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Bambi's Revenge

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Abstract:

Conflicts between humans and wild animals are emerging from the recovery of once endangered animals populations, and the intrusion of humans into formerly natural areas. As a response to these threats, animal control programs are generally designed with the objective of establishing and maintaining a stable population. This paper challenges this view by studying the management of urban deer in a suburb of Rochester, NY. Benefits and damages imposed by animal populations, as well as the costs of control measures are incorporated. Pulsing controls can be far more efficient than steady state regimes under a wide range of conditions but potential gains can be dissipated by management constraints. The effect of citizen opposition to the killing of animals is investigated.

Keywords:
Deer management, wildlife conflicts, pest control, pulsing, cycles, dynamic programming, renewable resources.

***JEL* Classification:** Q28

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Bambi's Revenge

"When deer populations are regulated by cars,
the costs are shared by everyone through their insurance premiums
and these costs are significant. Any other method of deer control is safer,
more humane and more cost-effective." Curtis (1992, p.8)

I. Introduction and Overview

The white-tailed deer population in North America is thought to have reached its historical low around 1900, when perhaps as few as 300,000 deer remained (Downing 1987, McCabe and McCabe, 1984). Like many other wild species, the number of white-tailed deer had been reduced by the drainage of wetlands, the conversion of forested land to agriculture, and through subsistence, commercial, and sport hunting. The depletion of once abundant stocks of wild game, along with the closing of the western frontier, started the U.S. conservation movement in the late 19th century; a movement that blossomed in the early 20th century. The Lacey Act of 1900 placed restrictions on the interstate commerce and traffic in wild game. This was followed by the establishment in 1903 of the National Wildlife Refuge system; and in 1905, of the National Forest system. Both measures contained provisions to protect habitat critical to waterfowl, migratory songbirds, and larger vertebrates. In 1916 the National Park Service was created, expanding the number of parks where hunting was prohibited. For the white-tailed deer, however, the turning point probably occurred with the abandonment of farms during the Great Depression. The conservation movement, championed by Theodore Roosevelt in the early 20th century, was continued into the New Deal of Franklin Delano Roosevelt. In 1937, the Federal Aid in Wildlife Restoration Act (or the Pittman-Robertson Act) earmarked a portion of federal taxes from the sale of firearms, ammunition and archery equipment for wildlife restoration projects.

The intensification of agriculture on a smaller land base, the emergence of vigorous and well funded habitat enhancement programs, and sharp declines (or eradication) in populations of natural predators resulted in the white-tailed deer re-colonizing forested and rural areas and invading suburban and urban areas across the Northeast and Midwest during the later part of this century. As a result, it is now estimated that between 16 and 20 million white-tailed deer browse within the continental United States (McCabe and McCabe, 1997; Winter, 1999).

At the population densities found in many suburbs today, the white-tailed deer has moved well beyond its endangered status of the early 20th century. For many, deer are no longer the symbol of a threatened species living in a romantic natural environment, as portrayed by the skilled animators who made the Walt Disney classic, *Bambi* (1942)[1]. Collisions with cars, [1.5 million per year in the United States (Winter, 1999)], the browsing of flowers, vegetables, and ornamental plants, and the role played by white-deer in the spread of Lyme's disease has caused many people to now view them as a public nuisance.

This paper is inspired by the efforts of one town, Irondequoit (a suburb of Rochester, New York), to manage its deer population. While the management problem we discuss is specific to this town, the general model is relevant to the management of other deer herds and rebounding wildlife populations, from alligators in Florida to cougars in California. Furthermore, the form of the optimal solution we identify challenges the commonly held view that deer and other wildlife populations are best managed with the objective of maintaining a steady state population.

In the next section we provide some background information on the deer population in Irondequoit, the civic response to the damage caused by the current population, their approach to determining a desirable herd size, and their search for an acceptable method of herd reduction. In the third section a model of the Irondequoit deer herd is specified and discussed. In the fourth and fifth sections, the model is calibrated, and the results are presented

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[1] The animated film *Bambi* portrayed the newborn fawn, Bambi, his parents, and many forest animals (most notably the rabbit Thumper) as talking, sentient creatures, subject to emotions such as love, hate and fear. Bambi loses his mother to a hunter's rifle and barely escapes a forest fire started by another group of careless hunters. From such a perspective, perhaps Bambi's descendants are extracting a bit of revenge on their present-day, human antagonists.

along with their implication for optimal wildlife management. The final section seeks to place the Irondequoit story in a broader perspective of wildlife management.

II. Irondequoit

The town of Irondequoit is located on the shores of Lake Ontario, in Monroe County, New York. It is a predominantly residential, suburban area of 43 km^2, approximately 6 km north of the city of Rochester. A small decline in the number of (human) residents in the 1980's has been followed by a stable population of approximately 53,000 in the 1990's (U.S. Bureau of Census, 1997).

By the late 1800's the white-tailed deer had been extirpated from this part of New York State. Re-colonization of the area by deer drifting north from Pennsylvania occurred slowly. Deer hunting was resumed in Monroe County in 1945, although no deer were present in Irondequoit or the greater Rochester area at that time (Hauber, 1993).

The first reports of car collisions and complaints of deer-damaged orchards were made in 1974. By 1976, hunting was allowed in most of Monroe County and sport hunting in the rural areas of the county slowed the growth of the deer population. In the town of Irondequoit, however, a by-law prohibiting the discharge of firearms and bows allowed for the continued growth of the herd within the town's boundaries. The number of deer-vehicle accidents (DVAs) grew steadily from 61 accidents in 1985 to 237 collisions in 1992 (Table 1). In response to the growing number of DVAs and complaints about damage to residential shrubs and gardens, a Citizen Task Force (CTF) was formed in 1992 to advise Town, County, and State officials on deer management issues. The CTF consisted of 11 members and included representatives of animal welfare and conservation groups, as well as citizens advocating a sharp reduction in the number of deer. The mandate given the CTF was first to establish an acceptable number of animals for the town of Irondequoit and second, to recommend a method of population control. Proposals had to be supported by at least 10 of the 11 members to become official recommendations of the CTF (Curtis, 1992).

In the winter of 1993, University of Syracuse biologists estimated the deer population in Irondequoit to be about 850 animals (a density of approximately 20 deer for "each" square kilometer of the Town). The CTF ultimately recommended a density of 8 to 10 deer per km^2 of "suitable habitat." After considering the feasibility and costs of alternative removal options, the CTF recommended that a culling program be initiated with the goal of reaching the recommended density within five years.

From 1993 to 1998, a total of 712 deer have been removed from the herd by sharpshooters using rifles. Another 27 animals have been killed with bow and arrow by private hunters (Table 1). Under the culling program, designated areas are baited with corn and sharpshooters fire on feeding deer from elevated platforms. This is a typical approach to the reduction of urban deer herds. Thus far, only law enforcement officers have acted as sharpshooters and the culling activities have taken place in Durand Eastman Park, a natural area of approximately 4 km^2 boasting the best deer habitat in Irondequoit. These restrictions have been motivated by safety concerns and a desire to minimize political opposition to culling.

The agreement on a desirable deer density and the use of sharpshooters to cull the deer herd was a major accomplishment on a highly charged, emotional issue. We certainly do not wish to downplay this achievement or detract from the ability of the community to find a practical solution to deer overabundance. However, the mandate given to the CTF and the question asked of its members may have been too narrowly defined to permit a search for an optimal solution. Asking the CTF to identify an "ideal population target" first, and only subsequently, to investigate and recommend a method to achieve this target ignores control costs in the determination of an optimal management regime. This approach also pre-supposes that a steady-state deer population is desirable in the long run. The analysis presented in this paper stresses the role and importance of control costs and demonstrates the sub-optimality of the steady state objective adopted for the management of the Irondequoit deer herd.

III. The Model

By seeking the advice of community members, State, County and Town officials sought to balance the costs of deer overabundance (car collisions, damage to property) against the recreational and other non-consumptive benefits of deer (wildlife observation, amenity/landscape value), and the costs of controlling the population. We therefore postulate that Town officials wish to maximize the welfare provided by the deer herd to the community by choosing a sequence of animal removals $\{Y_t\}$ to solve:

$$\underset{\{Y_t\}}{Maximize} \sum_{t=0}^{\infty} \rho^t \left[\frac{B\left(X_{t-1}\right)}{\rho} + \left(p - \frac{c}{X_{t-1}}\right) Y_t - FC \right]$$

$$\begin{aligned} \text{Subject to} \quad & X_t - X_{t-1} = F\left(X_{t-1}\right) - Y_t \left[1 + \frac{F\left(X_{t-1}\right)}{X_{t-1}}\right] \\ & Y_t \leq \alpha X_{t-1} \\ & FC = \begin{cases} 0 \text{ if } Y_t = 0 \\ q > 0 \text{ if } Y_t > 0 \end{cases} \\ & X_{-1} \text{ given} \\ & B\left(X_{-1}\right) = 0 \end{aligned} \qquad \text{(P)}$$

where

t	an index of time denoting years
X_t	the number of animals in the population at year t
Y_t	the number of animals removed in a culling operation in year t
ρ	the annual discount factor
p	the value derived from the killing of deer (meat consumption)
α	the proportion of the herd X_t that can physically be removed in year t
FC	the fixed cost (0 or q if culling takes place) of management actions in year t
$B(X_t)$	the net benefits derived from population X_t in year t. It is assumed that $B(X)$ is a strictly concave, single peaked function reaching its maximum at $X=\overline{X}$ with $B_x>0$ for $X\in[0,\overline{X})$, $B_x(\overline{X})=0$ and $B_x<0$ for $X\in(\overline{X},\infty)$

c a per unit cost of harvesting parameter

$F(X_{t-1})$ represents the net change in the animal population occurring without deliberate human intervention between periods t-1 and t

It is assumed that the function $B(X)$ is a parabola that measures both the positive benefits and negative costs of the deer herd. For a stock lower than $\bar{X}$, the net benefits from the stock are increasing, while for stocks exceeding this critical value they are decreasing[2]

The particular timing of events adopted for this model reflects the typical approach to deer culling. A population estimate is obtained in early winter (X_{t-1}), after which culling takes place. Since a proportion of the animals removed during the winter are pregnant females, the effective impact of culling on the herd's population in the next period is assumed equal to $Y_t\ [1 + F(X_{t-1}) / X_{t-1}]$.[3] Thus, the net change in the number of deer between years t-1 and t is given by subtracting this "effective take" from the natural growth $F(X_{t-1})$. The resulting population generates benefits and costs measured by $B(X_t)$. This timing is consistent with greater opportunities to see deer in the spring and fall seasons, increased damage to personal gardens in spring, summer and fall, and the occurrence of roughly 52% of all DVA's between September and December (Irondequoit Deer Action Committee, 1997).

Safety concerns severely restrict how and where population control activities can take place. Since only part of the territory can be covered by culling operations, a constraint exists

2 Recent efforts to measure benefit functions for wildlife and habitat support the assumption of decreasing marginal benefits. See, for example, Rollins and Lyke (1998) and Layton et al. (1999).

3 Since $F(X)$ measures annual mortality (for instance, high mortality of young fawns) as well as fecundity, this assumption underestimates the number of embryos taken by the cull and the effectiveness of winter hunting. This formulation assumes that sharpshooters do not select deer on the basis of age or sex. This is not entirely supported by facts. Animals that can be positively identified as males have been spared. However, since most males have lost their antlers by the time culling takes place, assessing the sex of animals at night is difficult and most deer seen at culling stations have in fact been removed.

on the number of animals that can be taken in a given year. Based on discussions with biologists, it is assumed that a constant proportion (α) of the stock resides in and around the areas where culling is permissible.

Historical data from Irondequoit suggest that culling costs are linear in Y and decreasing in X (further details are presented below), justifying the cost structure incorporated into the model. Because of its linearity, the model admits the possibility of a bang-bang control solution whereby the wildlife manager should adjust the stock as rapidly as possible to bring it to its long term steady state[4]. This management strategy is not unlike the recommendation of the Irondequoit CTF and reflects the general tendency to design urban deer control programs around a fixed population target (Jordan et al., 1995). Unfortunately, the presence of fixed costs and stock-dependent variable culling costs result in violations of the Spence-Starrett (1975) theorem guaranteeing the optimality of most rapid approach paths.[5]

The practical implication of this technical failure is that the popular steady state approach to deer and other wildlife management may be sub-optimal. The presence of fixed costs and the form of the variable cost function individually raise the possibility of optimal "pulsing" whereby the population is cyclically reduced to a low number and allowed to grow naturally for a number of years until it reaches the threshold $X^*(t)$ that triggers resumption of culling.

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[4] The bang-bang control for the unconstrained problem (α=1) is obtained by forming the present value Hamiltonian

$$H = \frac{B(X_{t-1})}{\rho} + \left[p - \frac{c}{X_{t-1}}\right]Y_t + \rho\lambda_t\left[F(X_{t-1}) - Y_t\left(1 + \frac{F(X_{t-1})}{X_{t-1}}\right)\right].$$

It can be verified that the steady state of this system is given by the stock X that solves:

$$\delta = F_x - \left(\frac{F(X)}{X + F(X)}\right)\left(\frac{X F_x(X) - F(X)}{X}\right) + \frac{B_x}{\rho}\left(\frac{X + F(X)}{\left(p - c/X\right)X}\right) + \frac{cF(X)}{X^2\left(p - c/X\right)}$$

[5] Fixed costs violate the quasi-concavity requirements of the theorem and stock dependent variable costs violate the separability requirement. See Spence and Starrett (1975) for details.

Everything else equal, fixed costs can induce pulsing because they introduce economies of scale in harvesting. Taking a smaller steady state number of deer each year implies greater average cost per deer removed than can be achieved by harvesting a greater number of deer on alternate years for instance. Stock dependent marginal costs of culling can also induce pulsing because harvesting a deer reduces future stocks and increases future marginal and average culling costs. If this dynamic externality is sufficiently strong, allowing the stock to grow for some time before proceeding with large scale culling can increase the effectiveness of sharpshooters and reduce the total variable costs of culling over time.

The potential advantages of pulsing in the harvesting of renewable resources has been noted by Clark et al. (1973), Pope (1973) and Hannesson (1975) for certain commercial fisheries. Clark (1990) also demonstrates the possible optimality of pulsing when marginal fishing costs are decreasing with the size of the catch. This situation shares similarities with the decreasing marginal cost observed here, although the problem studied by Clark does not contain a dynamic stock externality.

Jaquette (1972 a, b) found that under a set of restrictive assumptions on the form of the functions measuring damage, control costs, and population dynamics, it is optimal to manage certain pest populations that vary stochastically over time under a pulsing policy. These results and the methods employed to derive them follow the footsteps of Arrow et al. (1958), Scarf (1959) and Stokey and Lucas (1989) who demonstrate that pulsing is an optimal solution to a broad class of inventory management problems.[6]

Since the problem of interest to this paper has an objective function linear in the control variable, is non-monotone in the state (and therefore the stock has a shadow price which can be either positive or negative), has a nonlinear law of motion for the stock, and decreasing marginal costs in X; an analytical solution to this problem remains elusive. We therefore proceed

[6] A considerable amount of evidence has also been produced demonstrating the optimality of pulsing in dynamic models of advertizing when consumer response functions have convex parts. For a survey of this literature, see Mesak and Darrat, 1992.

empirically to gain insight into critical relationships, parameter values, and the form of the solution.

IV. Calibration and Results

1. Biology

In 1993, University of Syracuse biologists estimated the Irondequoit deer population at 850 animals and later concluded (Nielsen et al., 1997) that the annual change in the level of the Irondequoit deer herd is given by $F(X) = 0.5703X(1\text{-}X/858)$, where X is the pre-culling population and K=858 is the maximum number of animals that can be sustained in Irondequoit (its carrying capacity). Accordingly, we write the law of motion for the stock as

$$X_t - X_{t-1} = F\left(X_{t-1}\right) - Y_t\left[1+\frac{F\left(X_{t-1}\right)}{X_{t-1}}\right] = 0.5703X_{t-1}\left(1-\frac{X_{t-1}}{858}\right) - Y_t\left(1.5703-\frac{0.5703X_{t-1}}{858}\right)$$

2. Stock Benefits and Damage

2.1 Benefits

Benefits stemming from the existence of free roaming deer in suburban areas include wildlife observation and the nonuse or amenity benefits accruing to those who have preferences for living in an environment that retains natural features. While no direct data exists on the magnitude of such benefits to the residents of Irondequoit, the deliberations and final recommendations of the CTF provide some insights.

Recall that the CTF first selected an ideal deer density by weighting only the benefits and damage of alternative deer populations. Since the task force ignored management costs in determining this target, we adopt the view that its recommendation corresponds to the maximum ($\bar{X}$) of the net benefit function *B(X)*. The most conservative interpretation of the CTF recommendation translates into an objective deer population of 45 to 55 animals. However, because of uncertainties about the actual amount of "suitable" deer habitat at the time of the CTF decision, the recommendation is generally interpreted as a mandate to reduce the population to between 10% and 20% of the 1993 level (Curtis, Hauber, personal communications). In what follows, we assume a target population of $\bar{X}$=100 animals (11.8% of the 1993 population).

2.2 Damage from Car Collisions

Despite a slight decrease in human population, the number of DVAs grew steadily over the 1985-1997 period, exceeding 100 collisions per year since 1987 and reaching a high of 237 in 1992 (Table 1). Figure 1 presents a plot of the relationship between the estimated number of deer-vehicle accidents (DVA_t) and the estimated population in the fall of that year (when most collisions occur), after culling and new births have taken place.

The risk of collision associated with a marginal deer was estimated via a constrained linear model with a dummy variable placed on observations prior to 1991, to account for a changes of methodology in recording DVAs. The estimated relationship is DVA_t= -35.418 $Dummy_t$ + 0.2605 X_t . The t-statistics for the two parameters are respectively -1.893 and 11.651. The F-statistic for the regression is 9.068 and 80.3% of the total variance found in the data is explained by the model.

A study of insurance claims resulting from DVAs in Tompkins County, NY during 1988 (Decker, 1990) indicates average claims amounting to $1,904 (constant dollars of 1998) including deductible. This number is consistent with the figure of $2,000 cited by the AAA Foundation for Traffic Safety (AAA, 1996). The value of insurance claims certainly overstates true damage to vehicles since the existence of deductibles in most automobile insurance contracts imply that minor collisions are less likely to result in a claim than serious ones. On the other hand, these insurance claims do not account for medical costs nor for the loss of productivity resulting from personal injuries. Neither do they account for the potential loss of human life. While no human deaths have yet been attributed to DVAs in Irondequoit, 5 to 6% of collisions in a neighboring county do result in personal injuries (Decker et al., 1990). In the absence of additional data, we limit the estimate of damage to property losses. Multiplying the constant probability of collision per deer (0.2605) by the value of claims per collision ($1,904) puts the expected annual damage per marginal deer at $495.

2.3 Damage to Vegetation and Property

In a survey of Western New York residents, 17% of homeowners reported property damage caused by deer in the previous year (Sayre and Decker, 1990). Losses estimated at $203 (dollars of 1998) per damaged property translate into an average loss over all residences of approximately $34.50. Assuming similar losses from a herd at carrying capacity in Irondequoit, the damage caused by deer to the 20,941 residential and small commercial properties of the Town (Town of Irondequoit Taxes and Assessment Division - personal communication) would amount to $720,000 per year, or an average damage of $842 per deer per year. Adding this figure to marginal losses from DVAs yields an estimate of total marginal damage at X=858 of -$1,337. On this basis we set $m = B_x(K) = -\$1{,}300$.

2.4 Net Benefits Function

We assume that net benefits can be represented by a function of the Gompertz family:

$$B(X) = \begin{cases} 0 & \text{if } X = 0 \\ aX\,Ln(b/X) & \text{if } X > 0 \end{cases}$$

where a>0, b>0. This particular functional form is strictly concave and meets the assumptions of the model ($B_x>0$ for $X<\bar{X}$, $B_x<0$ for $X>\bar{X}$). Based on the data developed above we solve the system of equations $B_x(\bar{X}) = 0$ and $B_x(K) = m$ and simplify to obtain

$$a = \frac{m}{Ln\left[\bar{X}/k\right]} = 604.81; \qquad b = e\bar{X} = 271.83$$

Accordingly, a deer herd of 100 animals would procure annual net benefits to the community of $B(100)$ = $60,481 or an average of approximately $1.11 per resident. Similarly, culling the last deer from Irondequoit would result in losses to the community of approximately $B(1)-B(0)$ = $3,390. While these numbers appear "reasonable", no convincing argument can be made that this function accurately reflects the true welfare value of the deer population to the residents of Irondequoit. For this reason, the analysis presented below focuses on the form and relative efficiency of alternative management regimes rather than on absolute welfare measures.

3. Culling Costs and Benefits

3.1 Culling Costs

Table 2 and Figure 2 illustrate the relationship between the cost of culling and the proportion of the stock harvested from 1993 to 1997.[7] Harvesting costs vary from \$28,448 for the removal of 80 deer from an estimated initial population of 850 in 1993 to \$91,900 for the removal of 215 deer form a stock of 643 in 1995. Based on this limited data, the cost of a culling is taken to be $C\ (X_{t-1},\ Y_t) = \$7{,}763 + \$231{,}192\ (Y_t/X_{t-1})$. This estimate is based on known fixed costs averaging q=\$7,763 per year, and regressing variable costs on the proportion of the stock taken. This yields c=\$231,192 (t=5.206).[8] This parameter can be interpreted as the total variable cost that would have to be incurred if the entire herd was removed in a single year using the current technology. As will be shown later, the results are insensitive to wide variations in q and c.

3.2 Value of Venison Meat (p)

An average of 28.8 lbs. of stew meat per deer harvested has been donated to charitable organizations since 1993. Given the availability of close substitutes, a replacement cost of \$2.49 per pound is imputed to set p = \$72 per deer.

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[7] Costs incurred once [e.g. purchase of riffles, a helicopter fly-over in 1996, carcass inspection (for research purposes only), meat inspection costs in 1993 (no longer required)] have been removed to provide a basis to predict future culling costs.

[8] Several other specifications have been attempted. Simultaneous estimation of q and c yielded very similar estimates of c and statistically insignificant estimates of q. Given known fixed costs and the very limited data available, we adopt this ad hoc procedure for the baseline estimate. The qualitative results are insensitive to wide variations in these parameters.

4. Proportion of the Herd That Can Be Removed in a Given Year (α)

Aerial population surveys place approximately one third of the Irondequoit herd in Durand-Eastman park (where culling takes place) during winter months (Hauber, Porter, personal communications). Based on this indication and accounting for the fact that an estimated 33.4% of the total population was removed in 1995, we initially set α=0.35. The effect of this constraint is studied extensively below.

5. Initial Condition and Discount Rate

An initial population (X_{-1}) of 331 animals is used throughout the analysis. This stock corresponds to the predicted pre-culling population of the Irondequoit deer herd in the winter of 1998-99. A real annual discount rate δ=4% [$\rho=1/(1+\delta)=0.9615$] is adopted. Results for δ=0 and δ=0.08 are also reported.

V. Results

The optimal management regime is obtained by recursively solving the Bellman equation

$$V\left(t, X_{t-1}\right) = \underset{Y_t}{Max}\left\{\rho^t\left[B\left(X_t\right) + \left(p - \frac{c}{X_{t-1}}\right)Y_t - FC\right] + V\left(t+1, X_t\right)\right\} \quad (1)$$

subject to the constraints in (P) and with the added terminal condition $V(T+1, X_t)=0$. A finite time horizon T=70 years was imposed. Duplication of some simulations with T=100 confirmed that the results reported below extend to longer time horizons and are likely stationary solutions that solve the infinite horizon problem.[9] In order to solve this problem via dynamic programming, it was discretized, first by rounding the state variable before each new calculation of the value function, and second, by imposing that Y_t be an integer in the interval $[0, \alpha X_{t-1}]$.

1. The Irondequoit Scenario

The optimal program for the management of the Irondequoit deer herd is illustrated in Figure 3. During an initial adjustment phase, the stock level is brought down from 331 to 113

[9] The Mathematica program used to compute the solutions is available from the authors.

animals, at which point a stable cycle of natural growth and culling is established. Because of the constraint on harvesting and the rapid natural regeneration of the deer population, the initial adjustment phase lasts 19 years. By then, the restriction on the stock that can be harvested results in an effective take barely greater than the natural growth of the stock and only very small population reductions can be achieved by pursuing additional culling. Once the population reaches 113 animals, it is allowed to grow naturally for a single year. This returns the stock to 169 animals and marks the beginning of a stationary management cycle.[10] This stable cycle consists of 11 consecutive years of culling at the constrained rate of $0.35X_{t-1}$ followed by a single year in which no harvesting takes place. This optimal harvesting rule can be written formally as

$$Y_t^* = \begin{cases} 0 & \text{if} \quad X_{t-1} \leq X^* \\ \alpha X_{t-1} & \text{if} \quad X_{t-1} > X^* \end{cases} \tag{2}$$

For stocks smaller than or equal to X*, the optimal strategy is to let the stock grow at its natural rate, while for stocks above X*, the maximum number of animals allowed by the constraint on harvesting are removed. The critical stock X* can be determined empirically by iterating the initial condition X_{-1} until the point where increasing the initial stock by one unit causes the period zero harvesting to go from zero to the positive amount Y*. For the baseline calibration, X*=113. Thus, the deer population of Irondequoit is never brought down to the target of 100 animals because it is simply too expensive to do so given the severe constraint on harvesting. Over the twelve years of the optimal cycle, an average of 136 deer roam the community, causing on average 35 car collisions per year, but as many as 44 when the population reaches its maximum.

[10] Strictly speaking, applying (10) to continuous variables X and Y yields chaotic trajectories whereby no value of X is ever exactly repeated. However, cycle stability always results when harvesting levels and stocks are constrained to integer values. The reader interested in a discussion of deterministic chaos as it applies to the management of animal populations can consult Grafton and Silva-Échenique (1997).

The net present value of the optimally managed herd is -$833,000.[11] In contrast, the option recommended by the CTF would require culling for 24 years in order to stabilize the population, and has a NPV of -$894,000. The Bang-Bang solution (footnote 4) has a steady state population of 126 animals and a NPV of -$846,000. Finally, an unmanaged population reaching and remaining at carrying capacity would have a net present value of -$14.3 million. Therefore, while the optimal cycle only offers marginal improvements in welfare relative to the CTF approach, both control methods yield substantially greater benefits than laissez-faire.

Foregoing culling on certain years has several consequences. Obviously, no culling costs need to be incurred on the off years, resulting in direct savings for the community. Refraining from culling results in an increase in the number of deer, which in turn produces greater viewing opportunities and recreational benefits, but raises the number of car collisions and the rate of property damage. Most importantly, allowing the population to increase reduces the marginal cost of future culling through higher population density.

Pulsing is the mechanism that allows the community to capitalize on naturally diminishing marginal costs of harvesting over time. Over the length of the cycle, the cost savings from pulsing outweigh the lower benefits received in years of low populations and the higher damage incurred in years of more abundant wildlife.

2. Relaxing the Constraint on the Territory Covered by Culling (α).

As Table 3 demonstrates, the meager advantage of pulsing over a most rapid approach to the CTF (or bang-bang) target is largely attributable to the severe restrictions imposed on the scope of culling operations. Relaxing this constraint (α) results in substantial increases in the

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11 Numerical solutions of the finite horizon problem exhibit the effects of terminal conditions. Stable repeated cycles become slightly distorted as $t \rightarrow T$. These terminal distortions are ignored since tests using periods of analysis of up to 100 years indicate that the adjustment period is unaffected by a change in the length of the analysis. Furthermore, increasing T does not modify the stable cycle, it only increases the number of repetitions that are observed. Thus, we are confident that the stable cycles are stationary solutions to the infinite horizon problem. The net present value of programs reported in the paper are the sum of 1) the NPV for the adjustment period and 2) the NPV of an infinite number of repetitions of the stable cycle.

net value of the managed herd to the point where positive net benefits can be obtained when α is greater than approximately 0.8. This stands in sharp contrast to the small improvements in welfare observed for the CTF target program. Increasing α increases the speed and efficiency of the initial population adjustment phase for both cyclical and MRAP management regimes. However, more flexible culling confers additional gains only under a pulsing regime. This is due to the fact that maintaining the population in a steady state requires harvesting the same number of animals every year. As long as α is large enough to compensate for natural growth, no gain is made by increasing α since this harvesting capacity goes unutilized. In contrast, a greater α enables a fine tuning of cyclical harvesting programs that translates into increased amplitude of optimal cycles (and potentially creating complex sequences of harvesting years and natural management years), reduced mean population over the cycle years, a systematic reduction of the minimum population, a general increase in the maximum population and a reduction of the optimal frequency of culling (Figure 4). With greater flexibility, the population can effectively be reduced to levels lower than the steady state target. The community is then in a position to take advantage of additional years when *B(X)* is increasing (early in the cycle) and c/X is decreasing (throughout), it can also delay the rapid increase in damage to vegetation and property, and avoid fixed costs.

It is worth noting that the general solution to the problem is obtained when α=1, and that the exact form of this solution differs slightly from Equation 2. For any value of α>0.94, the optimal policy function is given by the rule $\left(\tilde{x}, X^*\right)=\left(18, 181\right)$, whereby the herd is allowed to grow provided it does not exceed 181 animals, and it is reduced to $\tilde{x}=18$ if it exceeds 181 animals. In the unconstrained problem (with α=1), it would technically be possible to exterminate the herd. However, it is not optimal to do so. The benefits of a herd averaging 90 animals over a seven year cycle exceed the costs of 23 car collisions per year on average, and of the other damage imposed by deer to produce a positive NPV of $284,000.

In Irondequoit, current restrictions on culling eliminate all flexibility and virtually dissipate all potential gains from pulsing. Yet, other areas frequented by deer exist where safe culling could take place (Hauber, personal communication). Extending the culling operation to these areas would afford greater latitude to wildlife managers and increase the gains to be realized from adopting a cyclical management regime.

3. Fixed Costs: a Second Order Effect.

As discussed previously, both the stock dependent variable cost of culling and the presence of fixed costs can give rise to the pulsing solution we observe. Further investigation into the causes of pulsing reveals that at current levels, fixed costs have little effect on the solution. Without a constraint on the scope of culling (α=1) removing all fixed costs (setting q=0) induces a cycle with an optimal policy $(\tilde{x}, X^*) = (18, 176)$. Although X* is reduced by the removal of fixed costs, the resulting optimal cycle is identical in all other respects to the optimal cycle with fixed costs of $7,763. Increasing fixed costs to $25,000 produces the policy rule $(\tilde{x}, X^*) = (18, 182)$ but the same optimal cycle once again. It takes an increase in fixed costs to $33,000 to extend the cycle to eight years under the policy $(\tilde{x}, X^*) = (11, 183)$. The longer cycle is obtained entirely through a reduction in $\tilde{x}$, the maximum population of this being 219, slightly smaller than the 223 previously observed.

4. Variable Costs

With culling costs averaging $534 per deer over the period 1993-97, the Irondequoit operation is more expensive than other similar bait-and-shoot programs where typical average costs vary from $200 to $400 per deer (Revkin, 1998; Jones and Witham, 1995; Drummond, 1995; Stradtmann et al., 1995). Reasons for this difference may be explained in part by relative deer density or by the terrain where bait and shoot operations take place. However, the high level of security measures taken in Irondequoit (e.g. two police officers per station, but only one shooter) is a more likely explanation for the difference in costs. Nonetheless, even in the absence of fixed costs, optimal culling continues to be cyclical with a variable cost parameter as small as 1/8 of its estimated value. As c is further reduced, the effect of culling on future marginal costs decreases, the savings associated with harvesting from large stocks vanish, as does the advantages of pulsing. The reduction in costs required to eliminate pulsing would bring the average historical cost per deer removed in Irondequoit to approximately $60. Since this level is more than three times smaller than the lowest level reported in the literature for any bait and shoot operation, it is very likely that the efficiency gains associated with cyclical herd reduction programs extend well beyond the Irondequoit situation to a wide range of urban and other wildlife management situations.

5. Discount Rate

Variations of the discount rate between 0% and 8% do not affect the qualitative properties of the solution regardless of the presence of a constraint on harvesting. For $\alpha=0.35$, setting $\delta=0$ generates a 15 year cycle with 14 years of culling and a threshold value $X^*=103$. The minimum and maximum populations over the cycle are reduced to 101 and 152 respectively. Alternatively, a larger rate of discount ($\delta=0.08$) increases the tolerated deer population to 115 animals, creating a cycle that is otherwise identical to the solution obtained for $\delta=0.4$ (11 years of culling in a 12 year cycle, minimum and maximum population at 113 and 169 respectively). Similar results are obtained when the constraint on harvesting is removed ($\alpha=1$). For $\delta=0$, the cycle is elongated to 8 years under the policy function (11, 169). The resulting time paths for the number of deer harvested and the stock level are otherwise identical to those arising from the solution of the problem with large fixed costs (>$33,000). When δ is increased to 8%, the rule (18, 181) yields an optimal cycle in all respects identical to the optimal solution described earlier for $\delta=0.04$.

Reducing the discount rate to its minimum value extends the length of optimal cycles and reduces their respective perigees. Identical results were observed after large increases in fixed culling costs. Reducing the discount rate gives far away events a greater impact on the NPV of the optimal program. In a cyclical management regime, the distant future (years preceding the next culling) are characterized by larger stocks and marginal animals that impose net welfare costs. The community can trade away some future costs either by culling at an earlier future date, or by increasing the length of the cycle through more drastic reductions of the population when culling takes place. The empirical evidence from Irondequoit indicates that this second strategy is more beneficial. The lower population initially provides fewer benefits to the community, but these losses are more than compensated by the benefits obtained from delaying high damage rates associated with larger populations and increasing the interval before culling is again necessary.

6. The Social Cost of Lethal Controls: The Bambi ("In My Back Yard") Effect.

The culling of deer and other wild animals can be a subject of considerable controversy. Even when a near consensus emerges on the need to adopt lethal methods of control, some individuals may still oppose such policies on economic, ethical or philosophical grounds. According to a survey of 890 Irondequoit residents conducted in 1998, only 24.3% of the population considers lethal techniques to be the most appropriate control method even though 64.5% of respondents wanted a decrease in dear population (Lauber et al. 1998). In Irondequoit and other communities, court injunctions have been sought, bait stations have been spiked with gasoline and other products that repel deer, demonstrations and vigils have been held and even bomb threats have been made to prevent the killing of deer [Revkin (1998a), Hauber, personal communication)].

Opposition to the killing of deer comes naturally from those who are forced to forego the benefits and enjoyment they would otherwise receive from a more abundant deer population, as well as from others who simply object to the actual killing of animals. In terms of our model, the killing of deer imposes disutility that can be accounted for by subtracting a disutility term $d(Y_t)$ from the objective function. If people are opposed to public agencies engaging in culling operations but are not affected by the scope of the operation, $d(Y_t)$ may enter the objective function much like a fixed cost (*i.e.* $d(Y_t)=0$ if $Y_t=0$; $d(Y_t)=z$ otherwise). Alternatively, the disutility may grow with the number of deer killed [$d'(Y)>0$].

Either way, $d(Y)$ adds to the cost of deer removal and *may lead to the eradication of the herd if it is a technically feasible option* (if $\alpha=1$). For instance, if $d(Y_t)\geq 3{,}328\ Y_t$ or if $d(Y_t)$ is a fixed cost greater than $100,000 (a disutility of merely $2.00 per Irondequoit resident) it would be optimal to eradicate the herd entirely since keeping a positive population results in a rebounding of the stock and the need for future culling and future disutility costs.[12] Thus, the additional disutility imposed on those opposed to killing deer may have the rather perverse effect of making it optimal to remove *all* deer rather than to remove fewer. This undoubtedly runs contrary to the preferences of community members who object to culling. Yet, when taking their welfare into account, extermination becomes optimal.

Of course, complete eradication is only feasible if the entire herd can be captured. It remains true, however, that if the disutility from killing animals enters the objective function as a cost, its impact on even severely constrained optimal cycles is to increase their length, and to reduce the minimum and mean population levels. However, as the disutility cost of lethal methods increases, we should expect non-lethal alternatives such as live-trapping and relocation or the administration of contraceptives to emerge as economically preferable management options.

VI. Concluding Remarks

The data reported in this paper demonstrates that the objective of managing urban animal populations for a steady state can lead to substantial inefficiencies and reduced community welfare. When constrained by the urban setting in which animal control activities

[12] As long as the variable disutility cost is not so large that it becomes more beneficial to let the population grow naturally and incur the -$14.3 million NPV of this laissez-faire policy, the solution becomes $(0, X^*)$. Eradication occurs at t=0 if $X>X^*$. For smaller initial stocks, the population is allowed to grow for some time, providing temporary stock benefits and postponing the cost of eradication. In the course of this research, laissez-faire has never been found to be optimal.

must take place, the optimal management of the Irondequoit deer herd dictates that authorities harvest as many animals as the constraint will allow, when (and only when) the stock exceeds an endogenously determined threshold level X*. This constrained policy is a specific case of the unconstrained $\left(\tilde{x}, X^*\right)$ regime in which the animal population is immediately reduced to $\tilde{x}$ whenever it exceeds X*. This result, optimal for a wide range of parameter values, empirically extends Jacquette's (1972) result to situations where stock growth is deterministic, non-linear and stock dependent; and where the animal population provides benefits in addition to the damage it imposes.

The dynamic externality from culling on future harvesting costs is the primary cause of optimal pulsing. In contrast to the sizeable potential gains of cyclical management identified in this research, pulsing has more generally been viewed as a technical curiosity in the management of fish stocks (Clark, 1990). In large scale commercial fisheries, it may be technically difficult or unprofitable to harvest a large proportion of the stock in any given year. As a result, the dynamic cost externality tends to be very weak and does not favor pulsing. The opposite is true in wildlife management situations, especially for large mammals such as deer, where the herd is relatively confined and a significant portion of the population (if not the entire stock) can be removed in a single season. For the management of deer, cyclical management would remain efficient even if marginal culling costs were far below levels currently observed in urban and sub-urban population control programs. This suggests that many of the communities wrestling with the control of deer populations could benefit from the implementation of cyclical programs.

The welfare gains that can be achieved depend critically on the constraints on harvesting imposed by patterns of land development and safety concerns. The more stringent the constraint, the least latitude is afforded to the wildlife manager and the smaller the advantages of pulsing. In Irondequoit, current restrictions dissipate all potential gains from pulsing. Serious considerations should therefore be given to extending the scope of culling operations to areas outside of Durand-Eastman Park and adopt a cyclical management regime that would reduce the frequency of culling.

Two phenomena could eliminate the potential advantages of cyclical harvesting. First is the presence of transactions costs associated with switching from culling on a certain year to not-culling the next. Such costs may take the form of strong public opposition to the resumption or interruption of culling, or from the need to debate the issues every time new approvals or budgets must be sought in order to resume culling. Sufficiently large switching costs could eliminate the gains from cyclical management and return optimality to a steady state regime. In continuous variational problems, such transactions costs can in fact be used to eliminate chattering and ensure the existence of a solution (Romer, 1986).

The "Bambi Effect" could also eliminate the optimality of pulsing if the disutility imposed by the killing of animals is sufficiently important. Depending on the magnitude of the disutility, complete eradication or laissez-faire become the optimal strategies, ensuring that future culling (and the cost it imposes) will not be required. On the other hand, the Bambi Effect could lead to the adoption of more expensive non-lethal methods. In this case, it is likely that the costs of capturing live animals or delivering contraceptives will also be dependent on the stock size. Careful planning of non-lethal control programs would therefore entail giving serious consideration to pulsing since cyclical management policies would likely emerge as optimal ways of dispelling conflicts between humans and wild animals.

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Table 1
Irondequoit Deer Population, Number Culled and Car Collisions

Year (t)	Winter Deer Population (X_{t-1})	Deer Culled (Y_t)	Deer-Vehicle Accidents
1985*	258		61
1986*	361		93
1987*	481		108
1988*	601		109
1989*	704		163
1990*	776		177
1991*	818		n/a
1992*	840		237
1993	850	80	206
1994	773	160	135
1995	643	215	122
1996	534	66	134
1997	525	120	150
1998	341	71	

- Population data for 1985 - 1992 are estimates based on Nielsen et al.'s (1997) model of population dynamics for the Irondequoit herd.
- DVA's for the period 1985 to 1990 is based on accident reports obtained from police records. Since 1992 however (no data is available for 1991), the number of collisions includes the number of dead animals found (mostly) on roadsides with signs of physical injury consistent with a car collision but that cannot be matched with an accident report. This procedure adjusts for unreported collisions.

Table 2

Culling Costs - Town Of Irondequoit

Culling Year	Total Costs Net of Single Items	Total Costs Net of single Items and known fixed costs	Proportion of the pre-Cull Stock Removed Y_t/X_{t-1}
1993	\$28,448	\$20,951	9.41%
1994	\$59,402	\$57,424	20.70%
1995	\$91,900	\$84,822	33.44%
1996	\$51,904	\$44,098	12.36%
1997	\$61,223	\$51,066	22.86%

Table 3

Characteristics of Optimal Steady State Cycles

α	X* (x,X*)	Total Number of Years	Number of Years with Culling	Minimum Population	Maximum Population	Amplitude	Mean Population	NPV of Infinite Optimal Program*	NPV of MRAP to Xbar
0.35	113	12	11	113	169	56	135.58	-$833,634	-$894,567
0.40	109	9	7	103	163	60	130.22	-$663,669	-$784,985
0.50	117	7	4	92	170	78	128.14	-$435,647	-$726,966
0.70	125	3	1	70	160	90	112.33	-$74,363	-$680,108
0.90	166	11	2	26	211	185	93.90	$229,266	-$659,023
1.00	(18,181)	7	1	18	223	205	90.14	$284,543	-$659,023

* Includes the benefits and costs incurred during the adjustment phase

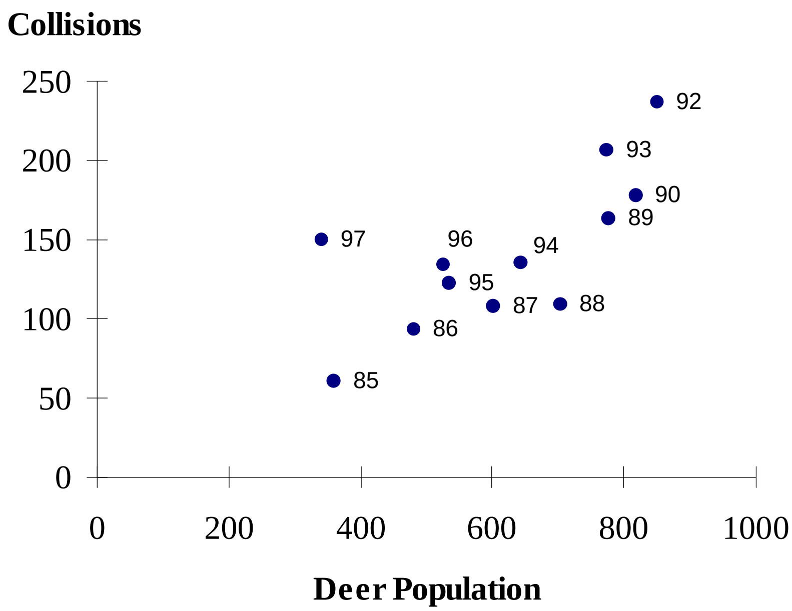

Figure 1
Animal Population and Deer-Vehicle Accidents, 1985 - 1997

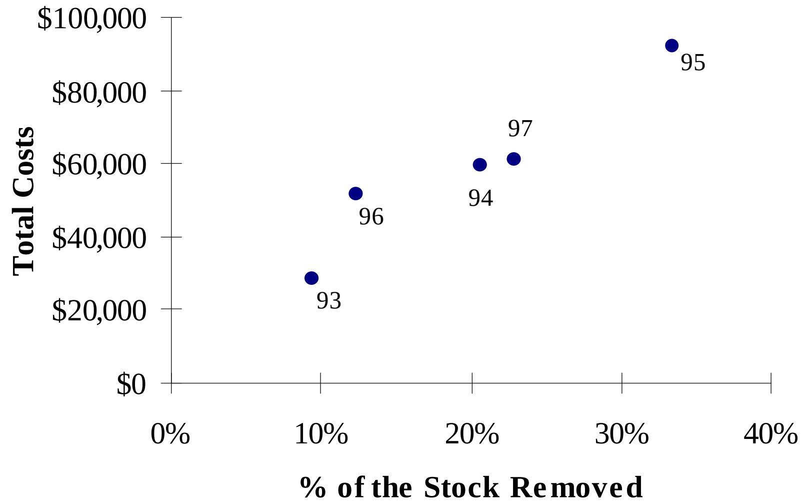

Figure 2
Culling Costs and Proportion of the Stock Removed, 1993 - 1997

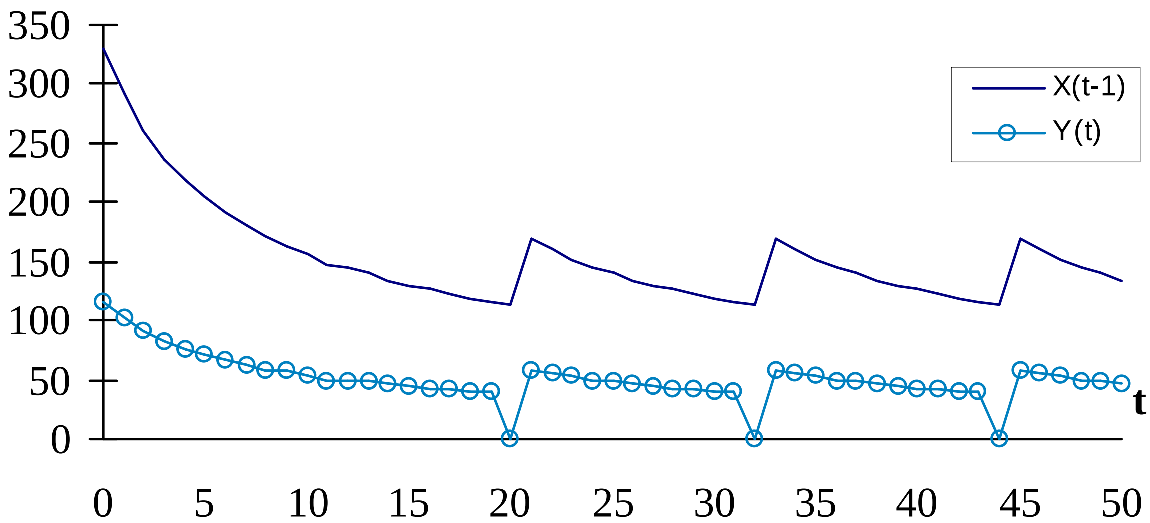

Figure 3

Optimal Management of The Irondequoit Deer Herd

α=0.35

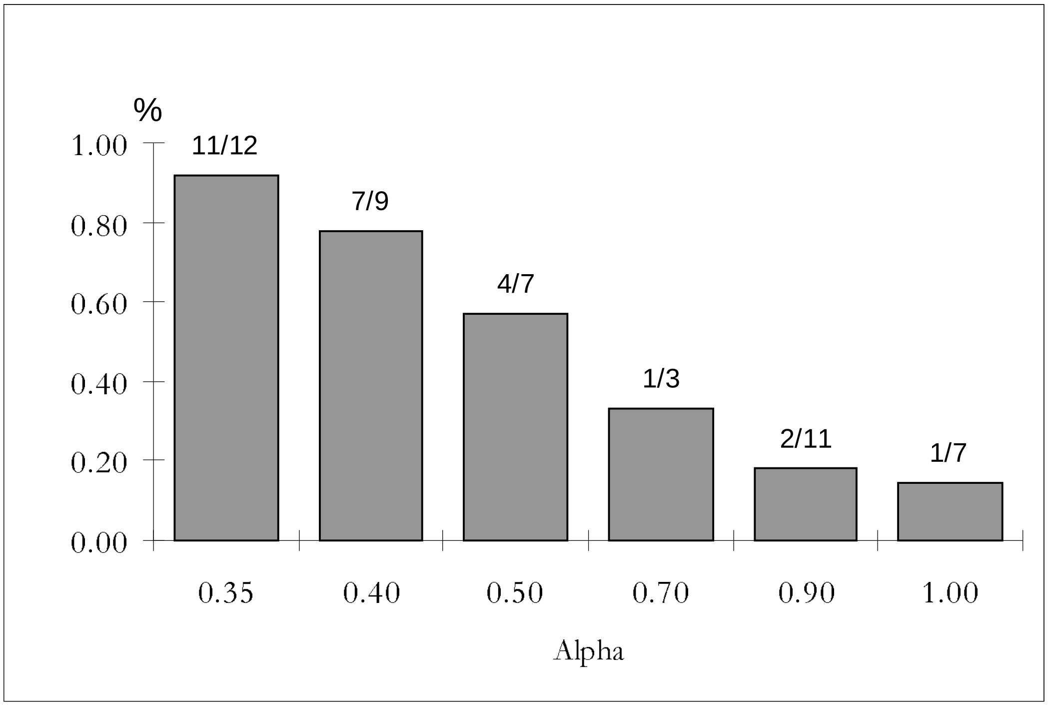

Figure 4
Frequency of Culling in Optimal Cycles