DESIGNING NONPOINT SOURCE POLICIES WITH LIMITED INFORMATION ABOUT BOTH RISK ATTITUDES AND PRODUCTION TECHNOLOGY

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Designing Nonpoint Source Pollution Policies with Limited Information about Both Risk
Attitudes and Production Technology

by

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Abstract

A pollution reduction program is designed where information about both technology and risk preferences is asymmetric. Program costs and the distribution of payments depend on the amount of information known to the policy maker. Empirically testable conditions for self-selection are derived; the method is applied to reducing nitrate contamination.
Designing Nonpoint Source Pollution Policies with Limited Information about Both Risk Attitudes and Production Technology

Regulating nonpoint source pollution remains one of the most difficult challenges in agricultural environmental policy. Some recent studies have produced policy schemes with theoretical appeal, but no single proposal has emerged as the clear answer to the nonpoint problem in practice. This difficulty arises from a combination of two kinds of problems. First, pollution is unobservable and depends on many site-specific factors that vary spatially, implying that the socially efficient policy is potentially different for each farm. Second, the unavoidable production risk in agriculture means that the relationship between incentive policies and input decisions is complex. As Leathers and Quiggin (1991) have shown, a change in the price of a polluting input has an ambiguous effect on its use, and in general, the policy response cannot be predicted without knowledge of risk preferences as well as the effect of inputs on risk.

At the most basic level, many policy difficulties stem directly from inherent asymmetric information between farmers and regulators on differences in technology and risk preferences. At one extreme, the government could collect enough information to regulate farmers individually, whereby each farm’s production plan, including the use of all polluting inputs, would be approved and enforced by government officials. Yet this approach is usually considered too intrusive to be politically feasible, and is not consistent with the voluntary nature of past farm policies (Chambers, 1992). Further, it ignores the fact that information can only be gathered and used at some cost.

Accordingly, there have been recent investigations into incentive-based policies that allow producers to self-select appropriate regulations, where the government does not know (or does not employ its knowledge of) each farmer’s resources. In Wu and Babcock’s (1995, 1996)
policy setting, the government is aware of various types of farm technologies but cannot match them to individual farmers. Farmers choose among different levels of abatement, each in exchange for a payment that is set to induce each farmer to choose the abatement level designed for his type. Though Wu and Babcock’s proposal is conceptually promising, their model does not consider production risk and has not been adequately tested for empirical feasibility.¹

This paper is the first to develop a mechanism design for a self-selecting program that incorporates simultaneously asymmetric information about both technology and risk preferences, where both production and pollution potential differ by technology. Both production and pollution are stochastic, affected by uncontrollable random inputs such as weather. The analysis assumes that the government knows technology and risk attitudes differ, but does not know the exact distribution of types across the farm population. Following Wu and Babcock (1995; 1996) and Peterson and Boisvert (2001), differences in technology are modeled in the policy design by separating farmers into discrete groups.

The policy design is subject to several constraints, which require that farmers would be willing to participate in the program, and that each type of farmer will self-select the appropriate policy. If risk preferences differ and their distribution across farmers is unknown, the analyst’s difficulty is that the policy constraints cannot be evaluated. The unique feature of our program design is to incorporate asymmetric information over risk attitudes through stochastic efficiency rules on the distribution of net returns. By evaluating the participation and self-selection constraints in stochastic efficiency terms, we derive necessary and sufficient conditions for program feasibility, which can be used to identify situations where self-selecting payments cannot exist, as well as those where payments are guaranteed to exist. These conditions, which

¹ Segerson (1988) showed the policy incentives necessary for nonpoint source polluters to internalize uncertain levels of environmental damage.
apply for all risk-averse farmers, depend on the marginal risk effect of the polluting input, and can be implemented empirically from estimated production relationships for each technology type. We further demonstrate that the stochastic efficiency approach leads to a simplified empirical problem that can be solved with linear programming methods.

Though the model is applicable to any voluntary environmental program, we demonstrate it empirically for the case of nitrate leaching and runoff in New York. Besides illustrating the proposed methods, the application to New York allows the cost of an incentive-based program to be compared with the cost of other farm programs. Further, the proportion of program payments due to each separate dimension of asymmetric information can be isolated. One interpretation of these cost differences is the value of information to the government, since they represent the most that could be rationally spent to collect more information on farmers.\(^2\) Alternatively, they are the taxpayers’ cost of allowing farmers to choose their own policies even though the government has enough information to assign regulations.

Below, a discussion of the proposed theoretical framework is followed by sections that describe the use of stochastic efficiency criteria and the empirical application to New York.

**Theoretical Framework**

Following Leathers and Quiggin (1991), we consider a farmer who must choose an input that affects both output and environmental quality in a random setting. Letting \(\theta \in \Theta\) represent an index on technology type, profits for a farmer with technology \(\theta\) are \(\pi^\theta(x, b_\theta) \equiv p_y y_\theta(x, b_\theta) - p_b b_\theta\), where \(p_y\) is the price of output, \(y_\theta\) is the technology-specific production function, \(x\)

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\(^2\) Rapid advances in information and GIS technologies will continue to bring down the cost of collecting data about differences in production practices and the quality of land and other resources at the farm level. With respect to risk attitudes, the most we could ever expect is to narrow the range, with little hope of knowing how preferences are actually distributed across the farm population. Another advantage of our model is the capacity to estimate the reductions in program costs if risk preferences are known to lie within some specific range.
represents a random input beyond the farmer’s control, \( b_0 \) is the controllable input with price \( p_b \). Emissions of pollution \( e \) are a function of the same inputs, so that \( e = g^\theta(x, b_0) \). For common cases of agricultural pollution such as runoff, soil and topographic conditions define the technologies in \( \Theta \), \( x \) may be uncertain weather or pest outcomes, and \( b_0 \) is a polluting input such as fertilizer.\(^3\) Let the support of \( x \) be the interval \([x, \bar{x}]\), and assume that \( x \) and \( b_0 \) are defined in such a way that \( \pi^x_\theta \geq 0 \) and \( \pi^b_\theta \geq 0 \) for all \( \theta \).

Farmers are assumed to maximize the expected utility of profit. A farmer with a von Neumann-Morgenstern utility function \( u \) and technology of type \( \theta \) will select \( b_\theta \) by solving the problem: \( \max E u(\pi^\theta(x, b_\theta)) \), where \( E \) is the expectation with respect to \( x \). Assume the function \( u \) belongs to a known set \( \Omega \) of continuous real-valued functions. Assume also that a solution to the farmer’s problem exists, and denote it \( b_\theta^0 \). If emissions are a negative externality, this unregulated input level, and therefore \( e \), exceeds the socially optimal level; suppose the government therefore wishes to implement \( b_\theta^* \leq b_\theta^0 \) as a regulation on technology \( \theta \).\(^4\)

To implement a different regulation on each technology through self-selection, the government must in effect devise a policy “menu,” where each item on the menu is a regulation on \( b \) with a corresponding compensation payment. Such a scheme can be viewed as a two-staged game of imperfect information, where the government chooses a set of policies in the first stage, and farmers select from these policies in the second stage (Smith and Tomasi, 1999).\(^5\) The

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\(^3\) Alternatively, \( b_0 \) may be a discrete variable that represents some production practice such as conservation tillage.

\(^4\) Because of the difficulties in estimating the social cost of pollution, it is generally not possible to determine a socially optimal regulation. In practice, this choice is usually made through the standards approach, where the government sets some maximum emission level based on scientific judgement, and determines a regulation on \( b_\theta \) so that the environmental standard is met (Baumol and Oates, 1988). Where environmental outcomes are uncertain, Lichtenberg and Zilberman (1988) show that an efficient way of setting these regulations is through chance constraints, whereby the probability of exceeding some severe level of pollution is restricted.

\(^5\) Here, there is imperfect information because the government knows the various types of farmers (i.e., the elements of \( \Theta \) and \( \Omega \)) but cannot assign individual farmers to these types. In general, this game involves the government and
government must solve this game by backward induction; it must determine how a farmer with each technology would respond to various combinations of payments and regulations, and then incorporate these responses in assigning a payment $s_\theta$ to the environmentally “safe” level of input $b_\theta^*$. The goal is to set each payment so that farmers of type $\theta$ choose the policy $(b_\theta^*, s_\theta)$ but those with technology $\theta'$ select $(b_{\theta'}^*, s_{\theta'})$.

If the government’s objective is to minimize the cost of implementing the regulations $b_\theta^*$ through self selection, it must solve the problem:

$$\min_s \sum_{\theta \in \Theta} a_\theta s_\theta$$

subject to:

$$Eu(\pi^\theta(x, b_\theta^*) + s_\theta) \geq Eu(\pi^\theta(x, b_\theta) + s_\theta) \quad \forall \theta \in \Theta, \forall u \in \Omega$$

(1)

$$Eu(\pi^{\theta'}(x, b_{\theta'}^*) + s_{\theta'}) \geq Eu(\pi^{\theta'}(x, b_{\theta'}^*) + s_{\theta'}) \quad \forall \theta, \theta' \in \Theta, \forall u \in \Omega$$

(2)

where $s$ is the vector containing the payments $s_\theta$, and $a_\theta$ is the number of producers with technology $\theta$. The first set of constraints (equation (1)) guarantees participation in the program. For each $\theta$, post-policy expected utility must exceed its pre-policy level for all permissible utility functions. The second set of constraints (equation (2)) is for self-selection; a farmer with soil $\theta$ and utility function $u$ must prefer the policy $(b_\theta^*, s_\theta)$ to $(b_{\theta'}^*, s_{\theta'})$ for all $\theta' \neq \theta$.

**Stochastic Efficiency Representation**

In the formulation above, the participation and self-selection conditions on each technology must be met for every utility function in $\Omega$. If all farmers have identical risk all producers, so that any farmers’ choice may depend strategically on the choices of all other farmers. If all policy options are available to any farmer regardless of others’ choices, this strategic interdependence can be ignored and the policy becomes a large number of two-player games between the government and each producer (Xepapadeas, 1997). In the language of the literature, this game can be interpreted either as an adverse selection model (where the government seeks to screen farmers based on their type), or as a principal-agent model with hidden information (the utility function and technology are known only to the farmer-agent). In either case, there is no hidden action; once farmers choose a level of regulation/compensation, it is assumed that their actions can be monitored and enforced.
preferences, $\Omega$ has a single element and the problem is one of finding separate policies on the basis of technology alone. If $\Omega$ contains many elements then a feasible policy could only be found by evaluating the constraint for each utility function, an infinite number of computations in the plausible case where each element of $\Omega$ is a point on the continuum of absolute risk aversion coefficients. The only way to avoid such an enumeration is through general criteria that imply the preference of $s_\theta$ over $s_{\theta'}$ for all relevant utility functions.

Stochastic efficiency criteria provide exactly the simplification required. For several specifications of $\Omega$, the statement that $Eu(m) \geq Eu(m')$ for all $u \in \Omega$ can be equivalently expressed by a single stochastic efficiency condition on the distributions of $m$ and $m'$. A particularly useful such rule is that of second-degree stochastic dominance (SSD). A cumulative distribution $G(m)$ dominates $H(m')$ by SSD if and only if the area under $G$ is nowhere more than that of $H$ and somewhere less than the area under $H$:

$$\int_{-\infty}^{\hat{m}} G(m) dm \leq \int_{-\infty}^{\hat{m}} H(m') dm'$$

for all $\hat{m}$, with strict inequality somewhere. Geometrically, this condition means two things: first, $G$ must start to the right of $H$ (i.e., the first non-zero point on $G$ must be larger than the first nonzero point on $H$), and second, the whole distribution $G$ must lie further to the right, in the sense that the accumulated area underneath it must be smaller. Hadar and Russel (1969) discovered that dominance by SSD is equivalent to greater expected utility for all utility functions that are increasing and concave; the SSD rule separates attractive alternatives from unattractive ones for all risk-averse decision-makers who prefer more to less.\(^6\)

\(^6\)Formally, if $G(m)$ dominates $H(m')$ by SSD, then $Eu(m) \geq Eu(m')$ for all $u(\cdot)$ such that $u' > 0$ and $u'' < 0$. Other stochastic efficiency criteria exist for other specifications of the utility set $\Omega$. First-degree stochastic dominance (FSD) assumes only that utility is increasing. Third degree stochastic dominance (TSD) applies for all utility
Here, define income $m$ to be the sum of profit and government payments, and denote the cumulative distribution function (cdf) of income for farmers with technology $\theta$ (or “group $\theta$”) as:

$$F_\theta(m; b, s) \equiv \Pr\{ \pi^\theta(x, b) + s \leq m \}$$

This definition says there is a distribution $F_\theta$ conditional on each combination of $b$ and $s$. Let $m$ and $\bar{m}$ represent the lowest and highest levels of $m$ with nonzero probabilities, respectively. To illustrate the use of SSD in the policy scheme, consider two groups; i.e., $\Theta = \{1, 2\}$. In this situation, the government must choose payments $s_1$ and $s_2$ to implement the input standards $b_1^*$ and $b_2^*$. The constraints (1) and (2), written in terms of SSD, require that $s_1$ and $s_2$ satisfy:

$$F_1(m; b_1^*, s_1) \succ F_1(m; b_1^0, 0), \quad F_2(m; b_2^*, s_2) \succ F_2(m; b_2^0, 0); \quad (4)$$

$$F_1(m; b_1^*, s_1) \succ F_1(m; b_2^*, s_2), \quad F_2(m; b_2^*, s_2) \succ F_2(m; b_1^*, s_1) \quad (5)$$

where “$\succ$” denotes dominance by SSD. The constraints in (4) state that payments must be selected so that farmers in both groups prefer to participate in the program— the post policy distribution $F_\theta(m; b_\theta^*, s_\theta)$ must dominate the pre-policy distribution $F_\theta(m; b_\theta^0, 0)$ for both groups. The constraints in (5) are the self-selection conditions. For farmers in group 1, the distribution under their “own” policy $(b_1^*, s_1)$ must dominate the distribution under the other policy $(b_2^*, s_2)$; a parallel interpretation applies to the constraint for group 2. If all the constraints are met, any risk-averse farmer in group $\theta$ will choose the policy $(b_\theta^*, s_\theta)$ over other alternatives.

functions that are increasing, concave, and have a positive third derivative (Whitmore, 1970). Meyer (1977) has discovered a set of criteria, named stochastic dominance with respect to a function (SDRF), that can order distributions when the Arrow-Pratt coefficient of absolute risk aversion of the utility function lies in a specified range. While policy rules could conceivably be developed for any of these cases, SSD is the most general rule that is applicable to agriculture. Empirical evidence suggests that farmers are risk-averse, but the degree of risk aversion and other properties of the utility function vary across studies. Moreover, as will be shown below, the the SSD ranking can be conveniently calculated from estimated production relationships.
Figure 1. Geometry of Policies that Satisfy SSD

A set of distributions that satisfy the policy constraints for group 1 is illustrated in Figure 1. The cdf labeled $F_1^0$ represents the pre-policy distribution $F_1(m; b_1^0, 0)$, where polluting inputs are set at $b_1^0$ and farmers receive no payment. If farmers are forced to reduce inputs to $b_1^*$ to meet environmental standards, the distribution of income shifts to the left because returns are smaller at every realization of the random input $x$; this distribution is labeled $F_1^*$ in the figure. If farmers now receive a nonrandom payment $s_1$ along with the regulation on $b_1$, their income distribution shifts to the right in a parallel fashion. To meet the participation constraint (4), the distribution must shift far enough so that it is preferred to $F_1^0$ by SSD. In the figure, the payment $s_1$ shifts the income distribution to $F_1^{**}$, which dominates $F_1^0$. The self-selection condition (5) imposes the additional requirement that $F_1^{**}$ dominate the distribution under group 2’s policy, $F_1(m; b_2^*, s_2)$. This distribution is labeled $F_1'$ in the figure and lies to the left of $F_1^{**}$; the self-selection condition is therefore met. If $F_1'$ lay to the right of $F_1^{**}, s_1$ would have to be enlarged.
Properties of the Solution

While the SSD formulation is conceptually appropriate and convenient, the conditions are too complex to solve the problem explicitly. To simplify it, we rely on the concept of simply related random variables (Hammond, 1971). Two random variables are simply related if their cdf’s cross at most once. Each of the SSD conditions in (4) and (5) compares some random variable of the form \( m = \pi^\theta(x, b) + s \) to another random variable \( m' = \pi^\theta(x, b') + s' \), where without loss of generality \( m \) represents returns at the lower input level (i.e., \( b < b' \)). We show in the appendix that the cdf’s of these random variables (\( F_m \) and \( F_{m'} \), respectively) can intersect only once, for any combination of \((b, s)\) and \((b', s')\). Formally:

**Result 1:** If the marginal value product of \( x \) is positive (i.e., \( \pi^\theta_x > 0 \)) and is monotonic in \( b \) (i.e., either \( \pi^\theta_{xb} > 0 \) or \( \pi^\theta_{xb} < 0 \) for all \( b \)), then \( F_m \) and \( F_{m'} \) intersect at most once. If the two distributions do cross, then \( F_{m'} \) intersects \( F_m \) from above (from below) if and only if \( \pi^\theta_{xb} > (<) 0 \).

Intuitively, the simply related property follows from the one-to-one correspondence between \( x \) and \( m \): each realization of the random variable \( m \) is associated with a unique value of \( x \), and larger \( m \)’s are associated with larger \( x \)’s because \( \pi^\theta_x > 0 \). If \( b \) is increased and \( \pi^\theta_{xb} > 0 \), then a given change in \( x \) causes a larger change in \( m \), so that the distribution \( F_{m'} \) is a “stretching” of \( F_m \). Such an increase in \( b \) means that \( F_{m'} \) is flatter than \( F_m \), as well as lying further to the right, since \( \pi^\theta_b > 0 \) (see Figure 2). The opposite case is where \( \pi^\theta_{xb} < 0 \), so that an increase in \( b \) “squeezes” the cdf and \( F_{m'} \) is steeper than \( F_m \) (Figure 3).

As suggested by the titles of the figures, the stretching/squeezing of the cdf’s in the case of simply related random variables is linked to the definition of relative riskiness proposed by Rothschild and Stiglitz. In particular, \( m' \) is defined to be riskier than \( m \) if the following set of equivalent conditions is met:
Figure 2. Distributions of Income Where $b$ is Risk-Increasing

Figure 3. Distributions of Income Where $b$ is Risk-Decreasing
1. If constants are added to $m$ and $m'$ so that their means are equal, then distribution of $m$ dominates the distribution of $m'$ by SSD.

2. The random variable $m'$ is $m$ plus “noise.”

3. The distribution of $m'$ has more weight in its tails than that of $m$.

Following Ramaswami (1992), define $b$ to be a *risk-increasing (risk-decreasing) input* if $m' = \pi^\theta(x, b') + s'$ is riskier (less risky) than $m = \pi^\theta(x, b) + s$ (where $b < b'$).

By Result 1, the two cdf's can cross only once, and at the intersection point one cdf must be steeper than the other. The next result relates technological conditions to riskiness (proof in appendix):

**Result 2:** If $\pi^\theta_{xb} > (<) 0$, then $b$ is a *risk-increasing (risk-decreasing) input*.

If $\pi^\theta_{xb} > 0$ then the distribution $F_{m'}$ is flatter than $F_m$, so that $F_{m'}$ intersects $F_m$ from above, as shown in Figure 2. In this case, if constants are added to the random variables so that the means of the two distributions are equal, then $F_m$ dominates $F_{m'}$ by SSD because area $A$ equals area $B$. If this is the case, Rothshild and Stiglitz’s proposition implies that $m'$ is riskier than $m$, which means that $b$ is a risk-increasing input.

In general, there are only two types of SSD constraints in the problem: the distribution $F_\theta(m, b^*_\theta, s_\theta)$ must dominate the pre-policy distribution $F_\theta(m; b^{0}_\theta, 0)$, as well as the distribution $F_\theta(m; b_\theta, s_\theta)$. Both of these constraints are of the form: $F_\theta(m, b^*_\theta, s_\theta) \succ F_\theta(m; b', s')$. There are two necessary conditions for any SSD condition to be satisfied (even if the distributions are not simply related): neither the mean of the dominant distribution nor its lowest observation may be smaller than those of the other distribution (Anderson et al., 1977). For the constraints in question, these requirements are: (1) $E[\pi^\theta(x, b^*_\theta) + s_\theta] \geq E[\pi^\theta(x, b') + s']$, and (2) $\pi^\theta(x, b^*_\theta) + s_\theta \geq$
The next result says that if the two variables are simply related, then one or the other of these conditions will also be sufficient for SSD, depending on which variable is riskier:

**Result 3:** If \( \pi^0(x, b^*_0) + s_0 \) is riskier (less risky) than \( \pi^0(x, b') + s' \), then the necessary and sufficient condition for the first distribution to dominate the second by SSD is:

\[
s_0 - s' \geq \pi^0(x, b^*_0) - \pi^0(x, b'_0) + (E\pi^0(x, b') - E\pi^0(x, b'_0))
\]

Geometrically, the riskier variable has a ‘flat’ cdf. If the first variable is riskier, then the only requirement for SSD is that the lowest observation be larger (\( m \geq m' \)); its flatter shape means that the cdf of \( m \) will lie to the right of \( m' \) for all realizations greater than \( m \) (Figure 4). If \( m \) is less risky, then the only requirement is for the expected value of \( m \) to be larger (\( Em \geq Em' \)), and the cdf of \( m \) will intersect that of \( m' \) from below (Figure 3). In both cases, the remaining necessary condition is automatically met.

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7 The two necessary conditions are derived by letting \( \hat{m} \) in equation (3) grow arbitrarily large and small, respectively. As \( \hat{m} \rightarrow \infty \), SSD implies that \( Em \geq Em' \); the mean of \( m \) must be no less than the mean of \( m' \). For “small” values of \( \hat{m} \), the SSD requires that the lower tail of \( F_m \) lie to the right of \( F_{m'} \).
The results above allow the SSD conditions, each of which implicitly includes an infinite number of constraints, to be written in terms a small number of inequalities. With this simplification, the policy problem in equations (1) and (2), for the case of two technologies, becomes:

\[
\min a_1 s_1 + a_2 s_2 \quad \text{(6)}
\]

Subject to:

\[
E \pi^1(x, b_1^*) + s_1 \geq E \pi^1(x, b_1^0); \quad \pi^1(x, b_1^*) + s_1 \geq \pi^1(x, b_1^0) \quad (P_1)
\]

\[
E \pi^2(x, b_2^*) + s_2 \geq E \pi^2(x, b_2^0); \quad \pi^2(x, b_2^*) + s_2 \geq \pi^2(x, b_2^0) \quad (P_2)
\]

\[
E \pi^1(x, b_1^*) + s_1 \geq E \pi^1(x, b_2^*); \quad \pi^1(x, b_1^*) + s_1 \geq \pi^1(x, b_2^*) + s_2 \quad (I_1)
\]

\[
E \pi^2(x, b_2^*) + s_2 \geq E \pi^2(x, b_1^*); \quad \pi^2(x, b_2^*) + s_2 \geq \pi^2(x, b_1^*) + s_1 \quad (I_2)
\]

In its most general form this problem has eight constraints, but the results above imply that only four of them are operative. Exactly which constraints are relevant depends on whether \( b \) is risk-increasing or risk-decreasing for each group, and whether \( b_1^* \) is larger or smaller than \( b_2^* \) (\( b_0^0 > b_0^* \) by assumption).

Table 1 lists the four possible cases for which we must establish conditions that ensure the existence of self-selecting policies and associated payments. Without loss of generality, assign the index \( \theta = 1 \) to the more polluting group, so that \( b_1^* < b_2^* \); i.e., group 1 must apply less input to meet some environmental standard. To see how some constraints can be ignored, consider the first case where \( b \) is risk-increasing for both groups (\( \pi_{\theta b}^0 > 0 \) for \( \theta = 1, 2 \)). Here, only the first constraint in \( (P_1) \) must hold because \( \pi^1(x, b_1^*) + s_1 \) is less risky than \( \pi^1(x, b_1^0) \); the other constraint is then automatically met (Result 3). Similarly, only the first condition in \( (P_2) \) is relevant. Self-selection for group 1 requires only the first condition in \( (I_1) \), since \( b_2^* > b_1^* \). On the other hand, group 2’s self-selection condition is the second in \( (I_2) \) because \( \pi^2(x, b_2^*) + s_2 \) is riskier than \( \pi^2(x, b_1^*) \).
<table>
<thead>
<tr>
<th>Case</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Necessary and Sufficient Condition for Payments to Exist</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk Increasing</td>
<td>Risk Increasing</td>
<td>( \pi^2_b(x, b) \geq E\pi^1_b(x, b) )</td>
</tr>
<tr>
<td>2</td>
<td>Risk Decreasing</td>
<td>Risk Decreasing</td>
<td>( E\pi^2_b(x, b) \geq \pi^1_b(x, b) )</td>
</tr>
<tr>
<td>3</td>
<td>Risk Decreasing</td>
<td>Risk Increasing</td>
<td>( \pi^2_b(x, b) \geq \pi^1_b(x, b) )</td>
</tr>
<tr>
<td>4</td>
<td>Risk Increasing</td>
<td>Risk Decreasing</td>
<td>( E\pi^2_b(x, b) \geq E\pi^1_b(x, b) )</td>
</tr>
</tbody>
</table>

In general, the four operative constraints define a linear programming problem with a feasible region in \( s_1 - s_2 \) space. As shown in Figure 5, \((P_1)\) constrains the choice of \( s_1 \) to lie on or to the right of the line at \( \hat{s}_1 \) (which equals either \( E\pi^1(x, b_1^0) - E\pi^1(x, b_1^*) \) or \( \pi^1(x, b_1^0) - \pi^1(x, b_1^*) \)), and \((P_2)\) constrains \( s_2 \) to lie on or above the line at \( \hat{s}_2 \). Once \( b_1^* \) and \( b_2^* \) and the profit functions are known, \((I_1)\) and \((I_2)\) take the form \( s_1 - s_2 \geq A \) and \( s_1 - s_2 \leq B \), respectively, where \( A \) and \( B \) involve either expectations or minimum observations. The appendix contains a complete description of the policy problem in each of the four cases, along with its solutions.

Existence of a solution in each case requires the feasible region to be nonempty (the shaded area in Figure 5). In general, this will be true as long as \( \hat{s}_1 \) and \( \hat{s}_2 \) are finite and \( A \leq B \). The first of these conditions holds by assumption, while the second depends on the technologies of the two groups. Table 1 reports these existence conditions for the four cases in terms of the derivatives of \( \pi^1 \) and \( \pi^2 \). In case 1, for example, the feasible region is nonempty if \( E\pi^1(x, b_2^*) - E\pi^1(x, b_1^*) \leq \pi^2(x, b_2^*) - \pi^2(x, b_1^*) \). For this to hold for an arbitrary choice of \( b_1^* \) and \( b_2^* \), the two profit functions must satisfy the condition \( E\pi^1_b(x, b) \leq \pi^2_b(x, b) \).
To interpret these existence conditions, note that all of them require some measure of group 2’s loss in returns (either in terms of the mean or the lower tail of the distribution) to exceed group 1’s loss. That is, for self-selection to be possible, the less polluting technology is required to be more “productive” in a stochastic sense. This requirement is consistent with Wu and Babcock’s result for the deterministic case, and an instance of the more general “single-crossing property” encountered in the literature (Mas-Collel et al., 1995). Given estimates of $\pi^1$ and $\pi^2$ and observations of $x$, the condition for any case can be checked empirically by comparing $E\pi^2_b$, $E\pi^1_b$, $\pi^2(x, b)$, or $\pi^1(x, b)$.

*Setting Policy Based on Soils Information*

In Figure 5, the cost function $a_1s_1 + a_2s_2$ is minimized at the intersection of the constraints $(P_2)$ and $(I_1)$ or point $d$. As the figure makes clear, this point is always the optimal solution as
long as \((I_1)\) intersects \((P_2)\) to the right of \((P_1)\); i.e., if point \(d\) lies to the right of point \(c\). The case-by-case solutions show that this is always true under the maintained assumptions in the model. Therefore, constraints \((P_2)\) and \((I_1)\) are binding while \((P_1)\) and \((I_2)\) are nonbonding. These facts mean that group 2 will be just as well off with the policy as without it \((P_2\) binds), but group 1 will be strictly better off \((P_1\) is nonbonding).

If the existence conditions for self-selecting payments are not met, then a policy at point \(d\) is impossible. Depending on available information, the alternatives are a uniform policy for all farms, or else assigned policies that differ by group. For the second alternative to be feasible, the government must have enough information to classify each farm. Since the government would assign the policies in this case, \((I_1)\) and \((I_2)\) can be ignored, and the minimum cost payments are the policy at point \(c\), where \(s_1 = \hat{s}_1\) and \(s_2 = \hat{s}_2\); farmers in both groups would be just as well off after the program as before.

Even if the self-selecting policies are possible, the government may still choose to assign policies because payments to farmers would be smaller (point \(c\) is cheaper than point \(d\)). There is a trade-off between government cost, on the one hand, and the amount of autonomy farmers can be given in selecting policies, on the other.

Information is valuable because program costs can be reduced if the government knows more about farmers’ technology. Yet this assumes that information on risk attitudes is fixed: the government knows only that farmers are risk-averse. While discovering every farmer’s risk attitude is unrealistic, several empirical studies have estimated risk coefficients of absolute risk aversion (CARA)\(^8\) from cross-sectional data, and collectively these studies represent a plausible set of utility functions that is smaller than the set assumed for SSD. Information on risk attitudes

\(^8\) The Arrow-Pratt coefficient of absolute risk aversion is defined as \(r(m) = -u’(m)/u”(m)\), and is positive for risk-averse decision-makers.
comes in the form of a narrower range of risk attitudes, and a procedure for valuing this information is the topic of following section.

**Valuing Information on Risk Attitudes**

Hammond also proved a result that allows various ranges in risk attitudes to be incorporated (Corollary 3-1, p. 1058): For two simply related random variables \( m \) and \( m' \), suppose that \( E[-\exp(-rm)] \geq E[-\exp(-rm')] \). If \( m \) is more (less) prone to low outcomes than \( m' \), then \( Eu(m) \geq Eu(m) \) for all \( u \) such that \( -u''(m)/u'(m) \leq (\geq) r \). Geometrically, \( m \) is more prone to low outcomes than \( m' \) if the cdf of \( m \) is first lies above the cdf of \( m' \) as the horizontal axis is scanned from left to right. Because this definition can only be met if the cdf of \( m \) is flatter, it is equivalent to \( m \) being riskier than \( m' \) by the definition above. The negative exponential utility function \(-e^{-rm}\) assumes that the CARA is constant and equal to \( r \). The corollary therefore says that if the riskier variable is preferred for a decision maker with a CARA of \( r_1 \), then it is also preferred for decision makers who are less risk-averse. Conversely, if the less risky variable is preferred with a CARA of \( r_0 \), then it is also preferred for decision makers who are more risk-averse.

If farmers’ CARA \( r(m) \) are known to be bounded within the range \([r_0, r_1]\), the policy problem can be written:

Minimize \( a_1 s_1 + a_2 s_2 \)  

Subject to:  
\[
E[-e^{-\eta_0 \pi^1(x, b_0^1)+s_1}] \geq E[-e^{-\eta_1 \pi^1(x, b_0^1)+s_1}] \geq E[-e^{-\eta_1 \pi^1(x, b_0^1)+s_1}]
\]

\[
(P_1)
\]

\[
E[-e^{-\eta_0 \pi^2(x, b_2^1)+s_2}] \geq E[-e^{-\eta_1 \pi^2(x, b_2^1)+s_2}] \geq E[-e^{-\eta_1 \pi^2(x, b_2^1)+s_2}]
\]

\[
(P_2)
\]

\[
E[-e^{-\eta_0 \pi^1(x, b_2^1)+s_1}] \geq E[-e^{-\eta_0 \pi^1(x, b_2^1)+s_1}] \geq E[-e^{-\eta_0 \pi^1(x, b_2^1)+s_1}]
\]

\[
(I_1)
\]
As above, only one constraint in each pair will be operative depending on the distribution of $x$. If the analytical form of this distribution allows the expected utility values to be written explicitly, the problem can be solved analytically. Otherwise, the expectations can be estimated from a random sample of observed values of $x$. The SSD formulation is the special case where $[r_0, r_1] = [0, \infty)$; in the opposite extreme where all farmers have an identical and constant CARA, then $r_0 = r_1$, and the constraints in each pair are redundant. Given estimates of the profit functions $\pi^1$ and $\pi^2$, payments can be calculated under various assumed limits $r_0$ and $r_1$ to trace out the relationship between better knowledge of risk attitudes and government cost.

### Empirical Application to Nitrate Loss from New York Corn Production

The model is applied empirically to the nitrate leaching and runoff problem from corn production in New York. Much of New York is predominated by multi-crop dairy farms, with about 30% of cropland devoted to corn production annually. Due in part to the use of nitrogen fertilizer on corn acreage, nitrate concentrations in some drinking water supplies have risen above their natural background levels. Differences in topography, farming practices, and soil conditions throughout the state imply that efficient limits on nitrogen application would differ. The objectives of the empirical analysis are: (1) to determine whether a self-selecting program of regulations on nitrogen fertilizer can be implemented, (2) if so, to estimate the cost of such a program, and (3) to discuss the implications for policy design if a self-selecting program cannot be identified.
In this empirical model, two specific soils (indexed by $\theta = 1, 2$) are chosen to represent different technologies, from Hydrologic groups A and B, respectively. Because these soils generate different amounts of nitrate residuals \textit{ceteris paribus}, the limits on fertilizer that meet environmental standards also differ. Whether these distinct regulations can be self-selected depends on the distribution of yields, and therefore net returns, from corn production on the two soils. Production and nitrate residuals are both random because they depend on unpredictable weather variables.

To study the effects of asymmetric information about risk attitudes, three alternative specifications are explored: (1) the government knows only that farmers are risk averse, (2) all farmers are risk neutral, and (3) risk-aversion coefficients are known to lie in a specified range based on information from previous empirical studies. By considering situation 2, we can determine the extent to which payment levels are set inappropriately if voluntary policy designs are based on the assumption that farmers are risk neutral, when in fact this may not be so. For each of these three alternatives, we assume initially that the information on soil types is also asymmetric, but we also present results for the symmetric case to determine the value of soils information. Before presenting the policy simulations and the procedure for finding payments in all the cases, the estimated yield functions, the pollution functions, and nitrogen standards are described.

\textit{The Yield Functions}

Data to estimate the yield functions are from field trials conducted by the Department of Soil, Crop, and Atmospheric Sciences at Cornell University. These field trials include 276

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9 Hydrologic group is a classification of soils based on their capacity to permit infiltration. Group A soils are generally lighter and more vulnerable to leaching than B or C soils (Thomas and Boisvert). As in other dairy producing regions, farmland in central New York is primarily made up of heavy soils situated on hillsides. Accordingly, the National Resources Inventory estimates a low proportion of A soils throughout the state.
observations of corn silage yield \((y)\), commercial fertilizer, and manure application at several sites around New York over several crop years; 52 of these observations are from group 1 soils and 224 are from group 2. To obtain a variable that represents total nitrogen applied \((N)\), manure was credited with 3 lbs. of nitrogen per ton and combined with the nitrogen in commercial fertilizer. The data were augmented with observations of rainfall in the growing season \((w)\), defined as accumulated precipitation from April through September, from weather stations near the experimental sites. Table 2 provides the descriptive statistics for the final data set.

To gain efficiency, the functions were estimated in a pooled regression using a quadratic specification. The model was fit by maximum likelihood, with the parameters bounded so that the derivatives in \(N\) and \(w\) are positive for both groups over the relevant range of the regressors. The estimated equation is:

\[
y = -15.12 + 0.699d_m + 25.71d_s + 0.1001N - 0.00024N^2 + 0.000057d_sN^2 + 1.51w \\
\quad -1.37d_s w - 0.0007Nw,
\]

where \(t\)-ratios are in parentheses, and \(d_m\) and \(d_s\) are dummy variables for manure application and soil group, respectively \((d_s = 1\) for group 2\). The interaction terms \(d_sN^2\) and \(d_s w\) allow the shape of the yield function in nitrogen and rainfall to differ by group. The estimated coefficients all have theoretically expected signs, and the fit also appears adequate. The estimated coefficients on \(d_sN^2\) and \(d_s w\) are both statistically different from zero, and their signs imply that group 2 has a higher marginal product of nitrogen but a smaller marginal product of rainfall. If weather is random, the negative coefficient on the interaction term \(Nw\) suggests that nitrogen is a risk-decreasing input for both groups, though the parameter was not estimated with great precision \((t = -1.40)\).
Evaluating the functions at average rainfall and nitrogen (20.9 in., and 131 lb./acre, respectively), a one-pound increase in nitrogen increases yield by 0.023 tons (45 lb.) and 0.038 tons (76 lb.) per acre for groups 1 and 2, respectively, while a one-inch increase in rainfall raises yield by 1.42 tons and 0.05 tons, respectively.

The Pollution Functions and Input Standards

Pollution is defined here as total nitrate loss (the sum of leaching and runoff) per acre of cropland. The pollution function is an estimated recursive system from Boisvert et al. (1997) that relates nitrate leaching and runoff to nitrogen from manure and commercial fertilizer,5 soil characteristics, and weather conditions. In logarithmic form, the predicted levels of leaching ($L$) and runoff ($R$) are given by:

$$\ln R = -4.40 - 0.569d_1 - 0.490d_2 + 0.628\ln N + 0.652\ln W_1 + 0.089\ln W_2$$
$$+ 0.023(\ln W_2)^2 + 0.005(\ln W_3)^2$$

$$\ln L = -75.33 + 38.31d_1 + 37.56d_2 - 6.74\ln R + 2.12(\ln R)^2 + 4.82\ln N + 5.77\ln W_1$$
$$+ 0.056(\ln W_2)^2 + 0.363\ln R \ln W_2 + 0.256\ln W_3 + 0.094(\ln W_3)^2 + 0.039\ln W_4$$

where $d_1$ and $d_2$ are dummy variables for groups 1 and 2, respectively, $N$ is total nitrogen applied, $W_1$ is total annual rainfall, and $W_2$, $W_3$, and $W_4$, are rainfall within 14 days of planting, fertilizer, and harvest, respectively.

For consistency, all three policy models find payments to implement the same restrictions on nitrogen use. To determine these restrictions, “environmentally safe” nitrogen levels are computed based on the leaching and runoff equations using chance constraints (Lichtenberg and Zilberman, 1988). Chance constraints require the regulation $N_0^*$ to satisfy

$$\Pr\{R(N_0^m + N_0^*, C_0, ...)$$

5 Throughout the empirical model, the policy variable is commercial fertilizer application. Because corn is grown primarily by dairy farmers who must dispose of animal waste, it is assumed that all corn acreage receives 20 tons of manure per acre. Where a measure of total available nitrogen is needed, manure application is credited with 3 lbs. of nitrogen per ton.
\[ W + L(N_0^m + N_0^*, C_0, W, R) > e^* \leq \alpha, \]
where \( C_0 \) and \( W \) represent the vectors of soil characteristics and weather variables in the leaching and runoff equations \( L(\cdot) \) and \( R(\cdot) \), \( e^* \) is a safety level on total nitrate emissions, and \( \alpha \) is some small probability.

In the simulations, the safety level \( e^* \) is varied over two alternative levels, 25 and 20 lb. per acre, and \( \alpha \) was set at 0.1. Distributions of nitrate losses for each soil were simulated from the weather observations at the Ithaca weather station over the 30-year period 1963-1992. These simulation data are summarized in Table 2. For each safety level \( e^* \), the appropriate regulation \( N_0^* \) was set so that nitrate loss exceeded \( e^* \) no more than 10% of the time (3 out of the 30 years).

### Table 2. Data and Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field trial observations, various New York sites</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Growing Season Rainfall (in./year)</td>
<td>20.91</td>
<td>3.94</td>
<td>14.87</td>
<td>27.00</td>
</tr>
<tr>
<td>Total Nitrogen Applied (lb./acre)</td>
<td>130.58</td>
<td>73.74</td>
<td>0.00</td>
<td>285.00</td>
</tr>
<tr>
<td>Corn Silage Yield (tons/acre)</td>
<td>20.93</td>
<td>4.51</td>
<td>9.10</td>
<td>30.00</td>
</tr>
<tr>
<td>Weather observations at the Ithaca weather station, 1963-92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual Rainfall (in./year)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>39.08</td>
<td>5.25</td>
<td>30.12</td>
<td>51.62</td>
</tr>
<tr>
<td>Rain within 14 Days of Planting (in./year)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.76</td>
<td>0.77</td>
<td>0.01</td>
<td>2.97</td>
</tr>
<tr>
<td>Rain within 14 Days of Fertilizer (in./year)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.17</td>
<td>1.28</td>
<td>0.01</td>
<td>6.46</td>
</tr>
<tr>
<td>Rain within 14 Days of Harvest (in./year)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>1.64</td>
<td>1.85</td>
<td>0.00</td>
<td>9.53</td>
</tr>
<tr>
<td>Growing Season Rainfall (in./year)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>20.35</td>
<td>3.90</td>
<td>14.17</td>
<td>27.35</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corn Silage Price (1992 $/ton)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>19.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen Price (1992 $/lb N)&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Nitrogen Variable Cost (1992 $/acre)&lt;sup&gt;d&lt;/sup&gt;</td>
<td>189.52</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Explanatory variable in the leaching and runoff equations.

<sup>b</sup> Explanatory variable in the yield equations.

<sup>c</sup> Mean price over the 1963-92 period.

<sup>d</sup> Source: Schmit; USDA-ERS, *Economic Indicators of the Farm Sector* (1992).
Table 3. Pre- and Post-Policy Fertilizer, Pollution, and Net Returns

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Pre-Policy</th>
<th>Post-Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSD</td>
<td>Bounded</td>
</tr>
<tr>
<td>Nitrogen fertilizer applied (lb./a)$^a$</td>
<td>92</td>
<td>90</td>
</tr>
<tr>
<td>Nitrate loss safety level (lb./a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen fertilizer applied (lb./a)</td>
<td>139</td>
<td>127</td>
</tr>
<tr>
<td>Nitrate loss safety level (lb./a)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The post-policy N levels may seem quite low, but these do not include the N implicit in the 20 tons of manure applied.

The estimated standards on fertilizer are reported in Table 3 (the last two columns). As expected, group 1 is more prone to nitrate losses, and for each safety level $N_1^* < N_2^*$.

Policy Simulations

Policies were found in each of the three cases by numerically solving different versions of problems (6) and (7) above for the payments $s_1$ and $s_2$, based on the profit function:

$$
\pi^\theta(w, N_\theta) = p_y y^\theta(w, N_\theta^{\text{m}} + N_\theta) - p_N N_\theta - v
$$

where, for group $\theta$, the uncontrollable random input is rainfall $w$, the policy variable is nitrogen fertilizer $N_\theta$, $p_y$ is the price of corn, $y^\theta$ is the estimated yield equation, $N_\theta^{\text{m}}$ is the nitrogen available from manure, $p_N$ is the price of nitrogen fertilizer, and $v$ is non-nitrogen variable cost. The values of parameters in the model are in Table 2. The random variable $w$ takes on values from a sample of growing season rainfall observations at the Ithaca weather station over the period 1963-1992. $N_\theta^{\text{m}}$ is set at 60 pounds per acre for both groups (assuming an application of...
20 tons of manure). The prices $p_y$ and $p_N$ are set at the mean of observed corn silage and nitrogen prices (in constant 1992 dollars), respectively, over the same 30 years. Other costs $v$ are based on enterprise budgets in USDA (1994) and Schmit (1994) (Table 2).

The general version of the SSD problem in equation (6) has eight constraints, four of which involve expectations of $\pi^0$, and four that are based on the lowest observations. In the risk-neutral case, all farmers would make their decisions based only on expected profits, so that the problem is (6) without the four constraints on lowest observations. If risk aversion coefficients are bounded, then the problem (equation (7)) again has eight constraints, four involving the utility function $-e^{-r\pi^0}$, and four involving $-e^{-r\pi^0}$; based on empirical evidence on farmers’ risk attitudes, $r_0$ is set at 0.001 and $r_1$ is set at 0.03 (Brink and McCarl, 1978; Buccola, 1982; Love and Buccola, 1991; Saha et al., 1994). In all three specifications, the policies were calculated on a per-acre basis ($a_1 = a_2 = 1$). All models were solved using GAMS.

The existence of self-selecting payments in the SSD case can be proven a priori from the conditions in Table 1. Because $\pi^0_{wN} < 0$ for both $\theta$, nitrogen is a risk decreasing input for both groups, corresponding to Case 2 in Table 1. The appropriate condition for self-selecting payments to exist is that $E\pi^2_b(x, b) \geq \pi^1_b(x, b)$ for all $b$. This condition holds for the estimated equations and sample of weather observations. Existence for the other cases must be shown by finding the solutions. To isolate the value of information in all three cases, another version of each model was solved with only participation constraints, corresponding to the case where the government assigns policies by farm.

---

11 Empirically, $\pi^1_b(\xi, b) = 19.57[0.1001 - 0.00048b - 0.0007 \times 14.17] - 0.34$; $E\pi^2_b = 19.57[0.1001 - 0.00037b - 0.0007 \times 20.35] - 0.34$. The condition $E\pi^2_b \geq \pi^1_b(\xi, b)$ holds for all $b \geq 2.01$. 
Another parameter required to solve each model is the pre-policy level of fertilizer. In each of the three risk specifications, these were set at the highest fertilizer level consistent with maximizing expected utility. Table 3 reports the pre- and post-policy levels of input and output for both groups. As one would expect, the input restrictions reduce net returns for both groups. Group 1’s returns are much more volatile because the yield on lighter soils is more sensitive to weather conditions. Nonetheless, nitrogen is a risk reducing input for both groups, implying that input restrictions require farmers to bear more risk.

The optimal payments for both groups of farmers are in Table 4. The participation payments represent the point of indifference between the pre- and post-policy outcomes, and are the amounts farmers would need to be paid to be willing to participate in the program. These payments are the government’s cost of a program that assigns policies by farm, and range from about $4 to $10 per acre for group 1 and $7 to $17 for group 2. Group 2’s payments are larger because returns fall more rapidly as nitrogen is reduced ($\pi_2 > \pi_1$). To put these payments in perspective, the average payment to all U.S. feed grain producers in 1992 was about $25 per acre.

The self-selection payments are those needed to ensure participation as well as incentive compatibility if farmers choose their own policies. These payments exceed the participation payments for group 1 but not for group 2, because group 1 needs an extra incentive to choose the appropriate policy. This extra payment is an “information premium,” since it represents the amount the government could save by using soils information to assign policies by farm. The information premium for group 1 ranges from $7 to $15 per acre, raising the self-selection payments to as high as $25 per acre.

---

12 For example, in the SSD case, the two candidates for the optimal pre-policy levels were the solutions to the problems $\max\{E[\pi^0(w, N_0)]\}$ and $\max\{\pi^0(w, N_0)\}$; the pre-policy level in the model was the higher of the two
As one would expect, the participation and self-selection payments, as well as the information premium, are larger for the more stringent environmental standard (a safety level of 20 lb. per acre). On average, payments for the 20 lb. standard are 62% larger than for the 25 lb. standard, representing the tradeoff between taxpayer cost and higher environmental quality.

Within each safety level, payments are highest for the SSD case (where the government knows only that farmers are risk averse). There are two important implications of this result. First, if farmers are in fact risk averse, but payment were set under the assumption of risk neutrality, payment levels would be from 30 to 40 percent too low.

Second, by comparing the results for SSD and “bounded” case, we see that payments would decrease if more information were available to narrow the range in farmers’ possible risk attitudes. This implies that information on risk attitudes is also valuable to the government. Interestingly, however, the lack of information does not make the policy prohibitively costly. For example, the highest payment of $25.50 (which is roughly equal to past commodity solutions. The risk-neutral case has the single candidate level that maximizes expected profit.

---

**Table 4. Optimal Payments**

<table>
<thead>
<tr>
<th></th>
<th>Safety level = 25</th>
<th>Safety level = 20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSD</td>
<td>Bounded$^a$</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------</td>
<td>------------</td>
</tr>
<tr>
<td>Group 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participation payment$^b$</td>
<td>6.33</td>
<td>5.75</td>
</tr>
<tr>
<td>Self-selection payment$^c$</td>
<td>17.50</td>
<td>12.89</td>
</tr>
<tr>
<td>Information premium$^d$</td>
<td>11.17</td>
<td>7.14</td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participation payment$^b$</td>
<td>11.62</td>
<td>7.43</td>
</tr>
<tr>
<td>Self-selection payment$^c$</td>
<td>11.62</td>
<td>7.43</td>
</tr>
<tr>
<td>Information premium$^d$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

$^a$ Arrow-Pratt coefficients of absolute risk aversion bounded in the range [0.001, 0.03]

$^b$ Payment required to for farmers to be willing to participate in the program

$^c$ Payment required for farmers to participate as well as self-select their own policy

$^d$ The difference between self-selection and participation payments, or the amount payments can be reduced if soils information is collected to assign policies by farm
payments) is the self-selection payment for group 1 under SSD, with a nitrate loss standard of 20 lb. If the government invests in research to learn more about farmers risk attitudes, it may be able to confirm that risk coefficients lie in a specified range, so that the payment can be reduced to $19.96 per acre. If instead the government invested in soils research to be able to classify farmers, the payment could be reduced to $10.69 per acre. Soils information appears to have a higher return to the government than risk information, which is significant because risk information is much more difficult to acquire.

**Policy Implications**

This paper demonstrates both the theoretical and empirical possibility of successfully designing a voluntary environmental program when the government’s information is limited. In particular, where both risk attitudes and technology are unknown, we identify the cases where payments can be associated with different environmental policies so that each farmer self-selects the appropriate one.

By accommodating unknown risk attitudes through the use of stochastic efficiency criteria, empirically testable conditions were derived to isolate the situations where self-selecting policies can exist. These conditions simplify to comparisons of either the lower tail or the mean of net return distributions, and differ depending on the marginal risk effect of the polluting input across technologies. In all cases, policies can be self-selected only if the least polluting technology is the most productive.

The conditions themselves lead to a linear-programming based procedure for finding actual payments, as well as an alternative method to be applied when the conditions cannot be verified. These methods were applied to nitrate contamination in New York, and the empirical results suggest that a self-selecting set of regulations could be successfully designed. Moreover,
substantial improvements in environmental quality are possible at modest cost; the estimated payments are generally less than $20 per acre, which is less than typical farm program payments in the past.

To make the regulations on fertilizer self-selecting, the group of farmers that is least prone to pollute needs to be compensated only for their loss in net returns, while the more polluting group receives an additional bonus. This extra payment is an “information premium” that could likely be avoided in New York, where the use-value assessment program already requires local officials to record each farm’s acreage in each of ten soil productivity groups (Thomas and Boisvert, 1995). This is clearly a case where information necessary to administer one agricultural policy can effectively reduce the cost of another.

The results of the New York application further suggest that information on risk attitudes is also valuable to the government. Payments can be reduced if the government has better knowledge of producers’ risk preferences. However, the benefit of obtaining this information does not appear to be large relative to the gain from soils information, and importantly, the lack of knowledge on risk attitudes does not render the program infeasible.
Appendix

Proof of Result 1

We will verify the claim by proving that if $\pi^\theta_{xb} > (\leq) 0$ and $F_{m'}$ intersects $F_m$, then $F_{m'}$ must be flatter (steeper) than $F_m$ at the intersection point, and the distributions cannot cross a second time. Formally, these distributions are:

$$F_m(m) \equiv \Pr\{\pi^\theta(x, b) + s \leq m\}$$

and

$$F_{m'}(m') \equiv \Pr\{\pi^\theta(x, b') + s' \leq m'\}$$

Let $F_x$ be the c.d.f. of $x$ (i.e., $F_x(a) \equiv \Pr\{x \leq a\}$), and define $X(m; b, s)$ as the inverse function of $\pi^\theta + s$, such that $X(\pi^\theta + s; b, s) = x$. Applying $X(\cdot)$ to both sides of the inequalities inside the definitions of $F_m$ and $F_{m'}$:

$$F_m(m) = \Pr\{x \leq X(m; b, s)\} = F_x(X(m; b, s))$$

$$F_{m'}(m') = \Pr\{x \leq X(m'; b', s')\} = F_x(X(m'; b', s'))$$

Let $m^*$ represent the first (smallest) level of $m$ where the distributions intersect: $F_m(m^*) = F_{m'}(m^*)$. By the relationships above, we also know that:

$$F_x(X(m^*; b, s)) = F_x(X(m^*; b', s')) \Rightarrow X(m^*; b, s) = X(m^*; b', s')$$

If $F_{m'}$ crosses $F_m$ from above at this point, then $\frac{\partial F_m(m^*)}{\partial m} < \frac{\partial F_{m'}(m^*)}{\partial m}$. In terms of $F_x$, this requirement can be written:

$$\frac{\partial F_x(X^*)}{\partial X} \frac{\partial X}{\partial m'} < \frac{\partial F_x(X^*)}{\partial X} \frac{\partial X}{\partial m}$$

where $X^* = X(m^*; b, s) = X(m^*; b', s')$. By the inverse function theorem, $\frac{\partial X}{\partial m'} = 1/\pi^\theta_x(x, b')$ and $\frac{\partial X}{\partial m} = 1/\pi^\theta_x(x, b)$. Therefore, the condition becomes:

$$\pi^\theta_x(x, b) < \pi^\theta_x(x, b')$$

Since $b < b'$ by assumption, this condition can only be satisfied if $\pi^\theta_{xb} > 0$. If the distributions cross a second time at some $m^{**} > m^*$, $F_{m'}$ must intersect $F_m$ from below so that $\frac{\partial F_m(m^{**})}{\partial m}$
∂F_m(m**)/∂m. By the same argument as above, this condition implies that \( \pi^{\theta}_{xb} < 0 \), which contradicts the assumption that \( \pi^{\theta}_x \) is monotonic in \( b \). Therefore, the distributions can cross at most once. The same set of arguments verifies that \( F_m' \) can only intersect \( F_m \) once from below when \( \pi^{\theta}_{xb} < 0 \).

*Proof of Result 2:*

Suppose that \( \pi^{\theta}_{xb} > 0 \). Result 1 implies that in this case \( F_m' \) is flatter than \( F_m \), so that \( F_m' \) crosses \( F_m \) from above (Figure 2). By the Rothschild-Stiglitz proposition, to show that \( m' \) is riskier than \( m \), it is sufficient to show that \( F_m \) dominates \( F_m' \) by SSD if a constant is added to \( m \) so that \( Em = Em' \). The adjusted distribution of \( F_m \) is represented by the dashed cdf in Figure 2. Since the means of \( m \) and \( m' \) are equal after this adjustment, \( \int_{m} m [dF_m - dF_m'] = 0 \). Integrating by parts, this expression becomes:

\[
m[ F_m(m) - F_m'(m) ]_m^\bar{m} - \int_{m}^{\bar{m}} [ F_m(m) - F_m'(m) ] dm = 0.
\]

Since \( F_m(m) = F_m'(\bar{m}) = 0 \) and \( F_m(m) = F_m'(\bar{m}) = 1 \), the first term in brackets equals zero.

Therefore, the net area between the two distributions equals zero—area \( A \) equals area \( B \) in Figure 2—and \( F_m \) dominates \( F_m' \) by SSD, since the area \( \int_{m}^{\bar{m}} \) starts negative and only reaches zero as \( \bar{m} \to \bar{m} \).

If \( \pi^{\theta}_{xb} < 0 \), then \( F_m' \) intersects \( F_m \) from below, as in Figure 3. Following the same argument as above, \( m' \) is less risky than \( m \) if and only if \( F_m' \) dominates \( F_m \) after the distributions have been adjusted so that \( Em = Em' \). After this adjustment, area \( A \) equals area \( B \), which implies that the SSD condition is met.
Solutions to the Policy Design Problem

Case 1: $\pi_{1,xb} \geq 0, \pi_{2,xb} \geq 0 (b$ is a risk-increasing input for both groups)

The problem is:

$$\begin{align*}
\text{min} & \quad a_1 s_1 + a_2 s_2 \\
\text{s.t.} & \quad E \pi_1^1 (x, b_1^*) + s_1 \geq E \pi_1^1 (x, b_1^0) \\
& \quad E \pi_2^2 (x, b_2^*) + s_2 \geq E \pi_2^2 (x, b_2^0) \\
& \quad E \pi_1^1 (x, b_1^*) + s_1 \geq E \pi_1^1 (x, b_2^*) + s_2 \\
& \quad \pi_2^2 (x, b_2^*) + s_2 \geq \pi_2^2 (x, b_1^*) + s_1
\end{align*}$$

The four constraints can be rewritten as:

$$\begin{align*}
s_1 & \geq E \pi_1^1 (x, b_1^0) - E \pi_1^1 (x, b_1^*) \equiv \tilde{s}_1 \\
s_2 & \geq E \pi_2^2 (x, b_2^0) - E \pi_2^2 (x, b_2^*) \equiv \tilde{s}_2 \\
s_1 - s_2 & \geq E \pi_1^1 (x, b_2^0) - E \pi_1^1 (x, b_1^0) \equiv A \\
s_1 - s_2 & \leq \pi_2^2 (x, b_2^0) - \pi_2^2 (x, b_1^0) \equiv B
\end{align*}$$

The last two constraints require that the quantity $s_1 - s_2$ lie in the range $[A, B]$, implying the existence condition for a solution is that $B \geq A$:

$$\pi_2^2 (x, b_2^*) - \pi_2^2 (x, b_1^*) \geq E \pi_1^1 (x, b_2^*) - E \pi_1^1 (x, b_1^*)$$

For this condition to hold for arbitrary choices of $b_1^*$ and $b_2^*$, the profit functions must satisfy:

$$\pi_2^2 (x, b) \geq E \pi_1^1 (x, b) \quad (8)$$

Since $\pi_{1,xb} \geq 0$ and $\pi_{2,xb} \geq 0$, $\pi_\theta^\theta (x, b) \geq \pi_\theta^\theta (x, b)$ for all $x \geq x$ ($\theta = 1, 2$), which implies in turn that:

$$E \pi_\theta^\theta \equiv \int \pi_\theta^\theta (x, b) dF_x \geq \pi_\theta^\theta (x, b), \quad \theta = 1, 2 \quad (9)$$

Combining (8) and (9):

$$E \pi_\theta^\theta (x, b) \geq \pi_\theta^\theta (x, b) \geq E \pi_\theta^\theta (x, b) \geq \pi_\theta^\theta (x, b) \quad (10)$$

The pre-policy fertilizer levels $b_\theta^0$ must be bounded between the solutions to two maximization problems:
The first-order conditions to these problems are:

\[
\max_{b_y} E\pi^y(x, b_y) \quad \text{and} \quad \max_{b_y} \pi^y(x, b_y) = 0
\]

Condition (10) above implies that \( b_2^0 \geq b_1^0 \).

The minimum-cost solution will occur at point \( d \) in Figure 5 as long as \( d \) does not lie to the left of \( c \). Point \( d \) is at the boundaries of \((I_1)\) and \((P_2)\), which are the equations \( s_1 - s_2 = A \) and \( s_2 = \hat{s}_2 \), respectively. By substitution, the value of \( s_1 \) at point \( d \) is \( \hat{s}_1 = A + \hat{s}_2 \). Point \( c \) is at the intersection of \((P_1)\) and \((P_2)\), where the value of \( s_1 \) is \( \hat{s}_1 \). Point \( d \) does not lie to the left of \( c \) as long as \( A + \hat{s}_2 \geq \hat{s}_1 \), or, rearranging:

\[ \hat{s}_1 - A \leq \hat{s}_2 \tag{11} \]

Substituting the values of \( \hat{s}_1 \) and \( A \) into the left side of (11):

\[
\hat{s}_1 - A = E\pi^1(x, b_1) - E\pi^1(x, b_1^*) - [E\pi^1(x, b_2) - E\pi^1(x, b_2^*)]
\]

\[
= E\pi^1(x, b_1) - E\pi^1(x, b_2^*)
\]

\[
\leq E\pi^1(x, b_1^*) - E\pi^1(x, b_2^*)
\]

\[
\leq E\pi^2(x, b_2^*) - E\pi^1(x, b_2^*) \equiv \hat{s}_2
\]

where the inequalities follow from the facts that \( b_2^0 \geq b_1^0 \) and \( E\pi^2_b \geq E\pi^1_b \), respectively.

Therefore, if the existence condition is met, the unique solution to the problem is at point \( d \), where \( s_1 = A + \hat{s}_2 \) and \( s_2 = \hat{s}_2 \).

\textbf{Case 2:} \( \pi_1^{1, x} \leq 0, \pi_2^{1, x} \leq 0 \) (\( b \) is a risk-decreasing input for both groups)

The problem is:
\[
\begin{align*}
\min & \quad a_1 s_1 + a_2 s_2 \\
\text{s.t.} & \quad \pi^1(x, b^*_1) + s_1 \geq \pi^1(x, b^*_1) \\
& \quad \pi^2(x, b^*_2) + s_2 \geq \pi^2(x, b^*_2) \\
& \quad \pi^1(x, b^*_1) + s_1 \geq \pi^1(x, b^*_1) + s_2 \\
& \quad E\pi^2(x, b^*_1) + s_2 \geq E\pi^2(x, b^*_1) + s_1
\end{align*}
\]

The four constraints can be rewritten as:
\[
\begin{align*}
s_1 & \geq \pi^1(x, b^*_1) - \pi^1(x, b^*_1) \equiv \hat{s}_1 \\
s_2 & \geq \pi^2(x, b^*_2) - \pi^2(x, b^*_2) \equiv \hat{s}_2 \\
s_1 - s_2 & \geq \pi^1(x, b^*_2) - \pi^1(x, b^*_1) \equiv A \\
s_1 - s_2 & \leq E\pi^2(x, b^*_2) - E\pi^2(x, b^*_1) \equiv B
\end{align*}
\]

The last two constraints require that the quantity \( s_1 - s_2 \) lie in the range \([A, B]\), implying the existence condition for a solution is that \( B \geq A \):
\[
E\pi^2(x, b^*_2) - E\pi^2(x, b^*_1) \geq \pi^1(x, b^*_2) - \pi^1(x, b^*_1)
\]

For this condition to hold for arbitrary choices of \( b^*_1 \) and \( b^*_2 \), the profit functions must satisfy:
\[
E\pi^2(x, b) \geq \pi^1(x, b)
\]

Since \( \pi^1_{\theta_1} \leq 0 \) and \( \pi^2_{\theta_2} \leq 0 \), \( \pi^0_{\theta}(x, b) \leq \pi^0_{\theta}(x, b) \) for all \( x \geq x^* (\theta = 1,2) \), which implies in turn that:
\[
E\pi^0_{\theta} \equiv \int_{x^*} \pi^0_{\theta}(x, b) dF_x \leq \pi^0_{\theta}(x, b), \quad \theta = 1,2
\]

Combining (12) and (13):
\[
\pi^2_{\theta}(x, b) \geq E\pi^2_{\theta}(x, b) \geq \pi^1_{\theta}(x, b) \geq E\pi^1_{\theta}(x, b)
\]

The pre-policy fertilizer levels \( b^0_{\theta} \) must be bounded between the solutions to two maximization problems:
\[
\max_{b^0_{\theta}} E\pi^0_{\theta}(x, b^0_{\theta}) \quad \text{and} \quad \max_{b^0} \pi^0_{\theta}(x, b^0)
\]

The first-order conditions to these problems are:
\[ E\pi^0_b(x, b_0) = 0 \quad \text{and} \quad \pi^0_b(x, b_0) = 0 \]

Condition (14) above implies that \( b_2^0 \geq b_1^0 \).

The minimum-cost solution will occur at point \( d \) in Figure 5 as long as \( d \) does not lie to the left of \( c \). Point \( d \) is at the boundaries of \((I_1)\) and \((P_2)\), which are the equations \( s_1 - s_2 = A \) and \( s_2 = \hat{s}_2 \), respectively. By substitution, the value of \( s_1 \) at point \( d \) is \( \hat{s}_1 = A + \hat{s}_2 \). Point \( c \) is at the intersection of \((P_1)\) and \((P_2)\), where the value of \( s_1 \) is \( \hat{s}_1 \). Point \( d \) does not lie to the left of \( c \) as long as \( A + \hat{s}_2 \geq \hat{s}_1 \), or, rearranging:

\[ \hat{s}_1 - A \leq \hat{s}_2 \quad (15) \]

Substituting the values of \( \hat{s}_1 \) and \( A \) into the left side of (15):

\[ \hat{s}_1 - A = \pi^1(x, b_0^0) - \pi^1(x, b_1^0) - [\pi^1(x, b_2^0) - \pi^1(x, b_1^0)] \\
\leq \pi^1(x, b_0^0) - \pi^1(x, b_1^0) \\
\leq \pi^1(x, b_0^0) - \pi^1(x, b_1^0) = \hat{s}_2 \]

where the inequalities follow from the facts that \( b_2^0 \geq b_1^0 \) and \( \pi^0_b(x, b) \geq \pi^0_b(x, b) \), respectively.

Therefore, if the existence condition is met, the unique solution to the problem is at point \( d \), where \( s_1 = A + \hat{s}_2 \) and \( s_2 = \hat{s}_2 \).

**Case 3:** \( \pi^{1, 0} \leq 0, \pi^{2, 0} \geq 0 \) (\( b \) is risk-decreasing for group 1 and risk-increasing for group 2)

The problem is:

\[
\begin{align*}
\min & \quad a_1 s_1 + a_2 s_2 \\
\text{s.t.} & \quad \pi^1(x, b_1^0) + s_1 \geq \pi^1(x, b_1^0) \\
& \quad E\pi^2(x, b_2^0) + s_2 \geq E\pi^2(x, b_2^0) \\
& \quad \pi^1(x, b_1^0) + s_1 \geq \pi^1(x, b_1^0) + s_2 \\
& \quad \pi^2(x, b_2^0) + s_2 \geq \pi^2(x, b_1^0) + s_1
\end{align*}
\]

The four constraints can be rewritten as:
\[ s_1 \geq \pi^1(x, b^0_1) - \pi^1(x, b^*_1) \equiv \hat{s}_1 \]
\[ s_2 \geq E\pi^2(x, b^0_2) - E\pi^2(x, b^*_2) \equiv \hat{s}_2 \]
\[ s_1 - s_2 \geq \pi^1(x, b^*_2) - \pi^1(x, b^*_1) \equiv A \]
\[ s_1 - s_2 \leq \pi^2(x, b^*_2) - \pi^2(x, b^*_1) \equiv B \]

The last two constraints require that the quantity \( s_1 - s_2 \) lie in the range \([A, B]\), implying the existence condition for a solution is that \( B \geq A \):

\[ \pi^2(x, b^*_2) - \pi^2(x, b^*_1) \geq \pi^1(x, b^*_2) - \pi^1(x, b^*_1) \]

For this condition to hold for arbitrary choices of \( b^*_1 \) and \( b^*_2 \), the profit functions must satisfy:

\[ \pi^2_b(x, b) \geq \pi^1_b(x, b) \quad (16) \]

Since \( \pi^1_{xb} \leq 0 \), \( \pi^1_b(x, b) \leq \pi^1_b(x, b) \) for all \( x \geq x \), which implies in turn that:

\[ E\pi^1_b \equiv \int \pi^1_b(x, b) dF_x \leq \pi^1_b(x, b) \quad (17) \]

Similarly, \( \pi^2_{xb} \geq 0 \) implies that \( \pi^2_b(x, b) \geq \pi^2_b(x, b) \), so that:

\[ E\pi^2_b \equiv \int \pi^2_b(x, b) dF_x \geq \pi^2_b(x, b) \quad (18) \]

Combining (16), (17), and (18):

\[ E\pi^1_b(x, b) \geq \pi^2_b(x, b) \geq \pi^1_b(x, b) \geq E\pi^1_b(x, b) \quad (19) \]

The pre-policy fertilizer levels \( b_0^0 \) must be bounded between the solutions to two maximization problems:

\[ \max_{b_0} E\pi^0(x, b_0) \quad \text{and} \quad \max_{b_0} \pi^0(x, b_0) \]

The first-order conditions to these problems are:

\[ E\pi^0_b(x, b_0) = 0 \quad \text{and} \quad \pi^0_b(x, b_0) = 0 \]

Condition (19) above implies that \( b^0_2 \geq b^0_1 \).
The minimum-cost solution will occur at point \( d \) in Figure 5 as long as \( d \) does not lie to the left of \( c \). Point \( d \) is at the boundaries of \((I_1)\) and \((P_2)\), which are the equations \( s_1 - s_2 = A \) and \( s_2 = \hat{s}_2 \), respectively. By substitution, the value of \( s_1 \) at point \( d \) is \( \hat{s}_1 = A + \hat{s}_2 \). Point \( c \) is at the intersection of \((P_1)\) and \((P_2)\), where the value of \( s_1 \) is \( \hat{s}_1 \). Point \( d \) does not lie to the left of \( c \) as long as \( A + \hat{s}_2 \geq \hat{s}_1 \), or, rearranging:

\[
\hat{s}_1 - A \leq \hat{s}_2
\]

(20)

Substituting the values of \( \hat{s}_1 \) and \( A \) into the left side of (20):

\[
\hat{s}_1 - A = \pi^1(x, b_1^0) - \pi^1(x, b_1^*) - \left[ \pi^1(x, b_2^*) - \pi^1(x, b_1^*) \right] \\
= \pi^1(x, b_1^0) - \pi^1(x, b_2^*) \\
\leq \pi^1(x, b_1^0) - \pi^1(x, b_1^*) \\
\leq E\pi^2(x, b_2^0) - E\pi^2(x, b_2^*) = \hat{s}_2
\]

where the inequalities follow from the facts that \( b_2^0 \geq b_1^0 \) and \( E\pi^2_p(x, b) \geq \pi^2_p(x, b) \), respectively.

Therefore, if the existence condition is met, the unique solution to the problem is at point \( d \), where \( s_1 = A + \hat{s}_2 \) and \( s_2 = \hat{s}_2 \).

**Case 4:** \( \pi^1_{xb} \geq 0, \pi^2_{xb} \leq 0 \) (\( b \) is risk-increasing for group 1 and risk-decreasing for group 2)

The problem is:

\[
\begin{align*}
\min & \quad a_1 s_1 + a_2 s_2 \\
\text{s.t.} & \quad E\pi^1(x, b_1^*) + s_1 \geq E\pi^1(x, b_1^0) \\
& \quad \pi^2(x, b_2^*) + s_2 \geq \pi^2(x, b_2^0) \\
& \quad E\pi^1(x, b_1^*) + s_1 \geq E\pi^1(x, b_2^*) + s_2 \\
& \quad E\pi^2(x, b_2^*) + s_2 \geq E\pi^2(x, b_1^*) + s_1
\end{align*}
\]

The four constraints can be rewritten as:
\begin{align*}
    s_1 & \geq E\pi^1(x,b^0_1) - E\pi^1(x,b^*_1) \equiv \hat{s}_1 \\
    s_2 & \geq \pi^2(x,b^0_2) - \pi^2(x,b^*_2) \equiv \hat{s}_2 \\
    s_1 - s_2 & \geq E\pi^1(x,b^*_2) - E\pi^1(x,b^*_1) \equiv A \\
    s_1 - s_2 & \leq E\pi^2(x,b^*_2) - E\pi^2(x,b^*_1) \equiv B
\end{align*}

The last two constraints require that the quantity \( s_1 - s_2 \) lie in the range \([A, B]\), implying the existence condition for a solution is that \( B \geq A \):

\begin{align*}
    E\pi^2(x,b^*_2) - E\pi^2(x,b^*_1) & \geq E\pi^1(x,b^*_2) - E\pi^1(x,b^*_1)
\end{align*}

For this condition to hold for arbitrary choices of \( b^*_1 \) and \( b^*_2 \), the profit functions must satisfy:

\begin{equation}
    E\pi^2_\alpha(x,b) \geq E\pi^1_\alpha(x,b) \tag{21}
\end{equation}

Since \( \pi^1_{xb} \geq 0 \), \( \pi^1_\alpha(x,b) \geq \pi^1_\alpha(x,b) \) for all \( x \geq x^* \), which implies in turn that:

\begin{equation}
    E\pi^1_\alpha \equiv \int \pi^1_\alpha(x,b) dF_x \geq \pi^1_\alpha(x,b) \tag{22}
\end{equation}

Similarly, \( \pi^2_{xb} \leq 0 \) implies that \( \pi^2_\alpha(x,b) \leq \pi^2_\alpha(x,b) \), so that:

\begin{equation}
    E\pi^2_\alpha \equiv \int \pi^2_\alpha(x,b) dF_x \leq \pi^2_\alpha(x,b) \tag{23}
\end{equation}

Combining (21), (22), and (23):

\begin{equation}
    \pi^2_\alpha(x,b) \geq E\pi^2_\alpha(x,b) \geq E\pi^1_\alpha(x,b) \geq \pi^1_\alpha(x,b) \tag{24}
\end{equation}

The pre-policy fertilizer levels \( b^0_\theta \) must be bounded between the solutions to two maximization problems:

\[
\max_{b_\theta} E\pi^0(x,b_\theta) \quad \text{and} \quad \max_{b_\theta} \pi^0(x,b_\theta)
\]

The first-order conditions to these problems are:

\[
E\pi^0_\theta(x,b_\theta) = 0 \quad \text{and} \quad \pi^0_\theta(x,b_\theta) = 0
\]

Condition (24) above implies that \( b^0_2 \geq b^0_1 \).
The minimum-cost solution will occur at point \( d \) in Figure 5 as long as \( d \) does not lie to the left of \( c \). Point \( d \) is at the boundaries of \((I_1)\) and \((P_2)\), which are the equations \( s_1 - s_2 = A \) and \( s_2 = \hat{s}_2 \), respectively. By substitution, the value of \( s_1 \) at point \( d \) is \( \hat{s}_1 = A + \hat{s}_2 \). Point \( c \) is at the intersection of \((P_1)\) and \((P_2)\), where the value of \( s_1 \) is \( \hat{s}_1 \). Point \( d \) does not lie to the left of \( c \) as long as \( A + \hat{s}_2 \geq \hat{s}_1 \), or, rearranging:

\[
\hat{s}_1 - A \leq \hat{s}_2 \tag{25}
\]

Substituting the values of \( \hat{s}_1 \) and \( A \) into the left side of (25):

\[
\hat{s}_1 - A = E\pi^1(x, b^0_1) - E\pi^1(x, b^*_1) - [E\pi^1(x, b^*_2) - E\pi^1(x, b^*_1)] \\
= E\pi^1(x, b^0_1) - E\pi^1(x, b^*_1) \\
\leq E\pi^1(x, b^*_1) - E\pi^1(x, b^*_1) \\
\leq \pi^2(x, b^0_2) - \pi^2(x, b^*_2) \equiv \hat{s}_2 
\]

where the inequalities follow from the facts that \( b^0_2 \geq b^0_1 \) and \( \pi^2(x, b) \geq E\pi^1(x, b) \), respectively.

Therefore, if the existence condition is met, the unique solution to the problem is at point \( d \), where \( s_1 = A + \hat{s}_2 \) and \( s_2 = \hat{s}_2 \).
References


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