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DYNAMIC AGGREGATE MILK SUPPLY RESPONSE WITH BIOLOGICAL CONSTRAINTS ON DAIRY HERD SIZE

by

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Acknowledgements

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Abstract

A quarterly milk supply response function is derived that accounts for biological and economic factors in a system of equations representing heifer numbers, cow numbers, and production per cow. The heifer and cow numbers are estimated simultaneously by observing the age composition of heifers and the quarterly biological constraints between heifers and cows. Based on the results of a dynamic simulation, this model predicts the supply response components better than two other traditional specifications that ignore the biological processes. Short-, intermediate-, and long-run supply elasticities with respect to various exogenous variables justifies the dynamic adjustment of this model.

Key words: quarterly milk supply response, biological constraints, age composition, net offspring rate, retention rate.

Dynamic Aggregate Milk Supply Response with Biological Constraints on Dairy Herd Size

Chinhwa Sun, Olan D. Forker and Harry M. Kaiser

I. Introduction:

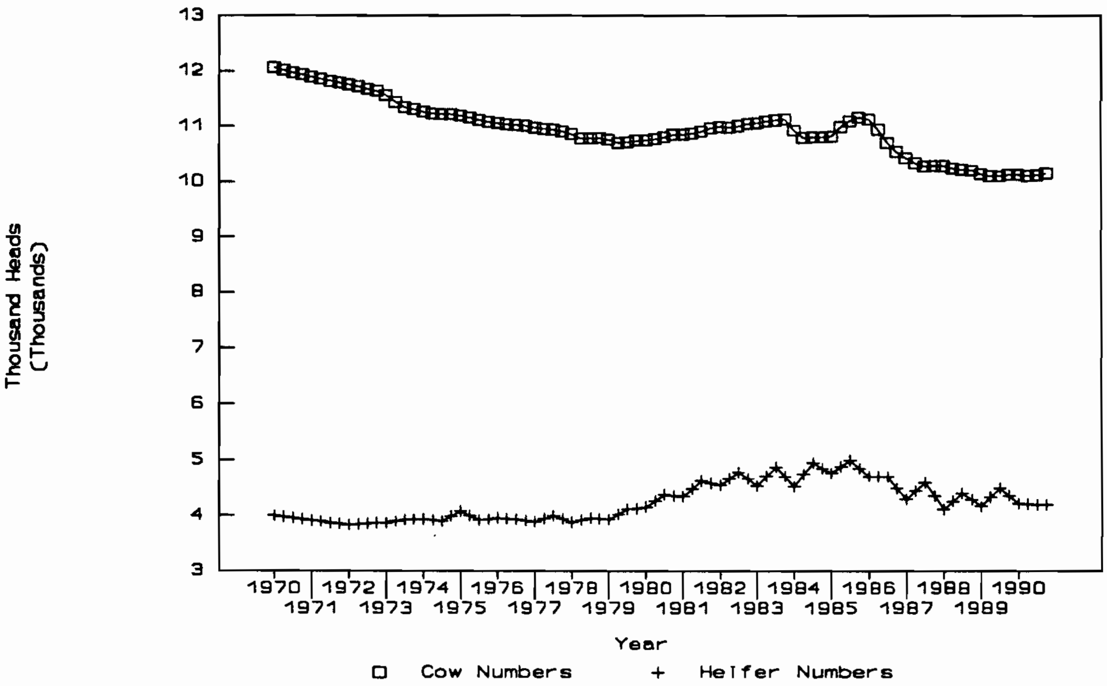
Dairy cow numbers are constrained by the biological reproduction and growth processes of cows. Since milk production is determined by the number of dairy cows and production per cow, the biological adjustment process of cows set a partial upper limit on the adjustment of milk production. Many research articles have formulated milk supply response models by estimating cow numbers and production per cow, but few of them considered the dynamic biological adjustment process (Cowling and Baker; Hallberg; Chavas and Klemme). However, the quarterly dynamic biological adjustment process has not been addressed in any research yet.

From 1970 through 1990, there was a downward trend in cow numbers, especially during the period that the Milk Diversion and the Dairy Termination Programs[1] were in effect. On the other hand, heifer numbers trended upward, except for the period immediately following the Dairy Termination Program. A time-series plot of cow and heifer numbers is shown in Figure 1. These two trends have been due to an increase in the culling rate of cows

[1] The Milk Diversion program (from January 1984 to March 1985) and Dairy Termination Program (from April 1986 to August 1987) were voluntary supply control policies armed at reducing surplus milk production. The Milk Diversion Program paid participants $10 Per hundredweight for reducing milk marketing from 5 to 30%. The Dairy Termination Program paid farmers who participated in the program to slaughter or export their dairy cattle and remain out of dairy farming for a period of five years.

Figure 1 Actural Cow and Heifer Numbers

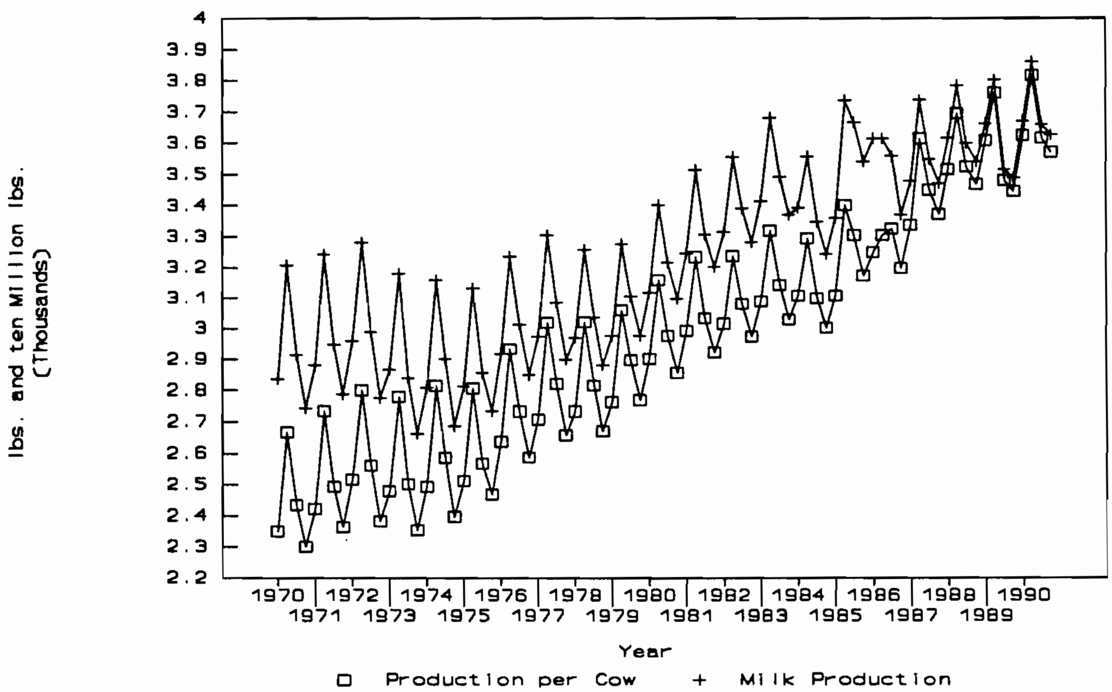
(1970, Quarter 1 to 1990, Quarter 4)



and an increase in the replacement rate of heifers. However, there is a prolonged effect of culling a cow and replacing it by a heifer because it changes not only the current cow numbers but also the age composition of the cohort which determine future heifer numbers. During the same period, a consistent increase in production per cow suggests a technology improvement which causes milk production to steadily trend upward (Figure 2).

The objective of this paper is to develop a quarter milk supply response function which accounts for biological constraints on cow numbers as well as the economic factors that influence dairy herd size. A system of equations explaining heifer numbers, cows numbers, and production per cow is constructed according to the constraints on the age structure of the cohort, the farmers' dynamic adjustment decision on culling and replacing cows, and improvements in technology. The following sections present the model

Figure 2 Actural Production per Cow and Milk Production



specifications and the estimation procedure. A comparison of the dynamic simulations across different models is also provided to illustrate the advantage of this model.

II. Model Specification:

According to dairy farmers' optimal investment scheme, cow numbers are adjusted by culling less efficient cows and/or adding replacement heifers. However, milk production per cow is determined conditional on the quality and quantity of all inputs and the current technology. Cow numbers, production per cow, and the manner in which dairy farmers formulate their expected prices determine the milk supply response.

Suppose dairy farmers maximize their discounted expected net profit flow from farm milk sales, net of variable costs, i.e.:

$$(1.1)\quad \max_{\text{w.r.t. } PPC_t \text{ and } COW_t} E\left\{\sum_{t=1}^{T} \rho^t[AMP^e_t \cdot PROD_t - C(PROD_t | COW_t, PFEED_t, FWAGE_t)]\right\}$$

s.t.

$$(1.2)\quad PROD_t = PPC_t \cdot COW_t$$

where E is the mathematical expectations operator, and ρ is the discount factor; variables are defined in Table 1. The variable, AMP^e_t, is the farmer's expectation of the stochastic milk price in period t, while the term, $C(PROD_t | COW_t, PFEED_t, FWAGE_t)$, is the cost function, which is conditional on current cow numbers, feed prices, and farm wages.

Table 1. List of Variable Definitions

AMP	= Milk prices received by farmers, all sold to plants, \$/cwt,
COW	= Number of milk cows, thousand head,
DPSCOW	= Prices for slaughter cows deflated by Farm Price Received,
DFWAGE	= Farm wages index deflated by Farm Price Paid,
DTP	= 1 if during Dairy termination program, 1986:2 to 1987:3, zero otherwise,
FRECE	= Index of farm prices received, all farm products, 1977=100,
FPAID	= Index of farm prices paid, commodities, services, interest, taxes and wage rates, 1977=100.
HEF	= Number of replacement heifers, thousand heads,
hefi	= Number of replacement heifers at i quarters age old,
MFP	= AMP/PFEED, Milk-feed price ratio,
MDP	= 1 if during the Milk diversion program, 1984:1 to 1985:2, zero otherwise,
PFEED	= Feed price based on 16% dairy feed, \$/ton,
PPC	= Production per cow, lbs.,
PROD	= Total pounds of milk produced, million lbs.,
S1	= 1 if period is the first quarter of the year, zero otherwise,
S2	= 1 if period is the second quarter of the year, zero otherwise,
S3	= 1 if period is the third quarter of the year, zero otherwise,
TREND	= Time trend as a proxy for technology improvement, equal to 1 in 1970:1,

Data Sources: Dairy Situation and Outlook and Agricultural Prices.

The optimal solution for cow numbers represents a capital stock which is constrained by the dynamic biological reproduction processes under uncertainty of output and input prices

(Chavas and Klemme). By using annual data from 1960 to 1982, Chavas and Klemme assumed an eleven year life span of cows in the herd and formatted an annual retention rate for each age group of cows. They estimated current cow numbers by summing up the past eleven years of heifer numbers, which are weighted by the cumulative retention rate. However, it is necessary to relax the assumption about the life span of cows when we are using quarterly data instead of yearly data. Furthermore, assuming the yearly retention rate for each age group of cows is too restrictive than just assuming a retention rate for last quarter's cow. Since the last quarter's heifer and cow numbers are observed, then the estimate of current cow number could be obtained by observing the biological relationship between heifers and cows. The estimation of the current period's replacement heifers and the retention rate of last period's cows will be explained in the next section.

Model for Cow and Heifer Numbers

According to USDA count, dairy cows are at least two years old and have freshened at least once; heifers are female calves between four to eight quarters in age[2]. When a heifer freshens, it is counted in the cow number statistics and represents two years investment. WE define COW_t as the stock of milking cows which are at least two years old and have freshened at least once, and HEF_t as number of heifers in period t. The current cow number is the sum of the first-lactation cows and a portion of last quarter's cows, which will be

[2] According to the definition of the data series (Dairy Situation and Outlook), a heifer is a female calf which weighs over 500 pounds and has not yet freshened. From the Holstein and Brown Swiss heifer growth curves for weight at various ages (Hoard's Dairyman), the age of the age of heifers varies from 10 to 24 months old and the first-lactation cows freshen at 9 quarters old.

retained from last quarter. The number of nine quarter old heifers at the age nine quarters in the current quarter is unobserved. However, an estimate can be obtained from heifer numbers observed in the last quarter conditional on the heifers' age composition of heifers. Define $hef_{i,t-1}$ as the unobserved i quarter age heifer numbers in last quarter, where i = 4, 5, 6, 7 and 8. The number of eight quarter old heifers in the previous quarter, $hef_{8,t-1}$, is counted as the number of the first-lactation cows which freshen in this quarter. If one considers this as a dynamic cow inventory problem, the following identity should hold for all periods:

(2) $$COW_t \equiv hef_{8,t-1} + \gamma_t \cdot COW_{t-1},$$

where, γ_t is the retention rate of cows in period t-1 which survive from period t-1 to t, and $0 \leq \gamma_t \leq 1$. The sum of the female calves for 4, 5, 6, 7, and 8 quarter of age could be observed by the number of heifers last quarter, i.e.:

(3) $$HEF_{t-1} = hef_{4,t-1} + hef_{5,t-1} + hef_{6,t-1} + hef_{7,t-1} + hef_{8,t-1}.$$

Since the i quarter old heifers in period t-1 should have been born in period t-i quarters, it is determined by cow numbers at period t-i-3 as follows[3]:

(4) $$hef_{i,t-1} = \delta_{i,t-1} \cdot COW_{t-i-3}$$

where, $0 \leq \delta_{i,t-1} \leq 1$ for i=4, 5, 6, 7 and 8. The variable, $\delta_{i,t-1}$, is the net offspring rate of cows in period t-i-3 when them gave birth to female calves three quarters later. It is determined by the economic situation in period t-i when calves were born and decisions of whether to keep them and raise them as heifers were made.

[3] A lactation period of 305 days (3 quarters) is normal for dairy cows (Schmidt et al.).

From equation (2) - (4), the biological reproduction processes of heifer and cow numbers is explained. The unknown retention and net offspring rates of cows, γ_t and $\delta_{i,t-1}$, can be approximated by the logistic transformation functions which are bounded by 0 and 1. The general form of this function in period t is $f_t(\beta) = 1/[1+\exp(X_t \cdot \beta)]$, where X_t is the set of explanatory variables and β is the coefficient vector associated with X_t. Note that the derivative of the $f_t(\beta)$ with respect to a particular explanatory variable, X_t, is derived as follows:

(5) $$\partial f_t(\beta)/\partial x_t = -\beta \cdot x_t \cdot (1-f_t(\beta)) \cdot f_t(\beta)$$

where, $\partial f_t(\beta)/\partial x_t \geq 0$ if $\beta \leq 0$, and $\partial f_t(\beta)/\partial x_t \leq 0$ if $\beta \geq 0$

Based on the expected profitability of keeping cows in the current period, the set of explanatory variables to explain the retention rate includes the expected milk-feed price ratio, deflated farm wages, a trend variable as a proxy for technological change over time, deflated slaughter cow price, quarterly dummy variables to capture seasonality, and two intercept dummy variables to account for quarters that the Milk Diversion Program (MDP) and Dairy Termination Program (DTP) were in effect. The retention rate in equation (2) is modeled as follows:

(6) $$\gamma_t = 1/[1 + EXP(A0 + A1 \cdot MFP^e_t + A2 \cdot TREND_t + A3 \cdot DPSCOW_t + A4 \cdot DTP_t + A5 \cdot MDP_t + A6 * S1_t + A7 \cdot S2_t + A8 \cdot S3_t)]$$

where A0 - A8 are unknown coefficients.

However, the net offspring rate of cows in period t-i-3, $\delta_{i,t-1}$, which gave birth to calves in period t-i, is determined by the milk-feed price ratio, deflated farm wage, trend variable, DTP in period t-i, DTP in period t-1, and seasonal dummy variables as follows:

$$(7)\quad \delta_{i,t-1} = 1/\{1+EXP[H0 + H1\cdot MFP_{t-i} + H2\cdot DFWAGE_{t-i} + H3\cdot TREND_{t-i} + H4\cdot DTP_{t-i} + H5\cdot DTP_{t-1} + H6\cdot S1_{t-i} + H7\cdot S2_{t-i} + H8\cdot S3_{t-i}]\},$$

where H0 - H8 are unknown coefficients. By substituting equation (6) and (7) in (2) and (3), we can rewrite heifer and cow number equations as follows,

$$(8)\quad COW_t = COW_{t-11}/\{1+EXP[H0 + H1\cdot MFP_{t-8} + H2\cdot DFWAGE_{t-8} + H3\cdot TREND_{t-8} + H4\cdot DTP_{t-8} + H5\cdot DTP_{t-1} + H6\cdot S1_{t-8} + H7\cdot S2_{t-8} + H8\cdot S3_{t-8}]\} + COW_{t-1}/\{1+EXP[A0 + A1\cdot MFP^e_t + A2\cdot TREND_t + A3\cdot PSCOW_t + A4\cdot DTP_t + A5\cdot MDP_t + A6\cdot S1_t + A7\cdot S2_t + A8\cdot S3_t]\} + u_{1t},$$

$$(9)\quad HEF_{t-1} = \sum_{i=4}^{8} COW_{t-i-3} /\{1+EXP[H0+H1\cdot MFP_{t-i} + H2\cdot DFWAGE_{t-i} + H3\cdot TREND_{t-i} + H4\cdot DTP_{t-i} + H5\cdot DTP_{t-1} + H6\cdot S1_{t-i} + H7\cdot S2_{t-i} + H8\cdot S3_{t-i}]\} + u_{2t}.$$

Model for Production Per Cow

Production per cow is estimated as a function of the expected milk-feed price ratio; production per cow lagged four quarters, which indicates the biological production capacity in the current quarter; intercept dummy variables, which indicate the period that the Milk Diversion Program was in effect; trend variable, which captures the technological improvement; and seasonal dummy variables to account for seasonality in output per cow:

$$(10)\quad PPC_t = C0 + C1\cdot MFP^e_t + C2\cdot MDP_t + C3\cdot S1_t + C4\cdot S2_t + C5\cdot S3_t + C6\cdot PPC_{t-4} + C7\cdot TREND_t + u_{3t}$$

where C0 - C7 are unknown coefficients. The condition of output price and input price being homogenous of degree one is imposed. Assuming a naive price expectation scheme (Kaiser et al.; Liu et al.; Klemme and Chavas), the expected milk-feed price ratio, MFP^e_t, is approximated by the past quarter's observed ratio, MFP_{t-1}. Because production per cow is equal to total production divided by cow numbers, it is hypothesized that the error terms in equation (8) - (10) are contemporaneously correlated, i.e., u_{1t}, u_{2t}, and u_{3t} are not independent of each other. If this is true, then:

$$(11) \qquad E(PROD_t) \equiv E(COW_t \cdot PPC_t)$$
$$\equiv E(COW_t) \cdot E(PPC_t) + COV(COW_t, PPC_t)$$
$$\neq E(COW_t) \cdot E(PPC_t), \text{ if } COV(COW_t, PPC_t) \neq 0.$$

where $E(\cdot)$ is the mathematical expectations operator, and $COV(x,y)$ is the covariance of x and y. Hence, equations (8) - (10) will be estimated simultaneously by nonlinear seemingly unrelated regression (SUR) to get linear unbiased estimators for HEF_{t-1}, COW_t, and PPC_t.

III. Estimation and Results:

The estimation procedure is carried out in three steps. The first step is to estimate heifer numbers by equation (9). The second step is to use the estimates of H0 - H8 to simulate $hef_{8,t-1}$. The third step is to use the simulated value of $hef_{8,t-1}$ as one of the explanatory variables in equation (8) to estimate total cow numbers. Since nonlinear SUR is utilized to estimate equation (8) - (10) simultaneously, $\delta_{i,t-1}$ and γ_t in equations (6) and (7), serve as restrictions across equations (8) and (9).

Data are quarterly series from 1970, quarter 1 to 1990, quarter 4 and are obtained from the New York State Dairy Database, which complies data from the USDA and other agencies. The simulated model starts in 1972:4 because of the lag structure of the model. The results are reported in Table 2. The R^2s are 0.9396, 0.9847, and 0.9847 for the heifer numbers, cow numbers, and production per cow equations, respectively. These equations fit the data reasonable well and all coefficients have the expected signs. Also, the coefficients are significantly different from zero except for coefficients associated with slaughter cow

Table 2. Nonlinear SUR Estimation of Equations (8) - (10)

Equation	Variables		Parameter Estimate	't' ratio	
Retention Rate of Cows in	CONSTANT	A0	-2.32623	-25.07	
period t-1, γ_t:	MFP_{t-1}	A1	-3.65371	-2.28	
Equation (8)	$TREND_t$	A2	.00354167	8.06	
	$DPSCOW_t$	A3	0.10764	0.50	
	DTP_t	A4	0.21539	7.90	
	MDP_t	A5	0.10699	3.95	
	$S1_t$	A6	-0.04846	-1.17	
	$S2_t$	A7	-0.16354	-3.94	
	$S3_t$	A8	-0.08191	-1.97,	R^2=0.9847
Net Offspring Rate of	CONSTANT	H0	1.64568	20.50	
Cows in Period t-i-3, $\delta_{i,t-1}$:	MFP_{t-i}	H1	-2.41601	-4.58	
Equation (9):	$DFWAGE_{t-i}$	H2	1.14690	18.29	
	$TREND_{t-i}$	H3	-.0044719	-31.92	
	DTP_{t-i}	H4	0.09629	7.25	
	DTP_{t-1}	H5	0.03830	4.26	
	$S1_{t-i}$	H6	0.07026	2.09	
	$S2_{t-i}$	H7	0.15616	4.37	
	$S3_{t-i}$	H8	0.05637	1.67,	R^2=0.9396
Production per Cow in	CONSTANT	C0	1177.86	5.55	
Period t, PPC_t:	MFP_{t-1}	C1	784.74	0.77	
Equation (10)	MDP_t	C2	-92.11807	-4.32	
	$S1_t$	C3	53.54086	2.79	
	$S2_t$	C4	183.37	5.52	
	$S3_t$	C5	65.76539	3.45	
	PPC_{t-4}	C6	0.42518	4.93	
	$TREND_t$	C7	9.94647	6.93,	R^2=0.9847

price in the cow equation and milk-feed price ratio in the production per cow equation.

For H1=-2.42 < 0, the partial derivative of the net offspring rate for i quarter old heifers in period t with respect to the milk-feed price ratio in period t-i is positive, $\partial\delta_{i,t-}$

$_1/\partial MFP_{t-i} > 0$ in equation (5). Hence, the milk-feed price ratio (MFP) has a positive impact on the net offspring rate ($H1 < 0$), retention rate of cows ($A1 < 0$), and production per cow ($C1 > 0$). The deflated farm wage (DFWAGE) has a negative impact on heifer numbers ($H2 > 0$). The Dairy Termination Program (DTP) has a negative impact on both heifer numbers and cow numbers ($H4 < 0$, $H5 < 0$, and $A4 < 0$). Also, the Milk Diversion Program (MDP) has a negative impact on both cow numbers and production per cow ($A5 < 0$ and $C2 < 0$). The trend variable (TREND) has a positive impact on the net offspring rate of cows and also heifer numbers ($H3 < 0$), but a negative impact on the retention rate of cows and also cow numbers ($A2 > 0$). Further, the trend variable has a positive impact on production per cow which explains the technological improvement of production per cow overtime ($C7 > 0$). The four quarters lagged production per cow (PPC_{t-4}) has a positive impact on the production per cow this year ($C6 > 0$) which implies the quality of cow in the same quarter is increasing over years. For the analysis of seasonality, the fourth quarter is set as the base quarter for the seasonal dummy variables. In the fourth quarter, the net offspring rate, and production per cow is the lowest ($H7 > H6 > H8 > 0$ and $C5 > C7 > C6 > 0$), but the retention rate of cows is the highest in the year ($0 > A6 > A8 > A7$).

IV. Model Validation:

A dynamic within-sample simulation approach is used to validate the model. Starting from 1972:4, the simulated endogenous variables are substituted into the lag structure for the remaining periods and all exogenous variables are set to their actual historical values. The performances are judged in terms of the closeness of the predicted endogenous variables to their historic values. The root-mean-square percentage errors (RMS % Error) measure is computed as:

$$\text{RMS\% Error} = \{(1/T)\sum_{t=1}^{T}[(YS_t - YA_t) / YA_t]^2\}^{1/2}$$

where YS_t is the simulated value of endogenous variables Y, YA_t is the actual historic value for endogenous variable Y, and T is the number of periods in the simulation. The RMS Percentage Error for milk production, cow numbers, production per cow, and heifer numbers are 1.64%, 0.77%, 1.45%, and 1.93%, which are also reported as SFK model in Table 3. These results suggest that the model simulations have a very small deviations from the actual value. These deviations are considered very good under dynamic simulation test. The time-series plots of simulated heifer numbers based on their age composition are presented in Figure 3. The time-series plots of simulated heifer numbers are presented in Figure 4. The Retention rates of cows, γ_t, are shown in Figure 5. According to the computed retention rate, the quarterly culling rate[4] is around 7% to 10%, and the annual culling rate is around 28% to 40%. This is comparable to most empirical data on culling rates (Hoard's Dairyman)

For comparison of the simulation performance to different model specifications, we choose two other specifications which have been used conventionally. The first quarterly model is specified by Kaiser, Forker, Lenz, and Sun (referred as KFLS hereafter). They estimate the following log linear cow number and production per cow equations:

[4] The quarterly culling rate is one minus the quarterly retention rate. The annual culling rate is around four times the quarterly culling rate.

Figure 3. Estimates of 4, 5, 6, 7, and

8 Quarter old Heifer (1972:4 - 1990:4)

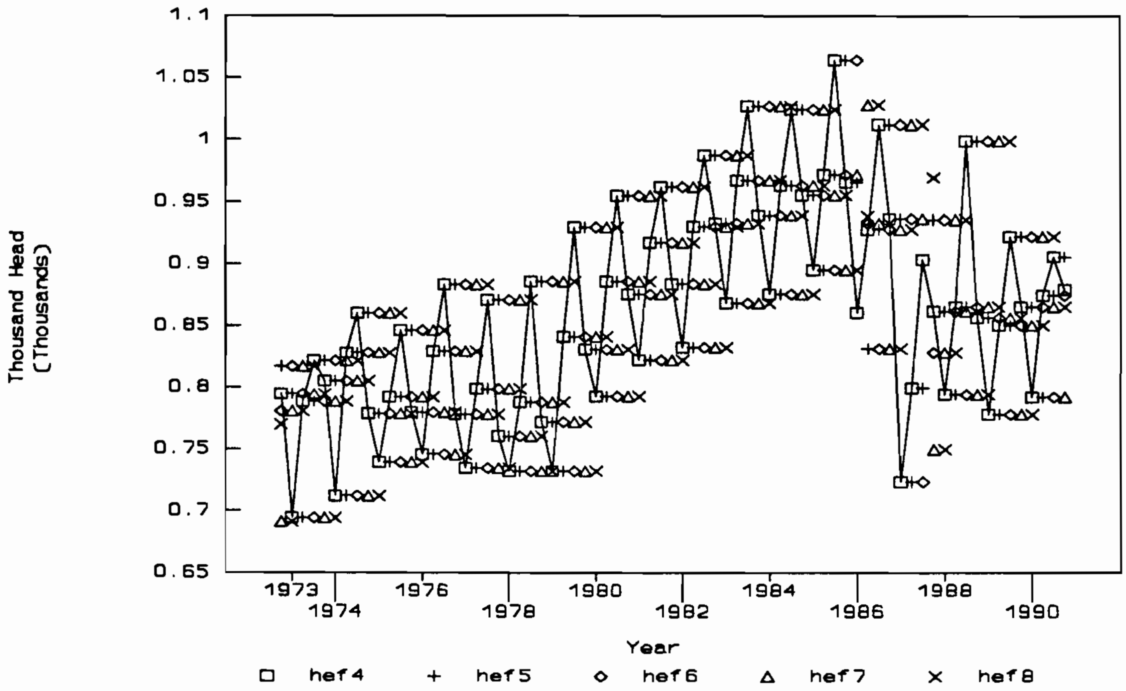


Figure 4. Dynamic Simulation of HEF

in SFK Model

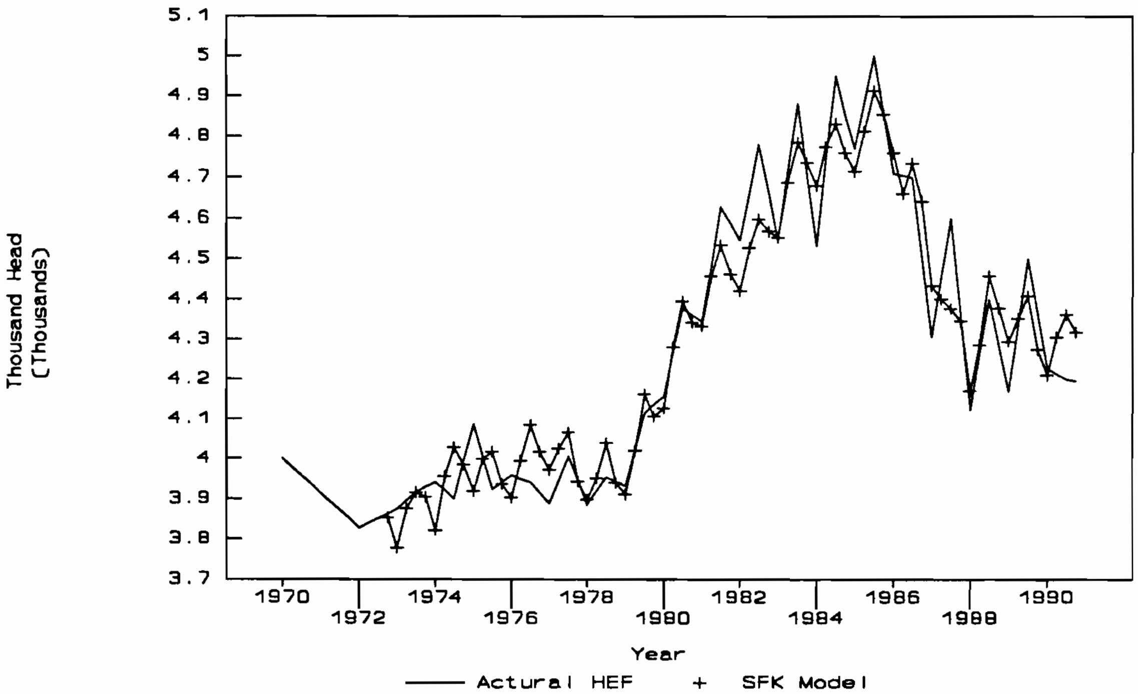
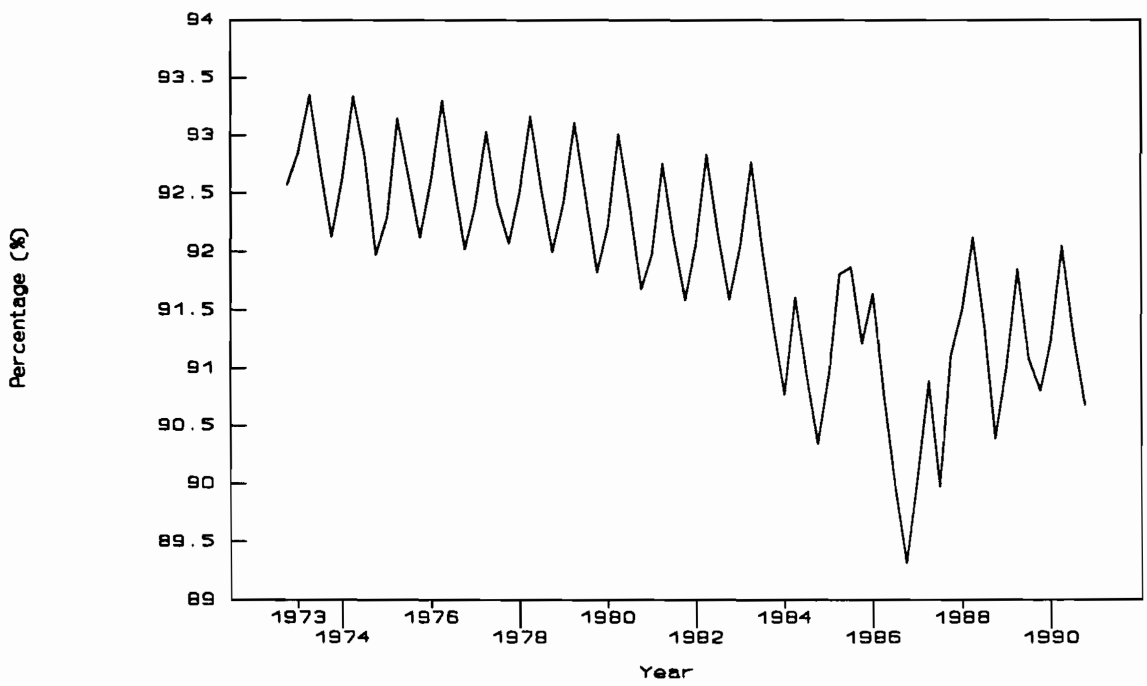


Figure 5. Retention Rate of Cows
(1972:4 - 1990:4)



$$\text{(12)} \quad \ln(COW_t) = .25 + 1.60 \cdot \ln(COW_{t-1}) - .93 \cdot \ln(COW_{t-2}) + .31 \cdot \ln(COW_{t-3})$$
(2.70) (13.77) (-3.94) (3.11)

$$+ .012 \cdot \ln(MFP_{t-1}) - .015 \cdot \ln(PSCOW_t) - .0092 \cdot DTP_t, \qquad R^2 = .9935$$
(1.85) (-1.34) (-4.35)

$$\text{(13)} \quad \ln(PPC_t) = 4.27 + .46 \cdot \ln(PPC_{t-1}) + .033 \cdot \ln(MFP_t) + .0028 \cdot TREND_t$$
(5.15) (4.36) (1.36) (5.03)

$$- .061 \cdot COS1_t + .012 \cdot COS2_t + .021 \cdot SIN1_t - .019 \cdot MDP_t. \qquad R^2 = .9804$$
(-19.3) (5.16) (3.13) (-2.17)

where t-ratios are given in parentheses; $COS1_t$ and $COS2_t$ are harmonic seasonal variables representing the first and second wave of the cosine function; $SIN1_t$, is the harmonic seasonal variable representing the first wave of the sine function.

The second quarterly model is specified by Liu, Kaiser, Forker, and Mount (referred as LKFM hereafter). It is a log linear equation of total milk production without considering cow

cow numbers and production per cow separately. They estimate the following equation for milk production:

$$(14)\quad \ln(PROD_t) = \underset{(4.08)}{5.82} + \underset{(8.20)}{.67}\cdot\ln(PROD_{t-1}) + \underset{(2.35)}{.068}\cdot\ln(MFP_{t-1}) + \underset{(.08)}{.0038}\cdot\ln(FWAGE_{t-1})$$

$$\underset{(-1.59)}{-.012}\cdot\ln(DTP_t+MDP_t) + \underset{(3.97)}{.0012}\cdot TREND_t + \underset{(5.08)}{.031}\cdot SIN1_t - \underset{(-17.6)}{.061}\cdot COS1_t$$

$$+\underset{(5.88)}{.014}\cdot COS2_t. \qquad R^2=.9563$$

The coefficients in equations (12), (13), and (14) are estimated by OLS with the same data set we used from 1970:1 to 1990:4. The comparisons across the different models are shown in Table 3.

Table 3. RMS % Error of Dynamic Simulations of KFLS, LKFM, and SFK Models

Variable	Model		
	SFK	KFLS	LKFM
Milk Production	1.64	2.31	2.69
Cow Numbers	0.77	1.17	-
Production per Cow	1.45	1.94	-
Heifer Numbers	1.93	-	-

The RMS percentage errors are calculated according to the simulated cow numbers, production per cow, and total milk production which is not in logarithms. The SFK model performs better than either KFLS or LKFM in predicting the historical values of the endogenous variables. Time-series plots of the dynamic simulations of total milk production, cow numbers, production per cow from KFLS, LKFM, and SFK models are shown in Figure 6, 7, and 8. In Figure 7, even a visual comparison of within-sample simulation indicates that the SFK model, matches the turning points of cow number better than the KFLS model.

Figure 6. Dynamic Simulation of PROD

In KFLS, LKFM, and SFK Models

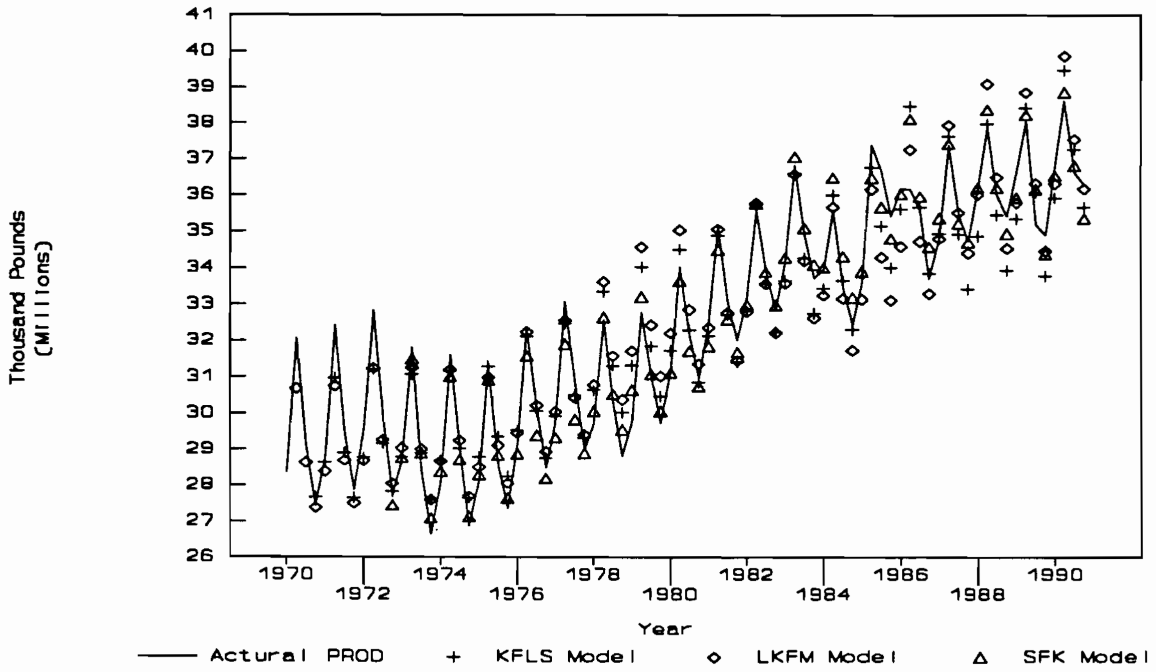


Figure 7. Dynamic Simulation of COW

In KFLS and SFK Models

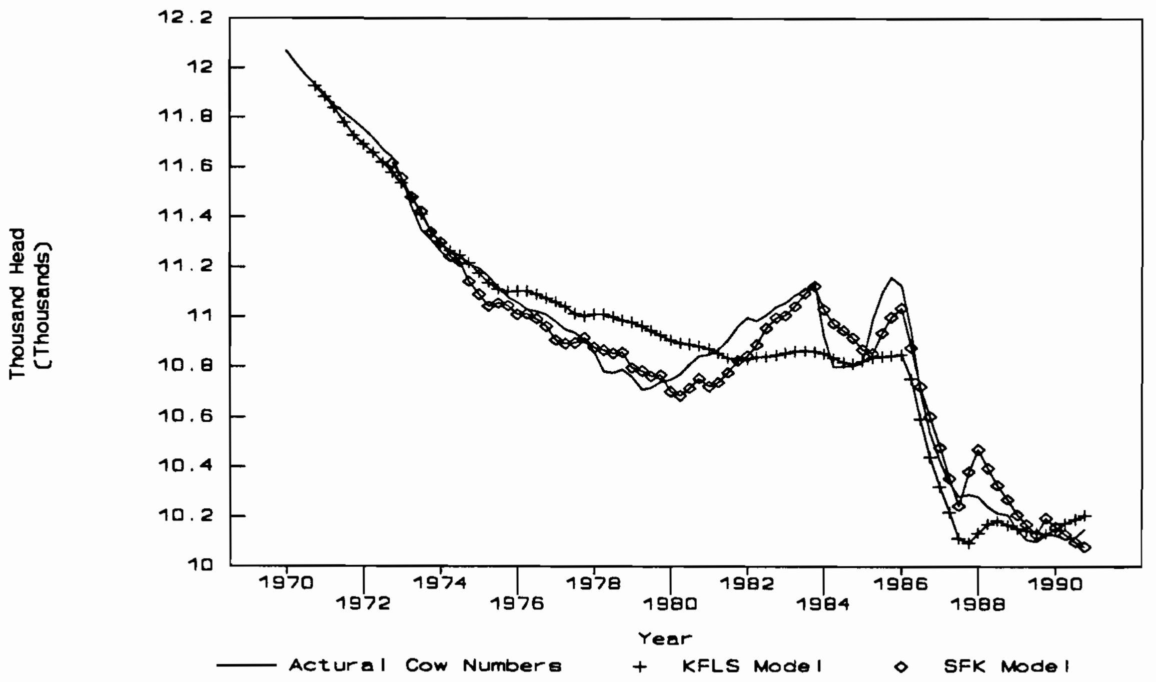
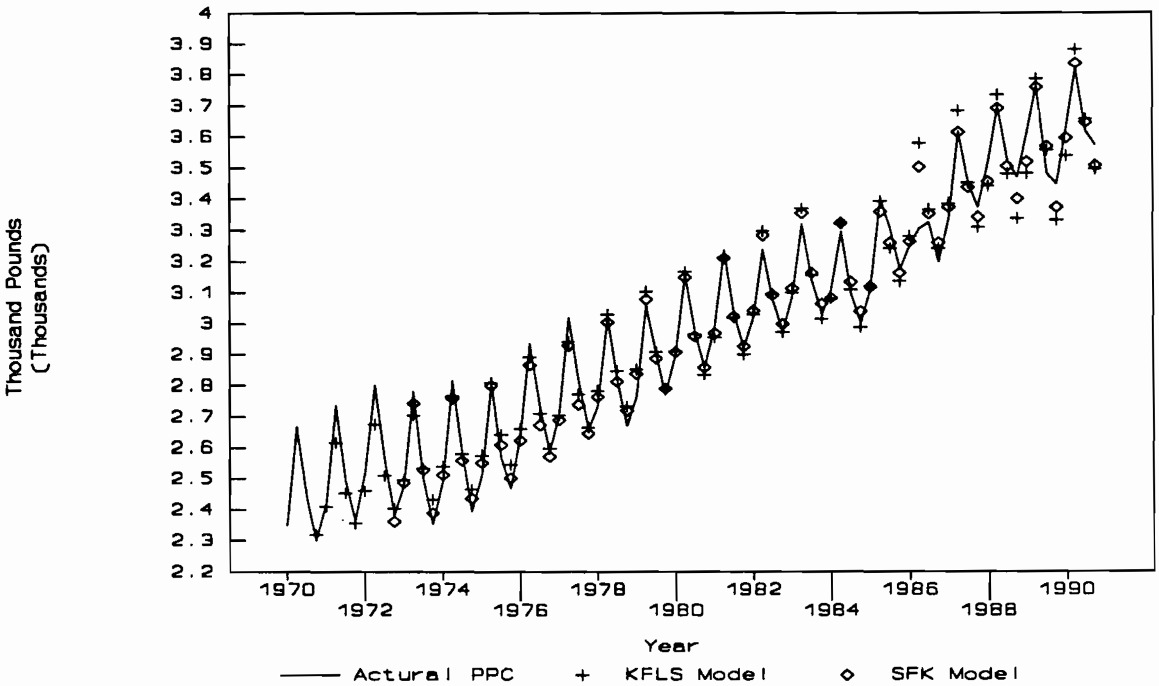


Figure 8. Dynamic Simulation of PPC

in KFLS and SFK Models



V. Estimated Elasticities:

Based on the present model, the elasticity measures the ratio of the percentage change of the simulated variable with a shock and without a shock to the percentage change of the specified exogenous variables. The following procedures are performed to calculate price elasticities. First, the estimated model is dynamically simulated from 1972:4 using the actual historical data. Second, the model is shocked in two ways: (i) a one period 10% temporary increase of the price variable, and (ii) a permanent 10% shock of the price variable. In each case, all other exogenous variables are set to their historical values. Figure 9, 10, and 11 show the impacts of a 10% one period increase in the milk-feed price ratio, deflated slaughter cow price, and the deflated farm wage in 1979:4, respectively. The elasticities calculated from (i) are stationary after 20 quarters, since the iterative feedback between

Figure 9. Elasticity of PROD, COW, PPC

& HEF w.r.t. One Period Increase of MFP

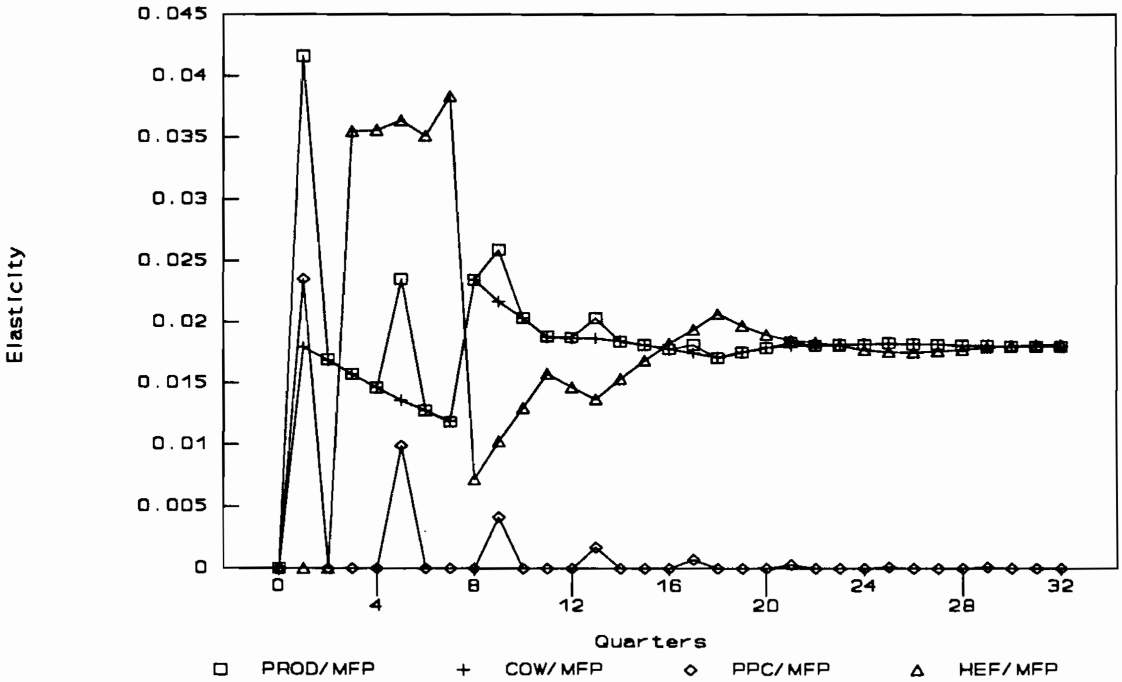
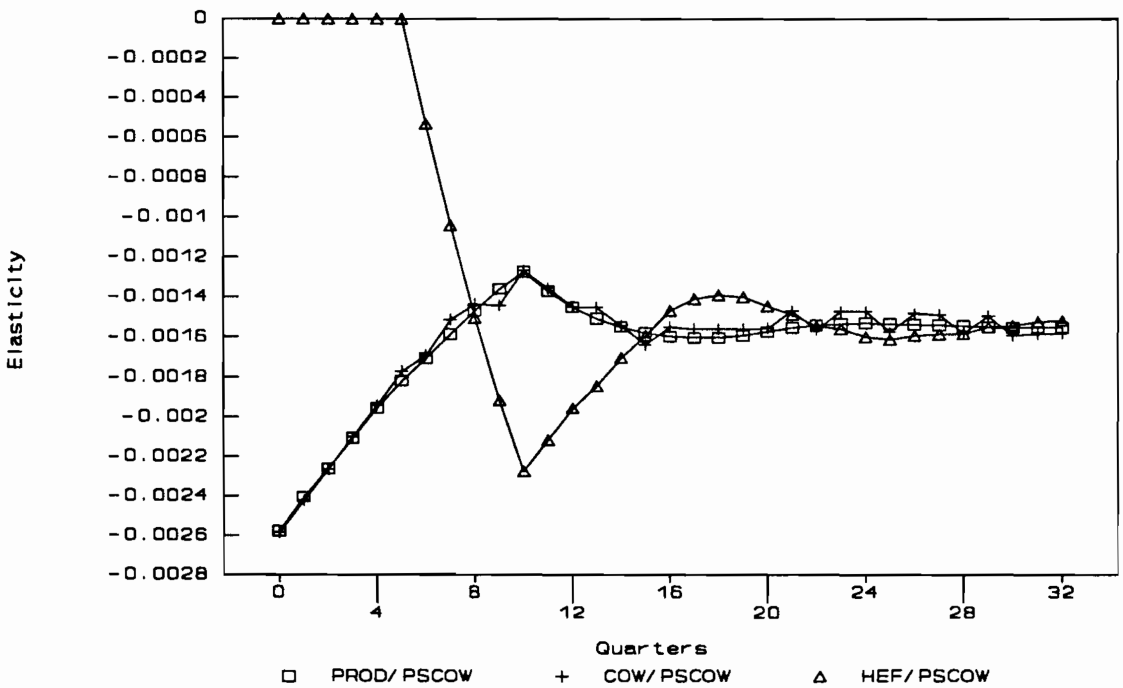


Figure 10 Elasticity of PROD, COW, and

HEF w.r.t. One Period Increase of PSCOW



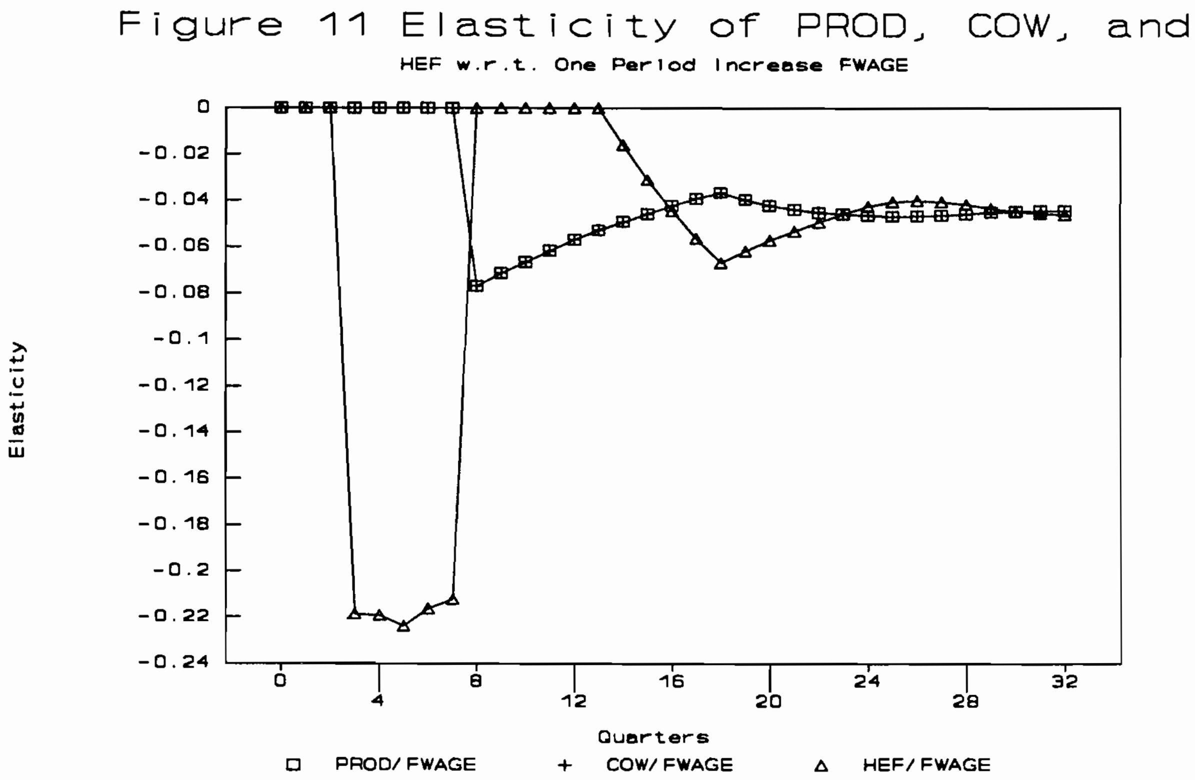


Figure 11 Elasticity of PROD, COW, and
HEF w.r.t. One Period Increase FWAGE
Elasticity
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
-0.2
-0.22
-0.24
0
4
8
12
16
20
24
28
32
Quarters
PROD/FWAGE
COW/FWAGE
HEF/FWAGE

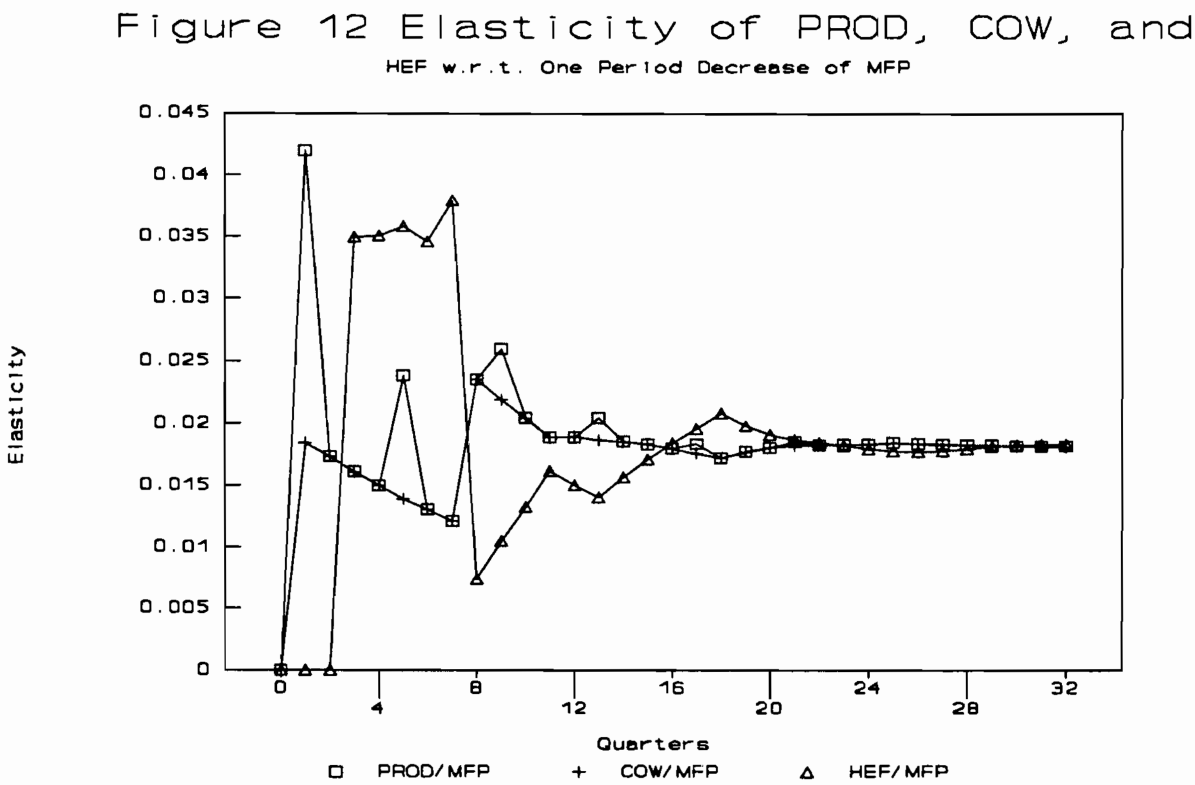


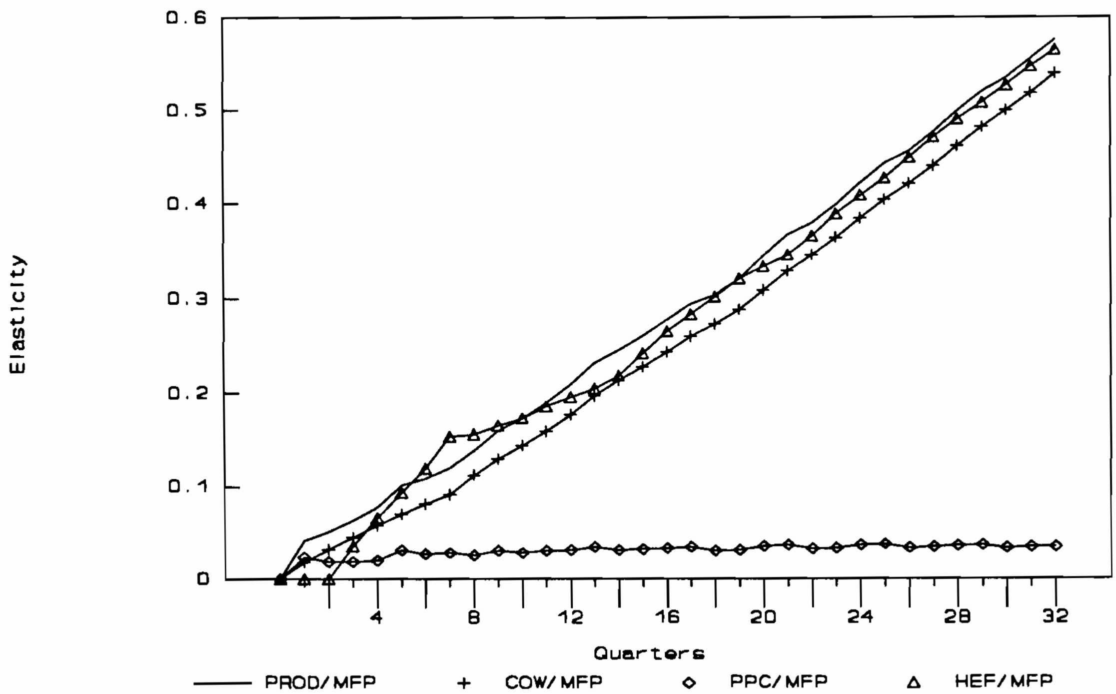
Figure 12 Elasticity of PROD, COW, and
HEF w.r.t. One Period Decrease of MFP
Elasticity
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
4
8
12
16
20
24
28
32
Quarters
PROD/MFP
COW/MFP
HEF/MFP

heifer and cow numbers takes almost 20 quarters to stabilize. A change in cow numbers will alter heifer numbers three quarters later and have a continuous feedback on cow numbers eight quarters later. A change in slaughter cow price alters cow numbers immediately, while it impacts heifer numbers five quarters later. The deflated farm wage has a large impact on heifer numbers and has a continuous feedback on heifer numbers from the change in cow numbers in the following year. For the hypothesis of irreversibility of milk supply response, Figure 12 shows the impact of a 10% one period decrease in the milk-feed price ratio. The hypothesis of irreversibility of supply response is not obvious by comparing Figure 9 and 12.

The elasticities calculated from a 10% permanent shock represent a cumulated impact by a 10% permanent shock in 1972:4 for j years. Figure 13 shows the time-series plot of the

Figure 13 Elasticity of PROD, COW, PPC,

& HEF w.r.t. a Permanent Inrease of MFP



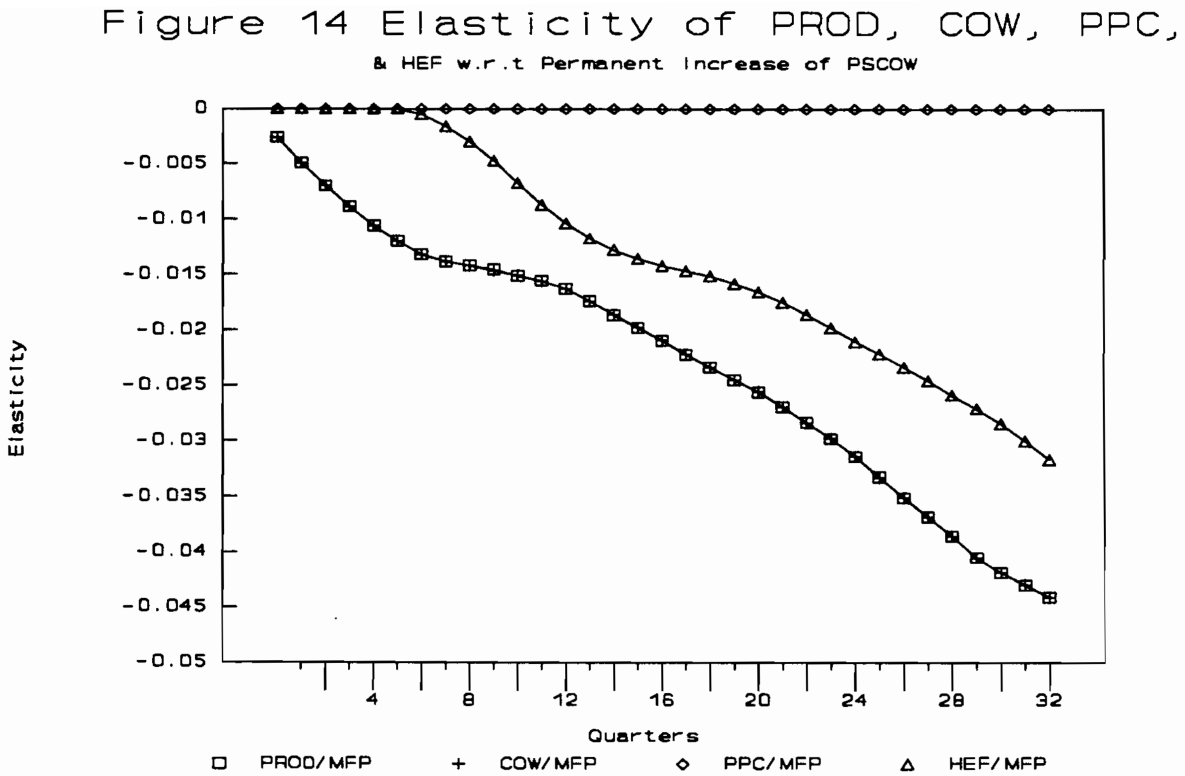


Figure 14 Elasticity of PROD, COW, PPC,
& HEF w.r.t Permanent Increase of PSCOW
Elasticity
0
-0.005
-0.01
-0.015
-0.02
-0.025
-0.03
-0.035
-0.04
-0.045
-0.05
4
8
12
16
20
24
28
32
Quarters
PROD/MFP
COW/MFP
PPC/MFP
HEF/MFP

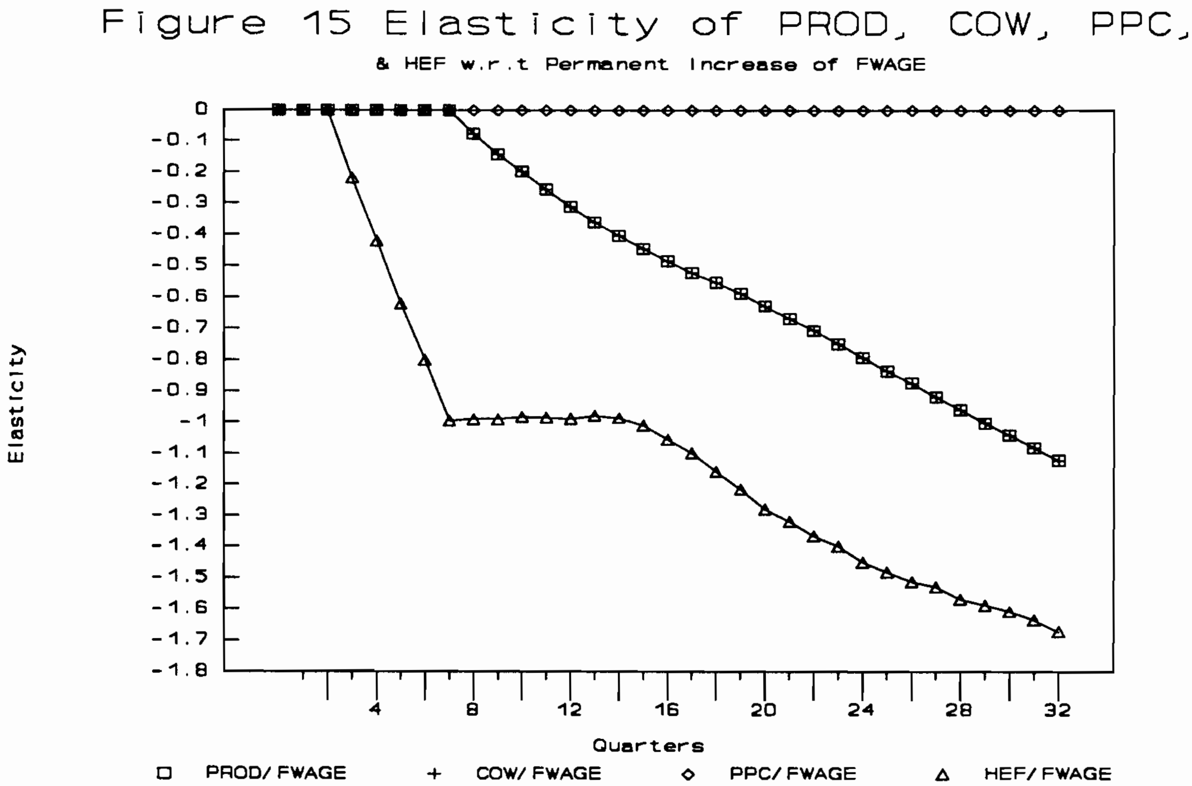


Figure 15 Elasticity of PROD, COW, PPC,
& HEF w.r.t Permanent Increase of FWAGE
Elasticity
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
-1.7
-1.8
4
8
12
16
20
24
28
32
Quarters
PROD/FWAGE
COW/FWAGE
PPC/FWAGE
HEF/FWAGE

cumulative elasticity of total production, cow numbers, production per cow, and heifer numbers with respect to milk-feed price ratio. Similarly, Figure 14 and 15 show the elasticity calculated for each endogenous variable with respect to the change of slaughter cow price deflated by farm price paid. Table 4 shows the cumulative elasticity of total production, cow numbers, and heifer numbers with respect to milk-feed price ratio, deflated slaughter cow price, and deflated farm wages.

As the length of run increases, the heifer number elasticities become closer to the cow number elasticities, which is evidence from Table 4. This implies that the herd size reaches a stationary point in the long run. A percentage change in the deflated farm wage has a greater percentage impact on the milk supply than the ones of milk price and slaughter cow price in the long run. This emphasizes that the farm wage plays an important role in determining the supply of milk.

The short-run supply elasticity of milk production with respect to milk price t is 0.08, which is comparable with the one reported in Chavas and Klemme (0.11), but is smaller than Chen et al. (0.38) and Dahlgran (0.3). The intermediate-run supply elasticities of milk production with respect to milk price for 3, 5, and 10 years are 0.21, 0.35, and 0.73 which are considerable smaller than those reported in Chavas and Klemme (0.22, 0,89, and 2.46). In the long-run (18 years), which corresponds to three complete life cycles of the dairy herd, the supply elasticity of milk production with respect to milk price (1.41) is well within the range of the ones reported in Hammond (0.14), Dahlgran (2.0), and Chen et al. (2.54) but extremely smaller than Chavas and Klemme (5.03).

For the hypothesis of irreversibility of milk supply response with respect to a 10%

Table 4. Short-Run, Intermediate-Run, and Long-Run Elasticities of U.S. Dairy Supply

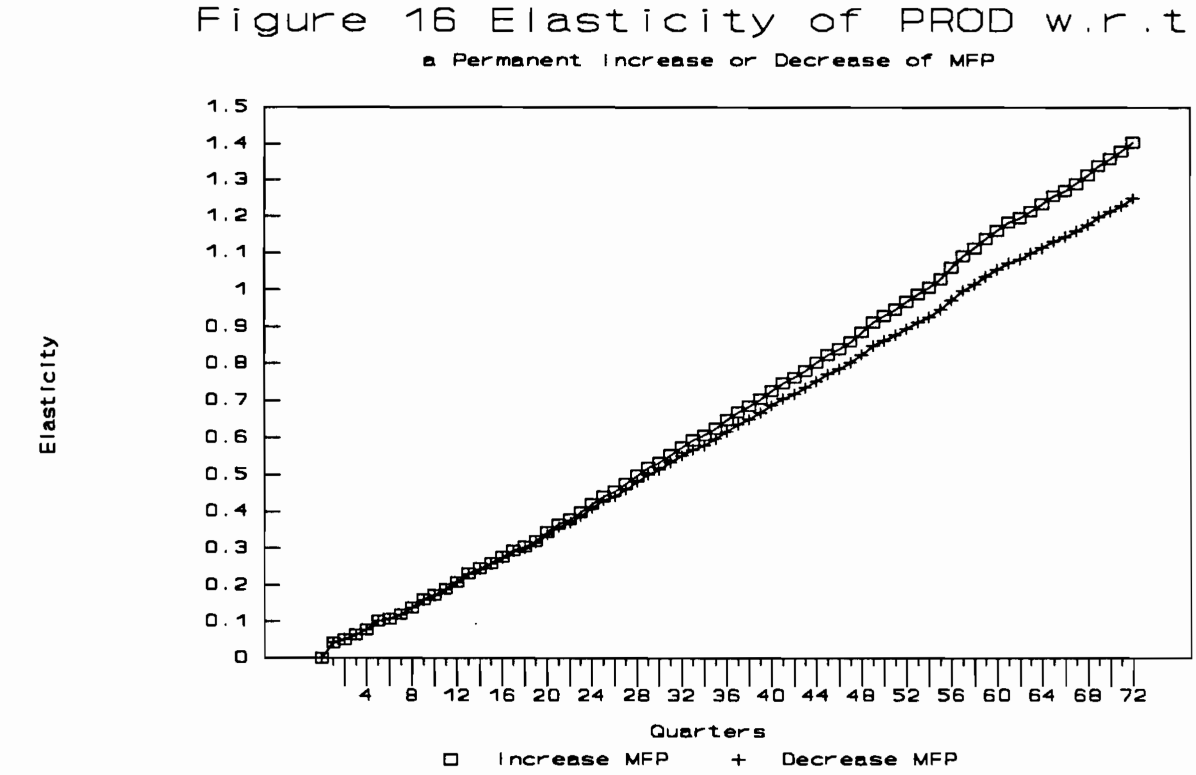
Elasticity[a]	Length of Run: (j Quarters) 0	1	2	3	..	(j Years) 1	2	3	4	5	8	10	14	18
$e_{HEF,MFP_{-j}}$	0	0	0	.04	..	.07	.16	.20	.27	.33	.57	.71	1.04	1.38
$e_{COW,MFP_{-j}}$	0	.02	.03	.04	..	.06	.11	.18	.25	.31	.54	.69	1.03	1.37
$e_{PROD,MFP_{-j}}$	0	.04	.05	.06	..	.08	.14	.21	.28	.35	.58	.73	1.06	1.41
$e_{HEF,PSCOW_{-j}}$	0	0	0	0	..	0	-.003	-.011	-.014	-.017	-.031	-.044	-.063	-.09
$e_{COW,PSCOW_{-j}}$	-.002	-.005	-.007	-.009	..	-.01	-.014	-.016	-.021	-.024	-.044	-.053	-.075	-.10
$e_{PROD,PSCOW_{-j}}$	-.002	-.005	-.007	-.009	..	-.01	-.014	-.016	-.021	-.026	-.044	-.053	-.075	-.10
$e_{HEF,FWAGE_{-j}}$	0	0	0	-.22	..	-.42	-.99	-.99	-1.05	-1.28	-1.67	-1.94	-2.50	-3.11
$e_{COW,FWAGE_{-j}}$	0	0	0	0	..	0	-.08	-.31	-.49	-.63	-1.12	-1.42	-2.01	-2.61
$e_{PROD,FWAGE_{-j}}$	0	0	0	0	..	0	-.08	-.31	-.49	-.63	-1.12	-1.42	-2.01	-2.61

[a]All elasticities are evaluated at the 1972:4 data point

one period decrease in the milk-feed price ratio is not obvious by comparing Figure 9 and 12. However, the hypothesis of irreversibility of the milk supply response function can be observed by comparing the elasticity simulated from a 10% permanent increase or decrease on milk-feed price ratio. There exists an asymmetric response on heifer numbers, cow numbers, and total production with respect to a permanent increase or decrease of milk-feed price ratio after several years. The elasticity for total production with respect to an increase in the milk feed price ratio is higher than that with respect to a decrease in the milk-feed price ratio. This result is illustrated in Figure 16.

VI. Summary:

This paper presents a model which is an improvement over previously published models in its ability to predict milk cow numbers, milk production per cow and total



milk production. It does this by taking into account the biological constraints on cow numbers simultaneously with consideration of economic factors. The model is specified to account for the lag structure between the time when economic decisions are made and the consequence of these decisions. The estimated net offspring rate of cows in period t-i-3 and retention rate of cows in period t-1 account for the effect of dairy farmers' current adjustment decisions on cow numbers.

A comparison of within-sample simulation with two different model specifications indicates that the present model is more accurate in estimating cow numbers, and matches the turning points better than the previously specified supply response models. Hence, a dynamic simulation of the model is conducted to calculate the short-run, intermediate-run, and long-run elasticities with respect to different exogenous variables. For a one period shock of exogenous variable, there is a iterative feedback scheme between cow numbers and heifer numbers and cow numbers takes almost 20 quarters to stabilize. For a permanent shock of milk-feed price ratio, there is an asymmetric milk production response after several years which recognizes the quasi-fixity of the inputs.

VII. References:

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Appendix : The Data and Sources

The data used to estimate the equations of the supply response model are presented in Table A. The sources for the data are listed below. In the table, the number in parentheses corresponds to the sources that the data were collected from.

(1). U. S. Department of Agriculture, Economic Research Service, *Dairy Situation and Outlook*, Washington D. C., 1970-91.

(2). U.S. Department of Agriculture, National Agricultural Statistics Service, *Agricultural Prices*, Washington D. C., 1970-91.

(3). *Dairy Situation and Outlook* for January 1 (Quarter 1) and July 1 (Quarter 3) estimates. Quarter 2 estimate is the average of Quarter 1 and Quarter 3. Quarter 4 estimate is the average of Quarter 3 and next year's Quarter 1.

TABLE A. DATA FOR DYNAMIC AGGREGATE MILK SUPPLY RESPONSE WITH BIOLOGICAL CONSTRAINTS ON DAIRY HERD SIZE

Variable Names	PRODUCTION PER COW	COW NUMBERS	REPLACE HEIFERS	ALL MILK PRICE	SLAUGHTER COW PRICE	16% DAIRY FEED	WAGE PAID BY FARMERS	MILK DIVERSION PROGRAM	DAIRY TERMINATION PROGRAM	FARM PRICE RECEIVED	FARM PRICE PAID
(Units)	(LBS)	(1000)	(1000)	($/CWT)	($/CWT)	($/TON)	(77=100)	(DUMMY)	(DUMMY)	(77=100)	(77=100)
	PPC	COW	HEF	AMP	PSCOW	PFEED	FWAGE	MDP	DTP	FRECE	FPAID
I 1970	2350	12070	4000	5.74	20.97	74.00	55.0	0	0	61	55
II	2668	12017	3979	5.45	21.63	73.00	57.0	0	0	60	55
III	2435	11970	3957	5.62	20.17	74.33	56.0	0	0	60.66	55.33
IV	2300	11931	3936	6.06	18.97	77.67	58.0	0	0	58.33	56
I 1971	2422	11891	3915	5.94	20.23	80.33	58.0	0	0	61	57
II	2735	11851	3894	5.62	21.03	80.00	60.0	0	0	62.33	58
III	2494	11819	3873	5.79	20.87	78.33	59.0	0	0	61	58
IV	2365	11790	3852	6.15	20.83	76.00	61.0	0	0	62	58.66
I 1972	2517	11755	3828	6.09	22.90	77.67	60.0	0	0	65.33	60
II	2801	11718	3840	5.79	24.33	77.33	64.0	0	0	66.66	61.33
III	2561	11671	3851	6.00	25.13	79.33	63.0	0	0	69.66	62
IV	2384	11642	3863	6.48	25.03	86.67	65.0	0	0	72.33	63.33
I 1973	2480	11559	3874	6.57	29.73	100.33	65.0	0	0	84.33	67.33
II	2780	11439	3896	6.41	32.83	105.00	70.0	0	0	94.66	70.33
III	2503	11348	3918	7.21	35.43	118.67	69.0	0	0	109.3	73.33
IV	2355	11309	3930	8.59	30.87	126.33	71.0	0	0	103.3	73.33
I 1974	2494	11265	3942	8.92	32.30	133.33	78.0	0	0	111.6	78.33
II	2815	11227	3921	8.26	28.10	125.33	77.0	0	0	98	79.66
III	2587	11218	3900	7.82	23.07	142.00	79.0	0	0	104.6	82
IV	2397	11212	3994	8.37	17.67	150.00	80.0	0	0	106.3	84
I 1975	2512	11197	4087	8.34	17.77	138.33	84.0	0	0	96.66	86
II	2808	11162	4006	8.08	21.33	132.00	84.0	0	0	99.66	88.66
III	2569	11118	3924	8.71	20.13	133.33	85.0	0	0	106	90.33
IV	2469	11079	3941	10.00	19.90	134.33	86.0	0	0	102.6	90.66
I 1976	2638	11060	3958	9.87	24.63	136.00	94.0	0	0	101.3	93.66
II	2934	11031	3950	9.26	28.17	138.33	92.0	0	0	104.6	94.33
III	2734	11023	3942	9.66	24.60	145.67	94.0	0	0	103.6	95.33
IV	2588	11011	3915	9.86	21.50	144.33	91.0	0	0	96.66	95

TABLE A. (cont.) DATA FOR DYNAMIC AGGREGRATE MILK SUPPLY RESPONSE WITH BIOLOGICAL CONSTRAINTS ON DAIRY HERD SIZE

Variable Names	PRODUCTION PER COW	COW NUMBERS	REPLACE HEIFERS	ALL MILK PRICE	SLAUGHTER COW PRICE	16% DAIRY FEED	WAGE PAID BY FARMERS	MILK DIVERSION PROGRAM	DAIRY TERMINATION PROGRAM	FARM PRICE RECEIVED	FARM PRICE PAID
(Units)	(LBS)	(1000)	(1000)	($/CWT)	($/CWT)	($/TON)	(77=100)	(DUMMY)	(DUMMY)	(77=100)	(77=100)
I 1977	2708	10983	3888	9.54	24.33	148.67	101.0	0	0	102.3	99
II	3019	10951	3947	9.40	25.90	149.67	99.0	0	0	105	101
III	2821	10937	4005	9.71	24.13	133.67	102.0	0	0	96	100
IV	2659	10907	3946	10.17	23.33	129.67	97.0	0	0	96.66	100
I 1978	2734	10860	3886	10.20	29.83	135.00	108.0	0	0	105.3	104.3
II	3021	10784	3921	10.07	36.50	137.67	109.0	0	0	116.6	108
III	2817	10779	3955	10.50	36.97	137.33	107.0	0	0	117	109.3
IV	2671	10791	3944	11.57	40.07	142.00	105.0	0	0	122	111.3
I 1979	2765	10762	3932	11.87	51.03	148.67	117.0	0	0	141	118.3
II	3061	10710	4024	11.53	54.30	150.33	117.0	0	0	134.3	122.6
III	2898	10719	4115	11.97	48.60	160.33	117.0	0	0	132.3	125
IV	2770	10741	4137	12.77	47.10	163.67	117.0	0	0	130	127.3
I 1980	2902	10752	4158	12.80	50.37	164.33	126.0	0	0	129.6	133.6
II	3160	10771	4268	12.60	44.37	165.33	126.0	0	0	125	136
III	2977	10811	4377	12.87	44.43	179.33	126.0	0	0	139.3	139.3
IV	2856	10846	4361	13.93	43.47	198.33	126.0	0	0	143.6	139.6
I 1981	2992	10851	4345	13.97	43.93	200.00	137.0	0	0	144	148.3
II	3235	10871	4487	13.53	43.13	198.00	137.0	0	0	142.3	150.6
III	3034	10906	4628	13.53	42.07	188.67	137.0	0	0	138.3	151.3
IV	2922	10964	4588	14.00	36.83	181.33	137.0	0	0	129.3	150.6
I 1982	3017	10995	4547	13.83	38.53	180.00	144.0	0	0	132.6	158.6
II	3239	10985	4664	13.30	41.17	179.67	144.0	0	0	136.6	159.6
III	3082	11007	4780	13.37	39.50	176.67	144.0	0	0	136.3	160.3
IV	2974	11040	4663	13.87	35.57	172.33	144.0	0	0	127.6	159
I 1983	3090	11058	4545	13.77	40.37	175.67	148.0	0	0	131.3	158.6
II	3321	11089	4713	13.37	41.83	183.33	148.0	0	0	135.3	160.3
III	3144	11112	4880	13.33	37.73	189.67	148.0	0	0	134.3	160.6
IV	3030	11131	4706	13.83	34.00	203.00	148.0	0	0	137.6	162.3

TABLE A. (cont.) DATA FOR DYNAMIC AGGREGATE MILK SUPPLY RESPONSE WITH BIOLOGICAL CONSTRAINTS ON DAIRY HERD SIZE

Variable Names	PRODUCTION PER COW	COW NUMBERS	REPLACE HEIFERS	ALL MILK PRICE	SLAUGHTER COW PRICE	16% DAIRY FEED	WAGE PAID BY FARMERS	MILK DIVERSION PROGRAM	DAIRY TERMINATION PROGRAM	FARM PRICE RECEIVED	FARM PRICE PAID
(Units)	(LBS)	(1000)	(1000)	($/CWT)	($/CWT)	($/TON)	(77=100)	(DUMMY)	(DUMMY)	(77=100)	(77=100)
I 1984	3108	10925	4532	13.40	38.83	201.67	151.0	1	0	145	164
II	3296	10799	4741	12.97	39.77	197.00	151.0	1	0	144.3	165
III	3100	10804	4950	13.20	36.67	188.00	154.0	1	0	142.6	164
IV	3003	10806	4855	14.10	34.43	177.33	150.0	1	0	136	163.6
I 1985	3109	10816	4770	13.63	39.30	174.33	154.0	1	0	134.3	163
II	3403	10987	4885	12.53	38.97	169.67	158.0	1	0	129.6	162.6
III	3305	11099	5000	12.17	34.90	165.33	154.0	0	0	122.3	161.3
IV	3174	11162	4855	12.60	32.97	163.33	150.0	0	0	125.6	161.6
I 1986	3251	11126	4709	12.37	35.90	167.00	150.0	0	0	122.6	160
II	3305	10943	4705	12.00	35.07	164.00	164.0	0	1	122	159
III	3327	10703	4700	12.37	35.80	159.00	166.0	0	1	124.3	159
IV	3199	10541	4503	13.33	35.20	151.00	159.0	0	1	122.3	158
I 1987	3340	10424	4305	12.97	41.00	153.00	159.0	0	1	121.6	158
II	3617	10339	4453	12.07	43.33	152.00	160.0	0	1	128	162
III	3453	10283	4600	12.30	44.20	154.00	161.0	0	1	127.3	163
IV	3375	10291	4361	12.87	43.60	156.00	162.0	0	0	129	165
I 1988	3519	10285	4122	12.20	48.00	166.00	171.8	0	0	130.3	165
II	3697	10244	4261	11.43	46.70	166.00	174.9	0	0	135	168
III	3526	10218	4400	11.87	45.73	199.00	179.1	0	0	144	172
IV	3471	10208	4285	13.30	44.97	197.00	181.2	0	0	143.6	174
I 1989	3611	10149	4169	13.07	48.40	196.00	182.4	0	0	149.6	176
II	3763	10110	4335	12.20	47.30	192.00	184.5	0	0	147.6	178
III	3484	10101	4500	12.41	48.97	184.00	185.5	0	0	145.3	179
IV	3448	10127	4364	14.50	47.77	182.00	184.5	0	0	146.6	178
I 1990	3627	10128	4227	14.63	51.77	186.00	188.4	0	0	151.3	181
II	3820	10111	4214	13.57	53.37	181.00	190.5	0	0	152	183
III	3620	10119	4200	14.03	52.83	181.00	191.5	0	0	149	184
IV	3575	10151	4197	12.50	49.13	181.00	194.6	0	0	144.3	187
Source	(1)	(1)	(3)	(1)	(1)	(2)	(1)	(1)	(1)	(2)	(2)

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