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CONSISTENT ESTIMATION OF LONGITUDINAL CENSORED DEMAND SYSTEMS: AN APPLICATION TO TRANSITION COUNTRY DATA

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Abstract

In this paper we derive a joint continuous/censored demand system suitable for the analysis of commodity demand relationships using panel data. Unobserved heterogeneity is controlled for using a correlated random effects specification and a Generalized Method of Moments framework used to estimate the model in two stages. In the first stage reduced form parameters are obtained through either ordinary least squares or heteroscedastic Tobit estimation, followed by the identification of structural parameters and imposition of cross-equation restrictions using minimum distance techniques. The procedure, which is demonstrated on data from Romania, yields elasticity estimates that lie within an intuitively pleasing range and reveals strong cross-substitution patterns between many commodity groups.

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1. Introduction

The availability of comprehensive individual and household level microeconomic data has greatly improved our understanding of how public policy instruments such as taxes, subsidies, and social programs impact consumer behavior. These information sources allow modeling opportunities not possible with aggregate level data, such as the analysis of complex substitution patterns between individual commodities and expenditure, and the ability to forecast demand patterns for segments of the population specifically targeted by government programs. Unfortunately, the econometric and computational techniques often required to extract such vital information from microeconomic data can be arduous, limiting exploitation of the data for policy analysis. In particular, the high proportion of zero expenditure levels for individual commodities hampers the estimation of large, theoretically consistent disaggregated consumer demand models.

There are several reasons why zero expenditure levels manifest in microeconomic data, the two most common of which are households at a corner solution for the commodity in question and limited survey periods leading to infrequency of purchase (IFP) errors. The former occurs when the market price for a good is equal to or greater than the consumer’s reservation price, while the latter results from the high cost of extending survey periods to the point where purchases of all goods and services are observed. More recent surveys have attempted to minimize IFP errors by extending data collection periods beyond the two-week norm and using multiple-year recall to enumerate infrequently purchased durable goods. Because of the basis for corner solutions in economic theory, most of the econometric techniques developed thus far are designed to capture economic non-consumption.

Both benchmark studies in the consumer demand literature addressing the corner solution problem in a theoretically consistent manner are grounded in Amemiya's work on simultaneous equation models with truncated normal endogenous variables. The first of these is by Wales and Woodland who derive a theoretically plausible censored demand system using the Kuhn-Tucker conditions that result from constrained maximization of a stochastic direct utility function. Subsequently, Lee and Pitt (1986, 1987) demonstrate a dual approach to modeling non-consumption at a corner solution based on virtual (reservation) prices. Each method leads to the formulation of a likelihood function based on composite distributions. Direct maximum likelihood estimation, therefore, requires evaluation of a partially integrated multivariate normal probability density function, which is often impractical for larger systems of more than a handful of equations.

Much effort has since been exerted to derive an estimator that circumvents the "curse of dimensionality", making the estimation of the large, disaggregated demand systems feasible. Several solutions have been proposed in the literature, each having unique advantages and drawbacks. Shonkwiler and Yen develop a two-step approach based on earlier work by Heien and Wessells that is general enough to model IFP errors as well as other processes generating zero expenditures. Nonetheless, its application to zero expenditure levels resulting from corner solutions has been criticized by Arndt and Arndt, Lui, and Preckel for an inability to account for the role of reservation prices. Instead, Arndt proposes the use of maximum entropy (ME) techniques to explicitly account for reservation prices and generate a simpler framework for the imposition of coherency conditions. Limiting this estimator's feasibility, however, is the fact that its asymptotic properties are unknown in non-linear applications such as the censored demand problem.

More recently, Perali and Chavas have proposed a consistent approach to the problem based on generalized method of moments (GMM) techniques, while Yen, Lin, and Smallwood formulate a quasi-maximum likelihood approach they claim is more efficient in small to moderately sized samples. Although all of these techniques provide a means of obtaining consistent estimates of disaggregated demand models, they are designed for cross sectional data, which suffers from a number of shortcomings. Chief among these are the limited ability to control for heterogeneous preferences and lack of significant real price variation. The development of an estimator able to exploit the greater price, expenditure, and demographic variability of increasingly available panel data stands to improve the reliability of demand estimates much more than efficiency improvements to existing cross sectional estimators. Panel data also provides an opportunity to reduce bias through more effective controls for household or individual level heterogeneity.

In order to facilitate the use of panel data in demand analysis, we develop a methodology for consistently estimating large, theoretically plausible longitudinal censored demand systems using GMM techniques. The approach, which yields parameter estimates that are consistent in a finite sample and asymptotically efficient, is demonstrated on a three-year panel data set from Romania. Although the specific model presented is theoretically appropriate only when zero expenditure levels represent corner solutions, the estimation framework is easily adapted to other parametric specifications.[1]

2. Specification and Estimation

Fixed vs. Random Effects

There are two different methods available for dealing with unobserved heterogeneity using panel data: the fixed and random effects approaches. Applying either to non-linear models is nontrivial, and several tradeoffs must be considered. Although the fixed effects approach places no distributional assumptions on the form of the unobserved heterogeneity, it requires restrictions on the disturbance covariance matrix to ensure tractability, and the estimation results cannot be used to make predictions for cross-sectional units outside of the sample. More importantly, consistency of the estimates requires the time series dimension of the panel be large, a rather uncommon feature of longitudinal surveys.

By contrast, the random effects approach is consistent when the time series is short, allows the specification of an unrestricted disturbance covariance matrix, and permits the investigator to make out of sample predictions. Its principle drawback is the need to assume the household specific effect (or unobserved heterogeneity) follows a specified distribution, and if the distributional assumption is invalid, random effects is no longer consistent.[2] Because the panel used in this study contains only three time periods and the model must be capable of producing out of sample predictions to conduct future policy analysis, the random effects approach is used below.

Theoretical and Empirical Specification

It is assumed the household preference structure can be described by a direct utility function weakly separable into two sub-utility functions, with the first containing leisure (L), and saving (S), and the second *N* commodity groups consisting of food, nonfood goods, services, and durables. The latter sub-utility function is also assumed to be strongly separable over time and take the form $U(q_{1jt},\ldots,q_{Njt},\varepsilon_{1jt},\ldots,\varepsilon_{Njt};A_{jt},c_j)$, where $t=1,\ldots,T$ indexes time periods, $j=1,\ldots,J$ indexes households, $q_{1jt},\ldots,q_{Njt}$ are household *j*'s demand levels for the *N* commodity groups in time period *t*, $\varepsilon_{1jt},\ldots,\varepsilon_{Njt}$ are the random disturbances associated with consumption of each commodity group, A_{jt} is a vector of household demographic variables (not all of which are time varying), and c_j is a time invariant household specific effect representing unobserved heterogeneity across households. The disturbance vector is placed directly into the utility function to make the specification consistent with the random utility hypothesis, reflecting the view that stochasticity in the consumer's optimization problem is derived from the econometrician's inability to observe factors known to the consumer that influence decisions.

In order to derive an empirical specification with the necessary flexibility to accurately characterize consumer demand patterns, economic decisions are modeled using the PIGLOG class of preferences. This leads to the familiar cost function corresponding to Deaton and Muellbauer's Almost Ideal Demand System (AIDS);

$$\begin{aligned}\log c(u,p,\varepsilon;A,c) &= \alpha_{0t} + \sum_k \alpha_{kt}\log p_{kt} + \tfrac{1}{2}\sum_k\sum_i \gamma^*_{kt}\log p_{kt}\log p_{it} \\ &+ \sum_k\sum_l \eta_{klt}\log p_{kt}a_{ljt} + u_{jt}\beta_0\prod_k p_{kt}^{\beta_k} + \sum_k \delta_{kt}\log p_{kt}c_j + \sum_k \log p_{kt}\varepsilon_{kjt}\end{aligned}, \qquad (1)$$

where u_{jt} is a reference level of utility and p_{jt} are the prices faced by household *j* in time *t*. Random disturbances have been incorporated into the cost function in the same manner as demographic variable entered through the procedure of translation (Pollack and Wales), so their inclusion does not affect the integrability of the resulting demand system or indirect utility function. Interestingly, this specification is analogous to the log-additive general error model (GEM) derived by McElroy for the cost function from production theory.[3] GEMs are consistent with the random utility hypothesis and produce more efficient and internally consistent parameter estimates than models simply "embedded in a stochastic framework" without any theoretical justification.

Since the household specific effect is akin to an unobserved vector of demographic variables, it is included in the empirical specification via demographic translating like the demographic variables in A_t. It can be shown (1) satisfies the requirements of a theoretically plausible cost function. Some of these must be checked empirically, such as concavity, but it is clear $\log c(u, p, \varepsilon; A, c)$ is continuous in *u* and *p*. Homogeneity of degree one in prices requires the following parameter restrictions:

$$\sum_k \alpha_{kt} = 1, \sum_k \gamma^*_{ki} = \sum_i \gamma^*_{ki} = \sum_k \beta_k = \sum_k \eta_{klt} = \sum_l \eta_{klt} = \sum_k \delta_k = \sum_k \varepsilon_{kjt} = 0 \ \ \forall j, t \ .$$

Inverting (1) to get the indirect utility function and applying Roy's Identity produces the following Marshallian uncompensated demand functions in budget share form:

$$s_{njt} = \alpha_{nt} + \sum_l \eta_{nlt} a_{ljt} + \sum_i \gamma_{ni} \log p_{it} + \beta_n \left(\log x_{jt} - \log g(P, \cdot) \right) + \delta_{nt} c_j + \tilde{\varepsilon}_{njt} \qquad (2)$$

where

$$\log g(P,\cdot) = \alpha_{0t} + \sum_k \alpha_{kt} \log p_{kt} + \tfrac{1}{2}\sum_k \sum_i \gamma^*_{ki} \log p_{kt} \log p_{it} + \sum_k \sum_l \eta_{klt} \log p_{kt} a_{ljt} + \sum_k \delta_{kt} \log p_{kt} c_j \quad , \tag{3}$$

$n = 1,..,N$ indexes the commodity groups or individual goods, $\gamma_{ki} = \tfrac{1}{2}(\gamma^*_{ki} + \gamma^*_{ik})$, and $\tilde{\varepsilon}_{njt} = \varepsilon_{njt} - \beta_n \sum_k \log p_{kt} \varepsilon_{kjt}$. There are several characteristics of the demand system in (2) and (3) that differ from the conventional AIDS model. The first, of course, is the inclusion of a household specific effect to measure unobserved heterogeneity in a demand systems context. Allowing the impact of the household specific effect to vary over time leads to a more flexible specification than typically found in applied work, where the fixed effects specification is frequently used to account for unobserved heterogeneity in linear models. Fixed effects implicitly assume $\delta_{nt} = 1 \;\forall\, n,t$, but if this restriction does not hold and the fixed effect is not orthogonal to the regressors, the parameter estimates of the slope coefficients will be biased. One of the advantages of the GMM approach used below is that it allows explicit testing of such restrictions.

Another uncommon feature of the model in the demand system is the form of the error term, $\tilde{\varepsilon}_{njt}$. Deriving the AIDS demand system consistently from a random utility function leads to individual disturbances that are heteroscedastic and correlated across equations. While the latter characteristic is often accounted for through seemingly SUR or joint ML estimation, the former property is usually ignored, despite the fact that Chavas and Segerson have shown heteroscedasticity in share equation disturbances to be a general property of all specifications derived from a random objective function.[4]

The specification of parameters in equations (2) and (3) requires some explanation. In particular, the coefficients on demographic variables, the household specific effect, and the share equation intercept are permitted to vary over time, while the coefficients on the economic variables, prices and total expenditure, are specified as time invariant. The theory gives us no guidance on how changes in demographic variables and unobserved characteristics influence demand patterns and the most flexible specification entails letting their impact change over time.[5] The coefficients on prices and total expenditure, however, define the structure of demand for the given commodities, which is generally assumed to be constant over time in lieu of major shifts in macroeconomic conditions or preference structures, neither of which are thought to occur in our data sample. Only when demand systems are estimated on long time series does the issue of structural change in the parameter estimates merit concern and statistical investigation. Nevertheless, an attempt to control for minor changes in macroeconomic conditions over time is made by allowing a different intercept for each share equation in every time period.

As stated above, the Romanian data's small time series dimension necessitates modeling c_j as a random effect, given that some of the demand equations must be estimated using non-linear methods. Assuming c_j is independent of the other regressors in the model leads to a non-linear generalization of the conventional variance components model (Balestra and Nerlove; Maddala; Butler and Moffitt). Unfortunately, due to the manner in which household level data in Romania and many other countries are collected, this assumption cannot be made in general. Rarely do surveys contain the exogenous market prices called for in theory, rather, prices are often computed as unit values, where the household's expenditures on a certain item are divided by the physical quantity purchased. These unit values are correlated to the household's preferences for goods of different quality, and consequently, with the household specific effect. By modeling this correlation, the endogenous nature of the unit values is accounted for, something that is very difficult to do with cross sectional data. In addition, c_j is likely correlated to the observable demographic variables in the model, and single equation Hausman tests conducted on the non-censored equations reject the null hypothesis that the household specific effect is orthogonal to the regressors.

Jakubson (1988) demonstrates the application of a "correlated random effects" approach to single equation Tobit estimation on panel data, based of previous work by Chamberlain in the linear (1982) and probit context (1984). The correlation between the household specific effect and regressors is modeled as a linear projection of c_j on all the right hand side variables, which in the context of equations (2) and (3) can be written as:

$$c_j = \sum_l \sum_t \lambda_{lt}^1 a_{ljt} + \sum_k \sum_t \lambda_{kt}^2 \log p_{kt} + \sum_t \lambda_t^3 \log x_{jt}^D + \upsilon_j \tag{4}$$

where υ_j is assumed to be independent of both the exogenous regressors and $\tilde{\varepsilon}_{njt}$, x_{jt}^D is expenditure deflated by some price index to be chosen later, and υ_j is distributed $\mathrm{N}(0, \sigma_\upsilon^2)$. For notational convenience, define $x_t' = \left[A_t' \mid \log p_t' \mid \log x_t^D\right]$ as a row vector of length K that includes all the regressors in (2) less the intercept.[6] Jakubson notes there are certain cases in which the assumption of independence between υ_j and $\mathbf{x} = \left[x_1 \cdots x_T\right]$ may be invalid. For example, since the effect of the regressors outside the sampling period is contained in υ_j, if $\mathbf{x}$ exhibits strong serial correlation, the independence assumption is violated.

When the expression in (4) is used to integrate the household specific effect out of the demand equations, the resulting correlated random effects demand system disturbances are heteroscedatic within each equation and correlated across equations through both the $\varepsilon's$ and υ. Furthermore, if the addition assumption is made that the $\varepsilon's$ are normally distributed, then the $u's$ are also normally distributed.

In its current form, the correlated random effects AIDS model is non-linear in the parameters and susceptible to severe multicollinearity during estimation due to the sharing of variables between the AIDS price index, $\log g(P,\cdot)$, and the rest of the model. A common simplification meant to reduce the complexity of the model is a linearization whereby $\log g^j(P,\cdot)$ is replaced with some approximating price index, often the Stone index. Unfortunately, despite its popularity, the Stone index is not an appropriate approximating index and has been shown by several researchers to induce bias in the linearized almost ideal demand system (LAIDS).[7] A scale-invariant price index that does not bias the LAIDS and has good approximation properties (Moschini; Buse (1998)) is the log linear Laspeyres, which is equivalent to the geometrically weighted average of prices $\log P_t^G = \sum_k s_{k0} \log p_{kt}$ when s_{k0} is calculated for some base level.

This price index can be substituted for $\log g(P,\cdot)$ to produce a LAIDS demand system consistent with the underlying specification. In addition, let it be the index used to deflate expenditures entering the specification of c_j, and define $\boldsymbol{\pi}_{nt}$ as the reduced form parameter vector of demand equation *n* in time period *t*. The reduced form LAIDS specification is then

$$s_{njt} = x_1'\pi_{n1} + x_2'\pi_{n2} + \ldots + x_T'\pi_{nT} + u_{njt}\,, \tag{5}$$

as the regressors from all time periods enter the reduced form demand equations through their correlation with the random effect. If the *N* demand equations are partitioned into a subset, N_1, containing goods not suffering from a high proportion of zero budget shares in the data, and a subset N_2, containing censored equations, then the system of demands in N_1 can be estimated consistently via equation-by-equation OLS. In fact, these reduced form estimates are the same ones that would be obtained by estimating the equations in N_1 jointly using SUR, since GLS re-weighting has no effect on a system of equations with identical right hand side variables and no cross-equation restrictions (Goldberger and Olkin). The estimates are less efficient, however, than those obtained by joint estimation of the censored and uncensored equations together.

The demands in N_2 can be specified as

$$s_{njt} = \begin{cases} x_1'\pi_{n1} + x_2'\pi_{n2} + \ldots + x_T'\pi_{nT} + u_{njt} & \text{if RHS} > 0 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

and estimated efficiently as a system of correlated Tobit equations. Since joint estimation requires the evaluation of N_2 dimension normal integrals, which is infeasible for large N_2, a consistent approach is adopted to obtain the reduced form parameters using equation-by-equation Tobit estimation. In contrast to the uncensored equations, the non-linearity of the Tobit model implies some efficiency loss from single equation estimation, despite the fact that the separate censored equations have identical right hand side variables with no restrictions. Of course, single equation Tobit estimation of (6) is only consistent if u_{nt} has the classical properties, but derivation of the model from a random utility function implies u_{nt} is heteroscedastic. Under heteroscedasticity, the reduced form Tobit estimates are biased and inconsistent (e.g., Pudney, p.148), so a modification to the conventional Tobit model is necessary.

A fairly general way of modeling the heteroscedasticity plaguing u_{nt} is to specify $E(u_{nt}^2) = \sigma_{nt}^2(w) = \sigma_{nt}^2 \exp(w'_{jt}\zeta_{nt})^2$, where w_{jt} is a vector of exogenous variables responsible for unequal dispersion of the individual error terms, ζ_{nt} is a vector of estimable parameters, and σ_{nt}^2 is an estimable common parameter in the covariance matrix. The ML Tobit routine can be modified to jointly estimate the parameters $(\pi_{nt}, \sigma_{nt}, \zeta_{nt})$ in each of the censored regression equations.

Stacking the equations in (5) and (6) over time sequentially by good defines a system of *NT* continuous/censored demand equations with correlated disturbances and the $NT \times (K+1)T$ reduced form coefficient matrix:

$$\mathbf{\Pi} = \begin{bmatrix} \mathbf{\Pi_1} \\ \vdots \\ \mathbf{\Pi_N} \end{bmatrix} \tag{7}$$

$$\mathbf{\Pi_n} = \left[\boldsymbol{\alpha} \mid \boldsymbol{\eta}_{\mathbf{nL_1}} \mid \left\langle \mathbf{diag}\{\eta_{n1t}\} \cdots \mathbf{diag}\{\eta_{nL_2t}\} \mid \gamma_{n1}\mathbf{I} \cdots \gamma_{nN}\mathbf{I} \mid \beta_n \mathbf{I} + \boldsymbol{\delta}_{\mathbf{n}} \boldsymbol{\lambda}' \right\rangle\right], \tag{8}$$

$\boldsymbol{\alpha}$ is a $T \times 1$ vector of intercepts, $\boldsymbol{\eta}_{\mathbf{nL_1}}$ is a $T \times L_1$ matrix of coefficients on the L_1 time invariant observable demographic variables, $\mathbf{diag}\{\eta_{nlt}\}$ are $T \times T$ matrices corresponding to L_2 coefficients on the time varying observable demographic variables, $\mathbf{I}$ is a $T \times T$ identity matrix, $\boldsymbol{\delta}'_{\mathbf{n}} = [\delta_{n1} \cdots \delta_{nT}]$ is a $1 \times T$ vector of parameters multiplying the household specific effect, and $\boldsymbol{\lambda}' = [\lambda^1_{11} \cdots \lambda^1_{LT} \mid \lambda^2_{11} \cdots \lambda^2_{NT} \mid \lambda^3_1 \cdots \lambda^3_T]$ is a $1 \times KT$ vector of parameters from the correlated random effect specification. While (8) represents the hypothesized structure of the underlying system, other specifications are possible and will be tested against this one is the next section. Finally, note that not all the parameters in (8) are identifiable as written. In particular, the δ parameters are only identified up to a scale factor, requiring the following normalization: $\delta_{n1} = 1 \ \forall n$.

Before proceeding to identification of the structural parameters, we make a slight change in notation. As written, the $\mathbf{\Pi}$ matrix only contains structural parameters from the mean function and must be modified to include the unrestricted parameters from the covariance matrix of the heteroscedatic Tobit models (the σ and ζ parameters) as well as the error variance from each uncensored equation. This is done by simply adding columns to $\mathbf{\Pi}$ and denoting the new reduced form parameter matrix as $\mathbf{\Pi}^*$ with dimension $NT \times K^* T$. For notational simplicity in the following sub-section, $\mathbf{\Pi}^*$ needs to be transformed from a matrix to a vector, so additionally, define $\boldsymbol{\pi} = vec(\mathbf{\Pi}'^*)$ as a NTK^*-vector of unrestricted, reduced form coefficients.[8]

Generalized Method of Moments Estimation Framework

Efficient estimation of the structural parameters in (8) requires a ML estimator be constructed from the joint distribution of all *NT* error terms in the full system. Unfortunately, such an approach is infeasible if the number of censored demand equations in the system is large, and made even more difficult by the non-linear manner in which the c_j enters the specification. Under the correlated random effects approach a separate equation is estimated for each good in every time period, so if just one commodity in the system is censored, *T* integrations must be performed on the likelihood function during joint ML estimation. Thus, even for short panels numerical evaluation of the likelihood function quickly becomes infeasible.

It is possible to circumvent the difficulties inherent in estimating the structural parameters using the joint distribution of the data by building an estimator that works off the marginal distributions of the *NT* error terms. This approach, developed by White and generalized by Jakubson (1998), called quasi-maximum likelihood estimation (QMLE) uses method of moments techniques to approximate joint ML estimation.[9] It relies on the asymptotic properties of GMM estimators, established by Hansen, to generate a consistent estimator for problems where the likelihood function can be written down in theory, but not calculated directly.

QMLE can be broken down into two stages, with the first involving consistent estimation of the reduced form parameters. As stated above, this is accomplished via OLS estimation on the non-censored equations and heteroscedastic Tobit estimation of the censored equations. The second stage entails using minimum distance techniques (Malinvaud) to impose the proper restrictions on the reduced form parameter estimates, including restrictions identifying the structure of correlated random effects, and demand theory restrictions on the price and expenditure coefficients, such as homogeneity and symmetry.

The minimum distance estimator is essentially a form of GMM estimator that minimizes the distance between a set of sample moments, the estimated reduced form $\mathbf{\Pi}^*$ matrix in this case, and their corresponding population moments, which are the underlying structural parameters. A critical piece of the estimation framework is the metric used to measure the distance between the sample and population moments. It is widely agreed the proper norm is the inverse covariance matrix of $\hat{\mathbf{\Pi}}^*$ (Jakubson, 1986), however, this matrix must be calculated taking into account the fact that $\hat{\mathbf{\Pi}}^*$ is estimated from the marginal distributions of the time period and good-specific demand equations and not through the joint likelihood function. A detailed derivation of the covariance matrix of the reduced form parameter estimates is given in Meyerhoefer (2002), where it is shown the matrix takes the form $\mathbf{\Omega} = D_1^{-1} D_2 D_1^{-1}$ with

$$D_1^{-1} = \begin{bmatrix} [\mathrm{E}(H_{1j1})]^{-1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & [\mathrm{E}(H_{NjT})]^{-1} \end{bmatrix}; \tag{9}$$

$$D_2 = \begin{bmatrix} \mathrm{E}(S_{1j1}^2) & \mathrm{E}(S_{1j1}S_{1j2}) & \cdots & \mathrm{E}(S_{1j1}S_{NjT}) \\ \mathrm{E}(S_{1j2}S_{1j1}) & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathrm{E}(S_{NjT-1}S_{NjT}) \\ \mathrm{E}(S_{NjT}S_{1j1}) & \cdots & \mathrm{E}(S_{NjT}S_{NjT-1}) & \mathrm{E}(S_{NjT}^2) \end{bmatrix}. \tag{10}$$

The matrix in (10) is the cross product of quasi-scores within and across equations, derived from the univariate continuous and censored demands in each time period, while the inverse diagonal elements in (9) are the derivatives of these quasi-scores, or the quasi-hessian. Therefore, the diagonal elements of (9) are composed of the asymptotic variance matrix of each good and time period specific univariate demand equation in second derivative form. If $\mathbf{\Omega}$ were the covariance matrix produced by maximizing the joint likelihood function, D_1^{-1} and D_2 would multiply to an identity by the information matrix equality, but given $\mathbf{\Omega}$ is derived from the marginal distributions of the data, both D_1^{-1} and D_2 enter the $NTK^* \times NTK^*$ asymptotic covariance matrix.

A consistent estimator of $\mathbf{\Omega}$ is obtained by replacing the population quantities with their sample counterparts. Therefore, let $\hat{\mathbf{\Omega}} = \hat{D}_1^{-1}\hat{D}_2\hat{D}_1^{-1}$ where $\hat{D}_1 = \mathbf{diag}\{\hat{d}_{1j1},...,\hat{d}_{NjT}\}$ with $\hat{d}_{njt} = (1/J)\sum_j^J H_{njt}(\hat{\boldsymbol{\pi}}_{nt}) \;\; \forall n,t$, and

$$\hat{D}_2 = \begin{bmatrix} (1/J)\sum_j^J \hat{S}_{1j1}^2 & (1/J)\sum_j^J \hat{S}_{1j1}\hat{S}_{1j2} & \cdots & (1/J)\sum_j^J \hat{S}_{1j1}\hat{S}_{NjT} \\ (1/J)\sum_j^J \hat{S}_{1j2}\hat{S}_{1j1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & (1/J)\sum_j^J \hat{S}_{NjT-1}\hat{S}_{NjT} \\ (1/J)\sum_j^J \hat{S}_{NjT}\hat{S}_{1j1} & \cdots & (1/J)\sum_j^J \hat{S}_{NjT}\hat{S}_{NjT-1} & (1/J)\sum_j^J \hat{S}_{NjT}^2 \end{bmatrix}. \quad (11)$$

With all the required ingredients at hand, estimation of the underlying structural parameters can proceed through construction of the minimum distance estimator. Recall that $\hat{\boldsymbol{\pi}}$ is the NTK^*-vector of reduced form parameter estimates and define $\boldsymbol{\psi}$ as the Q-vector of structural parameters ($Q < NTK^*$).[10] To minimize the distance between the estimated reduced form and structural parameters it is necessary to define a function mapping $\boldsymbol{\psi}$ into $\boldsymbol{\pi}$, denoted $h(\boldsymbol{\psi})$. In general, $h(\cdot)$ is a non-linear function, although if the restrictions placed on the reduced form parameter vector $\boldsymbol{\pi}$ are linear, $h(\boldsymbol{\psi}) = H\boldsymbol{\psi}$

where H is a matrix. Accordingly, the minimum distance estimator for $\boldsymbol{\psi}$ solves the following problem

$$\min \mathbf{D}(\boldsymbol{\psi}) = [\hat{\boldsymbol{\pi}} - h(\boldsymbol{\psi})]'\hat{\boldsymbol{\Omega}}^{-1}[\hat{\boldsymbol{\pi}} - h(\boldsymbol{\psi})], \tag{12}$$

where the substitution of $\hat{\boldsymbol{\Omega}}^{-1}$ for $\boldsymbol{\Omega}^{-1}$ in (12) does not change the estimator's asymptotic properties. Under the null hypothesis that the restrictions are correct $J\mathbf{D}(\hat{\boldsymbol{\psi}})$ is a chi-squared distributed random variable with df $= NTK^*$-Q. This Wald statistic can be used to formulate tests (nested and non-nested) of the underlying specification of structural parameters.

As written, the minimization problem in (12) yields consistent estimates of all the structural parameters as well as the variance parameters. However, given the main objective of this study is to calculate price and expenditure elasticities for the censored and uncensored commodities, it is possible to reduce the dimensions of the problem. Separate identification of coefficients on the demographic variables and intercepts, although possible, is not necessary since they are only included as controls, allowing the model to produce unbiased price and expenditure effects. Therefore, minimum distance estimation can proceed on the subset of the $\boldsymbol{\Pi}^*$ matrix corresponding to the γ, β, and univariate variance parameters, while leaving the intercepts and coefficients on the observable demographic variables unrestricted (Chamberlain, 1984).

The non-linear function $h(\cdot)$ is specified to disentangle the coefficients on the economic variables, γ and β, from the parameters of the random effect specification, δ and λ. Additionally, $h(\cdot)$ is used to impose restrictions on the structural parameters implied by the theory of consumer demand. Under the LAIDS specification three sets of theoretical restrictions are possible on the estimated demand functions: adding up, homogeneity, and symmetry. A final restriction, negativity, cannot be imposed on the parameters during estimation, but must be checked empirically to ensure that the estimates are coherent. Imposition of homogeneity is done by restricting $\sum_i \gamma_{ni} = 0 \;\; \forall n$, while symmetry requires $\gamma_{ni} = \gamma_{in} \;\; \forall n, i$. Ensuring the system adds up is much more complicated due to the fact that although the observed shares add up, the latent shares need not. Furthermore, imposing adding up through parameter restrictions requires all structural parameters in the system be identified, greatly increasing the dimensions of the minimum distance problem. To avoid these difficulties, adding up is not imposed on the structural parameter estimates. This should have little impact on the price coefficients since the combination of symmetry and homogeneity restrictions implies the $\gamma's$ sum to zero across equations by default. Rather, the predicted budget shares are not required to sum exactly to unity, though they remain consistent estimates of the true shares.

Elasticity Formulae

There has been some debate regarding the appropriate derivation of elasticities for the LAIDS model (see, for e.g., Alston *et al*.; Buse (1994)). Edgerton *et al*. conclude the formulae proposed by Chalfant seem to work best, in accordance with the findings of Green and Alston's (1990, 1991) survey of various derivations found in the literature. Since Chalfant's formulae are derived using the Stone index approximation, they must be modified slightly to reflect our use of the geometric price index in the LAIDS model. This is done by replacing s_i with s_{i0} (the budget share computed over the entire sample of households in all time periods) in the numerator of the price elasticity formula.[11] Thus, the LAIDS expenditure and uncompensated price elasticities for the uncensored equations become

$$E_n = \frac{\beta_n}{s_n} + 1, \tag{13}$$

$$e_{ni} = \frac{\gamma_{ni} - \beta_n s_{i0}}{s_n} - \delta^*_{ni}. \tag{14}$$

With the censored demand equations, it is possible to compute two different types of elasticities. The conditional elasticities correspond to households consuming non-zero amounts of the good in question, while the unconditional elasticities apply to consumers and non-consumers alike. Generally, it is the unconditional elasticities that are of interest and these are the ones calculated in this study. First, however, it is necessary to derive the unconditional marginal effects and expected budget shares for the heteroscedastic Tobit model. This is done below for the expenditure elasticity by adapting the derivations given in McDonald and Moffitt to the heteroscedastic Tobit model. The unconditional expectation of each budget share takes the form

$$E(s_{njt}) = \mathbf{x}'_j \boldsymbol{\pi}_{nt} \boldsymbol{\Phi}(z) + \sigma_{nt} \exp(w'_{jt} \zeta_{nt}) \boldsymbol{\varphi}(z), \tag{15}$$

while the marginal effect with respect to log of total expenditures is

$$\frac{\partial E(s_{njt})}{\partial \ln x_{jt}} = \boldsymbol{\Phi}(z) \frac{\partial E(s^*_{njt})}{\partial \ln x_{jt}} + E(s^*_{njt}) \frac{\partial \boldsymbol{\Phi}(z)}{\partial \ln x_{jt}}, \tag{16}$$

$$E(s^*_{njt}) = E(s_{njt} \mid s_{njt} > 0) = \mathbf{x}'_j \boldsymbol{\pi}_{nt} + \sigma_{nt} \exp(w'_{jt} \zeta_{nt}) \frac{\boldsymbol{\varphi}(z)}{\boldsymbol{\Phi}(z)}. \tag{17}$$

The above expressions are used to construct price and expenditure elasticities for the heteroscedastic Tobit equations in the usual manner.

3. Data and Results

Data used in the estimation of the joint continuous/censored demand system are drawn from the nationally and regionally representative 1994-96 Romanian Integrated Household Survey (RIHS). The RIHS contains three individual cross sections composed of 24,523 households in 1994, 31,558 households in 1995, and 32,013 households in 1996, as well as an embedded panel data set of 6,940 households. The survey does not contain market prices collected from vendors at the point of purchase, but households do report total expenditures and physical quantity purchased, allowing unit values to be computed. Monthly market prices are approximated in each of the survey's forty-seven counties by the median unit value calculated from the sample of purchasing households.[12]

Half of the goods in the twelve equation demand system are composite commodities: Nonfood goods, grains, fruits and vegetables, gasoline and diesel fuel, and a group containing meats, dairy, oils, and fats, and other foods, while the other six goods in the system are single commodities, namely, bread, coffee, beer, wine, liqueur, and tobacco products. The only commodity whose price is not computed from survey data is tobacco, for which the survey does not report a physical quantity purchased. Instead, the monthly national consumer price index for tobacco products derived by the Romanian National Institute for Statistics (NIS) is used. Finally, all the prices are put in real terms using a monthly consumer price index (CPI) constructed from a regional food CPI based on unit values purged of endogeneity, and the NIS's national level nonfood and service CPIs.[13]

Total consumption expenditure is computed by aggregating information on food, nonfood goods and services, collected over a one-month period, or a retrospective one-year time frame in the case of durables. For many households, especially in rural areas, a significant share of food consumption is derived from own production, in-kind payments, and gifts. These are valued at household specific open market price if the household purchases some of the own-consumed product in the market, and the regional market price if the household makes no market purchases of the product. Monetized home consumption is then added to purchased food, nonfood goods, services, and the flow of services from durable goods (based on a constant ten-year depreciation schedule) to create the total consumption expenditure variable.

The RIHS contains a wealth of demographic information that is exploited to control for varying preference structures and heterogeneity across households. Regional location and seasonality commonly have a very large impact on consumption patterns, so eight regional and four seasonal variables are constructed. The same is true of household composition, which is accounted for using three variables: the number of young children in the household between the ages of zero and four, the number age five through seventeen, and the number of adults eighteen years of age or older. Characteristics of the household head are used to proxy for household level preference controls and include the head's age, a dummy variable indicating whether the head is female, and four dummies denoting educational attainment at either the primary level, lower secondary or technical school, upper secondary, or university/college level.

The final variables needed to estimate the demand model are the left hand side budget shares. Table 1 lists the average budget shares and percentage of zero expenditures for each commodity group in the demand system. The grouping of goods in the model is primarily policy driven, as commodities subject to differential value added (VAT) or excise tax rates during Romania's transition are all treated separately.[14] Nevertheless, every attempt was made to place goods that are close substitutes in the same group whenever possible, in accordance with the composite commodity theorem. The fact that the budget shares of each commodity vary little from 1994 through 1996 lends credence to our assertion that the structure of commodity demand in Romania was stable during this period. The degree of censoring is naturally much higher for the individual commodities (with the exception of bread) than larger commodity groups, which are also generally composed of necessities and staple foods. Indeed, the sharp contrast in censoring levels across the commodity groups highlights the need for a joint continuous/censored approach to the modeling the full system of equations. As noted by Perali and Chavas and Pudney, instances of zero expenditure levels due to non-consumption are more likely in developing countries than wealthier societies. The same is true of transition countries, such as Romania, where many households live below the poverty line and the removal of communist-era price subsidies has lead to large real price increases during the transition period.[15] In addition, the survey period of the RIHS is long enough to make the possibility of systematic IFP errors in the data remote, so most of the observed zero expenditure levels are attributable to economic non-consumption.

Table 1: Average Budget Shares and Degree of Censoring

	Average Budget Share			% of Zero Budget Shares		
Commodity	94	95	96	94	95	96
Bread	7.2	6.8	7.0	2.4	0.6	0.6
Grains	3.3	2.8	3.2	1.9	0.7	1.2
Fruits, Vegetables	12.6	15.7	14.7	0.2	0.1	0.1
Meat, Dairy, Oils, Fats	26.0	27.0	27.3	0.1	0.0	0.0
Other foods	7.8	8.3	8.4	0.2	0.0	0.0
Coffee	1.2	1.3	1.2	50.8	45.3	44.0
Beer	0.6	0.7	0.6	73.4	69.8	72.6
Wine	1.8	2.0	2.2	56.8	56.0	57.1
Liqueur	1.1	1.1	1.1	57.8	51.9	53.4
Tobacco products	1.4	1.3	1.5	62.9	64.1	66.3
Gasoline, Diesel fuel	0.1	0.04	0.1	86.1	85.6	85.0
Nonfoods	36.4	32.9	32.1	1.1	1.1	1.0

Specification Tests

Although the theoretical derivation of the censored demand equations from a random utility function indicates the presence of heteroscedasticity in the error term, it is advisable to confirm the implications of the theory empirically before corrective action is taken. The Lagrange multiplier (LM) test detailed in Greene (2000) presents a convenient method for investigating the presence of heteroscedasticity in the Tobit model, since only estimation of restricted (homoscedastic) model is necessary to construct the test statistic. The test is also fairly general, assuming the variance of the unrestricted model is specified as $\sigma_{nt}^2(w) = \sigma_{nt}^2 \exp(w'_{jt}\zeta_{nt})$. It is hypothesized that the unequal dispersion of error terms in each univariate Tobit model is related to household size and the log of total expenditures, making w_{jt} a vector of length two. Consequently, the LM statistic has a limiting chi-squared distribution with two degrees of freedom.

Test statistics for each of the univariate Tobit models are given in Table 2, and can be compared to the 5 percent chi-squared critical value of 5.99 and the 1 percent critical value of 9.21. For most of the censored commodities the null hypothesis of homoscedasticity is rejected at both the 5 percent and 1 percent levels of significance in each time period, though there are several exceptions. Homoscedasticity cannot be rejected in the beer equation in 1994 and the gasoline/diesel fuel equation in 1994 and 1996 at either level of significance, while the null is rejected for wine in 1996 at the 1 percent level, but not at the 5 percent level. Although the LM statistic is large for gasoline and diesel fuel in 1995, the average budget share of this good is much lower in 1995 than the other two years. If one assumes this large drop in consumption is due to some external shock, the test's rejection of homoscedasticity in 1995 is more likely a reflection of the model's inability to capture the underlying dynamic than an accurate characterization of the population disturbances.[16] Therefore, we model the demand for gasoline and diesel fuel using a conventional homoscedastic Tobit model and the demand for all other censored commodities with the heteroscedastic specification given above.[17]

Table 2: Lagrange Multiplier Test for Heteroscedasticity

		LM statistic		
Commodity		94	95	96
Coffee		13.96	69.67	84.01
Beer		2.34	19.06	9.56
Wine		18.47	16.00	6.45
Liqueur		16.29	28.78	38.10
Tobacco products		15.98	24.94	20.43
Gasoline, Diesel fuel		2.56	65.71	2.07

It is also possible to test whether more parsimonious models can be used to characterize the data generating process, and in particular, the specification of the correlated random effect. For example, fixed effect models typically make the implicit assumption that $\delta_{nt} = 1 \ \forall n,t$, while most nonlinear applications of the random effects approach impose the additional restriction that all the λ parameters are equal to zero. Both of these nested specifications can be tested using the minimum distance framework by subtracting the distance function of the incrementally restricted model (B) from that of the less restricted model (A). The resulting test statistic $JD(\hat{\boldsymbol{\psi}}_{\mathrm{B}}) - JD(\hat{\boldsymbol{\psi}}_{\mathrm{A}})$ follows a chi-squared distribution with df = df_B - df_A.

Restricting the impact of the random effect to be constant over time leads to the test statistic $\chi^2_{(24)} = 1121$, which is considerably larger than the critical value 36 at the 5% level of significance. Likewise, the restriction that the random effect is orthogonal to the regressors in the model is also soundly rejected at the 5% level of significance. In that case the test statistic is $\chi^2_{(63)} = 1379$ and critical value 83. Therefore, both of the incremental restrictions on the model commonly assumed to hold in other studies are rejected under on our data sample.

Elasticity Estimates

The parameter estimates used to calculate the expenditure and price elasticities are given in Table A.1 of the Appendix. While the γ and β parameters are estimated with precision in most cases, the δ and λ parameters have low individual significance and irregular scaling, although they are jointly significant. In particular, the λ's are very small and the δ's large in many cases. The multiplicative effect of these parameters, defining the total impact of the random effect on the budget shares is correctly scaled, but the estimator has difficultly separately identifying the component parameters of the random effect. One likely reason is multicollinearity among the individual prices and expenditure in each of the three time periods as well as across the set of prices. In the presence of collinearity and a flat objective surface, identification of parameters entering the model in a nonlinear fashion, such as the δ's, can be difficult. Accordingly, when the nonlinearity is removed by setting $\delta_{nt} = 1 \ \forall n,t$ the λ's increase in magnitude and become statistically significant.[18] Nonetheless, elasticity estimates are derived from the model allowing the impact of the random effect to vary over time since chi-squared tests reject more restrictive specifications. Fortunately, multicollinearity has only a marginal impact on the economic parameters of interest and does not hinder the model's forecasting potential.

Expenditure and price elasticities are reported in Table 3. The total expenditure elasticities reveal that all goods in the system are normal goods and most are economic necessities in the sense that they have elasticities less than unity. The two exceptions are gasoline/diesel fuel and nonfood goods, which exhibit very high demand responsiveness to changes in total expenditure, placing them soundly in the luxury category. This is intuitively appealing since the nonfood category contains durable service flows from items such as televisions, household appliances, and automobiles, which are prohibitively expensive for many Romanians. The same is true of gasoline and diesel fuel, especially after the government's initial removal of fuel subsidies. Likewise, it is not surprising to find the goods deemed most necessary are the two staple foods, bread and grain.

The uncompensated own-price elasticities range from the least price responsive commodity, bread, with an elasticity of –0.482 to the highly own-price responsive good, beer, having an elasticity of –1.246. Other commodities falling into the price inelastic category are tobacco products, gasoline and diesel fuel, and the meats, dairy, oils, and fats group. Grains, other foods, nonfood goods, coffee, and fruits and vegetables all have price elasticities close to unity, while the alcoholic beverages are own-price elastic. The only food elasticity estimate of unexpected magnitude is the own-price elasticity of grains. Usually staple commodities tend to be less price elastic, but separating bread from grains increases the proportion of flour for baked goods in the latter group, which could lead to greater price responsiveness. Alcohol and tobacco elasticities are generally within the range of those found in the commodity demand literature, although the own-price responsiveness of beer is higher in Romania than has been documented for other countries

Table 3: Expenditure, Own-Price, and Compensated Own-Price Elasticities (Standard Errors in Parenthesis)

Commodity n	Expenditure	Own-Price	Comp. Own-Price
Bread	0.339	-0.482	-0.458
	(0.009)	(0.023)	(0.023)
Grains	0.441	-1.039	-1.025
	(0.013)	(0.044)	(0.044)
Fruits, Vegetables	0.633	-0.922	-0.831
	(0.007)	(0.021)	(0.020)
Meat, Dairy, Oils, Fats	0.653	-0.717	-0.542
	(0.005)	(0.028)	(0.028)
Other foods	0.759	-0.964	-0.902
	(0.010)	(0.032)	(0.032)
Coffee	0.753	-0.955	-0.945
	(0.023)	(0.034)	(0.034)
Beer	0.851	-1.246	-1.242
	(0.041)	(0.101)	(0.101)
Wine	0.877	-1.195	-1.186
	(0.035)	(0.053)	(0.053)
Liqueur	0.804	-1.140	-1.133
	(0.029)	(0.053)	(0.053)
Tobacco products	0.939	-0.666	-0.657
	(0.031)	(0.146)	(0.146)
Gasoline, Diesel fuel	1.811	-0.812	-0.809
	(0.073)	(0.043)	(0.043)
Nonfoods	1.670	-0.994	-0.429
	(0.006)	(0.008)	(0.008)

(Leung an Phelps; Smith). The tobacco elasticity is larger than estimates for the U.S. and U.K., but falls within the -0.6 to -0.8 range reported for less developed countries (Chaloupka and Jha).

Most estimates of the own-price elasticity of gasoline are based on aggregate data, making it difficult to find an appropriate comparison for household level panel estimates.

Typically, aggregate time series data is thought to capture short run responses to price changes whereas household level data, due to its greater regional and demographic variability, yields estimates more similar to predictions made from aggregate cross sections or panels. The latter information sources reflect adjustment to the greater variability in exogenous factors across regions, thereby capturing long run effects. A survey by Dahl and Sterner of various studies broken down by data source and model type finds short run estimates are typically less than –0.4, while long run own-price elasticities of gasoline vary between –0.6 and –1.2, a range encompassing the price response found in Romania.

The income compensated own-price elasticities are reported to infer whether the continuous/censored random effects demand system satisfies coherency conditions, or curvature restrictions on the underlying cost function, required by economic theory. Coherency can be verified by confirming that the eigen values of the Slutsky substitution matrix are non-positive, but this is a computationally cumbersome process. Instead, Edgerton *et al*. suggest checking that the compensated own-price elasticities are all negative and significant, a necessary condition for coherency that is satisfied by all of the compensated own-price elasticities in Table 3.

Uncompensated cross-price elasticities for each commodity in the system are reported in Table 4, with their standard errors. These as well as the standard errors in Table 3 were computed using the delta method.[19] The price elasticity matrix contains many statistically significant elements, indicative of complex cross-substitution patterns among a variety of goods in Romania. All of the alcoholic beverages are strong gross substitutes for one another, while tobacco is complementary to beer and liqueur, but a substitute for wine.

Table 4: Price Elasticity Matrix (Standard Errors in Parenthesis)

$\%\Delta Q_i$ / $\%\Delta P_j$	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
Q_1	-0.482	0.079	0.011	0.018	0.041	-0.019	-0.001	-0.006	0.020	-0.034	0.005	0.030
	(0.023)	(0.013)	(0.021)	(0.037)	(0.021)	(0.009)	(0.017)	(0.012)	(0.012)	(0.025)	(0.008)	(0.010)
Q_2	0.171	-1.039	0.161	-0.023	0.110	0.072	-0.158	-0.002	-0.012	0.352	-0.021	-0.052
	(0.029)	(0.044)	(0.035)	(0.067)	(0.038)	(0.013)	(0.030)	(0.017)	(0.022)	(0.045)	(0.011)	(0.015)
Q_3	-0.015	0.029	-0.922	0.121	0.120	-0.005	-0.028	-0.009	0.007	-0.010	0.037	0.041
	(0.010)	(0.008)	(0.021)	(0.024)	(0.014)	(0.006)	(0.010)	(0.009)	(0.009)	(0.016)	(0.005)	(0.009)
Q_4	-0.017	-0.009	0.062	-0.717	0.027	0.013	0.044	0.004	0.004	-0.054	-0.012	0.004
	(0.010)	(0.008)	(0.013)	(0.028)	(0.012)	(0.005)	(0.009)	(0.006)	(0.007)	(0.015)	(0.004)	(0.007)
Q_5	0.006	0.032	0.194	0.060	-0.964	-0.001	-0.044	0.062	0.025	-0.005	-0.046	-0.078
	(0.018)	(0.015)	(0.024)	(0.040)	(0.032)	(0.011)	(0.017)	(0.013)	(0.015)	(0.028)	(0.007)	(0.012)
Q_6	-0.107	0.111	-0.082	0.120	-0.020	-0.955	0.002	0.055	-0.001	0.002	-0.052	-0.094
	(0.038)	(0.023)	(0.053)	(0.079)	(0.050)	(0.034)	(0.038)	(0.033)	(0.030)	(0.060)	(0.020)	(0.029)
Q_7	-0.066	-0.334	-0.355	0.533	-0.279	-0.006	-1.246	0.212	0.278	-0.339	0.160	-0.165
	(0.075)	(0.060)	(0.095)	(0.153)	(0.091)	(0.042)	(0.101)	(0.061)	(0.058)	(0.118)	(0.036)	(0.051)
Q_8	-0.077	-0.028	-0.144	-0.156	0.114	-0.156	0.114	-1.195	0.027	0.115	-0.004	0.185
	(0.029)	(0.018)	(0.045)	(0.061)	(0.037)	(0.020)	(0.033)	(0.053)	(0.025)	(0.048)	(0.026)	(0.040)
Q_9	0.018	-0.040	-0.036	-0.101	0.060	-0.006	0.229	-0.007	-1.140	-0.036	-0.105	-0.228
	(0.047)	(0.036)	(0.066)	(0.100)	(0.063)	(0.027)	(0.048)	(0.038)	(0.053)	(0.074)	(0.025)	(0.036)
Q_{10}	-0.122	0.356	-0.110	-0.607	-0.045	-0.002	-0.180	0.193	-0.023	-0.666	0.005	-0.057
	(0.060)	(0.049)	(0.078)	(0.134)	(0.077)	(0.035)	(0.063)	(0.047)	(0.047)	(0.146)	(0.026)	(0.037)
Q_{11}	-0.051	-0.057	0.099	-0.373	-0.228	-0.050	0.105	-0.128	-0.094	-0.004	-0.812	-0.218
	(0.024)	(0.015)	(0.036)	(0.054)	(0.027)	(0.015)	(0.025)	(0.033)	(0.021)	(0.032)	(0.043)	(0.058)
Q_{12}	-0.087	-0.043	-0.131	-0.093	-0.269	-0.013	-0.002	-0.019	-0.013	-0.007	0.001	-0.994
	(0.002)	(0.001)	(0.004)	(0.003)	(0.006)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.004)	(0.008)

Gasoline and diesel fuel exhibit a predictable complementarity with nonfood goods, and a less obvious complementary relationship with the meat, dairy, oils, and fats group and other foods.

4. Summary and Conclusions

This study develops a framework to exploit the rich information content of longitudinal data in the estimation of large, disaggregated demand systems. Censoring of the dependent variables makes maximum likelihood estimation of these systems difficult with cross sectional data and infeasible for panels with even a small number of time periods. Therefore, a consistent and asymptotically efficient GMM estimator is developed to identify the parameters of an empirical specification consistent with the random utility hypothesis and flexible enough to nest a variety of different models of household heterogeneity. First, estimates of reduced form parameters are obtained from linear regressions and non-linear heteroscedastic Tobit models. The minimum distance estimator is then used to identify the underlying structural parameters, impose economic restriction on the model, and test for more restrictive specifications of the household specific effect. The most appropriate model allows the impact of the household specific random effect to vary over time, a generalization rarely tested for in the applied literature.

The estimation framework is well suited to modeling consumer demand patterns in transitional and developing countries where observed zero expenditure levels are typically due to economic non-consumption rather than IFP errors, provided the collection horizon of survey data is sufficiently long. Estimation results from Romania during a three-year period demonstrate the framework's use in characterizing consumers' demand for individual commodities within the context of a large, comprehensive demand system. Such estimates are crucial to the analysis of taxes and subsidies levied on individual commodities, when it is necessary to know both the own-price responsiveness of the good in question as well as cross substitution effects with other commodities making up the consumer's market basket.

Endnotes

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[1] These include Double Hurdle models where a separate binary censor is estimated jointly with a Tobit equation to capture IFP errors as well as zero expenditures due to corner solutions (Deaton and Irish; Blundell and Meghir).

[2] It is the distributional assumption that allows predictions for an out-of-sample household in the random effects model. If the distribution of the household specific effect is known, then so is the expected value of this effect conditional on the data, allowing its magnitude to be predicted with new data. Under fixed effects, however, there is no information on the household specific effect for an out-of-sample household.

[3] Under McElroy's formulation of the GEM model, the direct sub-utility function in could have been written as $V_2(q_{1jt} - \varepsilon_{1jt}, \ldots, q_{Njt} - \varepsilon_{Njt}; A_{jt}, c_j)$.

[4] A rare exception is the Perali and Chavas study where the heteroscedastic disturbances of their AIDS specification are treated through ML estimation of single equation heteroscedastic Tobit models.

[5] For example, shifting consumer attitudes towards second hand smoke will cause the marginal effect of changes in the number of young children in the household on the budget share for tobacco products to be time varying.

[6] To avoid confusion in the notation, total expenditures will always be referred to in logarithmic form as $\log x_{jt}$, while the vector x_t' is as defined above.

[7] Pashardes interprets the resulting bias as an omitted variable problem where the omitted variable is correlated with the regressors, while Buse (1994) shows an errors in variables model in which the bias cannot be corrected by instrumenting achieves the same result. A different interpretation is offered by Moschini, who demonstrates how the induced bias stems from the Stone index's lack of invariance to changes in the units of measure.

[8] The $vec(\cdot)$ operator stacks the columns of its argument, so the $\mathbf{\Pi}^*$ matrix is transposed to stack its rows.

[9] The term "quasi-maximum likelihood" has become more general since its use in the White reference. While our QLME approach falls into the class of GMM estimators, Yen, Lin, and Smallwood's quasi-maximum likelihood estimator does not.

[10] The exact length of $\boldsymbol{\psi}$ depends on number of restrictions placed on the $\boldsymbol{\Pi}^*$ matrix.

[11] This modification only impacts the calculation of censored price elasticities, which must be computed for each household in every time period and then averaged.

[12] In rare cases where no households in a given county purchase a commodity in the specified month, the median unit value is computed across a larger region and/or longer time period.

[13] Details on the construction of this CPI are given in Meyerhoefer (2001, p. 25). The approach taken to purge unit values of endogeneity is taken from Chen and Ravallion.

[14] During the first ten years of Romania's transition there were several changes to commodity tax rates, including the addition of new taxes and multiple rate changes on the same commodities. These are given a detailed review in Meyerhoefer (2001).

[15] Headcount estimates from Meyerhoefer (2001) put the percentage of the population living in poverty at 25 to 30 percent during the survey period, depending on the method used to compute the poverty line.

[16] One of the coldest winters in the century for Romania occurred during 1995, and the availability of gasoline and diesel fuel may have been restricted as petroleum resources where shifted to municipal authorities responsible for domestic heating.

[17] As an empirical check, the fuel equation was estimated using the heteroscedastic specification. Although the routine converged, the parameters in the variance function were very poorly identified.

[18] This does not mean identification of the total impact of the random effect on the budget shares is improved, only the identification of the individual parameters used to integrate out the random effect.

[19] We follow the suggested simplification of Edgerton *et al*. and Chalfant to calculate the standard errors while assuming the predicted budget shares are non-stochastic, in which case the elasticities of the uncensored equations reduce to linear combinations of the parameters. However, the derivatives of the censored elasticities with respect to the parameter vector used in the delta method calculation are still too complex to solve analytically. Therefore, they are solved numerically using a finite differences method.

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Appendix

Table A.1: Correlated Random Effects LAIDS Parameter Estimates (t-values in Parenthesis)

Commodity n	γ_{n1}	γ_{n2}	γ_{n3}	γ_{n4}	γ_{n5}	γ_{n6}	γ_{n7}	γ_{n8}	γ_{n9}
Bread	0.0330	0.0041	-0.0059	-0.0111	-0.0009	-0.0019	-0.0004	-0.0014	0.0009
	(20.39)	(4.53)	(-4.02)	(-4.31)	(-0.59)	(-2.93)	(-0.32)	(-1.66)	(1.02)
Grains	0.0041	-0.0018	0.0025	-0.0054	0.0020	0.0020	-0.0050	-0.0004	-0.0006
	(4.53)	(-1.27)	(2.31)	(-2.57)	(1.72)	(5.15)	(-5.45)	(-0.80)	(-0.83)
Fruits, Vegetables	-0.0059	0.0025	0.0037	0.0033	0.0130	-0.0014	-0.0043	-0.0023	0.0004
	(-4.02)	(2.31)	(1.27)	(0.99)	(6.64)	(-1.58)	(-2.91)	(-1.87)	(0.32)
Meat, Dairy, Oils, Fats	-0.0111	-0.0054	0.0033	0.0510	-0.0004	0.0023	0.0111	-0.0009	0.0001
	(-4.31)	(-2.57)	(0.99)	(6.84)	(-0.11)	(1.69)	(4.68)	(-0.52)	(0.04)
Other foods	-0.0009	0.0020	0.0130	-0.0004	0.0013	-0.0003	-0.0037	0.0047	0.0018
	(-0.59)	(1.72)	(6.64)	(-0.11)	(0.50)	(-0.39)	(-2.62)	(4.47)	(1.54)
Coffee	-0.0019	0.0020	-0.0014	0.0023	-0.0003	0.0008	0.00003	0.0010	-0.00002
	(-2.93)	(5.15)	(-1.58)	(1.69)	(-0.39)	(1.43)	(0.05)	(1.79)	(-0.04)
Beer	-0.0004	-0.0050	-0.0043	0.0111	-0.0037	0.00003	-0.0039	0.0036	0.0045
	(-0.32)	(-5.45)	(-2.91)	(4.68)	(-2.62)	(0.05)	(-2.45)	(3.83)	(5.07)
Wine	-0.0014	-0.0004	-0.0023	-0.0009	0.0047	0.0010	0.0036	-0.0057	0.00002
	(-1.66)	(-0.80)	(-1.87)	(-0.52)	(4.47)	(1.79)	(3.83)	(-3.79)	(0.03)
Liqueur	0.0009	-0.0006	0.0004	0.0001	0.0019	-0.00002	0.0045	0.00002	-0.0027
	(1.02)	(-0.83)	(0.32)	(0.04)	(1.54)	(-0.04)	(5.07)	(0.03)	(-2.68)
Tobacco products	-0.0031	0.0107	-0.0021	-0.0159	-0.0007	0.00004	-0.0053	0.0058	-0.0006
	(-1.74)	(7.59)	(-0.94)	(-4.05)	(-0.31)	(0.04)	(-2.87)	(4.29)	(-0.43)
Gasoline, Diesel fuel	0.0001	-0.0008	0.0051	-0.0037	-0.0039	-0.0009	0.0026	-0.0027	-0.0020
	(0.24)	(-2.27)	(6.62)	(-3.29)	(-6.47)	(-2.79)	(4.69)	(-3.60)	(-4.24)
Nonfoods	-0.0136	-0.0076	-0.0119	-0.0304	-0.0130	-0.0016	0.0006	-0.0018	-0.0019
	(-17.88)	(-15.27)	(-9.21)	(-15.51)	(-13.26)	(-3.10)	(0.79)	(-1.59)	(-2.78)

Table A.1 (Continued)

γ_{n10}	γ_{n11}	γ_{n12}	β_n	δ_{n2}	δ_{n3}	$\lambda_1^{price = n}$	$\lambda_2^{price = n}$	$\lambda_3^{price = n}$	$\lambda_1^{total\ exp.}$	$\lambda_2^{total\ exp.}$	$\lambda_3^{total\ exp.}$
-0.0031	0.0001	-0.0136	-0.0463	33.7886	21.0817	0.00011	0.00003	-0.00006	-0.00006	0.00000	-0.00006
(-1.74)	(0.24)	(-17.88)	(-71.94)	(1.20)	(1.18)	(1.19)	(1.11)	(-1.18)	(-1.20)	(-0.81)	(-1.20)
0.0107	-0.0008	-0.0075	-0.0174	-5.9503	3.7050	0.00024	0.00011	0.00009			
(7.59)	(-2.27)	(-15.27)	(-43.24)	(-1.02)	(0.80)	(1.20)	(1.18)	(1.18)			
-0.0021	0.0051	-0.0119	-0.0525	67.2856	46.1001	-0.00005	0.00015	0.00012			
(-0.94)	(6.62)	(-9.21)	(-55.50)	(1.19)	(1.17)	(-1.15)	(1.20)	(1.19)			
-0.0159	-0.0037	-0.0304	-0.0928	42.6104	-5.5242	-0.00008	-0.00019	-0.00002			
(-4.05)	(-3.29)	(-15.51)	(-63.88)	(1.11)	(-0.35)	(-1.13)	(-1.19)	(-0.74)			
-0.0007	-0.0039	-0.0130	-0.0196	-27.0519	11.3827	-0.00009	-0.00005	-0.00015			
(-0.31)	(-6.47)	(-13.26)	(-24.26)	(-1.12)	(0.98)	(-1.19)	(-1.12)	(-1.19)	σ_n	ζ_{1n}	ζ_{2n}
0.00004	-0.0009	-0.0016	0.0004	-14.5568	-7.7422	0.00003	-0.00003	-0.00002	0.0711	-0.1435	-0.3170
(0.04)	(-2.79)	(-3.10)	(1.11)	(-1.12)	(-1.02)	(1.18)	(-1.16)	(-1.14)	(26.96)	(-26.75)	(-39.37)
-0.0053	0.0026	0.0006	0.0097	-0.3790	10.8113	0.00001	-0.00005	0.00007	0.0852	-0.1059	-0.3248
(-2.87)	(4.69)	(0.79)	(18.87)	(-0.06)	(1.01)	(0.90)	(-1.18)	(1.19)	(25.62)	(-17.58)	(-33.78)
0.0058	-0.0027	-0.0018	0.0144	-24.7714	-26.7477	-0.00002	0.00002	0.00004	0.1665	-0.1326	-0.3679
(4.29)	(-3.60)	(-1.59)	(16.64)	(-1.11)	(-1.13)	(-1.17)	(1.13)	(1.18)	(28.33)	(-24.70)	(-49.42)
-0.0006	-0.0020	-0.0019	0.0077	10.3540	7.3969	-0.00003	0.00001	-0.00002	0.1386	-0.1679	-0.4578
(-0.43)	(-4.24)	(-2.78)	(16.54)	(1.08)	(1.01)	(-1.13)	(1.01)	(-1.13)	(29.38)	(-31.39)	(-52.54)
0.0100	0.0002	0.0009	0.0076	15.2579	16.5353	-0.00037	-0.00027	-0.00002	0.0983	-0.0993	-0.1816
(2.34)	(0.24)	(0.80)	(9.02)	(0.95)	(0.95)	(-1.17)	(-1.14)	(-0.89)	(24.99)	(-17.39)	(-22.38)
0.0002	0.0046	0.0013	0.0193	-242.991	-199.680	0.00001	0.00005	0.00034	0.0544		
(0.24)	(4.63)	(1.03)	(10.64)	(-1.20)	(-1.18)	(1.17)	(1.17)	(1.20)	(34.80)		
0.0009	0.0013	0.0788	0.2267	-56.7096	4.5283	0.00000	-0.00009	-0.00004			
(0.80)	(1.03)	(28.37)	(104.00)	(-1.09)	(0.20)	(0.61)	(-1.19)	(-1.16)			