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The Role of Curvature in Estimating Accurate Dual Functions

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The Role of Curvature in Estimating Accurate Dual Functions

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Abstract

A production function highly concave in inputs will have a dual profit function less convex in prices. Even large variations in prices will cause little variation in input usage and output, resulting in poor profit function estimates. This is demonstrated using Monte Carlo simulation with various curvatures and error structures.

Introduction

In a production economics course I ask students to estimate an empirical profit function. Since few data sets exist with profit and price observations, I generate these data from empirically estimated production functions, mostly per acre crop production functions. From the production function, I derive the empirical profit function, and into that function I insert reasonable market prices for the outputs and inputs. Then I append a random normal distribution error term on the computed profit values. Since the crop production functions are usually quadratic with two inputs, I instruct the students to estimate a normalized quadratic profit function from those data.

I have observed that if I append anything but a very small error term onto the profit variable, the coefficients estimated for the profit function will produce demand functions by Hotelling's lemma that deviate greatly from the true demand functions. This occurs even when the R-squared value is still respectable. Since I have students calculate the demand functions directly from the production function in an earlier problem

exercise, I append small random errors to the dependent profit variable to ensure that they get comparable coefficients for the demand equations using both the primal and dual approaches. This results in an R-squared value for the profit function that is typically greater than .95, instilling an unreasonable expectation for any empirical estimation students might do later.

This phenomenon is due to the curvature of the production function and the resultant profit function. If the production function is highly concave, then the profit function will be less convex in prices. In contrast, if the production function is only slightly concave, then the profit function will be more convex in prices. If the production function is highly concave, then even large variations in prices will cause little variation in input usage and output production. In contrast, if the production function is slightly concave, those same variations in prices will produce greater variation in input usage, output, and thus profits. A greater variability in profits should provide a better econometric estimate of the profit function and resultant demand functions.

Lusk, Abdulkadri, and Featherstone investigated primal and dual estimation accuracy under different price variability and measurement error, but did not examine the role of curvature. In this paper I measure the impact of curvature and measurement error under constant price variability. This is done by specifying single input production functions that vary in their degree of concavity. Then, using the identical range of prices on all production functions, profits are computed. Various amounts of random errors are appended to all profits, and a profit function is estimated for each of the technologies. The result is that for any level of error, the least concave production function produces better profit function estimates.

Approach

From the single input quadratic production function:

$$y = x - cx^2$$

the normalized dual profit function is:

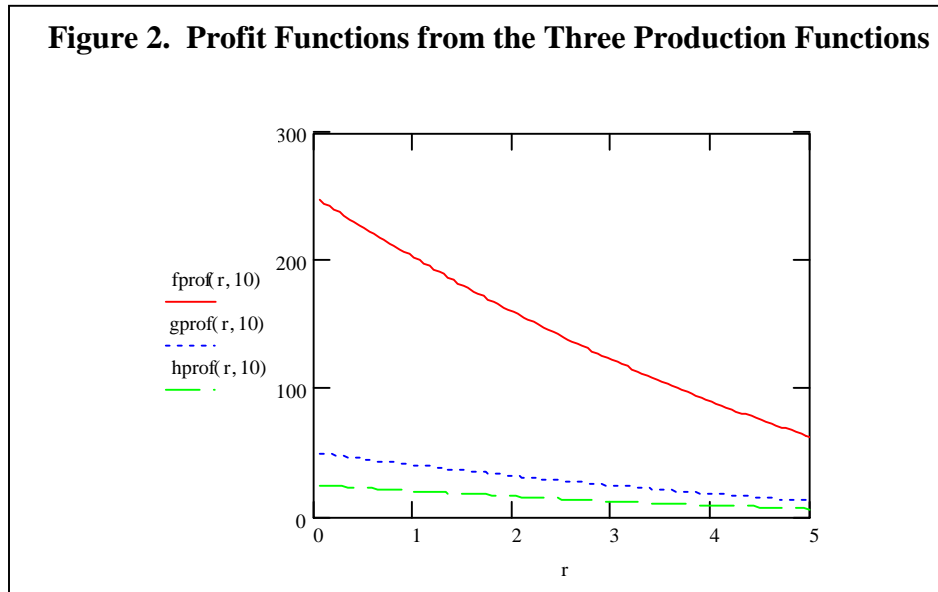
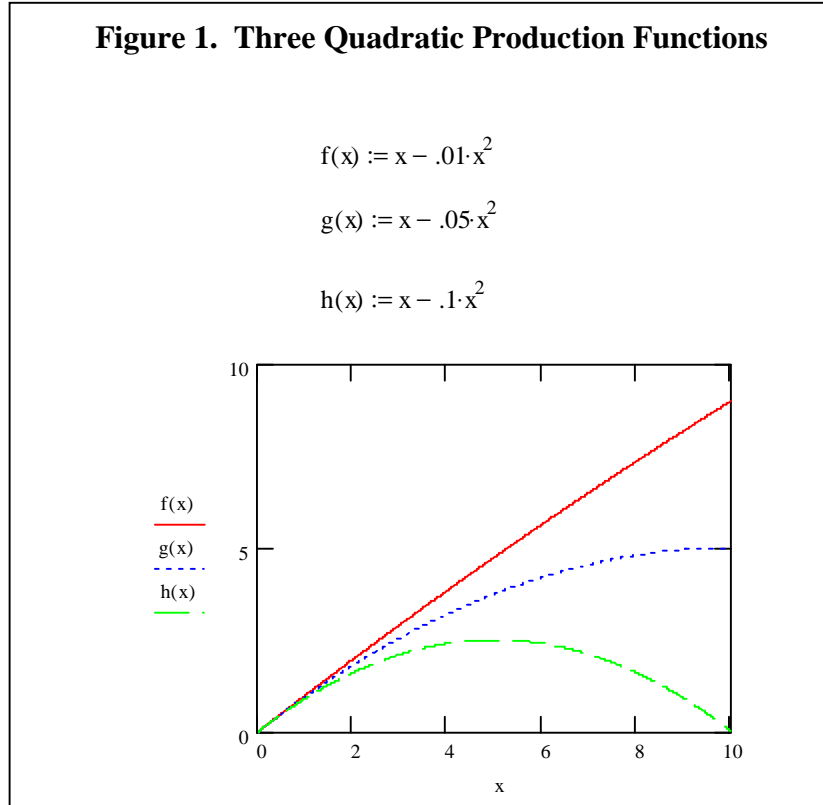
$$\frac{\Pi}{p} = \frac{1}{4c} - \frac{1}{2c} \left(\frac{r}{p}\right) + \frac{1}{4c} \left(\frac{r}{p}\right)^2$$

Where y is output, x is input, Π is profit, p is output price, r is input price, and c is the parameter of the production function.

Parameter c sets the degree of curvature of the production function. The greater the absolute value of c , the more concave is the production function over any given input range. Values of 0.01, 0.05, and 0.10 are illustrated in Figure 1. The associated profit functions for each of these three production functions are illustrated in Figure 2. The production function that is less concave with a c parameter value of 0.01 produces the profit function that is the most convex.

Ten quadratic production functions are simulated with $c=0.01, 0.02, \dots, 0.10$. For each of these values, a quadratic profit function is specified. To generate 100 profit observations for each of these ten technologies, the price of output is set equal to 10, and 100 input prices are generated over the range of 1 to 6 in increments of .05. Normal random errors were added to each normalized profit value. Ten levels of error were added by multiplying the standard unit normal random draw starting with 1 and increasing in increments of 1 to 10. Each experiment was replicated 40 times. The experimental design then consisted of 10 different technologies, with 100 observations

per technology and with 10 levels of error per technology. This numbers 100 regression models. Each regression is replicated 40 times.



Results

Table 1 (page 7) shows the average estimated coefficients of the regressions with the standard deviation of the 40 replicates below each average. The first column lists the ten c values that were simulated. The second column lists the multiple of the standardized normal distribution that was added to the computed normalized profit before the regression. The third, fourth, and fifth columns are the estimated b_1 , b_2 , and b_3 coefficients, which represent the intercept, linear real input price, and the quadratic real input price. Finally, the last column is the estimated R-squared value. Standard deviations of the 40 regressions are listed in parentheses below the average of each estimate or statistic.

Each line where the error term was multiplied by zero ($m=0$) represents the true parameters with no errors in estimation. With any curvature, as reflected by a constant parameter c value, as the error is increased, the validity of the estimates decreases. Although each estimate is still unbiased, the standard deviations of the estimates are quite high, often times reflected in an average of the 40 replicates being quite different from the true estimate. However, accuracy deteriorates at any error term augmentation when the curvature of the production function is increased. That is most noticeable when the results of $c=.01$ are compared with the results of $c=.10$. Within an error multiple of 2 times the unit normal, the average coefficient for the quadratic term is 24.26 for the $c=.01$ technology compared with the true estimate of 25.00, while the average coefficient for the quadratic term is 3.26 for the $c=.10$ technology compared with the true estimate of 2.50. The coefficient of variation (standard deviation/mean) for that estimate for the

$c=.01$ technology is 0.53, but the coefficient of variation for that estimate for the $c=.10$ technology is much greater at 3.16.

Conclusions

The profit functions estimated from different curved technologies with various error terms on observed profits clearly show that technologies that are less concave produce profit function estimates that are more concise over any range of prices. The reason is that any range of prices will map out a greater variability of input usage, output produced, and resultant profits for less concave profit functions.

The issue is whether a valid comparison is presented by using the identical price variation regardless of the curvature of the technology. After all, it is the market that determines input and output prices. If a production function is more concave, then it would be necessary for output and input prices to be more variable to generate the same range of output as a less concave production function. For the market to experience the same output variability, it would be necessary for the market to produce more price variability. That relationship needs to be explored.

References

Lusk, Jayson L., Abdullahi O. Abdulkadri, and Allen M. Featherstone. "An Examination of the Empirical Properties of Duality between the Restricted Profit, Unrestricted Profit, and Production Functions." Paper presented at the 1999 American Agricultural Economics Association Meetings, Nashville, TN, August 8-11, 1999.

Table 1. Estimates of Normalized Quadratic Profit Functions with Various Curvature and Error Terms (parameters defined at end of table)

c	m	b_1	b_2	b_3	R^2
0.01	0.00	25.00 (0.00)	-50.00 (0.00)	25.00 (0.00)	1.00 (0.00)
0.01	1.00	25.06 (0.57)	-50.03 (3.61)	24.72 (5.15)	0.96 (0.01)
0.01	2.00	24.76 (1.23)	-49.27 (8.57)	24.26 (12.79)	0.85 (0.02)
0.01	3.00	24.94 (1.66)	-49.20 (10.05)	23.30 (13.72)	0.71 (0.04)
0.01	4.00	24.30 (2.55)	-45.55 (15.96)	19.06 (21.75)	0.59 (0.07)
0.01	5.00	24.49 (2.70)	-47.57 (17.54)	22.62 (24.77)	0.47 (0.06)
0.01	6.00	24.99 (4.11)	-49.58 (25.07)	24.79 (34.90)	0.40 (0.07)
0.01	7.00	25.70 (4.73)	-53.18 (29.98)	27.98 (42.18)	0.34 (0.07)
0.01	8.00	25.63 (5.33)	-54.87 (32.32)	32.01 (45.25)	0.27 (0.07)
0.01	9.00	25.32 (5.02)	-52.60 (31.56)	28.84 (44.57)	0.23 (0.07)

0.02	0.00	12.50 (0.00)	-25.00 (0.00)	12.50 (0.00)	1.00 (0.00)
0.02	1.00	12.53 (0.53)	-25.01 (3.39)	12.45 (4.91)	0.86 (0.02)
0.02	2.00	12.36 (1.07)	-24.45 (7.08)	11.81 (10.40)	0.59 (0.05)
0.02	3.00	12.82 (1.76)	-26.68 (11.51)	14.51 (16.23)	0.40 (0.06)
0.02	4.00	12.06 (2.21)	-22.41 (15.43)	8.66 (23.30)	0.28 (0.07)
0.02	5.00	12.62 (3.16)	-26.23 (19.40)	14.09 (26.25)	0.20 (0.08)
0.02	6.00	12.96 (3.93)	-28.16 (24.05)	17.27 (33.45)	0.15 (0.07)
0.02	7.00	12.94 (4.09)	-27.16 (26.53)	15.38 (37.09)	0.12 (0.06)
0.02	8.00	11.51 (4.89)	-18.59 (32.14)	3.11 (45.96)	0.10 (0.05)
0.02	9.00	12.09 (5.71)	-22.73 (32.58)	9.42 (44.25)	0.08 (0.04)

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Table 1. Estimates of Normalized Quadratic Profit Functions with Various Curvature and Error Terms (cont.)

c	m	b_1	b_2	b_3	R^2
0.03	0.00	8.33 (0.00)	-16.67 (0.00)	8.33 (0.00)	1.00 (0.00)
0.03	1.00	8.35 (0.66)	-16.74 (4.52)	8.39 (6.56)	0.73 (0.04)
0.03	2.00	8.10 (0.91)	-14.72 (5.12)	5.66 (7.03)	0.39 (0.06)
0.03	3.00	7.96 (1.85)	-14.04 (12.08)	4.86 (17.56)	0.22 (0.07)
0.03	4.00	9.00 (2.25)	-20.99 (13.23)	15.04 (18.79)	0.14 (0.06)
0.03	5.00	9.34 (2.44)	-23.94 (16.46)	18.26 (24.21)	0.11 (0.04)
0.03	6.00	7.80 (3.69)	-12.94 (22.36)	3.80 (30.11)	0.07 (0.04)
0.03	7.00	8.34 (4.91)	-18.51 (27.50)	12.55 (35.58)	0.06 (0.06)
0.03	8.00	8.56 (4.36)	-17.67 (29.24)	8.87 (42.06)	0.06 (0.04)
0.03	9.00	8.07 (5.86)	-13.45 (38.37)	2.59 (53.89)	0.05 (0.03)

0.04	0.00	6.25 (0.00)	-12.50 (0.00)	6.25 (0.00)	1.00 (0.00)
0.04	1.00	6.25 (0.70)	-12.45 (4.42)	6.24 (6.07)	0.59 (0.05)
0.04	2.00	6.15 (1.38)	-11.76 (8.86)	5.28 (12.32)	0.26 (0.06)
0.04	3.00	6.08 (2.14)	-12.73 (13.25)	6.64 (18.17)	0.15 (0.06)
0.04	4.00	6.15 (2.09)	-11.36 (11.95)	4.36 (15.95)	0.09 (0.05)
0.04	5.00	5.84 (2.82)	-10.19 (17.66)	3.80 (24.71)	0.06 (0.04)
0.04	6.00	7.75 (3.56)	-23.03 (20.51)	22.27 (27.48)	0.05 (0.04)
0.04	7.00	5.49 (4.22)	-8.51 (27.70)	1.01 (38.65)	0.05 (0.03)
0.04	8.00	5.26 (4.17)	-8.31 (27.00)	1.91 (38.54)	0.03 (0.03)
0.04	9.00	6.72 (4.77)	-16.68 (31.84)	12.75 (46.32)	0.03 (0.03)

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Table 1. Estimates of Normalized Quadratic Profit Functions with Various Curvature and Error Terms (cont.)

c	m	b_1	b_2	b_3	R^2
0.05	0.00	5.00 (0.00)	-10.00 (0.00)	5.00 (0.00)	1.00 (0.00)
0.05	1.00	5.01 (0.62)	-9.95 (4.02)	4.87 (5.89)	0.49 (0.07)
0.05	2.00	4.82 (1.31)	-8.31 (8.03)	2.50 (11.35)	0.20 (0.08)
0.05	3.00	5.09 (1.80)	-10.56 (11.43)	5.49 (15.60)	0.11 (0.05)
0.05	4.00	5.16 (2.70)	-11.08 (17.17)	6.88 (24.75)	0.07 (0.04)
0.05	5.00	4.74 (2.55)	-8.68 (16.77)	3.31 (25.20)	0.05 (0.04)
0.05	6.00	5.48 (3.87)	-12.54 (24.18)	8.36 (33.15)	0.04 (0.03)
0.05	7.00	4.34 (4.29)	-5.86 (27.82)	-0.69 (38.96)	0.03 (0.03)
0.05	8.00	6.06 (4.62)	-17.99 (29.14)	16.50 (42.15)	0.03 (0.02)
0.05	9.00	5.83 (5.14)	-13.06 (32.62)	6.39 (45.29)	0.04 (0.03)

0.06	0.00	4.17 (0.00)	-8.33 (0.00)	4.17 (0.00)	1.00 (0.00)
0.06	1.00	4.15 (0.61)	-8.11 (3.94)	3.82 (5.57)	0.39 (0.06)
0.06	2.00	3.92 (1.32)	-6.68 (8.75)	1.69 (12.64)	0.16 (0.06)
0.06	3.00	3.87 (1.68)	-6.65 (10.69)	1.39 (14.92)	0.09 (0.05)
0.06	4.00	4.09 (2.64)	-8.14 (15.60)	3.75 (21.83)	0.06 (0.04)
0.06	5.00	3.76 (3.09)	-7.13 (17.47)	4.18 (24.38)	0.03 (0.03)
0.06	6.00	4.47 (3.56)	-10.38 (24.50)	7.33 (35.67)	0.04 (0.03)
0.06	7.00	4.58 (4.31)	-12.29 (26.76)	9.65 (38.25)	0.04 (0.03)
0.06	8.00	3.16 (4.05)	-3.04 (29.47)	-1.75 (44.61)	0.03 (0.02)
0.06	9.00	6.14 (5.15)	-21.74 (32.68)	23.50 (47.50)	0.03 (0.03)

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Table 1. Estimates of Normalized Quadratic Profit Functions with Various Curvature and Error Terms (cont.)

c	m	b_1	b_2	b_3	R^2
0.07	0.00	3.57 (0.00)	-7.14 (0.00)	3.57 (0.00)	1.00 (0.00)
0.07	1.00	3.60 (0.66)	-7.03 (4.27)	3.18 (6.06)	0.35 (0.06)
0.07	2.00	3.40 (1.23)	-5.82 (7.38)	1.85 (9.97)	0.11 (0.06)
0.07	3.00	3.83 (1.93)	-8.86 (12.27)	5.78 (17.66)	0.07 (0.05)
0.07	4.00	3.53 (2.07)	-7.04 (12.90)	3.74 (18.18)	0.04 (0.03)
0.07	5.00	3.70 (2.51)	-8.04 (17.07)	5.03 (25.25)	0.03 (0.03)
0.07	6.00	3.99 (4.09)	-10.26 (26.75)	7.84 (37.40)	0.04 (0.03)
0.07	7.00	3.10 (4.96)	-4.85 (31.82)	0.26 (44.98)	0.03 (0.03)
0.07	8.00	3.15 (3.72)	-6.38 (24.36)	3.99 (36.32)	0.02 (0.03)
0.07	9.00	4.41 (5.60)	-14.05 (33.13)	15.59 (45.90)	0.02 (0.02)

0.08	0.00	3.13 (0.00)	-6.25 (0.00)	3.13 (0.00)	1.00 (0.00)
0.08	1.00	3.07 (0.59)	-6.06 (4.11)	2.75 (6.28)	0.27 (0.07)
0.08	2.00	3.05 (0.97)	-5.69 (6.42)	2.44 (9.48)	0.09 (0.04)
0.08	3.00	3.03 (1.66)	-6.34 (10.98)	3.65 (15.85)	0.05 (0.04)
0.08	4.00	3.38 (2.83)	-7.88 (17.81)	5.24 (24.62)	0.05 (0.03)
0.08	5.00	2.51 (3.35)	-2.46 (19.85)	-2.67 (26.88)	0.04 (0.03)
0.08	6.00	1.97 (3.15)	2.31 (22.35)	-9.44 (34.49)	0.03 (0.03)
0.08	7.00	2.96 (4.49)	-4.31 (30.35)	-0.58 (43.21)	0.03 (0.02)
0.08	8.00	2.03 (4.84)	5.96 (30.58)	-17.26 (43.72)	0.03 (0.03)
0.08	9.00	4.08 (6.32)	-9.04 (39.21)	5.18 (54.13)	0.03 (0.03)

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Table 1. Estimates of Normalized Quadratic Profit Functions with Various Curvature and Error Terms (cont.)

c	m	b_1	b_2	b_3	R^2
0.09	0.00	2.78 (0.00)	-5.56 (0.00)	2.78 (0.00)	1.00 (0.00)
0.09	1.00	2.75 (0.66)	-5.18 (4.03)	2.13 (5.77)	0.23 (0.07)
0.09	2.00	2.77 (1.40)	-5.30 (9.18)	2.48 (13.09)	0.09 (0.05)
0.09	3.00	2.66 (1.68)	-4.95 (11.27)	1.97 (16.35)	0.05 (0.04)
0.09	4.00	2.37 (2.36)	-2.49 (14.18)	-1.21 (19.46)	0.04 (0.03)
0.09	5.00	3.05 (3.19)	-6.35 (20.05)	3.08 (27.96)	0.03 (0.03)
0.09	6.00	3.42 (3.50)	-8.23 (21.63)	5.65 (30.07)	0.03 (0.02)
0.09	7.00	3.56 (4.36)	-10.37 (29.20)	8.93 (41.16)	0.03 (0.03)
0.09	8.00	3.23 (4.61)	-8.64 (29.47)	8.16 (41.63)	0.02 (0.02)
0.09	9.00	2.80 (5.65)	-9.00 (35.25)	10.03 (49.99)	0.03 (0.02)

0.10	0.00	2.50 (0.00)	-5.00 (0.00)	2.50 (0.00)	1.00 (0.00)
0.10	1.00	2.59 (0.58)	-5.78 (3.96)	3.76 (5.83)	0.19 (0.06)
0.10	2.00	2.65 (1.29)	-5.67 (7.68)	3.26 (10.29)	0.07 (0.05)
0.10	3.00	3.17 (1.61)	-8.88 (10.28)	7.37 (14.35)	0.05 (0.03)
0.10	4.00	2.28 (2.10)	-3.21 (13.85)	-0.14 (20.01)	0.04 (0.03)
0.10	5.00	2.16 (2.69)	-1.81 (17.73)	-2.23 (25.72)	0.03 (0.02)
0.10	6.00	2.54 (4.38)	-5.52 (26.69)	3.52 (35.84)	0.03 (0.02)
0.10	7.00	2.23 (3.51)	-3.76 (22.03)	0.98 (31.88)	0.02 (0.02)
0.10	8.00	3.28 (5.50)	-8.12 (34.85)	4.97 (48.84)	0.03 (0.03)
0.10	9.00	3.60 (5.06)	-12.49 (33.43)	12.53 (47.86)	0.02 (0.03)

c is the parameter of production function $y = x - cx^2$; m is scalar multiple of unit random normal error; b_1 terms are averages of 40; regression of $\frac{\Pi}{p} = b_1 + b_2\left(\frac{r}{p}\right) + b_3\left(\frac{r}{p}\right)^2$. Standard deviation of estimates in parentheses; 40 replications.

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