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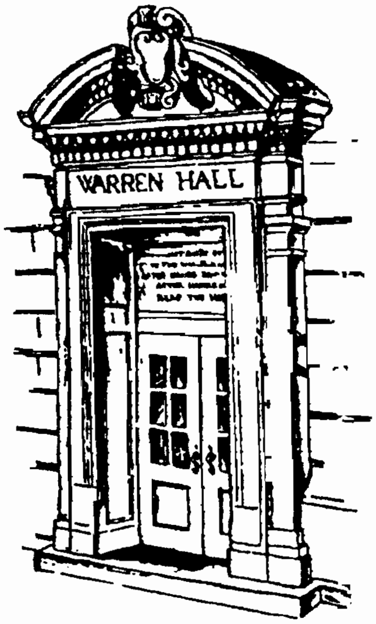

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Fishing in Stochastic Waters

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by

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Abstract

We consider the fishery management problem when the harvested resource evolves according to $dX = [rX(1 - X/K) - Y]\,dt + \sigma X\,dz$, where X is biomass, Y is the rate of harvest and z is a standard Wiener process. The expected drift rate is $[rX(1 - X/K) - Y]$, where $r > 0$ is the species' intrinsic growth rate and $K > 0$ is environmental carrying capacity. Thus, the drift rate corresponds to logistic growth less harvest. The variance rate is σ^2X^2. We seek to maximize the expected present value of net revenue. If net revenue is given by $\pi = (p - c/X)Y$, where $p > 0$ is the per unit price for fish and $c > 0$ is a cost parameter, we obtain a closed form solution for optimal biomass that contains Clark's singular solution as a special case ($\sigma=0$). Further, we observe a range where increases in the discount rate *increase* optimal biomass. The model is calibrated for the Pacific halibut, and stochastic and deterministic solutions are compared. The paper closes with a discussion of how the model might be used within a program of individual transferable quotas (ITQs) which have been adopted or are being considered by coastal countries trying to correct for the chronic and widespread problem of overfishing.

Fishing in Stochastic Waters

I. Introduction

One of the important applications of modern capital theory has been in the field of resource economics where methods of dynamic optimization have been used to solve for optimal rates of harvest and steady-state biomass in a single-species fishery. When maximizing the present value of net revenue from a fishery, Clark (1990, p.40), has shown that optimal steady-state biomass must satisfy the equation

$$F'(X) - \frac{c'(X)F(X)}{p - c(X)} = \delta \qquad (1)$$

where X is biomass (the fish stock), F(X) is a concave natural growth function, net revenue is given by $\pi = [p - c(X)]Y$, where c(X) is the stock-dependent average cost of harvest, p is a constant per unit price for harvested biomass, Y, and δ is the rate of discount. The average cost function is assumed to be convex with derivatives $c'(X) < 0$ and $c''(X) > 0$. The current-value Hamiltonian is linear in Y and the approach to X^*, the optimal biomass, is "most rapid," with $Y = Y_{MAX}$, if $X > X^*$ and $Y = 0$, if $X < X^*$ [see Spence and Starrett (1975)].

When (a) natural growth is logistic, so that $F(X) = rX(1 - X/K)$, where $r > 0$ is called "the intrinsic rate of growth," and $K > 0$ is "the environmental capacity," and (b) the average cost function takes the form c/X, where $c > 0$ is a cost parameter, then equation (1) yields an explicit solution for X^* given by

$$X^* = \frac{K}{4}\left[\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right) + \sqrt{\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{pKr}}\right] \qquad (2)$$

Along an approach path to X^* the fish stock changes according to

$$dX/dt = \dot{X} = rX(1 - X/K) - Y \qquad (3)$$

In this paper we consider the fishery management problem when natural growth, net of harvest, is subject to a continuous stochastic process of the form

$$dX = [rX(1 - X/K) - Y]\,dt + \sigma X\,dz \qquad (4)$$

where the expected "drift rate" in biomass is $rX(1 - X/K) - Y$, the "variance rate" is $\sigma^2 X^2$, and dz is the increment of a standard Wiener process. Biomass becomes and Itô variable and future net revenues are uncertain. [See the Appendix to Pindyck (1991) for a compact discussion of Wiener and Itô processes.]

We are able to obtain an explicit solution for optimal biomass (which becomes a "size barrier" or "stopping rule") that contains equation (2) as a special case when $\sigma = 0$. For $\delta > 2\sigma^2 > 0$ the optimal barrier, X_S^*, is less than X^*. In contrast to the deterministic model, there is a range of variation in δ where increases in δ cause the optimal stock to *increase.*

In the next section we formulate the stochastic optimization problem and solve for the optimal barrier. In section three we present some numerical results for the Pacific halibut fishery and compare X_S^* with X^* for alternative values of δ. The fourth section concludes with a brief discussion of the potential application of size barriers like X_S^* to real world fisheries management.

II. The Model

The problem of maximizing expected discounted net revenue from the fishery will be more tractable if we employ the transformation $q = \ln X$, where *ln* is the natural log operator. Using Itô's Lemma it can be shown that

$$dq = [r(1 - e^q/K) - Y\,e^{-q} - \sigma^2/2]\,dt + \sigma\,dz \tag{5}$$

where $X = e^q$. Discounted net revenue at instant t can be written as $\pi(q,Y) = (p - ce^{-q})Y\,e^{-\delta t}$, and the transformed optimization problem is stated as

$$\text{Maximize} \quad E_0 \int_0^{\infty} (p - ce^{-q})Y\,e^{-\delta t}\,dt$$

$$\text{Subject to} \quad dq = [r(1 - e^q/K) - Y\,e^{-q} - \sigma^2/2]\,dt + \sigma\,dz$$

$$Y_{MAX} \geq Y \geq 0, \quad X(0) = X_0 \text{ given}$$

where E_0 is the expectation operator taken at t=0. Since discounting is the only nonautonomous feature in this problem the Hamilton-Jacoby-Bellman (H-J-B) equation requires

$$\delta V = \text{Max}\,\{(p - ce^{-q})Y + [r(1 - e^q/K) - Y\,e^{-q} - \sigma^2/2]V' + (\sigma^2/2)V''\} \tag{6}$$

where $V = V(q)$ is an unknown (current) value function. The expression to be maximized is linear in the control, Y, and there exists a switching function $\omega(t) = p - ce^{-q} - V'e^{-q}$ where if $\omega(t) < 0$, $Y^* = 0$, if $\omega(t) > 0$, $Y^* = Y_{MAX}$ and on the rare occasions when $\omega(t) = 0$, $Y^* = rX_S^*(1 - X_S^*/K)$, where $X_S^* = e^{q^*}$.

The size barrier is a boundary and the continuity condition requires

$$\delta V = [r(1 - e^q/K) - \sigma^2/2][pe^q - c] + (\sigma^2/2)\, V'' \qquad (7)$$

at $X_S^* = e^{q^*}$. This is an ordinary differential equation.

As a candidate try $V = k_1 e^{2q} + k_2 e^q + k_3$, where k_1, k_2 and k_3 are unknown constants. Substituting V and V´´ into equation (7) and collecting the terms involving e^{2q}, e^q and the remaining parameters and constants, one can show that the equation is satisfied when

$$k_1 = \frac{pr}{K(2\sigma^2 - \delta)},\quad k_2 = \frac{(2Kpr - Kp\sigma^2 + 2cr)}{K(2\delta - \sigma^2)},\quad k_3 = \frac{c(\sigma^2 - 2r)}{2\delta}$$

Substituting the expressions for k_1 and k_2 into the expression $V' = pe^q - c$, which is the "smooth-pasting condition for this problem, and replacing e^q with X and e^{2q} with X^2 results in a quadratic whose positive root is

$$X_S^* = \frac{K}{4}\left[\theta\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right) + \sqrt{\theta^2\left(\frac{c}{pK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c(\delta - 2\sigma^2)}{pKr}}\right] \qquad (8)$$

where $\theta = (\delta - 2\sigma^2)/(\delta - \sigma^2/2)$. Note: when $\sigma = 0$, $\theta = 1$ and equation (8) reduces to equation (2). For $\delta > 2\sigma^2 > 0$, $1 > \theta > 0$ and $X_S^* < X^*$. This result is consistent with Pindyck (1980) who found more rapid depletion of a nonrenewable resource when average extraction costs were nonlinear in "proven reserves," R, with $c''(R) > 0$, as in our model. In Pindyck's model, the stochastic drift in reserves induced cost increases, when reserves declined, that exceeded cost reductions, when reserves expanded. As a result, the competitive industry increased their rate of extraction and price started lower and rose more rapidly.

A similar effect is at work in our stochastic fishery model. By maintaining the stock in the neighborhood of $X_S^* < X^*$ the sole owner or manager can reduce the degree to which random fluctuations raise the expected cost of harvest. Thus, in a stochastic environment, where the average cost of harvest is c/X, it pays to maintain a lower stock with smaller variations. In the next section we explore the extent to which X_S^* lies below X^* via a numerical example calibrated to the Pacific halibut fishery. Sensitivity analysis reveals an interesting and unexpected effect when the discount rate, δ, is varied.

III. The Pacific Halibut Fishery

Clark (1990, p.47) briefly describes the history of the Pacific halibut, a large and commercially valuable fish that ranges over an extensive area of the North Pacific. Following a period of open access, this fishery was put under the regulation of the International Pacific Halibut Commission, in 1924. The Commission was successful in rebuilding the stocks until the 1950s when Russian and Japanese trawlers entered the fishery. During the 1960s and 1970s the combined rates of harvest caused the halibut stocks to decline far below estimates of the level that would support maximum sustainable yield. In the late 1970s both the U.S. and Canada enacted legislation which extended their exclusive fishing zones (the "200-mile limit"), and in the 1980s the management plans adopted by the U.S. and Canadian governments have been coordinated by the Commission.

Based on a previous study by Mohring (1973), Clark adopts the following parameter values: $r = 0.71$, $K = 80.5 \times 10^6$ kg, and $(c/p) = 17.5 \times 10^6$ kg, and uses equation (2) to solve for the optimal steady-state values X^* and Y^*, for different values of δ. The values of X^* have been recalculated and are reported in Table 1 under the column $\sigma = 0.0$, where $X_S^* = X^*$. For $\sigma = 0.1$ and $\sigma = 0.2$ Table 1 also reports the values of θ and X_S^* for δ ranging from 0.05 to 0.30. When $\sigma = 0.0$, increases in δ cause the optimal biomass to *monotonically* decline, and as $\delta \to \infty$, $X^* \to X_\infty = (c/p) = 17.5 \times 10^6$ kg, where X_∞ is the stock level at open access equilibrium. At X_∞ net revenue is zero, and early economists [Gordon (1954) and Scott (1955)] observed that fishery rent had been "dissipated."

In the stochastic model, increases in δ increase θ. For a given level of σ, and for values of δ where $\infty > \delta > 2\sigma^2 > 0$, Table 1 reveals how increases in δ affect X_S^*. Initially the increases in δ cause increases in θ that more than offset changes in the other terms that would reduce X_S^*. Thus, over some initial range, increases in δ, will *increase* X_S^*. At some point, however, further increases in δ induce changes in these other terms that cause X_S^* to decline. As $\delta \rightarrow \infty$, X_S^* and X^* both asymptotically approach $X_\infty = c/p$. This is shown in Figure 1 where for σ=0.2 and δ=0.1, $X_S^* = 12.4 < X_\infty = 17.5$. As δ increases, X_S^* increases above X_∞, peaks and then asymptotically declines back to X_∞.

For a given δ, increases in σ cause X_S^* to decline. Further, as σ increases the range where increases in δ increase X_S^* expands. For example, when $\sigma = 0.1$, X_S^* declines when δ goes from 0.15 to 0.20, whereas when σ = 0.2, X_S^* continues to increase from $\delta = 0.10$ through $\delta = 0.30$. The elasticity of X_S^* to changes in σ depends on the level of δ. For example, when $\delta = 0.10$ an increase in σ from 0.1 to 0.2 (100 percent) causes X_S^* to decline by 67.9 percent, whereas when $\delta = 0.30$ the same increase in σ elicits only a 15.1 percent decrease in X_S^*.

IV. Implications for Fisheries Management

Despite the extension of territorial waters outwards to 200 nautical miles, nation states have had difficulty in managing stocks within their exclusive economic zones (see *The Economist*, March 19th 1994). Coastal and distant water fleets that expanded during the 1970s and 1980s combined to overfish stocks of cod, haddock, pollock, herring and the various species of tuna. The Canadian government has recently imposed a moratorium on the harvest of cod, a mainstay of the small fishing villages in Newfoundland and the other maritime provinces. Moratoria are also being considered for other depleted stocks. The two major questions become: "When can fishing resume?" and "When fishing is resumed, how should the fishery be managed?"

The stochastic model presented in Section II has the potential to be estimated or calibrated for the major single-species fisheries. Estimates of X_S^* would provide a "starting rule" for depleted fisheries. In the U.S. and other developed countries, an estimate of current biomass is often required under current management plans. Suppose the current estimate of yellowtail flounder on Georges Bank is X_t and that the size barrier for yellowtail is X_S^*. Allowable catch for year t might be set at $Y_t = (X_t - X_S^*) \geq 0$. Allowable catch might then be distributed among eligible fishers according to the prevailing distribution of ownership shares. If the ith fisher held title to a share s_i, the biomass that he or she could catch in year t would be $Y_{i,t}$ = $s_i Y_t$. If this individual quota was transferable, in that the individual could sell all or a portion of $Y_{i,t}$, the fishery is said to be managed under a system of individual transferable quotas (ITQs). Given the stochastic nature of future net growth, a new Y_t would be determined each year.

Landings taxes or ITQs have long been touted by economists as the most efficient way to manage a common property fishery [Christy (1973) and Brown (1974)]. New Zealand, Iceland, Canada, Australia and the United States are now experimenting with variations of the ITQ program described above. The range of transferability is often limited to preclude locational or noncompetitive concentration of quota. New Zealand's experiment is the longest running and the U.S. experiment is limited to a small shellfishery on the mid-Atlantic continental shelf. Results to date are encouraging and there seems to be a growing realization on the part of managers, biologists, and most importantly, fishers, that ITQs can provide a more rational framework for managing single-species fisheries.

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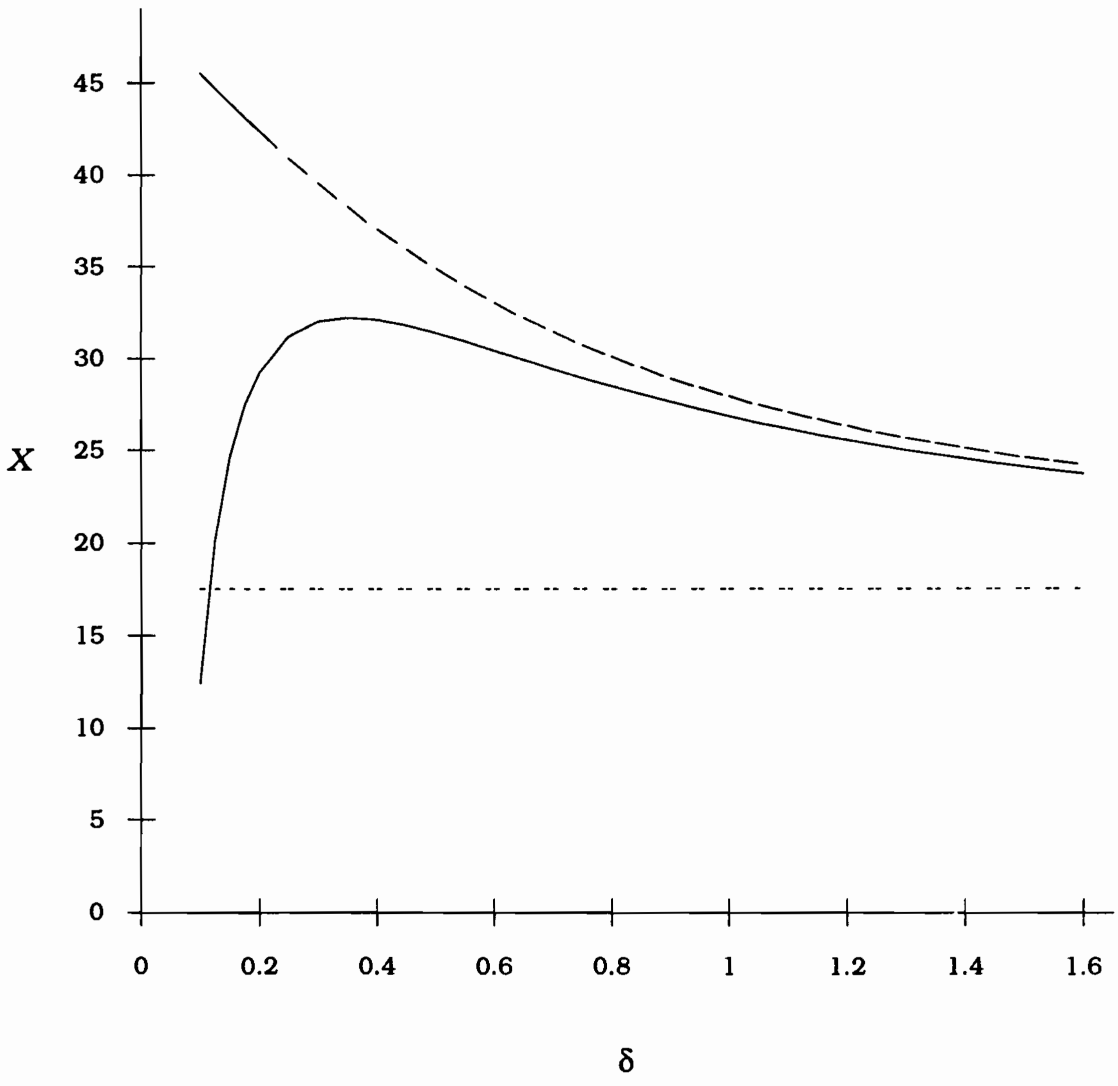
TABLE 1 - OPTIMAL BIOMASS IN THE STOCHASTIC FISHERY

	$\sigma = 0.0$	$\sigma = 0.1$		$\sigma = 0.2$	
δ	$X^* = X_S^*$	θ	X_S^*	θ	X_S^*
0.05	47.2	0.67	31.7	$\delta<2\sigma^2$	----
0.10	45.5	0.84	38.6	0.25	12.4
0.15	43.9	0.90	39.6	0.54	24.6
0.20	42.3	0.92	39.3	0.67	29.2
0.25	40.9	0.94	38.6	0.74	31.2
0.30	39.9	0.95	37.7	0.79	32.0

FIGURE 1: X^* AND X_s^* AS A FUNCTION OF δ WHEN $\sigma = 0.2$, r = 0.71, K = 80.5, AND X_∞ = c/p = 17.5.



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