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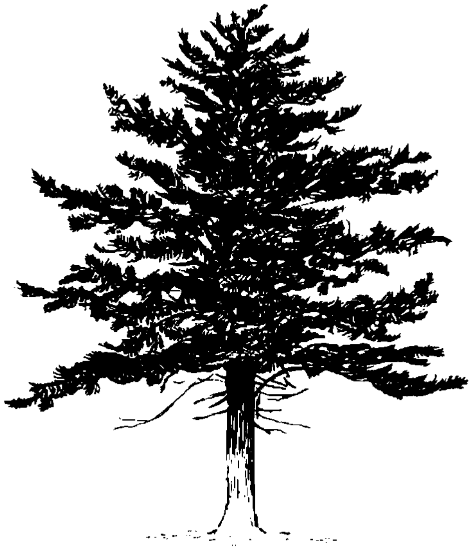

Wilderness: Options to Preserve, Extract or Develop

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Abstract

Wilderness is characterized by the presence of extractable resources [$R(t)=1$] and the absence of development [$D(t)=0$]. In its unexploited, undeveloped state, wilderness provides a flow of amenity services at rate $E=E(t)$, where E is assumed to evolve according to a process of geometric Brownian motion, $dE = \gamma E dt + \sigma_E E dz_E$. If resources are extracted, or if the wilderness is developed at $t=\tau$, environmental amenities are lost forever [$E(t)=0$ for $t \geq \tau$]. If the wilderness is developed at $t=\tau$, before resource extraction, the resources are no longer available, and the option to extract is lost [$R(t)=0$ for $t \geq \tau$]. Suppose that the value of the resources at instant t [$P=P(t)$] and the return on a completed development [$V=V(t)$] also evolve according to geometric Brownian motion, with $dP = \mu P dt + \sigma_P P dz_P$, and $dV = \alpha V dt + \sigma_V V dz_V$, and where the increments dz_E, dz_P, and dz_V are assumed to be uncorrelated. Let the cost of resource extraction be C, the cost of development with resources present [$R(t)=1$] be K_1 and the cost of development with resources absent [$R(t)=0$] be K_0. The option value of wilderness is identified along with the stochastically evolving barriers, $P^*(t)$ and $V_1^*(t)$. Wilderness will be preserved provided that $P(t)$ never catches $P^*(t)$ and $V(t)$ never catches $V_1^*(t)$. A numerical example illustrates how to calculate the barriers $P^*(t)$ and $V_1^*(t)$ given the discount rate (δ), the drift rates (γ, μ, α), standard deviation rates (σ_E, σ_P, σ_V), the cost of extraction (C), the alternative costs of development (K_1, K_0), and the realizations $E(t)$, $P(t)$, and $V(t)$.

Wilderness: Options to Preserve, Extract or Develop

I. Introduction

The extension of option theory to the analysis of real investments has provided economists with new insights into the proper way to evaluate decisions which are risky and costly to reverse (Dixit and Pindyck, 1994). This approach has the potential for widespread application to problems in the field of resource and environmental economics. Brennan and Schwartz (1985) have used option theory to examine the optimal time to develop and abandon a copper mine. The stylized problem introducing the paper by McDonald and Siegal (1986) was to determine the value and optimal timing of a project to extract petroleum from shale. Dixit's (1989) model of exit and entry provided a rich framework from which to evaluate decisions that were costly or impossible to reverse. Clarke and Reed (1989) consider the forest rotation problem when tree growth and timber price are stochastic. Reed (1993) considers the decision to cut or preserve an old-growth forest when timber value and amenity value are evolving according to geometric Brownian motion. Conrad has applied option theory to determine the timing of an investment to slow global warming (1997).

This paper applies option theory to the decision to extract resources and/or develop a wilderness area. As such, it follows in the now extensive and somewhat confusing literature on option value which sprang from the seminal article by Weisbrod (1964). [See Chavas and Mullarkey (1997) for a partial survey of this literature and an attempt to "reunite the children of Weisbrod."] The model in this paper takes the perspective of a social planner or public lands manager trying to determine the desirability and timing of decisions to extract resources from and/or develop a wilderness area. The model, while similar in spirit to the model of land development in Clarke and Reed (1990) and the sequential investment models in Dixit and Pindyck (1994), exhibits important differences. Wilderness amenity value, the future value of the extractable resource, and the return on a completed development all evolve according to uncorrelated geometric Brownian motions. If resources are extracted, or the wilderness is developed, amenity value is lost forever. The resources could be extracted first, and development could take place at a later date. Alternatively, development could take place with resources still *in situ,* but if this happens, the resources are presumed lost or unavailable for future extraction. The cost of development will depend on whether the resources have been previously extracted. Starting with wilderness, the decision to extract or develop will involve the loss of some options and the acquisition of others. The option value of preservation is shown to be a separable function of amenity value [$E=E(t)$], resource value [$P=P(t)$], and the return on development [$V=V(t)$]. The value-matching and smooth-pasting conditions imply stochastically evolving barriers, $P^*(t)$ and $V_1^*(t)$, which must be determined simultaneously. Preservation remains optimal provided that neither $P(t)$ nor $V(t)$ reach (or catch) $P^*(t)$ and $V_1^*(t)$, respectively.

The rest of the paper is organized as follows. The next section formally presents the model, emphasizing the contemporaneous possibilities and potential irreversibilities. This is followed, in Section III, with the derivation of the option value of wilderness preservation and the conditions under which resources would be extracted or the wilderness directly developed. Section IV presents a numerical example to illustrate how the barriers $P^*(t)$ and $V_1^*(t)$ are calculated. The paper concludes with Section V.

II. The Model

Consider a wilderness area containing natural resources (perhaps timber or coal) that could be extracted. The area could also be developed into a resort community. The state of the area will be described by two indicator variables, $R(t)=0,1$ and $D(t)=0,1$. $R(t)=1$ indicates that the resources are still present and available for extraction, while $R(t)=0$ indicates that the resources have been previously extracted or, because of previous development, are unavailable for extraction. $D(t)=0$ indicates that the area is undeveloped, while $D(t)=1$ indicates that the area has been developed. The model assumes that extraction is different than development and that the preservation of wilderness requires both $R(t)=1$ and $D(t)=0$.

In its unextracted, undeveloped state, wilderness is assumed to provide a flow of amenity services at rate $E=E(t)$. If resources are extracted or the area developed, the amenity flow is lost forever. Symbolically, if at $t = \tau$, $R(\tau)=0$ or $D(\tau)=1$, $E(t)=0$ for $t \geq \tau$. If the wilderness is preserved, future amenity value is assumed to evolve according to a process of geometric Brownian motion given by

$$dE = \gamma E dt + \sigma_E E dz_E \tag{1}$$

where γ is the mean rate of drift, σ_E is the standard deviation rate and dz_E is the increment of a standard Wiener process.

Let $P=P(t)$ denote the value of the resources indicated by $R(t)=1$. It is assumed that P also evolves according to geometric Brownian motion as given by

$$dP = \mu P dt + \sigma_P P dz_P \tag{2}$$

where μ is the mean drift in the value of resources contained in the wilderness area and σ_P is the standard deviation rate. If a decision is made to extract the resources at $t = \tau$, the net revenue will be $P(\tau) - C$, where C is the known and constant cost of extraction.

It is possible to develop the area after the resources have been extracted or to develop the wilderness *directly*, leaving the resources *in situ*. Development is regarded as irreversible, and the direct development of wilderness results in not only the permanent loss of amenities, but also "kills" the extraction option. Symbolically, if the wilderness is developed at $t = \tau$, $D(\tau)=1$ and $R(t)=E(t)=0$ for $t \geq \tau$. The rate of return from the completed development is denoted as $V=V(t)$ and is also assumed to follow geometric Brownian motion as given by

$$dV = \alpha V dt + \sigma_V V dz_V \tag{3}$$

with mean drift rate α and standard deviation rate σ_V. The cost of development is assumed to depend on whether the resources have been extracted. If the wilderness is developed directly [from $R(t)=1$] the cost is K_1. If the resources have previously been extracted [$R(t)=0$] the cost of development is K_0. Depending on the resources and the type of development, K_1 could be greater than or less than K_0.

It seems plausible that while E, P and V are all evolving according to geometric Brownian motion, they are uncorrelated. Formally, it is assumed that $E\{dz_E, dz_P\} = E\{dz_E, dz_V\} = E\{dz_P, dz_V\} = 0$.

If the initial state is wilderness [$R(0)=1$, $D(0)=0$], there are four possible scenarios: (a) permanent preservation [R(t)=1, D(t)=0, $t > 0$], (b) extraction at $t = \tau$ but no development [R(t)=0, D(t)=0, $t \geq \tau$], (c) development at $t = \tau$ without prior extraction [R(t)=0, D(t)=1, $t \geq \tau$], and (d) extraction at $t = \tau_1$ and subsequent development at $t = \tau_2$ [R(t)=0, D(t)=0, $\tau_2 > t \geq \tau_1$ and R(t)=0, D(t)=1, $t \geq \tau_2$]. Preservation of wilderness provides a dividend in the form of E(t) and also preserves the option to extract or develop. If resources are extracted first, one looses the amenity dividend and the option to develop at K_1, but one gains the one-time net revenue, P(t) - C, and the option to develop at cost K_0. In the next section we will solve for these option values and the barriers when they would be exercised.

III. Option Values and Barriers

As with the sequential investments considered by Dixit and Pindyck (1994), it will be necessary to consider the timing and value of terminal options first. In our case, this means determining when to exercise the option to develop, given that resources have been extracted previously, [R(t)=0, D(t)=0]. This is a standard option which has been thoroughly covered by Dixit and Pindyck (1994). It is reviewed here because it will be needed when solving for the extraction and development options from a state of wilderness [R(t)=1, D(t)=0].

The option to develop when R(t)=0 is denoted as $F_0(V)$ and must satisfy the Bellman equation $\delta F_0(V) = (1/dt)E_t\{dF_0\}$, where δ is the discount rate and $E_t\{\bullet\}$ is the expectation operator [not to be confused with wilderness amenities, E=E(t)]. Itô's Lemma is used to take the stochastic differential and given equation (3) implies

$$\delta F_0(V) = \alpha V F_0'(V) + (\sigma_V^2/2)V^2 F_0''(V) \qquad (4)$$

where F_0' is the first derivative and F_0'' is the second derivative of $F_0(V)$. This familiar ordinary differential equations is satisfied when $F_0(V) = A_0V^\beta$, where A_0 is a constant to be determined and

$$\beta = (1/2 - \alpha/\sigma_V^2) + \sqrt{(\alpha/\sigma_V^2 - 1/2)^2 + 2\delta/\sigma_V^2} \qquad (5)$$

For the development to have a finite value, $\delta > \alpha$, in which case $\beta > 1$.

The value-matching and smooth-pasting conditions are boundary conditions which must hold at V_0^*, the critical value that "triggers" development. These conditions imply

$$A_0V^\beta = \frac{V}{(\delta - \alpha)} - K_0 \qquad (6)$$

$$\beta A_0V^{(\beta-1)} = \frac{1}{(\delta - \alpha)} \qquad (7)$$

Equation (6) is the value-matching condition and requires that the option to develop at V_0^* must equal the expected present value of the completed project less the development cost, K_0. Equation (7) is obtained from the smooth-pasting condition which requires that the slopes of the value functions, at V_0^*, be the same. This implies that $F_0' = 1/(\delta - \alpha)$. Equations (6) and (7) can be solved for V_0^* and A_0 yielding

$$V_0^* = \frac{\beta(\delta - \alpha)K_0}{(\beta - 1)} \qquad (8)$$

and

$$A_0 = \frac{\left[\frac{\beta(\delta-\alpha)K_0}{(\beta-1)}\right]^{(1-\beta)}}{\beta(\delta-\alpha)} \tag{9}$$

To summarize, if the resources have been extracted from the wilderness area, the decision to develop is simply the irreversible investment (or entry) decision as described by Dixit (1989) or Dixit and Pindyck (1994), and requires that the stochastically evolving V(t) reach V_0^*, which can be calculated from estimates of α, σ_V, δ, and K_0. Knowing how to optimally behave if resources have been previously extracted, we can now consider the more complex options from a state of wilderness.

With the wilderness still intact [R(t)=1,D(t)=0], there are three options available: continued preservation, extraction, or direct development. The value of continued preservation should intuitively depend on E(t), P(t), and V(t). Let H(E,P,V) denote the value function for wilderness. On the continuation region (where preservation is optimal) the Bellman equation requires that $\delta H(E,P,V) = E + (1/dt)E_t\{dH\}$ and, given our assumption of uncorrelated geometric Brownian motions, Itô's Lemma implies

$$\begin{aligned}\delta H(E,P,V) = E + \gamma E H_E + (\sigma_E^2/2)E^2 H_{EE} + \mu P H_P + (\sigma_P^2/2)P^2 H_{PP} + \\ \alpha V H_V + (\sigma_V^2/2)V^2 H_{VV}\end{aligned} \tag{10}$$

where $H_E = \partial H(\bullet)/\partial E$, $H_{EE} = \partial^2 H(\bullet)/\partial E^2$, and so forth. The solution to this partial differential equation, *interpreted as the value of wilderness,* is the separable form

$$H(E,P,V) = E/(\delta-\gamma) + BP^{\varepsilon} + A_1 V^{\beta} \tag{11}$$

The terms on the RHS of (11) have a straightforward interpretation. The term $E/(\delta - \gamma)$ is the expected present value of wilderness amenity flows, given that the current (observable) level is $E = E(t)$. Because $E(\tau)$ is log normally distributed with an expected value of $E(t)e^{\gamma(\tau-t)}$, $\tau > t$, a drift rate of $\gamma > 0$ causes a reduction the effective rate of discount when calculating expected present value.

The second term, BP^{ε}, is the value of the option to extract, where $B=B(t)$ will be a time-varying coefficient (discussed in greater detail below) and

$$\varepsilon = (1/2 - \mu/\sigma_P^2) + \sqrt{(\mu/\sigma_P^2 - 1/2)^2 + 2\delta/\sigma_P^2} \tag{12}$$

With $\delta > \mu$, $\varepsilon > 1$.

The third term, A_1V^{β}, is the value of the option to directly develop (foregoing extraction), where β is defined by equation (5), and $A_1 = A_1(t)$ is also a time-varying coefficient. At any instant, it will be possible to determine $B(t)$, $A_1(t)$ and the two barriers $P^*(t)$ and $V_1^*(t)$. These coefficients and barriers will depend on the currently observable values for $E(t)$, $P(t)$, and $V(t)$ and are determined simultaneously from the following value-matching and smooth-pasting conditions.

$$E/(\delta - \gamma) + BP^{\varepsilon} + A_1V^{\beta} = P - C + A_0V^{\beta} \tag{13}$$

$$E/(\delta - \gamma) + BP^{\varepsilon} + A_1V^{\beta} = \frac{V}{(\delta - \alpha)} - K_1 \tag{14}$$

$$B = \frac{P^{(1-\varepsilon)}}{\varepsilon} \tag{15}$$

$$A_1 = \frac{V^{(1-\beta)}}{\beta(\delta - \alpha)} \tag{16}$$

Equation (13) is the value-matching condition where the land manager would be indifferent between continued preservation and resource extraction. If the resources are extracted, the manager gives up the expected present value of wilderness amenities and kills the options to extract and to develop at a cost of K_1. In exchange, the manager would receive the one-time cash flow, P - C, from sale of the resources, and gain the option to develop at cost K_0.

Equation (14) is the value-matching condition for indifference between continued preservation and direct development. In exchange for the expected present value of wilderness amenities, the options to extract and to develop, one would obtain the expected present value of the completed project, $V/(\delta - \alpha)$ less development cost K_1.

Equation (15) is obtained from the smooth-pasting condition that requires $H_P = 1$, and which must hold at $P^*(t)$, where the land manager is indifferent between continued preservation and extraction. Equation (16) is obtained from the smooth-pasting condition requiring $H_V = 1/(\delta - \alpha)$ which must hold at $V_1^*(t)$, where the land manager is indifferent between continued preservation and direct development.

When the land manager observes E(t), P(t), and V(t) she must use equations (13) - (16) to instantaneously solve for $B = B(t)$, $A_1 = A_1(t)$, $P^*(t)$, and $V_1^*(t)$. Then a comparison of P(t) with $P^*(t)$, and V(t) with $V_1^*(t)$ must reveal that $P(t) < P^*(t)$, and $V(t) < V_1^*(t)$ for preservation to remain optimal. In this problem the barriers $P^*(t)$ and $V_1^*(t)$ stochastically evolve along with the realizations $E(t)$, $P(t)$, and $V(t)$. Wilderness is lost (through extraction or direct development) the first time that $P(t)$ catches $P^*(t)$ or $V(t)$ catches $V_1^*(t)$. A numerical example might be helpful to illustrate the calculation of $P^*(t)$ and $V_1^*(t)$ and to identify the effect on $P^*(t)$ and $V_1^*(t)$ from changes in the drift rates, standard deviation rates, the discount rate, the cost of extraction and the alternative costs of development, K_1 and K_0.

IV. A Numerical Example

Numerical analysis of the options to preserve, extract or develop requires the formulation of a discrete-time analog to the model of Section III. Equations (1) - (3) are approximated by

$$E_{t+1} = [1 + \gamma + \sigma_E Z_{E,t+1}]E_t \tag{17}$$

$$P_{t+1} = [1 + \mu + \sigma_P Z_{P,t+1}]P_t \tag{18}$$

$$V_{t+1} = [1 + \alpha + \sigma_V Z_{V,t+1}]V_t \tag{19}$$

where $Z_{E,t+1}$, $Z_{P,t+1}$, and $Z_{V,t+1}$ are independent standard normal variates. From the value-matching and smooth-pasting conditions given in equations (13) - (16), when can write P_t^* and $V_{1,t}^*$ as

$$P_t^* = \frac{\varepsilon[(A_{1,t} - A_0)V_t^\beta + C + E_t/(\delta - \gamma)]}{(\varepsilon - 1)} \tag{20}$$

$$V^*_{1,t} = \frac{\beta(\delta-\alpha)[K_1 + B_t P_t^{\varepsilon} + E_t/(\delta-\gamma)]}{(\beta-1)} \tag{21}$$

In equation (20), $A_{1,t}$ depends on the value of $V^*_{1,t}$, and in equation (21), B_t depends on the value of P^*_t. The equations for $A_{1,t}$ and B_t are given by

$$A_{1,t} = \frac{\left[\dfrac{\beta(\delta-\alpha)[K_1 + B_t P_t^{\varepsilon} + E_t/(\delta-\gamma)]}{(\beta-1)}\right]^{(1-\beta)}}{\beta(\delta-\alpha)} \tag{22}$$

$$B_t = \frac{\left[\dfrac{\varepsilon[(A_{1,t} - A_0)V_t^{\beta} + C + E_t/(\delta-\gamma)]}{(\varepsilon-1)}\right]^{(1-\varepsilon)}}{\varepsilon} \tag{23}$$

The algorithm used to generate P^*_t and $V^*_{1,t}$ worked as follows. First, generate the realizations E_{t+1}, P_{t+1}, and V_{t+1} according to equations (17) - (19), from initial values E_0, P_0, and V_0. Second, for each period (including t=0) make a *guess* for B_t and $A_{1,t}$. Third, substitute the guess for B_t and the realized values for P_t and E_t into equation (22) and the guess for $A_{1,t}$ and the realized values for V_t and E_t into equation (23). These equations yield *calculated values* for $A_{1,t}$ and B_t. Fourth, compare the calculated values to the guesses for B_t and $A_{1,t}$. Fifth, change the guesses until they numerically coincide with the calculated values. Sixth, calculate and save the values P^*_t and $V^*_{1,t}$. One can then plot P_t and P^*_t and V_t and $V^*_{1,t}$ to see if the realizations ever catch the barriers.

A MATLAB (Version 5) program, based on the above algorithm, is listed in the Appendix. The drift and standard deviation rates were assigned the values of γ=0.03, μ=0.01, α=0.02, σ_E=0.3, σ_P=0.2, and σ_V=0.1. The program then generates sample realizations from initial conditions $E_0 = 1$, $P_0 = 20$, and $V_0 = 3$. The remaining parameter values of $\delta = 0.05$, $C = 50$, $K_1 = 120$ and $K_0 = 100$ allow for the immediate calculation of β, ε, A_0, and the barrier V_0^*. These values are summarized in Table 1.

The program specifies a horizon of $T = 101$ periods, and was run $N = 1{,}000$ times. A sample realization (Run #5) is shown in Figure 1. In this run, P_t reaches P_t^* at $t = 64$ and resources are extracted. For $t > 64$ the relevant barrier for V_t becomes $V_0^* = 6$. This is reached at $t = 80$, at which time the area is developed at a cost of $K_0 = 100$.

In the program, if P_t reached P_t^* before V_t reached $V_{1,t}^*$ (so $t_P < t_V$), V_t was immediately compared with V_0^*. If $V_t \geq V_0^*$ it was arbitrarily assumed that extraction would take precedence, and development would be delayed one period to $t_V = t_P + 1$. If P_t failed to reach P_t^* and V_t failed to reach $V_{1,t}^*$ by $t=100$, then wilderness was preserved. In this case the program also sets $t_V = t_P + 1$, and wilderness preservation (WP) would be indicated by $t_p = 100$ and $t_v = 101$. Extraction followed by development (ED) would be indicated by $t_p < t_v < 100$, extraction with no development (END) is indicated by $t_p < t_v = 100$, and direct development (DD) is characterized by $100 \geq t_p > t_v$.

The results of the 1,000 realizations are also reported in Table 1. Extraction was followed by development in 727 runs. Extraction with no development occurred 27 times. Direct development took place 92 times and wilderness was preserved 154 times.

How are the barriers affected by changes in the underlying parameters? We can numerically address this question by seeing how the barriers would change in a single period, based on a change in a single parameter, while leaving the other parameters, *and the realizations* unchanged. Table 2 summarizes the comparative statics for P_t^* and $V_{1,t}^*$.

Table 1. Numerical Results from 1,000 Realizations

Parameters:

$\gamma = 0.03$ $\mu = 0.01$ $\alpha = 0.02$ $\sigma_E = 0.3$ $\sigma_P = 0.2$ $\sigma_V = 0.1$

$\delta = 0.05$ $C = 50$ $K_1 = 120$ $K_0 = 100$

Calculated Values:

$\beta = 2$ $\varepsilon = 1.8508$ $V_0^* = 6$ $A_0 = 2.7778$

Outcomes:

Extraction with Subsequent Development (ED)	727
Extraction with No Development (END)	27
Direct Development (DD)	92
Wilderness Preserved (WP)	154

Table 2. Comparative Statics of P_t^* and $V_{1,t}^*$

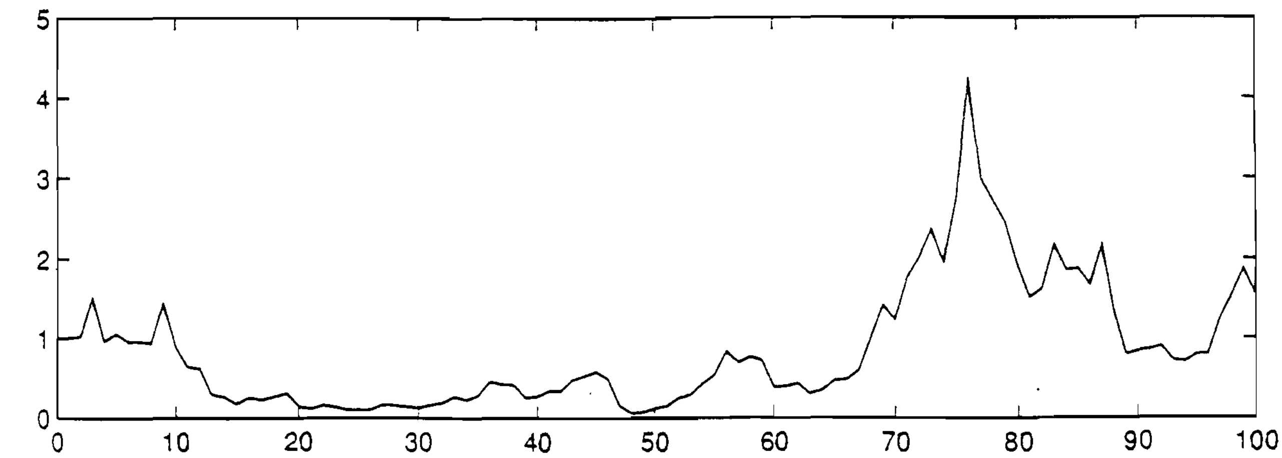
	γ	σ_E	μ	σ_P	α	σ_V	C	K_1	K_0	δ
P_t^*	+	0	+	+	-	+	+	-	+	-
$V_{1,t}^*$	+	0	+	+	-	+	-	+	-	+

The economic intuition behind these results is as follows. If γ increases, one expects the amenity value of wilderness to increase more rapidly in the future. It would require higher values for both P_t^* and $V_{1,t}^*$ before extraction or direct development would be optimal, since either causes a permanent loss of the amenity dividend. If one holds constant the realizations E_t, P_t, and V_t (the realizations are not recalculated with changes in drift or standard deviation rates), then a change in σ_E has no direct affect on P_t^* or $V_{1,t}^*$.

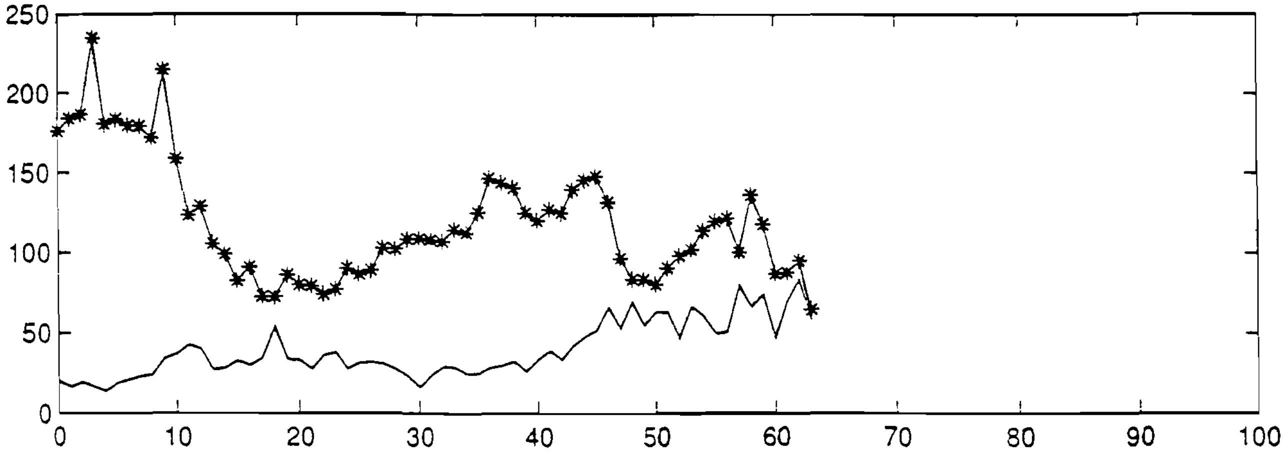
If μ increases both P_t^* and $V_{1,t}^*$ increase. This may at first seem counter-intuitive, since in the standard model of an irreversible investment,

Figure 1. The Sequence of Extraction and Development in Run #5

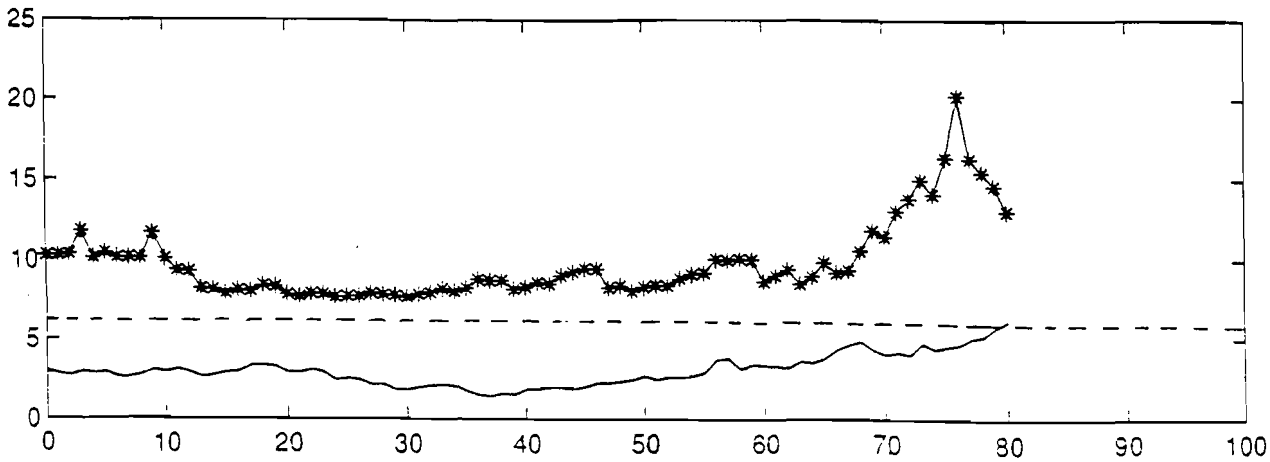
The Realization E_t.



The Realization P_t Reaches the Barrier P^*_t at t=64.



The Realization V_t and the Barriers $V^*_{1,t}$ and V^*_0. V_t reaches V^*_0 at t=80.



an increase in the drift rate lowers the critical barrier or trigger value. It is important to remember that P_t is the price of the resource, and its extraction provides only a "one-shot" net revenue. If the expected drift rate of the resource price increases, one would wish to postpone both extraction and direct development to see if there might be an above average run-up in price which would provide for a large, one-time, net revenue, $[P_t - C]$. Because direct development causes resources to become permanently unavailable, an increase in μ also increases $V^*_{1,t}$. An increase in σ_P increases both P^*_t and $V^*_{1,t}$ since a higher standard deviation rate might result in an unexpected run-up in P_t, and if it doesn't occur, then one still has the option of direct development plus the amenity dividend to partially cover the downside risk of small or negative changes in P_t.

If α increases, both P^*_t and $V^*_{1,t}$ decline. This is the standard result from irreversible investment theory for $V^*_{1,t}$. The decline in P^*_t is the result of the fact that the area can still be developed, at cost $K_0 < K_1$, even if resources are extracted first. An increase in σ_V causes both P^*_t and $V^*_{1,t}$ to increase since waiting for a run-up in the now more variable V_t is partially protected by the option to extract and the amenity dividend.

If the cost of extraction, C, goes up, one would require a higher resource price to induce extraction, while a lower critical $V^*_{1,t}$ would induce direct development. An increase in K_1 has the effect of lowering P^*_t while raising $V^*_{1,t}$, while an increase in K_0 (the cost of development after resources have been extracted) will raise P^*_t and lower $V^*_{1,t}$.

Finally, an increase in δ lowers P^*_t while raising $V^*_{1,t}$. The one-shot nature of $[P_t - C]$ means that an increase in the discount rate is likely to induce earlier extraction, while direct development, with a lower expected, discounted value, would necessitate a higher $V^*_{1,t}$.

V. Conclusions

This paper has examined the optimal timing of preservation, resource extraction, and development of a wilderness. The fact that resource extraction or development often results in an irreversible loss of wilderness, and that the benefit and opportunity cost of such actions are uncertain, means that wilderness preservation also preserves options. This does not mean that wilderness should never be disturbed, but that the decision to extract or develop needs to be formulated in terms of threshold or critical values, where *if* the price of extractable resources or the return on development reaches a critical level, it becomes optimal to extract or develop, based on expected present value.

If there are sequential possibilities, such as extraction followed by development, the options and critical values become more complex to determine. The model of this paper allowed wilderness to be directly developed or to be developed after resource extraction (at a possibly different cost). In this paper development was possible after resource extraction, but if direct development took place, the unextracted resources, along with the wilderness amenity flow, were presumed lost forever.

The option value of preservation depended on current amenity value, the price of extractable resources and the rate of return on the site, if it were developed. This resulted in an unusual feature: the critical values (stopping barriers) were stochastically evolving along with amenity value, resource price, and the return on the development project. At any instant, the critical barriers could be calculated if current amenity value, resource price and project return were known. Wilderness was preserved (it had a higher expected present value) provided that price and the project's rate of return remained below their stochastically evolving barriers.

While spot prices for wilderness resources (for example, coal, timber, copper, or gold) are readily observable, the current rate of return, or dividend, on a development project (say, a ski resort) would have to be based on observable returns to comparable and possibly neighboring projects (say other ski resorts in the vicinity).

A numerical example showed how the critical barriers could be computed. The numerical model also permitted an analysis of the sensitivity of price and return barriers to changes in the underlying parameters. An increase in the expected drift rate for the price of the resource caused both the price and return barriers to increase. This was a result of the "one-shot" nature of the net revenue from resource extraction, and implied that both extraction and development might be postponed if one expected resource prices to increase more rapidly in the future.

An increase in the cost of extracting resources from the wilderness would raise the price barrier (making extraction less likely), but lower the development barrier (making development more likely). An increase in the discount rate lowers the price barrier (making extraction more likely) while raising the development barrier (making development less likely).

Given the seminal work by Weisbrod (1964) and Arrow and Fisher (1974), it is not surprising that modern option theory should provide important conceptual insights and the appropriate methodology to evaluate decisions to preserve or develop wilderness. Empirical analysis of actual sites and projects will be hampered by a lack of time series data on site-specific amenities and the return on site-specific developments which would be needed to estimate mean drift and standard deviation rates. It would be possible to numerically explore the frequency of preservation, extraction, and development for alternative sets of parameters and initial conditions (by repeated use of a program such as that listed in the Appendix), and this may allow a more comprehensive analysis of the price and return conditions that would result in extraction or development.

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Appendix

```
% Wilderness Stopping Barriers
N=1000; % Number of Realizations
tp=[N:1];
tv=[N:1];
for k=1:N
gamma=0.03;mu=0.01;alpha=0.02;sigmaE=0.3;sigmaP=0.2;sigmaV=0.1;T=101;
E=[1:T];
P=[1:T];
V=[1:T];
E(1,1)=1;P(1,1)=20;V(1,1)=3; % Initial Conditions
for i=1:100 % Sample Realizations
E(1,i+1)=(1+gamma+sigmaE*randn)*E(1,i);
P(1,i+1)=(1+mu+sigmaP*randn)*P(1,i);
V(1,i+1)=(1+alpha+sigmaV*randn)*V(1,i);
end
delta=0.05;C=50;K1=120;K0=100;
beta=(1/2-alpha/(sigmaV^2))+sqrt((alpha/(sigmaV^2)-1/2)^2+2*delta/(sigmaV^2));
epsilon=(1/2-mu/(sigmaP^2))+sqrt((mu/(sigmaP^2)-1/2)^2+2*delta/(sigmaP^2));
Vstar=(beta*(delta-alpha)*K0)/(beta-1);
A0=((Vstar)^(1-beta))/(beta*(delta-alpha));
A1=[1:T];
B=[1:T];
A1hat=[1:T];
Bhat=[1:T];
Pstar=[1:T];
V1star=[1:T];
for j=1:100 % Solving for consistent B(t) and A1(t) and P*(t) V1*(t)
A1hat(1,j)=10;
Bhat(1,j)=1;
A1(1,j)=(((beta*(delta-alpha)*(K1+Bhat(1,j)*(P(1,j)^epsilon)+E(1,j)/(delta-gamma))/(beta-1)))^(1-beta))/(beta*(delta-
alpha));
B(1,j)=(((epsilon*((A1hat(1,j)-A0)*(V(1,j)^beta)+C+E(1,j)/(delta-gamma)))/(epsilon-1))^(1-epsilon))/epsilon;
while abs(A1(1,j)-A1hat(1,j))+abs(B(1,j)-Bhat(1,j))>0.000001
A1hat(1,j)=(A1(1,j)+A1hat(1,j))/2;
Bhat(1,j)=(B(1,j)+Bhat(1,j))/2;
end
Pstar(1,j)=(epsilon*((A1(1,j)-A0)*(V(1,j)^beta)+C+E(1,j)/(delta-gamma)))/(epsilon-1);
V1star(1,j)=(beta*(delta-alpha)*(K1+B(1,j)*(P(1,j)^epsilon)+E(1,j)/(delta-gamma)))/(beta-1);
end
tp(k,1)=100;
tv(k,1)=100;
for j=1:100 % Determining the first time P(t) catches P*(t)
if P(1,j)>=Pstar(1,j)
tp(k,1)=j-1; % The time period is the index less one
break
end
end
for j=1:100 % Determining the first time V(t) catches V1*(t)
if V(1,j)>=V1star(1,j)
tv(k,1)=j-1;
break
end
end
if tp(k,1)<tv(k,1) % If extraction first, then the crital barrier is Vstar
for j=tp(k,1)+1:100
if V(1,j)>=Vstar
tv(k,1)=j-1;
break
end
end
end
if tv(k,1)==tp(k,1) % If tp(k,1)=tv(k,1), then tv(k,1)=tp(k,1)+1
tv(k,1)=tp(k,1)+1;
end
disp([k tp(k,1) tv(k,1)])
end
DD=0;END=0;WP=0; % Counting the Outcomes
for k=1:N
if tv(k,1)==101
WP=WP+1; % WP=Wilderness Preserved
end
end
for k=1:N
if tv(k,1)==100
END=END+1; % END=Extraction but No Development
end
end
for k=1:N
if tp(k,1)>tv(k,1)
DD=DD+1; % DD=Direct Development
end
end
ED=N-(WP+END+DD),END,DD,WP, % ED=Extraction then Development
```

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