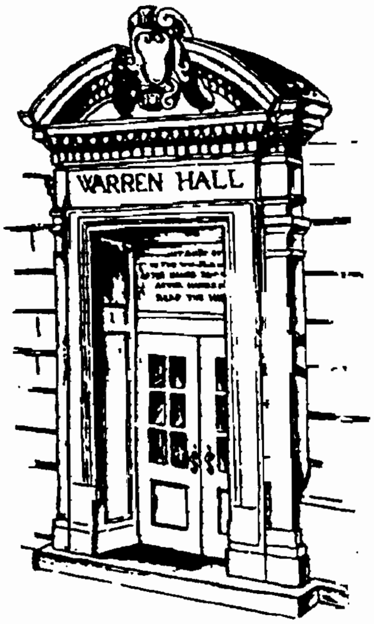
WP 95-09
August 1995



Working Paper

Department of Agricultural, Resource, and Managerial Economics
Cornell University, Ithaca, New York 14853-7801 USA

Global Warming: When to Bite the Bullet

Jon M. Conrad

It is the policy of Cornell University actively to support equality of educational and employment opportunity. No person shall be denied admission to any educational program or activity or be denied employment on the basis of any legally prohibited discrimination involving, but not limited to, such factors as race, color, creed, religion, national or ethnic origin, sex, age or handicap. The University is committed to the maintenance of affirmative action programs which will assure the continuation of such equality of opportunity.

Global Warming: When to Bite the Bullet

by

Jon M. Conrad
Professor of Resource Economics
Cornell University
Ithaca, New York 14853

The author would like to thank Jon Erickson for able research assistance, but is solely responsible for the modeling, analysis and interpretations contained in this paper.

Global Warming: When to Bite the Bullet

Abstract

A stopping-rule model is developed to determine the optimal timing of an investment or policy to slow global warming. Under a policy of "business-as-usual" temperature is assumed to be drifting upward according to a process of Brownian motion given by $dC = m_1\, dt + s_1\, dz$, where C is mean global temperature at instant t, m_1 is the mean drift rate, s_1 the standard deviation in drift, and dz is the increment of a standard Wiener process. Investments or policies to slow global warming are called "bullets," and are denoted by the ordered triple (K, m_2, s_2), where K is the present-value cost of making an investment (biting the bullet) which promises to change the drift in temperature to $dC = m_2\, dt + s_2\, dz$, $m_1 > m_2$ and $s_1 \geq s_2$. The damage from global warming is assumed to be given by the convex function $D = \beta\, e^{\gamma (C - C_0)}$, where D is damage in billions of dollars, β and γ are positive parameters, and C_0 is a reference temperature. Using Itô's Lemma one can show that $dD = \alpha_1 D\, dt + \sigma_1 D\, dz$ before biting the bullet and $dD = \alpha_2 D\, dt + \sigma_2 D\, dz$ after biting the bullet, where $\alpha_i = m_i\gamma + (s_i\gamma)^2/2$ and $\sigma_i = s_i\gamma$, $i = 1,2$. Thus, damage before and after biting the bullet evolves according to geometric Brownian motion. Value functions are derived which measure the expected discounted damage from a policy of business-as-usual and biting the bullet. The value-matching and smooth-pasting conditions permit the identification of a critical temperature or "trigger", C^*, which, if reached, causes the global economy to exercise its option and optimally bite the bullet. The value function while optimally waiting contains a term that measures the option value of a particular bullet, (K, m_2, s_2). By varying the cost, K, mean drift, m_2, and standard deviation, s_2, one can calculate how option value changes with different bullets. The model is calibrated based on time-series data for temperature "anomalies," and estimates of the damage from and the costs of slowing global warming. From a broader perspective, stopping-rule models appear to be well-suited to evaluating environmental policies and determining the optimal timing and value of investments or regulations to protect the environment.

Global Warming: When to Bite the Bullet

I. Introduction

The accumulation of greenhouse gases (carbon dioxide, methane, nitrous oxides, and chloroflurocarbons) and the prospect of global climate change has spawned a broad research program to identify, model and determine the impact of increases in mean global temperature. There are several volumes which summarize what we have learned and what we still don't know about global warming [See the Panel on Policy Implications of Greenhouse Warming (1992), Cline (1992) or Nordhaus (1994)]. While there are many physical, biological and social dimensions to global warming, rational public policy requires information in at least four areas: (1) evidence that global warming has occurred or is likely to occur in the future, (2) predictions on the magnitude and timing of global warming, (3) estimates of the damage from global warming and (4) estimates of the cost of taking actions to reduce global warming or to mitigate its damage.

By its very nature, global warming is a stochastic process. While anthropogenic activities, such as the burning of fossil fuels, the cutting of forests or the release of chloroflurocarbons, may influence the evolution of mean global temperature, other factors, such as volcanic activity, the frequency of El Niño, and changes in solar luminosity, make it difficult to identify "the strength of the greenhouse signal." This natural variability in the earth's climate in turn makes the formulation and timing of appropriate public policies difficult and controversial.

In the next section a model of global warming is developed which focuses on the issue of timing. The model assumes that if sovereign countries follow a "business-as-usual policy," and nothing is done to reduce the emission of greenhouse gases, that temperature will drift upward, subject to a random walk. Increases in the mean global temperature are associated with damages which increase at an increasing rate. There exists, however, a "bullet" (or policy) which, if bitten, will reduce the mean drift rate in temperature and possibly its variance. There is a cost to biting the bullet, and the question becomes, "when to bite the bullet?" Readers familiar with the literature on option value will recognize this as an optimal stopping (starting) problem. By casting public policy in this framework one can identify a critical temperature that triggers a biting of the bullet. It is also possible to estimate the value of having a particular bullet to bite.

In the third section the literature on global warming is reviewed in an attempt to estimate or calibrate the optimal stopping model. The fourth section provides base-case results and sensitivity analysis of the model. The fifth and final section offers some conclusions and caveats.

II. The Model

Let $C = C(t)$ denote the mean global temperature, at instant t, in degrees Celsius. Under business-as-usual it is assumed that global economic activity will cause C to evolve according to

$$dC = m_1\,dt + s_1\,dz \tag{1}$$

where $m_1 > 0$ is the mean drift in temperature, $s_1 > 0$ is the standard deviation about the drift, and dz is the increment of a standard Wiener process [see Dixit and Pindyck (1994)]. Equation (1) says that the change in temperature follows a process of Brownian motion. In the next section evidence is presented that is consistent with this assumption.

The increase in mean global temperature is assumed to inflict, on net, damage on the global economy. Increases in the atmospheric concentration of greenhouse gases may result in protracted periods of drought, more severe storms, higher sea levels and more frequent coastal flooding. The relatively rapid modification of certain ecosystems may preclude adaption on the part of many plant and animal species, thus accelerating the rate of extinction. These damages are difficult to assess and are perhaps the most speculative element of global warming research. Most studies presume a

Damage, with or without the policy, is a function of C, and its evolution can be determined using Itô's Lemma. Under business-as-usual, damage will evolve according to

$$dD = \alpha_1 D\ dt + \sigma_1 D\ dz \tag{4}$$

where $\alpha_1 = m_1\gamma + (s_1\gamma)^2/2$ and $\sigma_1 = s_1\gamma$. If the bullet is bit, Itô's Lemma will imply

$$dD = \alpha_2 D\ dt + \sigma_2 D\ dz \tag{5}$$

with $\alpha_2 = m_2\gamma + (s_2\gamma)^2/2$ and $\sigma_2 = s_2\gamma$. Equations (4) and (5) reveal that the damage from global warming follows a process of geometric Brownian motion. Future damage is log-normally distributed. If the bullet is not bit, the expected damage at instant t is $E\{D(t)\} = D_0\ e^{\alpha_1 t}$ with a variance of $Var\{D(t)\} = D_0^2\ e^{2\alpha_1 t}\ (e^{\sigma^2 t} - 1)$.

The fact that damage is log-normally distributed facilitates the calculation of discounted expected damages, which will converge to finite values provided $\delta > \alpha_1 > \alpha_2$. (This condition will be satisfied for estimates of m_1, m_2, s_1, s_2, and γ presented in the next section.) Suppose, at instant t, that the mean global temperature is $C(t) > C_0$ with an instantaneous damage of D(t) given by equation (2). Suppose further that no action has been taken to slow global warming and we wish to calculate the discounted expected damage if the bullet is *never* bit. This may be calculated as

$$\pi_1 = E_t \int_t^{\infty} D(\tau)\, e^{-\delta(\tau - t)}\, d\tau = D(t) \int_t^{\infty} e^{\alpha_1(\tau - t)}\, e^{-\delta(\tau - t)}\, d\tau$$

$$= D(t) \int_t^{\infty} e^{-(\delta - \alpha_1)(\tau - t)}\, d\tau = D(t) \left[\frac{e^{-(\delta - \alpha_1)(\tau - t)}}{-(\delta - \alpha_1)} \right]_t^{\infty}$$

$$= \frac{D(t)}{(\delta - \alpha_1)} \qquad (6)$$

Similar mathematics will reveal that the sum of discounted expected damages plus the cost of biting the bullet (at t) is given by

$$\pi_2 = \frac{D(t)}{(\delta - \alpha_2)} + K \qquad (7)$$

Assuming that no action has been taken to slow global warming, a rational environmental policy requires a continuous comparison of the expected cost of doing nothing with the expected cost of biting the bullet. On the continuation region, where it is optimal to do nothing, the value function, $V_1(D)$, must satisfy the Hamilton-Jacoby-Bellman (H-J-B) equation given by

$$\delta V_1 = D + \alpha_1 D V_1' + (1/2)\,\sigma_1^2 D^2 V_1'' \tag{8}$$

equation. The homogeneous portion has the well-known solution $V_1 = \xi D^{\varepsilon_1} + \eta D^{\varepsilon_2}$, where ξ and η are unknown constants, and with $\delta > \alpha_1$, $\varepsilon_1 < 0$ and $\varepsilon_2 > 1$ are the roots of the characteristic equation

$$\varepsilon = (1/2 - \alpha_1/\sigma_1^2) \pm \sqrt{(\alpha_1/\sigma_1^2 - 1/2)^2 + 2\delta/\sigma_1^2} \tag{9}$$

Given the definition of option value (to be discussed shortly), it turns out that $\xi = 0$, and the solution to the homogeneous portion may be written as simply $V_1 = \eta D^{\varepsilon}$, with $\varepsilon = \varepsilon_2 > 1$.

A particular solution to equation (8) is the expression for π_1 given in equation (6). A general solution to the H-J-B equation is obtained by adding the solution of the homogeneous portion to the particular solution yielding

$$V_1 = \eta\, D^{\varepsilon} + \frac{D}{(\delta - \alpha_1)} \tag{10}$$

If, at time t, a decision is made to bite the bullet, the value function, assuming the bullet is irreversible and $\delta > \alpha_1 > \alpha_2$, is simply given by π_2, which for completeness we rewrite as

$$V_2 = \frac{D(t)}{(\delta - \alpha_2)} + K \tag{11}$$

Then, on the continuation region it must be the case that $V_1 < V_2$; that is, the cost of doing nothing is less than the cost of biting the bullet. Of interest is the first time that the global economy would be indifferent between business-as-usual and biting the bullet; that is, the first time that $V_1 = V_2$. This is the value-matching condition requiring

$$\eta D^{\varepsilon} + \frac{D}{(\delta - \alpha_1)} = \frac{D}{(\delta - \alpha_2)} + K \tag{12}$$

In addition to the value-matching condition there is a "higher contact" or smooth-pasting condition requiring $\partial V_1/\partial D = \partial V_2/\partial D$ at the instant of indifference. [See Dixit and Pindyck (1994, pp 130-132) for a discussion.] This condition requires

$$\varepsilon\eta D^{\varepsilon - 1} + \frac{1}{(\delta - \alpha_1)} = \frac{1}{(\delta - \alpha_2)} \tag{13}$$

One can use equations (12) and (13) to solve for η and D^*, the critical damage level at which to bite. Given the damage function we can also solve for C^*, the critical temperature at which to bite. Equations (12), (13) and (2) can be used to show that

$$\eta = \frac{K}{(1 - \varepsilon)\, D^{\varepsilon}} \tag{14}$$

$$D^* = \frac{\varepsilon\, (\delta - \alpha_1)(\delta - \alpha_2)\, K}{(\varepsilon - 1)(\alpha_1 - \alpha_2)} \tag{15}$$

$$C^* = \ln\left[\frac{\varepsilon\, (\delta - \alpha_1)(\delta - \alpha_2)\, K}{(\varepsilon - 1)(\alpha_1 - \alpha_2)\, \beta}\right] / \gamma + C_0 \tag{16}$$

The last equation says we should bite the bullet if and when the mean global temperature reaches C^*. It must be remembered that $C(t)$ is evolving according to equation (1) and that from the perspective of $t = 0$ (with C_0) the "first-passage time" is a random variable. Given that $C(t)$ is evolving according to Brownian motion, the expected first-passage time can be calculated as simply $t^* = (C^* - C_0)/m_1$. If one can estimate or assign values to the parameters β, γ, m_1, s_1, m_2, s_2, δ, and K, one can calculate α_1, σ_1, α_2, σ_2, ε, η, D^*, C^*, and t^*.

III. Estimation and Calibration

Temperature Anomalies and Brownian Motion

One of the important areas of global climate research has been the construction of "temperature anomalies" which might be used to statistically test for the existence of the greenhouse effect and to estimate its magnitude. There are now several series of anomalies, available on diskette (see Boden *et al.* 1994). One of the more widely used is the "El Niño/Southern Oscillation - Subtracted" (or ENSO) series, constructed by Jones, Wigley and Briffa, covering the period 1868-1991. This series (contained in Boden *et al.* 1994) attempts to adjust for the temporary drop in mean global temperature during years when an El Niño occurred.

A temperature anomaly is the estimated deviation of temperature (at some instant of time, in some location) from a reference temperature. The construction of mean global temperature anomalies was a formidable task [see Jones and Wigley (1990)]. Let C_t be the mean global temperature in year t and R a constant reference temperature, both in degrees Celsius (°C). The anomaly in year t is defined as $A_t = C_t - R$. By taking a first-order difference one can calculate

$$D_{t+1} = A_{t+1} - A_t = C_{t+1} - C_t \tag{18}$$

and it is possible to calculate the mean annual rate of change in temperature and its variance for the period 1868-1991, or for selected subintervals. The ENSO series was used to calculate D_{t+1}, and to estimate m_1 and s_1 for the periods 1868-1991, 1868-1949, and 1950-1991. The results are reported in Table 1.

For the period 1868-1991 the average annual change in temperature was 0.00435484 °C with a standard deviation of 0.09768243 °C. If one calculates the same statistics for the period 1868-1949 one obtains a sample mean of 0.00170732 °C with a standard deviation of 0.09490482 °C. The statistics for the period 1950-1991 are 0.00952381 °C and 0.10387498 °C, respectively. While the average annual rate of increase went from about 0.0017 °C for the period 1868-1940, to 0.0095 °C for the period 1950-1991 (over a fivefold increase), neither rate is significantly different from zero. The anomalies themselves are significant to only two decimals, and given their variability, one cannot reject the null hypothesis that the 1868-1949 and 1950-1991 rates are equal. Neither the ENSO, nor other anomaly series allow one to confidently forecast the rate of increase in mean global temperature into the next century. Estimates of the likely increase in mean global temperature will have to come from another area of research.

The ENSO anomalies appear consistent with the assumption of Brownian motion, adopted in the model of Section II. The discrete-time analogue to equation (1) takes the form

where ε_{t+1} is an i.i.d. standard normal variate. Subtracting R, the constant ENSO reference temperature, from both sides of (19) leads to the regression $A_{t+1} = m_1 + \rho A_t + \mu_{t+1}$. Subtracting A_t from both sides will facilitate the test for a unit root and yields the "unrestricted" regression

$$D_{t+1} = m_1 + (\rho - 1)A_t + \mu_{t+1} \qquad (20)$$

Given the descriptive statistics in Table 1 and the model of Section II, it seems reasonable to test the joint null hypothesis H_o: $m_1 = 0$, $\rho = 1$. The appropriate F - test is discussed in Pindyck and Rubinfeld (1991, p. 461) or Hamilton (1994, p. 207) and takes the form

$$F = \frac{(T - k)(RSS_R - RSS_U)}{qRSS_U} \qquad (21)$$

where T is the number of observations, k is the number of parameters estimated in the unrestricted regression, RSS_R is the residual sum of squares in the restricted regression, RSS_U is the residual sum of squares in the unrestricted regression, and q is the number of restrictions. For the ENSO data T = 123, k = q = 2, RSS_R = 1.176, and RSS_U = 1.118, resulting in F = 3.139. The critical F - value, based on the Dickey - Fuller test of H_o at $\alpha = 0.05$ is $F^* = 4.70 > F = 3.139$, and thus one fails to reject a random walk (Brownian motion) in anomalies and temperature.

Figure 1 shows the time path for temperature when starting the ENSO anomalies from an initial condition of C_{1867} = 14.46 °C. (This initial value was arbitrarily chosen. Given the cumulative change of 0.54 °C, it leads to a temperature in 1991 of 15 °C.) Also shown in Figure 1 are three Brownian sample paths generated when m_1 = 0.00435484 and s_1 = 0.09768243; ie, the values for average annual increase and standard deviation for the period 1868-1991. The four paths tend to diverge as they approach 1991, one sample path ends at 14.27 °C, while the other two finish above the anomaly-based path, ending at 15.69 °C and 15.83 °C. The 66 percent forecast confidence interval (plus or minus one standard deviation) is given by

$$C_{1867+t} = C_{1867} + 0.00435484\,t \pm 0.09768243\sqrt{t} \qquad (22)$$

[see Dixit and Pindyck (1994, p. 67)]. Note that the interval grows with the square root of time. The 66 percent confidence interval for mean global temperature in 1991 would be 16.09 to 13.91 °C if C_{1867} = 14.46 °C.

Predicting Future Increases in Mean Global Temperature

The historical record implied by the ENSO temperature anomalies does not permit estimation of the likely rate of increase in mean global temperature over the next century. This rate will depend on the emissions of all the greenhouse gases, but perhaps most importantly on the burning of fossil fuels. Policy makers have relied on the simulation results from general circulation models (GCMs) to project the magnitude and timing of temperature change. These are large dynamic models, containing upwards of several hundred thousand equations. There are about a half-dozen such models, and they commonly simulate the equilibrium climatic conditions associated with an increase in greenhouse gas concentrations equivalent to a doubling of CO_2 above preindustrial levels. The preindustrial atmospheric concentration of CO_2 has been estimated at 280 parts per million (ppm).

After reviewing several studies, the Panel on Policy Implications of Greenhouse Warming (1992, p.21) concluded that a doubling of CO_2 would, in equilibrium, raise mean global temperature from 1 to 5 °C. Based on an analysis of the results of several GCMs, the Intergovernmental Panel on

Climate Change, or IPCC (1990), concluded that a doubling of CO_2 would induce a "best-guess" increase in mean global temperature of 2.5 °C, with likely bounds being 1.5 to 4.5 °C

energy used in developed and less developed countries. Cline (1992, Table 2.1) summarizes the results of three studies indicating that a CO_2 doubling, and its associated increase in temperature, might occur in the next century between the years 2050 and 2075. The IPCC (1990) forecasts an "effective doubling" of CO_2 equivalents, including other greenhouse gases, by the year 2025, even though the atmospheric concentration of CO_2 will be less than 560 ppm.

Table 2.1 from Cline (1992) also implies a range of from 2.9 to 10.0 °C as the possible increase in temperature by the end of the next century. This translates to an average annual rate of 0.029 to 0.1 °C, with perhaps a best guess of 0.05 °C per year. Recall that the ENSO anomalies for the period 1950-1991 implied an annual rate of increase of slightly less than 0.01 °C. Thus, the GCMs are calling for a best guess annual rate that is five times higher than what has been "experienced" in the past 40 years.

Damage from a Doubling in CO_2 Equivalents

Increases of 2.5 or 3 °C have in turn been used when considering the possible damage from global warming. Increased temperature and climate change may (1) cause a reduction in the output from agricultural and forest sectors, (2) increase coastal flooding (from a rise in sea level), (3) result in more severe and/or more frequent storms (with an increase in property damage and loss of human life), (4) increase the rate of extinction of plant and nonhuman animal species, and (5) induce an overall loss in "amenity value." This list is not exhaustive. [See the draft of Chapter 6 of the IPCC (1994)]]

Many of the components of the damage from climate change are difficult to assess because they are nonmarket in nature. Based on a review of existing studies, Cline (1992, p.6) speculates that the damage in the U.S. from an increase of 2.5 °C might be $60 billion per year. Nordhaus (1991b, Table 3) considers annual reductions of from one-quarter to two percent of world output as a result of a 3 °C increase in mean global temperature. In 1989, with a world output estimated at $20,000 billion, this would imply an annual loss of $50 to $400 billion.

Both Nordhaus (1991a) and Cline (1992, Chapter 5) provide estimates of the cost of reducing CO_2 emissions, by some percentage, from a baseline emission level. For example, Nordhaus (1991a) estimates the cost of abatement for percentage reductions from the 1985 CO_2 - equivalent emission level (estimated to equal 8.0 billion metric tons, carbon weight).

A comparison of the cost of reducing greenhouse gas emissions is given in Table 2. The numbers from Nordhaus (1991a) are in billions of 1989 dollars and come directly from his Table 9 (p. 63). Cline (1992), in Table 5, p. 229, gives estimates of abatement cost as a percentage of global GNP. Using the percentages that were adjusted for engineering and forestry options for the year 2050, and using a 1989 world GNP of 20,000 billion dollars (to make the dollar estimates comparable to Nordhaus), one obtains the implied costs given in Table 2.

Cline (1992, p. 232) concludes that a 20 percent reduction in GHG emissions might be achieved at zero cost. Initial energy-saving investments and substitutions might reduce emissions and energy bills by simply improving the efficiency of energy use. Changes in the way utilities price electricity may also reduce emissions at little or no cost to customers.

Nordhaus, while not finding "a free lunch," sees initial reductions as being relatively low cost. For reductions at or above 30 percent, the estimates are reasonably close and both imply a convex total cost curve.

After analyzing the benefits from damage reduction, Nordhaus (1991b), concluded that the optimal reduction in GHG emissions was 11 percent. This was the percentage reduction that equated marginal damage with marginal abatement cost. At that percentage reduction, abatement costs were estimated at $3 billion per year and the total benefit was estimated at $6 billion per year.

Cline (1992) does not solve for the optimal reduction in greenhouse gas emissions, but if he did it would presumably exceed 20 percent (obtainable at zero cost). Depending on the actual increase in temperature and associated damage, Cline recommends a two-stage policy approach, where "milder" remedies are taken immediately and other policies are set into place which would create stronger economic incentives to reduce emissions further, if justified by updated damage and abatement costs.

Base-Case Parameters

The inability of temperature anomaly data to provide evidence and estimates of the likely trend in mean global temperature, coupled with the speculative nature of damage and abatement costs, makes the estimation and/or calibration of the model developed in Section II problematic. It is still interesting, and hopefully useful, to explore the numerical implications of variations to a "base-case" parameter set. The base-case set assumes the following parameter values: $\beta = 1$, $\gamma = 1.90126082$, $m_1 = 0.01$, $s_1 = s_2 = 0.1$, $m_2 = 0.005$, $\delta = 0.05$, and $K = \$60$ billion. The values for γ, m_1, s_1, m_2, and s_2 imply $\alpha_1 = 0.03708657$, $\sigma_1 = \sigma_2 = 0.19012608$, $\alpha_2 = 0.02758027$, as per the formulas given in Section II.

The values for β and γ were actually determined from two points on a damage function of the form given in equation (2). These parameter values are based on the assumption that there is, already, \$1 billion in annual damage at $C = C_0 = 15$ °C, and that there will be \$300 billion in annual damage at $C = 18$ °C. This damage function is plotted in Figure 2.

The value $m_1 = 0.01$ is slightly larger than the mean annual rate of increase for the period 1950-1991, but, as noted before, is considerably less than the annual rate of increase of 0.05 °C per year which emerges from some of the GCMs. It is assumed that the bullet is a project or set of policies that would cut the drift rate to $m_2 = 0.005$ while leaving the standard deviation unchanged at 0.10.

The discount rate is initially set a $\delta = 0.05$. In the model of Section II, the expected discounted damage of *not* biting the bullet will become infinite if $\delta \le \alpha_1$. This implies that for low rates of discount or high rates of drift in damage it is optimal to bite the bullet immediately.

The value of K is a present-value cost of biting the bullet. It is not possible to associate a particular annual abatement cost with a reduction from m_1 to m_2. If it is initially assumed that the annual cost of the bullet is I = \$3 billion, then K = \$3/δ = \$60 billion.

Collectively, the base-case parameter set becomes: $C_0 = 15$, $m_1 = 0.01$, $s_1 = s_2 = 0.10$, $m_2 = 0.005$, $\delta = 0.05$, $\beta = 1$, $\gamma = 1.90126082$, I = \$3 billion and K = I/δ = \$60 billion. The implied values for α_1, α_2, σ_1, σ_2 are $\alpha_1 = 0.03708657$, $\sigma_1 = \sigma_2 = 0.19012608$, $\alpha_2 = 0.02758027$. Sensitivity analysis will be conducted to determine the change in the critical temperature that causes the bullet to be bit, and to determine the value of the option of biting a bullet with a different present-value cost or effectiveness (ie, a change in K, m_2 or s_2).

IV. Numerical Results

The critical values D^*, C^*, the expected first-passage time from C_0 to C^* (denoted t^*), and the option value, OV, for the base-case and seven alternative parameter sets are given in Table 3. In the base case, annual damages must reach \$10.19 billion, occurring at $C^* = 16.22$ °C, before the base-case bullet (60, 0.005, 0.1) is bitten. With $m_1 = 0.01$, it will take an expected $t^* = 122$ years to reach the trigger temperature. The option value of the base-case bullet is \$16.23 billion.

The base-case t^* may strike some as an unusually long expected first-passage time. With Brownian motion in temperature, the expected first-passage time is calculated as $t^* = (C^* - C_0)/m_1$. An increase in m_1 will, *ceteris paribus*, reduce C^* and shorten first-passage time. One must be careful, however, to always check the convergence condition $\delta > \alpha_1$, since increasing m_1 will increase α_1. For example, if $m_1 \geq 0.016792$, $\alpha_1 > \delta = 0.05$, when all other parameters are at their base-case settings. (We will return to this issue at the end of this section.)

If the discount rate is decreased to $\delta = 0.04$, D^* decreases to \$5.78 billion, associated with a C^* of 15.92 °C. An increase to $\delta = 0.06$ raises the requisite D^* to \$14.45 billion and C^* to 16.40 °C.. Changes in the discount rate drastically affect option value. At $\delta = 0.04$, OV = \$228.00 billion. If δ increases to 0.06 the option value of the base-case bullet plummets to a paltry \$3.47 billion. Given expected first-passage times of 90 years or longer, the significance of the discount rate in altering option value is not surprising.

The base-case damage at 18 °C was assumed to be \$300 billion per year. If this is revised upward to \$400 billion, the optimal D^* and C^* fall to \$8.82 billion and 16.09 °C, respectively. The option value of the base-case bullet increases to \$30.02 billion. Thus, a shift of the damage function, upward and to the left in Figure 2, causes the damage and temperature triggers to fall and while raising the option value of a particular bullet.

Suppose the annual cost of the bullet increased to \$50 billion per year so that K = \$50 billion/δ = \$1,000 billion. With such an expensive bullet, damage would have to reach \$169.86 billion, associated with a $C^* = 17.70$ °C, to induce action. Expected first-passage time is a lengthy $t^* = 270$ years. The option value of such an expensive bullet, when the current temperature is C = 15 °C [and D(C=15) = \$1 billion] is only \$8.78 billion.

The remaining cases consider the following three bullets: (60,0,0.10), (60,0.005,0), and (60,0,0). In the first case the bullet can reduce the mean drift in temperature to $m_2 = 0$, although the variance remains at $s_2 = 0.10$. This is a very effective bullet, although given that $\alpha_2 = m_2\gamma + (s_2\gamma)^2/2$, the actual change in the drift rate of damage only goes from the base-case $\alpha_2 = 0.02758027$ to $\alpha_2 = 0.01807396$ with $m_2 = 0$. This bullet is exercised when annual damage reaches $D^* = 7.26$ billion (at $C^* = 16.04$ °C) and has an option value of \$24.55 billion.

For the bullet (60,0.005,0) the mean drift rate in temperature is left unchanged at $m_2 = 0.005$, but the variance is reduced to zero. This has the effect of reducing α_2 to 0.0095063 and σ_2 to zero. This bullet has an option value of \$28.91 billion, exercised at $C^* = 15.97$ °C.

The last bullet is a combination of the previous two, and is the most effective in the sense that $m_2 = s_2 = 0$. This reduces the post-bite drift and variance of damages to zero. This bullet would be exercised at $C^* = 15.93$ °C and has an option value of $32.08 billion.

It should be emphasized that the base-case drift rate of $m_1 = 0.01$, while reflective of the annual increase from 1950 through 1991, is five times less than the rate of increase in temperature implied by an average of the general circulation models simulating a doubling of atmospheric CO_2. If $m_1 = 0.05$; indeed, for the other base-case parameters, if $m_1 > 0.016792$, the expected present value of damages is undefined, with the implication that the base-case bullet should be bitten immediately. Putting it another way, if $m_1 = 0.05$, and the other base-case parameters are the same, then $\alpha_1 = 0.113137$. The economy's real rate of discount would have to exceed α_1 for it to rationally postpone a biting of the base-case bullet.

V. Conclusions and Caveats

This paper has applied the relatively recent advancements in stopping rule theory to the timing of an investment or policy to slow global warming. The investments to slow or stop global warming were referred to as "bullets" and were defined by the ordered triple (K, m_2, s_2), where K was the present-value cost of biting the bullet, m_2 was the expected rate of drift in temperature and s_2 its standard deviation, after the bullet is bitten. The drift and standard deviation under "business-as-usual" were denoted by m_1 and s_1, respectively, and both pre- and post-bullet temperature processes were assumed to follow Brownian motion; an assumption which is consistent with the ENSO temperature anomalies.

If the damage from a mean global temperature of $C \geq C_0 = 15\ ^{\circ}C$ can be described by the convex function $D = \beta\, e^{\gamma (C - C_0)}$, where D is annual damage in billions of dollars and β and γ are positive parameters, then the drift in damage before and after biting the bullet will follow a process of geometric Brownian motion. It is then possible to identify value functions which define expected discounted damage before and after biting the bullet, and via the "value-matching" and "smooth-pasting" conditions, determine a critical damage (D^*) or temperature (C^*) which, if reached, "triggers" a biting of the bullet. The model also permits one to calculate the option value of a particular bullet; that is, the value of being able to optimally adopt a particular (K, m_2, s_2).

The model was calibrated after an analysis of the ENSO temperature anomalies from Jones *et al.* (1994) and a review of the previously published estimates of the economic damage from, and the cost of policies that might slow, global warming. Damage estimates and the likely abatement costs for reducing greenhouse gas emissions below a baseline emission rate came from Cline (1992) and Nordhaus (1991a, 1991b, and 1994).

There is much that we still don't know about global warming, and the numerical results of this paper are best taken with a grain of salt. The base-case bullet, with a present-value cost of K = \$60 billion and a promise to reduce the drift rate to $m_2 = 0.005$, with a standard deviation of $s_2 = 0.10$, would not be bitten until damage reached \$10.19 billion at $C^* = 16.22$. With $C_0 = 15$ °C and $m_1 = 0.01$, this would not be expected to occur until 122 years from now.

This base-case result may strike readers familiar with the literature on global warming as an unrealistically long time to wait, given that most studies predict a doubling of atmospheric carbon and a 2.5 or 3 °C temperature increase as early as the middle of the next century. These results are predicated on rates of fossil fuel consumption and circulation models that imply an $m_1 = 0.05$, more than five times larger than the base-case $m_1 = 0.01$. If one maintains the other base-case parameters, but increases the pre-bullet drift rate to $m_1 = 0.016792$, then the drift rate in damage (α_1) exceeds the discount rate, $\delta = 0.05$. If this happens, the integral of expected discounted damage will not converge, and it is optimal to bite the bullet immediately. If mean drift really will be $m_1 = 0.05$, and the other base-case parameters are applied, $\alpha_1 = 0.113137$ and the real rate of discount must exceed α_1 to justify a postponement of bullet-biting time.

An important feature of stopping rule models are the option values imbedded in the underlying value functions. In the model of this paper one can explore the value of alternative bullets, under the presumption that they will be optimally exercised. This feature would appear to have widespread applicability in the evaluation of environmental or other regulatory policies under uncertainty.

Returning to the plausibility of the numerical results of this paper, one should keep in mind that the predictions of mean global temperature in the next century are based on large scale simulation models which have not been validated in a classical econometric sense and which may be deficient in the omission of SO_2 emissions which tend to cool the earth's climate (see *The Economist*, April 1st, 1995). The next two decades should provide us with important observations on mean global temperature, observations that may permit physical and social scientists to validate or reject and revise the first generation of general circulation and economic models used to explore the magnitude, likelihood, and cost of global climate change.

References

Boden, T. A., Kaiser, D. P., Sepanski, R. J., and F. W. Stoss. 1994. *Trends '93: A Compendium of Data on Global Change*, Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Cline, W. R. 1992. *The Economics of Global Warming*, Institute for International Economics, Washington, D. C..

Dixit, A. K. and R. S. Pindyck. 1994. *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.

The Economist, 1995. "Reading the Patterns," April 1st, pp. 65-67.

Hamilton, James D. 1994. *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.

Intergovernmental Panel on Climate Change (IPCC). 1990. *Potential Impacts of Climate Change: A Report Prepared for IPCC by Working Group II*, World Meteorological Organization and United Nations Environment Programme, New York, New York.

Intergovernmental Panel on Climate Change (IPCC). 1994. "The Social Costs of Greenhouse Damage and the Benefits of Control," Chapter 6 (3rd Draft), World Meteorological Organization and United Nations Environment Programme, New York, New York.

Jones, P. D., Wigley, T. M. L., and K. R. Briffa. 1994. "Global and Hemispheric Temperature Anomalies - Land and Marine Instrumental Records," in *Trends '93: A Compendium of Data on Global Change*, T. A. Boden *et al.* editors, Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

Jones, P. D. and T. M. L. Wigley. 1990. "Global Warming Trends," *Scientific American*, (August):84-91.

Nordhaus, W. D. 1994. *Managing the Global Commons: The Economics of Climate Change*, The MIT Press, Cambridge, Massachusetts.

Nordhaus, W. D. 1991a. "The Cost of Slowing Climate Change: A Survey," *The Energy Journal,* 12(1):37-65.

Nordhaus, W. D. 1991b. "To Slow or Not to Slow: The Economics of the Greenhouse Effect," *The Economic Journal,* 101 (July):920-937.

Pindyck, R. S. and D. L. Rubinfeld. 1991. *Econometric Models and Economic Forecasts* (Third Edition), McGraw-Hill, New York.

Table 1. Average Annual Rate of Increase and Standard Deviation for Mean Global Temperature as Implied by the Jones *et al.* (1994) ENSO - Subtracted Anomalies, 1868-1991, 1868-1949, 1950-1991.

1868-1991

$\Sigma D_{t+1} = 0.54$,	$n = 124$	$\hat{m}_1 = 0.00435484$	$\hat{s}_1 = 0.09768243$

1868-1949

$\Sigma D_{t+1} = 0.14$,	$n = 82$	$\hat{m}_1 = 0.00170732$	$\hat{s}_1 = 0.09490482$

1950-1991

$\Sigma D_{t+1} = 0.40$,	$n = 42$	$\hat{m}_1 = 0.00952381$	$\hat{s}_1 = 0.10387498$

where $D_{t+1} = A_{t+1} - A_t = C_{t+1} - C_t$, since $A_t = C_t - R$ = the temperature anomaly in year t, C_t = mean global temperature in year t, R = a constant reference temperature.

Table 2. The Cost of Reducing GHG Emissions as Presented in Nordhaus (1991 a) and Modified from Cline (1992)

Percentage Reduction From Baseline Emission	Nordhaus ($ Billions, 1989)	Modified from Cline ($ Billions, 1989)
20	16.3	0.0
30	49.5	40.0
40	107.9	140.0
50	190.8	260.0
60	308.7	380.0
70	474.8	500.0
80	706.8	620.0

Table 3. Critical Damage Rate, D^*, Temperature Trigger, C^*, Expected First-Passage Time, t^*, and Option Value

	Base-Case	$\delta = 0.04$	$\delta = 0.06$	D(C=18)=$400 B	K=$1,000 B	$m_2 = 0$	$s_2 = 0$	$m_2 = s_2 = 0$
D^*	$10.19	$5.78	$14.45	$8.82	$169.86	$7.26	$6.34	$5.83
C^*	16.22	15.92	16.40	16.09	17.70	16.04	15.97	15.93
t^*	122	92	140	109	270	104	97	93
OV	$16.23	$228.00	$3.47	$30.02	$8.78	$24.55	$28.91	$32.08

Notes: (a) The base-case parameters are $C_0 = 15$, $m_1 = 0.01$, $s_1 = 0.1$, $m_2 = 0.005$, $s_2 = 0.1$, $\delta = 0.05$, I = \$3 B, D(C=15) = \$1 B, and D(C=18) = \$300 B. These parameters imply $\beta = 1$, $\gamma = 1.90126082$, $\alpha_1 = 0.03708657$, $\sigma_1 = \sigma_2 = 0.19012608$, $\alpha_2 = 0.02758027$, and $K = I/\delta$ = \$60 B.

(b) D^* and OV are in billions of dollars, C^* in °C, and t^* is in years from C(0) = 15 °C when $m_1 = 0.01$.

(c) $OV = -\eta D(15)^{\varepsilon} = -\{K/[(1-\varepsilon)(D^*)^{\varepsilon}]\}D(15)^{\varepsilon} = -K/[(1-\varepsilon)(D^*)^{\varepsilon}]$, since D(15) = 1.

Figure 1. Jones et al. (1994) ENSO Implied Temperatures and Three Brownian Sample Paths

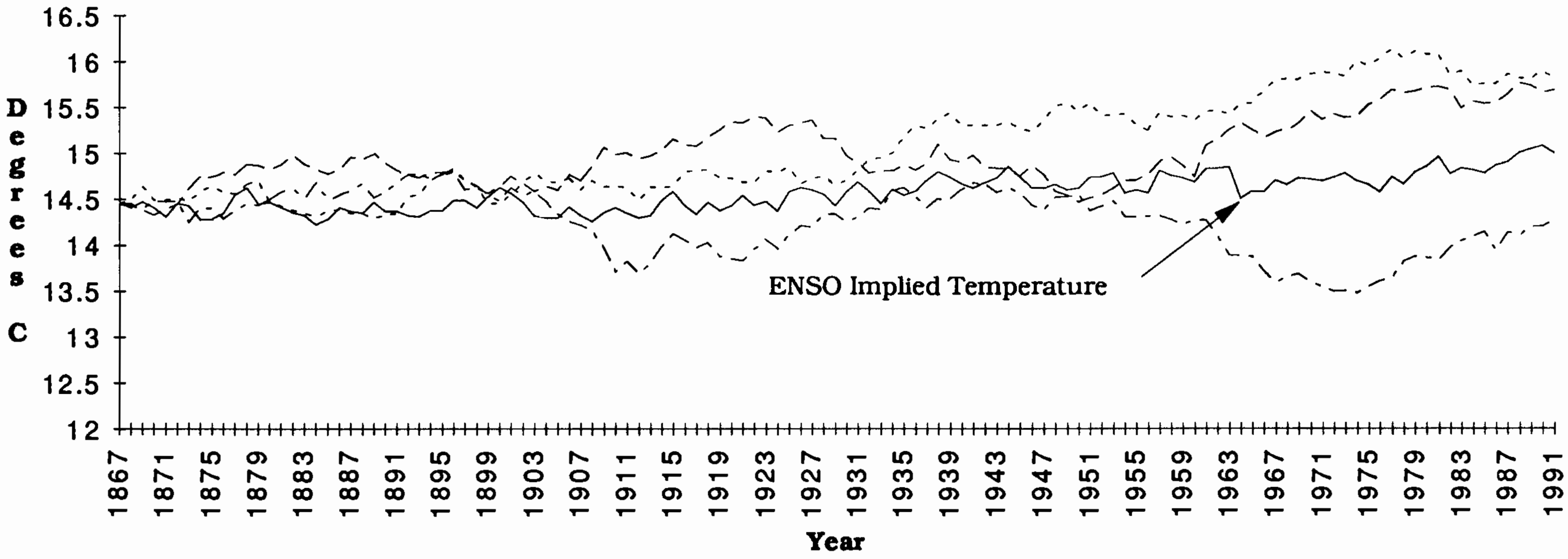
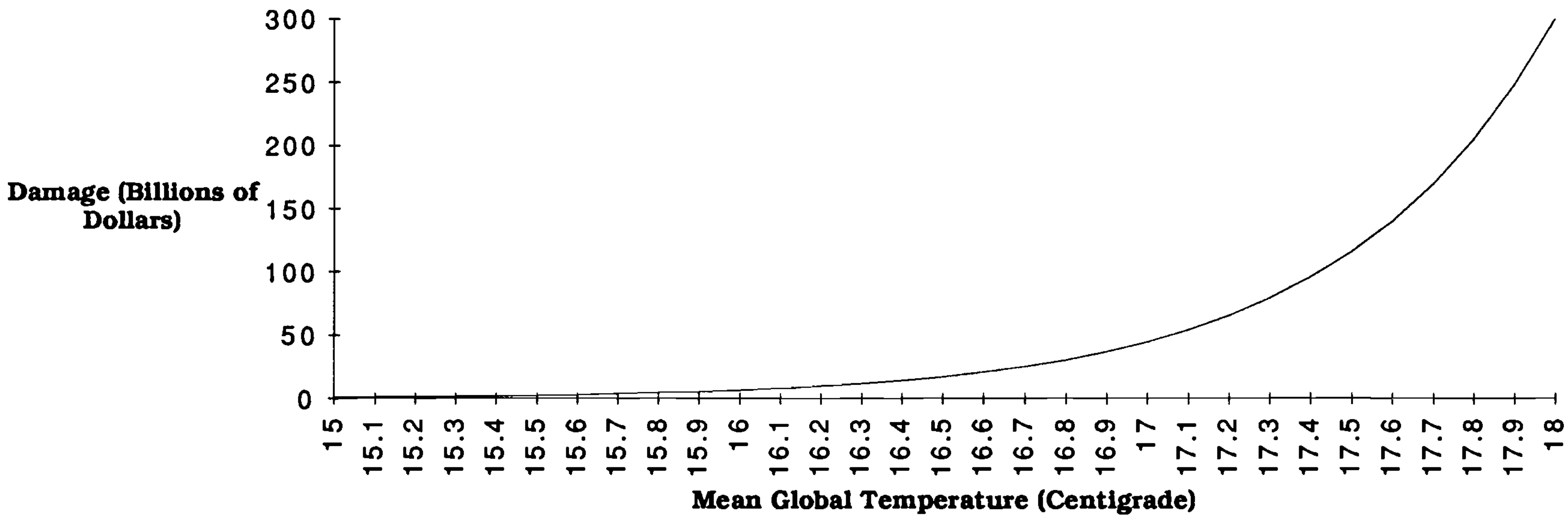


Figure 2. The Damage Function D=Beta*exp[Gamma*(C-15)] for Beta=1 and Gamma=1.90126082



OTHER A.R.M.E. WORKING PAPERS

No. 94-14	Using Variational Inequalities to Solve Spatial Price Equilibrium Models with Ad Valorem Tariffs and Activity Analysis	Charles F. Nicholson Phillip M. Bishop Anna Nagurney
No. 95-01	Assessing the Accuracy of Benefits Transfers: Evidence from a Multi-Site Contingent Valuation Study of Groundwater Quality	T.P. Vandenberg G.L. Poe J.R. Powell
No. 95-02	Measuring the Differences in E(WTP) when Dichotomous Choice Contingent Valuation Responses are Not Independent	Gregory L. Poe Michael P. Welsh Patricia A. Champ
No. 95-03	Timber Extraction in the Brazilian Amazon: Trends in International and Domestic Markets	Steven W. Stone
No. 95-04	Econometric Relationships for Estimating and Ranking Distributions of Nitrate Leaching from Corn Production	Richard N. Boisvert Anita Regmi Todd M. Schmit
No. 95-05	Modeling the Effects of Climate Change on Grain Production in the US: An Experimental Design Approach	Zhuang Li Timothy D. Mount Harry M. Kaiser Susan J. Riha Daniel S. Wilks Radha Sampath
No. 95-06	Estimating the Organizational Costs of Managing the Farm	Loren W. Tauer
No. 95-07	Global Hunger: The Methodologies Underlying the Myth	Thomas T. Poleman
No. 95-08	On the Option Value of Old-Growth Timber: The Case of the Headwaters Forest	Jon M. Conrad