irrational behavior under normal circumstances.

**2. The Modeling Risk Response**

The simplicity and ellegance of expected utility is that it sums up all response to risk as a result of DMUW, or concavity of the utility of wealth function. While it is certainly possible to model risk loving behavior, most applications have assumed risk aversion and hence a concave utility function. Further, estimation has centered around determining reasonable ranges for a very few measures of concavity. Most notable among these are Arrow and Pratt's measures of relative and absolute risk aversion.The thought was that individuals react similarly to all risk, but differ somewhat by wealth level. In particular, individuals should be less absolute risk averse, and more relative risk averse as their wealth increases. Thus estimating these ranges for various wealth levels could allow us to predict behavior in new environments. In addition to its simplicity, expected utility enjoys wide acceptance as a normative model. Being built on hard to dispute axioms requiring that decision makers be consistent in their preferences, it is often argued that markets should select out those who systematically fail to follow expected utility (see for example Green, 1987). Typically, behavior violating expected utility theory is called "irrational." Several violations of expected utility have been found in laboratory experiments (e.g. Allais, 195X; Kahneman and Tversky, 1979, see Starmer 2001 for a review). However, little has been done to examine the effects on competition from the particular types of irrationality uncovered in the laboratory.

Sandmo (1971) famously analyzed the impacts of rational risk aversion on competitive markets.  He outlines the impact of risk aversion, via the expected utility model, among competing firms on production, welfare, and competition. Among Sandmo's most prominent results are that risk averse firms will unambiguously produce less output than risk neutral firms when faced with price risk. Thus, risk averse firms are at a competitive disadvantage. For this reason, many have supposed that those displaying severe risk preferences would be sifted from the market through competition, eliminating the need to model risk in many circumstances. Here, we hope to outline the impacts of more general risk behaviors on competitive markets and behavior.

Two behaviors that have been found consistently when dealing with uncertainty are overconfidence (Alpert and Raiffa, 1982) and loss aversion (Kahneman and Tversky, 1979).

Overconfidence is typified by individuals underestimating the amount of uncertainty. When asked to construct 95% confidence intervals, individuals generally produce an interval that contains the truth much less often than expected (e.g. Alpert and Raiffa, 1982, find the truth is contained in respondents' intervals about one third of the time). This suggests that individuals perceive distributions that are very tightly packed around their mean, when in fact the distributions may be more dispersed. Loss aversion is a bit more complex on the surface. Individuals first compare all outcomes to a reference level of wealth, classifying all outcomes below the reference point as a loss, and all above as a gain. A loss averse decision-maker displays diminishing marginal utility of gains, and diminishing marginal pain from loss. Thus, Kahneman and Tversky (1979) have proposed representing the individuals preferences using a value function that is convex below the reference point, implying risk loving behavior, and concave above, implying risk averse behavior. Further, at the reference point, losses are much more painful than gains are pleasurable. Thus, the value function is kinked at the origin, with the marginal utility larger for losses than gains.

We represent overconfidence employing a Bayesian prior centered around the mean of a distribution. Suppose an individual faces a gamble with wealth outcomes distributed with probability density $f(s \mid \mu)$, where $s$ represents wealth outcome, and $\mu$ is a parameter representing the mean of the distribution. Then, let $g(s \mid \mu, \sigma_g)$ be a unimodal distribution with mode of $\mu$ represent the overconfidence function. Then, it must be the case that

(1)

$$\sigma_f^2 = \int_{-\infty}^{\infty} (s-\mu)^2 f(s \mid \mu) ds < \int_{-\infty}^{\infty} (s-\mu)^2 \frac{g(s \mid \mu, \sigma_g) f(s \mid \mu)}{\int_{-\infty}^{\infty} g(s \mid \mu, \sigma_g) f(s \mid \mu) ds} ds = \int_{-\infty}^{\infty} (s-\mu)^2 h(s \mid \mu, \sigma_g) ds = \sigma_h^2$$

.

To see this, suppose that $f(s\,|\,\mu) = f(s'\,|\,\mu)$ with $\mu < s < s'$. Then $h(s\,|\,\mu,\sigma_g) > h(s'\,|\,\mu,\sigma_g)$ because $g(s\,|\,\mu,\sigma_g) > g(s'\,|\,\mu,\sigma_g)$. Also, $(s-\mu)^2 < (s'-\mu)^2$. Thus probability weight is redistributed from points that are widely dispersed to those closer to the mean, reducing perceived variance, and narrowing all confidence intervals. Further, the smaller is $\sigma_g$ the greater the concentration of *h* around the mean. We call $\sigma_g$ the parameter of diminishing distance perception (DDP) because the smaller is $\sigma_g$, the lower is perceived probability as distance from the mean increases. The overconfidence function we describe is closely related to Stein's shrinkage estimator (1955) used to correct standard error estimates in maximum likelihood estimation of means. In Stein's seminal paper, he shows that the maximum likelihood estimator for the mean of a multivariate (at least three) normal distribution is inadmissible given a squared error loss function. Stein derived an alternative estimator displaying greater precision. This estimator is called a *shrinkage* estimator because it shrinks the expected squared error. Efron and Morris (1975) showed that Stein's estimator can be derived by Bayesian estimation, defining a prior over the distribution of the means that is concentrated around the means. Thus this prior increases probabilities near the mean, and decreases probabilities near the tails. In our context, Stein shrinkage is a reasonable behavioral model representing the mental process leading to overconfidence.

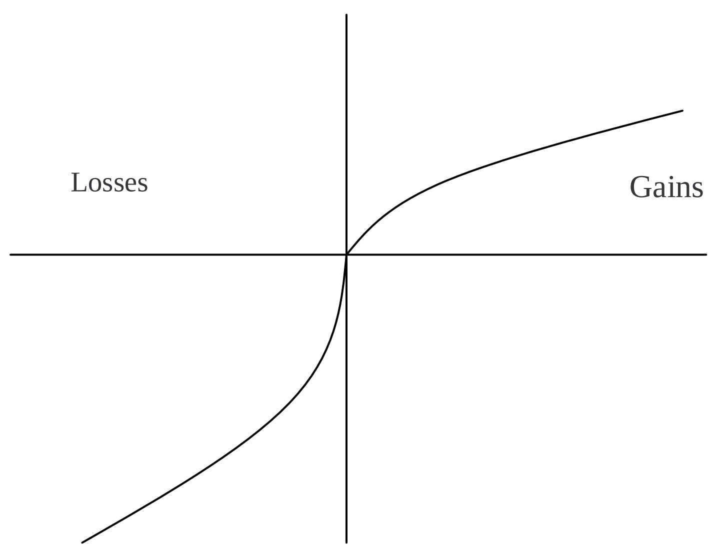
While the prospect model of decisions-making captures many important behavioral elements, it lacks the simple measures of behavior generated by the expected utility model (such as Arrow and Pratt measures). In fact the behavior described by prospect theory is complicated enough that one may think there is no simple way to characterize the properties of any particular specification. For this reason we seek a simple representation of the primary characteristics of prospect theory. Under a set of circumstances, our model of overconfidence can be combined with expected utility theory to represent loss averse behavior. Here we outline the conditions for equivalence.

Kahneman and Tversky's (1979, later revised in Tversky and Kahneman, 1992) prospect theory is based on a utility function that is reference dependent, rather than wealth dependent. Before a decision is evaluated, the individual compares the possible outcome to their reference point (most likely their current wealth situation) and classifies every outcome as either a gain or a loss. Let $s$ be the wealth outcome of some risky choice, and let $w$ be the individual's wealth level. In accordance with Prospect theory (Kahneman and Tversky, 1979) we can represent utility of outcome as

$$v(s\,|\,w)=\begin{cases} v^{+}(s-w) & \text{if} \quad s>w \\ v^{-}(s-w) & \text{if} \quad s\leq w \end{cases}$$

where $v^{+}(0)=v^{-}(0)=0$, so that utility is continuous, $v^{+}{}''(s)<0, v^{-}{}''(s)>0$, and $v^{+}{}'(0)<v^{-}{}'(0)$. Figure 1 displays a typical prospect theoretic value function.

**Figure 1. Prospect Theoretic Value Function**



Note the immediate steep decline for loss relative to a gain and the diminishing pain from a loss. Thus, individuals behave risk loving over losses, and risk averse over gains. Losses are also treated as having a much more severe impact than gains. Secondly, probabilities are distorted in the minds of the decision-maker. These distortions can be represented by a monotonically increasing weighting function $\pi\left(f(s)\right)$, where $\pi(0)=0$, $\pi(1)=1$, $\pi\left(f(s)\right)>f(s)$ for small probability density, and $\pi\left(f(s)\right)<f(s)$ for large probability density. Hence, the value function upon which decisions are based can be written

$$V(s\,|\,w)=\int_{-\infty}^{\infty}v(s\,|\,w)\pi\left(f(s)\right)ds.$$

For now, we will ignore the probability weighting function. A value function $v_a(s\,|\,w)$ is equivalent to (will generate the same decision predictions as) the prospect model for a given reference point if for any $s$ in the support,

$$v_a(s\,|\,w)\propto v(s\,|\,w).$$

**Proposition 1** For any prospect theoretic specification $v(s\,|\,w)$ that is continuously differentiable in the loss domain, there exists $u(s)$, a concave function, and $g(s\,|\,w)$, a uni-modal probability density function such that

$$v_a(s\,|\,w)=u(s)g(s\,|\,w)\propto v(s\,|\,w).$$

**Proof:** Let the support for $s$ be given by $(\underline{s},\overline{s})$, where $\underline{s},\overline{s}$ are possibly infinite. Let $u(s)$ be a concave function of wealth with $u(0)=0$ (without loss of generality). Thus, we have satisfied proposition 1 if we can find $g(s\,|\,w)$ that is unimodal, and has $\alpha g(s\,|\,w)u(s)=v(s\,|\,w)$.

Let $g(s\,|\,w)=\dfrac{v(s\,|\,w)}{u(s)\int_{-\infty}^{\infty}\frac{v(s\,|\,w)}{u(s)}ds}$, and $\alpha=\int_{-\infty}^{\infty}\dfrac{v(s\,|\,w)}{u(s)}ds$. We need to show that $g(s\,|\,w)$ is both a probability density function, and that it is unimodal. By construction, $g(s\,|\,w)$ integrates to 1.

Also by construction, for any $s$, both $u(s)$ and $v(s|w)$ have the same sign. Thus $g(s|w) \geq 0$ for all $s$. Thus we have a true probability density function. Differentiating the with respect to $s$ reveals

$$\frac{\partial g(s|w)}{\partial s} = \frac{v'(s|w)u(s) - u'(s)v(s|w)}{\alpha(u(s))^2}.$$

Thus, $g(s|w)$ will be unimodal if $v'(s|w)u(s) \underset{<}{\overset{>}{-}} u'(s)v(s|w)$ for $s \underset{>}{\overset{<}{-}} \tilde{s}$. Let $\tilde{s} = w$. Then this will be the case if $\frac{v'(s|w)}{v(s|w)} \leq \frac{u'(s)}{u(s)}$ whenever $\overline{s} > s > w$, and $\frac{v'(s|w)}{v(s|w)} \geq \frac{u'(s)}{u(s)}$ whenever $w > s > \underline{s}$. Note that $\frac{v'(s)}{v(s)}$ is declining in $s$ over both the loss and gain domains. With regards to the gain domain, it is simple to find a concave function satisfying $\frac{v'(s|w)}{v(s|w)} \leq \frac{u'(s)}{u(s)}$. The condition is weakly satisfied by $u(s) = v^+(s|w)$, and can be satisfied strictly by any slight reduction in the concavity of $u(\cdot)$. Conditions over the loss domain are significantly more complicated. Suppose we can represent both $v^-$ and $u$ using infinite Taylor series approximations, so $v^- = \sum_0^\infty v_i (s-w)^i$, and $u = \sum_0^\infty u_i (s-w)^i$. To satisfy continuity and concavity, we require that $u_0 = 0$, $u_1 = v^{+\prime}(0)$, and $\sum_0^\infty i(i-1)u_i(s-w)^{i-2} < 0$. We can approximate any point in the loss domain with a pair of lines $\hat{v}^-(s|w) = a + bs$, $\hat{u}(s) = c + ds$, where $b, d > 0$, $a \leq 0$ (for a convex function), and $c \geq 0$ (for a concave function). Using this approximation, the requirement for the loss domain is satisfied if $\frac{b}{a+bs} \geq \frac{d}{c+ds}$, or $\frac{c}{d} \geq \frac{a}{b}$, which must be true given the right hand side is negative and the left hand side is positive. Thus any concave function over the loss domain satisfying smoothness and continuity conditions will suffice. ■

Proposition 1 is useful, not only because it gives us an alternate formulation of prospect theory, but because it decomposes the risk behavior implied by prospect theory into two components: diminishing marginal utility, and diminishing distance perception. Likewise, the converse is true.

**Proposition 2** For any concave utility function $u(s)>0$, and continuous uni-modal DDP function $g(s|w,\sigma_g)$ with mode equal to $w$, $\lim_{s\uparrow w} g'(s|w), \lim_{s\downarrow w}|g'(s|w)|>0$,

$\frac{g''(s|w,\sigma_g)}{g(s|w,\sigma_g)}>-\frac{u''(s)}{u(s)}$ for $s<w$, $\frac{g''(s|w)}{g(s|w)}<\frac{u''(s)}{u(s)}+2$ for $s>w$, and

$\frac{u'(s)}{u(s)}>-\frac{g'(s|w,\sigma_g)}{g(s|w,\sigma_g)}$ for $s>w$ for will produce $v(s|w)=u(s)g(s|w,\sigma_g)$ that satisfies the properties of a prospect theory value function.

**Proof:** Let $u(s)$ be any concave utility function. Because both $g(s|w,\sigma_g)$ and $u(s)$ are continuous, their products must also be continuous. Because both are continuous and $\lim_{s\uparrow w} g'(s|w), \lim_{s\downarrow w}[-g'(s|w)]>0$, it must be that $\lim_{s\uparrow w} u(s)g'(s|w)+u'(s)g(s|w)=$

$\lim_{s\uparrow w} u(w)g'(s|w)+u'(w)g(w|w) \; > \lim_{s\downarrow w} u(s)g'(s|w)+u'(s)g(s|w)$

$=\lim_{s\downarrow w} u(w)g'(s|w)+u'(w)g(w|w)$, or $\lim_{s\uparrow w} v(s|w)>\lim_{s\downarrow w} v(s|w)$. If

$\frac{g''(s|w,\sigma_g)}{g(s|w,\sigma_g)}>-\frac{u''(s)}{u(s)}$ for $s<w$, then $2\frac{u'(s)}{u(s)}+\frac{g''(s|w,\sigma_g)}{g'(s|w,\sigma_g)}>-\frac{u''(s)}{u(s)}\frac{g(s|w,\sigma_g)}{g'(s|w,\sigma_g)}$,

because the DDP function must be positively sloped. Thus, $u''(s)g(s|w,\sigma_g)$

$+2u'(s)g'(s|w,\sigma_g)+u(s)g''(s|w,\sigma_g)>0$. Also, if $\frac{g''(s|w,\sigma_g)}{g(s|w,\sigma_g)}<-\frac{u''(s)}{u(s)}+2$ for $s>w$, then $-2\frac{u'(s)}{u(s)}+\frac{u(s)}{u'(s)}\frac{g''(s|w,\sigma_g)}{g(s|w,\sigma_g)}<-\frac{u''(s)}{u'(s)}$, or, because $\frac{u'(s)}{u(s)}>-\frac{g'(s|w,\sigma_g)}{g(s|w,\sigma_g)}$, it must be that $2\frac{g'(s|w,\sigma_g)}{g(s|w,\sigma_g)}+\frac{u(s)}{u'(s)}\frac{g''(s|w,\sigma_g)}{g(s|w,\sigma_g)}<-\frac{u''(s)}{u'(s)}$, or $u''(s)g(s|w,\sigma_g)+2u'(s)g'(s|w,\sigma_g)+u(s)g''(s|w,\sigma_g)<0$. Thus convexity and concavity requirements are met. Lastly, if $\frac{u'(s)}{u(s)}>-\frac{g'(s|w,\sigma_g)}{g(s|w,\sigma_g)}$ whenever $s>w$, then $u'(s)g(s|w,\sigma_g)+g'(s|w,\sigma_g)u(s)>0$, thus the value function is monotonically increasing.■

Proposition 2 suggests that for any concave utility function, an individual that displays DDP to a great enough degree will behave as if loss averse. It must be noted that in all cases satisfying either proposition 1 or 2, the DDP function implies a shrinking of the variance. The only DDP function that results in correct perceptions is one that is uniform on the entire support. The uniform distribution is the distribution that maximizes variance given the distribution is uni-modal. Thus, any representation in satisfying either proposition 1 or 2 results in overconfidence.

In the case of our behavioral model, the probability density function suggests that the individual overemphasizes probabilities that are near the reference point, and underemphasizes those further away. This can be thought of like a Bayesian prior, and should have an effect much like Stein's (1955) shrinkage estimators, drawing variance predictions closer to zero. More notably, diminishing marginal utility of wealth and DDP have opposing effects on behavior. Diminishing marginal utility of wealth causes individuals to be more risk averse, and be willing to pay more to insure against risks. DDP causes individuals to be less risk averse, as they perceive the risk to be less than it truly is. In the case of a behavioral model, a prospect theoretic type value function may result from exaggerating the precision of wealth information. In fact there is a long literature documenting *overconfidence*, or the exaggeration of precision, in more general situations arising in psychology experiments (see Oskamp, 1982; or Alpert and Raiffa, 1982).

It is important to note that the proof requires the utility function to display less concavity over the positive line than the prospect theoretic value function. Rabin has criticized expected utility representations for the absurd level of concavity necessary to rationalize responses to small risks. Nelson has discovered that the same problem is amplified in a prospect theoretic framework. It may be appealing to think of the prospect theoretic value function as the composite of some less concave utility function, and some measure of shrinkage that distorts this value.

At this point it is also important to say a word regarding the probability weighting function employed in prospect theory. To this point we have ignored probability weighting in our proofs. The proof of proposition 1 is trivially extended to the prospect model with probability weights by passing only the density of wealth through the probability weighting function, so that

$$V(s \mid w) = \int u(s)\, p(s \mid w)\, \pi\big(f(s)\big)\, ds \, .$$

In this case, the weighting function increases the density of low probability outcomes, and decreases the density of high probability outcomes. A possibly more appealing avenue to accomplish weighting is through the shrinkage process. The prior ensures overweighting probabilities that are near the center of a distribution (those near the reference point), and underweighting those near the tails in order to account for the shape of the prospect theoretic value function. In most applications (outside of the laboratory), the underlying uncertainty will also be described by a uni-modal distribution, with high probability density near the reference point, and low density near the tails. In this case, the weighting function can be accounted for by changing the shape of the shrinkage density, raising the tail and lowering the mode densities, to mimic the effect of the probability weighting function. Thus in many applications, estimation could be limited to the two competing effects we have outlined, risk aversion and DDP.

**4. Competing Explanations**

The use of a prior to represent risk response is not a new or unique concept. Viscusi (1989) proposed a model he called prospective reference utility employing a utility function, prior and the density function of the risk. In fact, this model has performed very well in empirical tests (Hey and Orme, 1994). Additionally, Chew (1983) proposed what has been called weighted utility theory. Weighted utility involves a multiplicative weight given to wealth outcomes that is very similar to a Bayesian prior, although much more restrictive in form. Weighted utility has also been the subject of many empirical tests (Chew and Waller, 1986; Hey and Orme, 1994).

Beyond the difference in motivation between standard prospect theory and the shrinkage representation, there are also important differences in ability to explain observed phenomena. While prospect theory has become the leading competitor to expected utility theory, there are several known weaknesses of prospect theory. Chief among these are the difficulties involved in estimating the parameters of a prospect model.

By their own admission, Tversky and Kahneman’s (1992) model requires either a cleverly designed experiment (like a *Trade Off* experiment, see Wakker and Deneffe, 1996), or specific prior knowledge of the parameters of the weighting function, to allow estimation of the value function. In applied risk research, it cannot be expected that natural experiments will offer subjects the convoluted decisions required to estimate both a value and probability weighting function. In fact, the majority of applied studies using cumulative prospect theory simply cite and use Tversky and Kahneman’s (1992) parameter estimates. These estimates were generated using only 15 subjects, consisting of UC Berkeley and Stanford undergraduates, answering hypothetical questions. By decomposing their theory into more basic components, it may be possible to produce a model that can be applied to standard data sets.

Secondly, there may be problems with interpreting the value function generated by estimating a prospect theoretic value function. Nielson (2001) has shown that while expected utility requires ridiculous levels of DMUW in small risk situations, prospect theory requires an even more concave (convex) value function, and often negatively sloped value functions. Thus, it is hard to imagine that such concavity could truly represent the diminishing value of a marginal dollar. If it does not represent the diminishing value of a dollar, then we are left once again to suppose their must be some other important determinant of behavior that we are ignoring, possibly the incorporation of experience with similar risks.

**5. DDP and Loss Aversion under Competition**

In this section, we follow the analysis of Sandmo (1971), applying the principles of DDP and expected utility maximization to examine the effects of competition on firms displaying DDP. We assume that the objective of the firm is to maximize expected utility of profits given the DDP function. The utility function of the firm's decision-maker is a concave, continuous and differentiable function of profits,

(2) $$u(\pi) > 0,\ u''(\pi) < 0 .$$

The cost function of the firm is given by

(3) $$F(x) = C(x) + B ,$$

where $x$ is output, $C(x)$ is the variable cost function, with $C(0) = 0, C'(x) > 0$, and $B$ is the fixed cost. The firm's profit function is thus given by

(4) $$\pi(x) = px - C(x) - B ,$$

where $p$ is the price of output, assumed to be random with true density $f(p)$, and expected value $E(p) = \mu$. The firm is subject to DDP function $g(p \mid \eta_g, \sigma_g)$, where $\eta_g$ is the mode (or price associated with the reference point). For the purposes of this exercise, we suppose that the decision-maker compares their resulting profit to the profit realized when the average price is realized. The firm thus maximizes the expected utility

(5) $$E\left[u\left(px-C\left(x\right)-B\right)|\,\mu,\sigma_g\right]=\int_0^\infty u\left(px-C\left(x\right)-B\right)\frac{f\left(p\right)g\left(p\,|\,\mu,\sigma_g\right)}{\int_0^\infty f\left(p\right)g\left(p\,|\,\mu,\sigma_g\right)dp}dp\,.$$

To proceed, we will use a Taylor series approximation of the utility function ,

(6) $$u\left(\pi\left(x,p\right)\right)=u\left(\pi\left(x,\mu\right)\right)+u'\left(\pi\left(x,\mu\right)\right)x\left(p-\mu\right)+\frac{1}{2}u''\left(\pi\left(x,\mu\right)\right)x^2\left(p-\mu\right)^2.$$

Thus, the maximization problem can be written as

(7) $$\begin{aligned}&\max_x E\left[u\left(\pi\left(x,\mu\right)\right)+u'\left(\pi\left(x,\mu\right)\right)x\left(p-\mu\right)+\frac{1}{2}u''\left(\pi\left(x,\mu\right)\right)x^2\left(p-\mu\right)^2|\,\mu,\sigma_g\right]\\&=u\left(\pi\left(x,\mu\right)\right)+u'\left(\pi\left(x,\mu\right)\right)x\left(\mu_h-\mu\right)+\frac{1}{2}u''\left(\pi\left(x,\mu\right)\right)x^2{\sigma_h}^2,\end{aligned}$$

where $\mu_h,\sigma_h^2$ are the perceived mean and variance of prices resulting from DDP. The first order condition associated with (7) can be written as

(8)

$$\begin{aligned}\frac{\partial EU}{\partial x}&=u'\left(\pi\left(x,\mu\right)\right)\left(\mu-C'\left(x\right)\right)+u''\left(\pi\left(x,\mu\right)\right)\left(\mu-C'\left(x\right)\right)x\left(\mu_h-\mu\right)\\&+u'\left(\pi\left(x,\mu\right)\right)\left(\mu_h-\mu\right)+\frac{1}{2}u'''\left(\pi\left(x,\mu\right)\right)\left(\mu-C'\left(x\right)\right)x^2{\sigma_h}^2+u''\left(\pi\left(x,\mu\right)\right)x{\sigma_h}^2=0,\end{aligned}$$

or, dividing by marginal utility

(9)

$$\left(\mu-C'\left(x\right)\right)+\left(\mu_h-\mu\right)-R_A\left[x{\sigma_h}^2+\left(\mu-C'\left(x\right)\right)x\left(\mu_h-\mu\right)\right]+\frac{1}{2}P_A\left(\mu-C'\left(x\right)\right)x^2{\sigma_h}^2=0,$$

where $R_A=\frac{u''}{u'}$ is the coefficient of absolute risk aversion and $P_A=\frac{u'''}{u'}$ is the coefficient of absolute prudence. Within this equation, $\mu_h$ and $\sigma_h^2$ are functions of the DDP process. More specifically, $\frac{\partial \sigma_h^2}{\partial \sigma_g^2} > 0$, while $\frac{\partial \mu_h}{\partial \tau_g} > 0$, where $\tau_g$ is the skewness of the distribution $g\left(p \mid \eta_g, \sigma_g^2\right)$, with $\mu_h = \mu$ if *g* is symmetric.

We will consider here the simple case where $g\left(p \mid \eta_g, \sigma_g^2\right)$ is a symmetric distribution. In this case, we can totally differentiate (9) to derive the comparative static result

(10)
$$\frac{dx}{d\sigma_g{}^2} = \frac{-\left[\frac{1}{2} P_A\left(\mu - C'(x)\right)x^2 - R_A x\right]\frac{\partial \sigma_h{}^2}{\partial \sigma_g{}^2}}{SOC} > 0 .$$

To see this note that dividing (9) by the perceived variance yields $\frac{\left(\mu - C'(x)\right)}{\sigma_h{}^2} - R_A x + \frac{1}{2} P_A\left(\mu - C'(x)\right)x^2 = 0.$ The model implies risk aversion on average (note risk neutrality obtains if $\sigma_h^2 = 0$, and the decision-maker acts as-if he knows with certainty that the price will be $\mu$), thus $\mu > C'(x)$. Thus, the firm displaying DDP will use more inputs on average and produce more on average. We can couple this with Sandmo's result showing that the greater is $R_A$ the less will be produced to find the tension between DDP and risk aversion in behavior under uncertainty. We will now turn our attention to the implications of DDP for competitive equilibrium.

*The Entry and Shut-down Decisions*

According to the classical model of competition, firms will enter the market if they can make a profit by doing so. In our model, entry will occur if the firm perceives that they will earn expected utility greater than $u(0)$, the profit earned prior to investing fixed costs. Further, a firm in the industry will shut down when $E\left(u(\pi)\right) < u(-B)$. Differentiating (7) with respect to $\sigma_h^2$ obtains

(11) $$\frac{\partial EU(\pi)}{\partial \sigma_g^2} = SOC + \frac{1}{2}u''\left(\pi(x,\mu)\right)x^2\frac{\partial \sigma_h^2}{\partial \sigma_g^2} < 0.$$

Thus firms with greater DDP (smaller $\sigma_g^2$) will enter the market while more rational firms that perceive correctly the risks they face would consider the expected profit too small considering the risk involved. This result mimics the result found by Camerer and Lovallo (1999) that overconfidence leads to greater rates of entrepreneurship. Further, this result is well supported by the entrepreneurship literature (e.g. Das and Teng, 1997; Barron, 2000) which has uniformly found that entrepreneurs are not more inclined to take risks, rather less inclined to take notice of the risks they face. Thus, as expected profit increases from zero, overconfident decision-makers will be the first into the market, and, as expected profits decline below zero, overconfident decision-makers will be the last to shut down.

*Competitive Equilibrium*

In order to evaluate the effects of DDP on competition, it is necessary to describe the market. Suppose demand is given by

(12) $$p = P(X) + \varepsilon,$$

where $X = \sum_{i=1}^{n} x_i$, $i$ is the index of (potential) firms, and $n$ is the number of firms producing, $P'(X) < 0$ and $\varepsilon$ is a random variable with mean 0. Thus, the standard equilibrium conditions dictate that

(13) $$E\left(U_i(\pi) \mid P(X), \sigma_g\right) < U_i(0)$$

for all firms $i$ that are not producing, and

(14) $$E\left(U_i(\pi) \mid P(X), \sigma_g\right) > U_i(0)$$

for all firms producing. From the previous discussion (and from Sandmo's result), we can specify $x_i = x\left(R_A, \sigma_g^2\right)$, where $x_{R_A} < 0, x_{\sigma_g^2} < 0$. We will represent perfectly rational (expected utility) behavior as resulting from the uninformative prior with $\sigma_g^2 = \infty$ (the improper uniform distribution over the entire real line), as is common in Bayesian theory.

**Proposition 3** Let $\mathbf{F} \subseteq \mathbf{R}^+ \times \mathbf{R}^+$ be the set of potential firms, and $\mathbf{F}_C \subseteq \mathbf{F}$ the set of firms producing under competitive equilibrium. Then, for any $\left(R_A, \sigma_g^2\right)$, $R_A > 0$ with $\left(R_A, \sigma_g^2\right) \in \mathbf{F}_C$, it must be the case that every firm with $\left(R_A, \sigma_{g'}^2\right) \in \mathbf{F}_C$ where $\sigma_{g'}^2 < \sigma_g^2$.

**Proof** The result follows directly from (11).

The result in proposition 3 suggests that as long as each decision-maker displays some level of risk aversion, at any level of risk aversion for which a rational actor produces, every actor with that level of risk aversion (or less) that misperceives the risk will operate. If all actors had identical levels of risk aversion, but varied by DDP, the market would necessarily be dominated by irrational actors. Rational actors would have a competitive disadvantage in being averse to risk, and recognizing the risk was there. Alternatively, those who could not see the risk would invest more heavily and drive more rational investors from the market.

A possibly more interesting question is what will happen when those with misperceptions begin to realize their results. Expected profit is given by

(15) $$P(X)x - C(x) - B.$$

An expected profit maximizer (risk neutral) will choose $x$ so as to solve

(16) $$P(X) - C'(x) = 0 .$$

From (9) a risk averter with DDP will solve

(17) $$P(X) - C'(x) + \left[\frac{1}{2} P_A \left(P(X) - C'(x)\right) x^2 - R_A x\right] \sigma_h^2 .$$

It is simple to see that, given the individual is not risk neutral, the perceived variance that maximizes expected profit is $\sigma_h^2 = 0$, or that resulting from the most overconfident (least rational) DDP function. Thus, the more overconfident (or loss averse) the firm, the greater the profits obtained on average. The overconfident firms are less likely to face a cash shortage given operation. Alternatively, the variance of profit is given by $\sigma_\varepsilon^2 x^2$. Thus, firms displaying overconfidence, which invest more heavily when operating, will necessarily face greater variance in profits, having a higher probability of substantial success, and a higher probability of spectacular failure. Finally, the skewness of profits is given by $\tau_\varepsilon x^3$, where $\tau_\varepsilon$ is the skewness of price. Thus, overconfidence will not alter the direction of skew in the profit distribution, but can substantially increase the skewness through increased investment.

*Welfare*

Finally, one may wonder about the welfare effects of overconfidence. This is easiest to consider by comparing equilibria consisting of identical actors. Clearly, because overconfidence leads to greater production for all levels of expected prices, consumers must benefit from the resultant lower equilibrium prices. On the other hand, producers necessarily obtain lower utility of profit on average than they anticipate, meaning they could be made better off. The ex post producer surplus must disregard overconfidence, calculating the true average net benefit. This necessarily declines as variance is misperceived, at a rate determined by the degree of risk aversion. If actors were truly risk neutral, misperceptions of variance would not matter to producers. Alternatively, if producers are very risk averse, misperceptions of variance could reduce producer surplus by more than the increase in consumer surplus leading to a market failure. Thus if firms are only mildly risk averse, there may exists some socially optimal level of overconfidence. On the other hand, if firms were severely risk averse, the government may play role in reducing overconfidence (through education, market publications, etc.) or reducing risk (through disaster relief) to improve welfare of producers.

*Non-Symmetric DDP*

If the *DDP* function alters the perceived mean, only a few of the preceding results differ. If DDP increases the mean, it will reinforce the results of reducing the variance, so long as it does not lead the firm to produce more than the risk neutral level of production. Firms begin to be at a competitive disadvantage once they produce more than the risk neutral amount. Alternatively, if decision-makers perceive a mean price that is below the true mean, this perception will work against the reduction in perceived variance, reducing the amount produced, and placing the firm at a competitive disadvantage.

## 6. Conclusion

While many have published proofs that competition forces rationality (see for example Green, 1987), this paper provides a rationale for why non-rational models may be relevant even in highly competitive industries. In fact, it seems clear that DDP, while irrational, creates a competitive advantage, and thus markets may be dominated by this particular brand of irrationality. DDP is consistent with both loss aversion models, and overconfidence. The fact that competition may encourage such behavior in the face of risk aversion makes it a little more understandable why such behavior may pop up in experimental settings. Further, empirical assessments in the entrepreneurship literature suggest that behavioral phenomenon such as DDP may play a larger role in entry decisions than factors like DMUW that are more commonly considered. There is little reason to believe that competition will sort DDP from the market, and thus DDP may also play a large role in production level decisions and exit from a competitive industry. The work in this paper provides a neo-classical economic argument for why this patently non-classical phenomenon should exist, persist and why behavioral effects may be important. Those who underestimate risk are likely to invest more, increasing their chances for greater success (or failure) than can be realized with a rational view of the world.

**References**

Alpert, M., and H. Raiffa. 1982. A Progress Report on the Training of Probability Assessors. In D. Kahneman, P. Slovic and A. Tversky (eds.) *Judgment Bias under Uncertainty: Heuristics and Biases*. New York: Cambridge University Press.

Baron, R. A. 2000. Counterfactual thinking and venture formation: The potential effects of thinking about 'what might have been.' *Journal of Business Venturing*, 15(1) 79 – 91.

Blake, D. 1996. Efficiency, Risk Aversion and Portfolio Insurance: An Analysis of Financial Asset Portfolios Held by Investors in the United Kingdom. *The Economic Journal*, 106(438) 1175 – 92.

Camerer, C. and D. Lovallo. 1999. Overconfidence and Excess Entry: An Experimental Approach. *American Economic Review* 89 (1) 306 – 318.

Chew, S. H. 1983. A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox. *Econometrica*. 51(4) 1065 – 92.

Chew, S.H. and W. Waller. 1986. Empirical Tests of Weighted Utility Theory. *Journal of Mathematical Psychology* 30(1) 55 – 72.

Cooper, A. C., C.Y. Woo, and W.C. Dunkelberg. 1988. Entrepreneurs' perceived chance of success. *Journal of Business Venturing*, 3(1) 97 – 108.

Das, T. K. and B.S. Teng. 1997 Time and entrepreneurial risk behavior. *Entrepreneurship Theory and Practice*, 22(2) 69 – 88.

Efron, B. and C. Morris. 1975. Data analysis using stein's estimator and its generalizations. *Journal of the American Statistical Association* 70 (350) 311 – 319.

Forlani, D. and J.W. Mullins. 2000. Perceived risks and choices in entrepreneurs' new venture decisions. *Journal of Business Venturing*, 15(4) 305 – 322.

Green, J. 1987. 'Making Book against Oneself,' the Independence Axiom, and Nonlinear Utility Theory. *Quarterly Journal of Economics* 102 (4) 785 – 96.

Hey, J.D. and C. Orme. 1994. Investigating Generalizations of Expected Utility Theory Using Experimental Data. *Econometrica.* 62(6) 1291 – 1326.

Just, D. R., and H. H. Peterson. 2003a. Expected Utility Calibration for Continuous Distributions. Department of Applied Economics and Management, Cornell University. Working Paper # 03-01.

Just, D. R., and Peterson, H.H. 2003b. Diminishing Marginal Utility of Wealth and Calibration of Risk in Agriculture. *American Journal of Agricultural Economics*, 85 (5) 1234 – 41.

Kahneman, D. and A. Tversky. 1979. Prospect Theory: an Analysis of Decision under Risk. *Econometrica.* 47(2) 263 – 92.

Keh, H.T., M.D. Foo and B.C. Lim. 2002. Opportunity Evaluation under Risky Conditions: The Cognitive Processes of Entrepreneurs. *Entrepreneurship Theory and Practice* 27(2) 125 – 48.

Low, M. B. and I.C. MacMillan. 1988. Entrepreneurship: Past research and future challenges. *Journal of Management* 14(2) 139 – 161.

Nielson, W. 2001. Callibration Results for Rank-Dependent Expected Utility. *Economics Bulletin* 4 (10) 1 – 5.

Oskamp, S. 1982. Overconfidence in Case-Study Judgments. In D. Kahneman, P. Slovic and A. Tversky (eds.) *Judgment Bias under Uncertainty: Heuristics and Biases*. New York: Cambridge University Press.

Palich, L. E. and D.R. Bagby. 1995. Using cognitive theory to explain entrepreneurial risk-taking: Challenging conventional wisdom. *Journal of Business Venturing*, 10(6) 425 – 438.

Saha, A., C. R. Shumway, and H. Talpaz. 1994. Joint Estimation of Risk Preference Structure and Technology Using Expo-Power Utility. *American Journal of Agricultural Economics* 76(5) 173 – 184.

Sandmo, A. 1971. On the Theory of the Competitive Firm Under Price Uncertainty. *American Economic Review*, 61(1) 65 – 73.

Simon, M., S. M. Houghton., and K. Aquino. 2000. Cognitive biases, risk perception and venture formation:How individuals decide to start companies. *Journal of Business Venturing*, 15(2) 113 – 134.

Sitkin, S. B. and A.L. Pablo. 1992. Reconceptualizing the determinants of risk behavior. *Academy of Management Review*, 17(1) 9 – 38.

Stein, C. 1955. Inadmissibility of the usual estimator for the mean of a multivariate normal distribution. In Proc. of the Third Berkeley symposium on mathematical statistics and probability, 197 – 206.

Tversky, A. and D. Kahneman. 1992. Advances in Prospect Theory: Cumulative Representation of Uncertainty. *Journal of Risk and Uncertainty* 5(4) 297 – 323.

Viscusi, W. Kip. 1989. Prospective Reference Theory: Toward an Explanation of the Paradoxes. *Journal of Risk and Uncertainty* 2(3) 235 – 64.

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