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Stock Pollutants and Risky Accumulation

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Abstract

This paper is concerned with pollution control when excessive levels of a stock pollutant might result in a discontinuous and permanent reduction in degradation - the environment's ability to decompose accumulated wastes. Optimal stopping rules are used to derive expressions for the optimal pollution stock in a deterministic model and a stochastic target in a model of risky accumulation. The stochastic target will be less than the optimal level of pollution in the deterministic model and reflects the expected regret to society if permanent environmental impairment occurs. While attainment of the target level of pollution is viewed as an acceptable risk, irreversible impairment could be triggered along the approach path. In the case where the degradation rate drops to zero, residual discharges may continue until the pollution stock reaches a lower optimum or they will cease immediately and society must endure an excessive pollution stock in perpetuity.

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I. Introduction

A stock pollutant is a residual waste that may accumulate or degrade over time. Such pollutants are often jointly produced with positively-valued services or commodities. The rate of residual discharge may depend on the level of commodity output and the resources devoted to treatment or pollution control. A degradable pollution stock may be broken down into less complex, benign compounds by biological, chemical or physical processes.

In the early and mid-1970s there were a series of papers which extended the theory of optimal economic growth to the management of stock pollutants. Papers by Plourde (1972), Smith (1972), Forster (1973, 1975 and 1977) and Cropper (1976) are examples of this earlier literature. More recently, Kitabatke (1989) has formulated a more general model having certain models of renewable resources and stock pollutants as special cases.

The purpose of this paper is to extend the earlier literature to the case where the degradation rate is subject to an uncertain, discontinuous and irreversible reduction. There are at least two examples where the level of a stock pollutant appears to have irrevocably altered system dynamics. Lakes that have undergone a significant reduction in pH from acid precipitation might be "limed" to bring the pH back up to former levels, but the dynamics of buffering within the lake appears to be permanently diminished compared to a pristine or marginally acidified lake (DePinto and Edzwald 1982).

The buildup of chloroflorocarbons (CFCs) in the stratosphere may have reached a level where the dynamics of the chemical cycle governing ozone generation have been altered. Two of the major pollutants, $CFCl_3$ and CF_2Cl_2, may persist in this altered cycle for 75 to 100 years (Stolarski 1988).

At issue in both of the above examples is a risk of permanent alteration of a dynamical system. How should society control pollution when faced with the risk of an abrupt and permanent (or very long-term) change to a less desirable dynamic process?

In the next section we construct a discrete-time version of a model developed by Forster (1975). The model provides a deterministic benchmark from which to evaluate pollution control with a risk of irreversible environmental impairment. In this model the degradation rate depends (smoothly) on the pollution stock. In contrast to Forster, an optimal stopping rule is used to derive an expression defining the optimal pollution stock. This approach is somewhat simpler than the maximum principle, although the resulting steady-state expressions are identical.

In the third section a model of risky accumulation is developed. Specifically, if the stock pollutant exceeds an unknown critical value, the relative level of degradation shifts permanently downward. This is a less extreme case than that examined by Cropper (1976). The analysis is again simplified by using a stopping rule to derive an expression that defines a "stochastic target." This equation is compared to the expression defining the optimal pollution stock in the deterministic model. The analysis permits a clear identification of the role "expected regret" plays in defining the stochastic target and also suggests an approach to regulatory policy for stock pollutants that risk permanent environmental impairment.

The fourth section presents an example using relatively simple functional forms. A deterministic problem, where the most rapid approach path is optimal, is solved and compared to the stochastic problem with an exponential cumulative distribution. Approach dynamics when the environmental system is altered are identified and displayed graphically. The fifth and final section offers some practical suggestions for pollution control in situations of risky accumulation.

II. The Deterministic Model

Consider an economy that in any period t uses a fixed resource to produce a consumption good C_t. The production of C_t may generate a residual waste, Z_t, that in turn may accumulate as the stock pollutant P_t. Specifically, the fixed resource may be used to produce C_t or *reduce* the amount of Z_t. In fact, we assume that there is a level, C_0, where allocation of the fixed resource to treatment implies a zero emission rate (that is, at C_0, $Z_t = 0$). More generally, we assume that the joint nature of production can be characterized by the convex function $Z_t = Z(C_t)$ over the domain $C_0 \leq C_t \leq C_{MAX}$, where C_{MAX} is the maximum possible level for C_t which can be achieved if all of the fixed factor is devoted to production. At C_{MAX}, however, the economy must endure emissions at Z_{MAX} (see Figure 1).

The emission rate in period t will contribute directly to the level of the stock pollutant in period t+1. The accumulated pollution stock, however, is subject to degradation at a level determined by $f(P_t)$. Following Forster (1975), $f(P_t)$ is assumed to be strictly concave over the domain $0 \leq P \leq \bar{P}$ (see Figure 2).

The emission and degradation functions give rise to the following difference equation to describe the dynamics of the stock pollutant

$$P_{t+1} - P_t = Z(C_t) - f(P_t) \tag{1}$$

Both the levels of consumption and the stock pollutant are assumed to affect utility in period t according to $U_t = U(C_t, P_t)$, where $U(\cdot)$ is strictly concave, with continuous derivatives $U_C > 0$, $U_{CC} < 0$, $U_P < 0$, $U_{PP} < 0$, $U_{CP} = 0$. Maximization of the present value of utility subject to pollution dynamics may be formally stated as

$$\begin{aligned} &\text{Maximize} && \sum_{t=0}^{\infty} \rho^t U(C_t, P_t) \\ &\text{Subject to} && P_{t+1} - P_t = Z(C_t) - f(P_t) \\ &&& P_0 \text{ given} \end{aligned}$$

where $\rho = 1/(1 + \delta)$ is a discount factor and δ is the periodic rate of discount.

The issues of existence, uniqueness and stability of steady state are discussed by Forster (1975) for the continuous-time version of this model. This paper will focus on the optimal level of pollution in the long run. An expression defining the optimal pollution stock can be derived from the optimal stopping rule.

Suppose a unique, saddle-point stable, equilibrium exists at (P,C). If $P_0 < P$, then it must have been optimal to accumulate to P and stop; maintaining the stock pollutant at its optimal level by producing $C_t = C$, where $Z(C) = f(P)$. The value function at (P,C) is

$$V = U(C,P) + \rho U(C,P) + \rho^2 U(C,P) + \ldots = (1 + \delta)U(C,P)/\delta \qquad (2)$$

The value to the economy of maintaining P must equal or exceed the value of maintaining a slightly larger pollution stock, $P + \Delta P$, which would result from a $C + \Delta C$. The value function for such a variation is

$$\begin{aligned} V_D = U(C + \Delta C,P) + \rho U(C + \Delta C,P + \Delta P) &+ \\ \rho^2 U(C + \Delta C,P + \Delta P) &+ \ldots \\ = U(C + \Delta C,P) + U(C + \Delta C,P + \Delta P)/\delta & \end{aligned} \qquad (3)$$

Note: In the period of expansion the economy benefits from the ΔC without experiencing the disutility of an increased pollution stock until subsequent periods. We may expand the right-hand-side (RHS) of (3) to obtain

$$V_D = U(C,P) + U_C\Delta C + [U(C,P) + U_C\Delta C + U_P\Delta P]/\delta \quad (4)$$

The initial increase in the pollution stock will be $\Delta P = Z'(C)\Delta C$. Solving for $\Delta C = \Delta P/Z'(C)$ and substituting into the second term on the RHS of (4) yields

$$V_D = U(C,P) + [U_C/Z'(C)]\Delta P + [U(C,P) + U_C\Delta C + U_P\Delta P]/\delta \quad (5)$$

Equating V with V_D, cancelling terms and dividing by ΔP will yield

$$\delta[U_C/Z'(C)] + [\Delta C/\Delta P]U_C = -U_P \quad (6)$$

Finally, at the new steady state we know that once again $Z(C) = f(P)$. Taking a total derivative we obtain $\Delta C/\Delta P = f\,'(P)/Z'(C)$. Substituting into the LHS of (6) and solving for U_C yields

$$U_C = -\frac{U_P\, Z'(C)}{[\delta + f\,'(P)]} \quad (7)$$

Equation (7) has a straight-forward interpretation. The LHS is the marginal utility of an increase in consumption. At the optimal state this will equal the marginal cost in perpetuity of the associated increase in the stock pollutant after accounting for transformation via $Z'(C)$ and the marginal change in degradation $f'(P)$. This same equation can be obtained from equations (7) and (8) in Forster (1975), when they are evaluated at steady state. In Section IV we will employ functional forms that permit us to derive an explicit expression for the optimal pollution stock from equation (7).

III. Risky Accumulation

Under risky accumulation we assume the existence of an unknown critical value for the stock pollutant, which if exceeded causes a permanent shift in the degradation function. Specifically, if $P_t \leq P_c$, then the level of degradation follows $f(P_t)$, while if $P_t > P_c$ the level of degradation follows $g(P_t) < f(P_t)$ for all $0 < P_t < \bar{P}$. If P_t exceeded P_c in an earlier period the impaired degradation rate may still allow the economy to naturally degrade the stock pollutant below P_c, but the $g(P_t)$ function now governs the level of degradation for all levels of P_t. The function $g(P_t)$ is also shown in Figure 2 for the case where $f(\bar{P}) = g(\bar{P})$. It is entirely possible that the positive root of $g(P_t)$ may lie to the left of $\bar{P}$.

While P_c is unknown it is assumed that a subjective cumulative distribution exists which defines the following two probabilities in every period where the critical value has not been exceeded.

$$\Pr(P_{t+1} > P_c \mid P_t \leq P_c) = \Gamma(P_{t+1})$$
$$\Pr(P_{t+1} \leq P_c \mid P_t \leq P_c) = 1 - \Gamma(P_{t+1})$$

for $P_{t+1} > P_t$, while $\Gamma(P_{t+1}) = 0$ for $P_{t+1} \leq P_t \leq P_c$. It is also assumed that $\Gamma'(P_{t+1}) > 0$; that is, an increase in P_{t+1} raises the probability of exceeding the critical value, when $P_{t+1} > P_t$.

The problem now is to maximize the present value of expected utility subject to the risk of an abrupt and permanent transition from $f(P_t)$ to $g(P_t)$. Suppose P_0 is zero or very small and the economy decides to allow the stock pollutant to increase, until at some point it is decided that further increases are too risky. What is implied by the stopping rule?

By staying at $P < P_c$ the economy is again settling for a present value of utility given by $V = (1 + \delta)U(C,P)/\delta$, previously derived in equation (2). This value must equal or exceed

$$V_S = U(C + \Delta C, P) + \rho\{[1 - \Gamma(P + \Delta P)]V_f + \Gamma(P + \Delta P)V_g\} \qquad (8)$$

where V_f is the value function if ΔP does <u>not</u> trigger the switch to $g(P_t)$, and V_g is the value function if ΔP causes the degradation function to permanently shift to $g(P_t)$. While $V_f = (1 + \delta)U(C + \Delta C, P + \Delta P)/\delta$ it is not possible to characterize V_g in general, since it will be determined by the best adjustment possible once it is realized that the dynamics of degradation have been permanently altered. V_g will depend in part on the level of the stock pollutant at the time of environmental impairment and the new optimal stock level for the function $g(P_t)$.

Expanding the RHS of (8) we obtain

$$V_S = U(C,P) + U_C\Delta C + \rho\{[1 - \Gamma(P) - \Gamma'(P)\Delta P](1 + \delta)[U(C,P) + U_C\Delta C + U_P\Delta P]/\delta + [\Gamma(P) + \Gamma'(P)\Delta P]V_g\} \tag{9}$$

The above expression is simplified somewhat by noting that if $P < P_c$, then $\Gamma(P) = 0$. By equating the simplified expression for V_S to V, cancelling terms, dividing by ΔP, noting $\Delta C/\Delta P = 1/Z'(C)$ in the period of expansion and $\Delta C/\Delta P = f\,'(P)/Z'(C)$ in steady state if environmental impairment is not triggered, it can be shown that

$$[U(C,P) - \delta\rho V_g]\Gamma'(P) = \delta U_C/Z'(C) + [1 - \Gamma'(P)\Delta P][U_C f'(P)/Z'(C) + U_P] \quad (10)$$

Suppose that ΔP, while positive, is very small, so that the term $\Gamma'(P)\Delta P$ is essentially zero. Then equation (10) implies that

$$U_c \approx -\frac{U_P\, Z'(C)}{[\delta + f'(P)]} + \frac{Z'(C)[U(C,P) - \delta\rho V_g]\Gamma'(P)}{[\delta + f'(P)]} \quad (11)$$

Compare equation (7) with approximation (11). Given the previous assumptions, the second term on the RHS of (11) is positive assuming that the utility just prior to triggering the switch to g(P) is greater than the annuity from the discounted value function V_g. This is likely to be the case, particularly for a severe drop in the degradation function. Under the extreme case where the degradation rate permanently drops to zero and the pollution stock cannot be reduced from its level of $P + \Delta P$, and if $P + \Delta P$ exceeds the deterministic optimum for g(P), then the economy will cease all residual emissions by adopting $C_t = C_0$, and settle for a utility flow of $U(C_0, P + \Delta P)$. In this case it can be shown that $V_g = (1 + \delta)U(C_0, P + \Delta P)/\delta$. The approximation then becomes

$$U_c \approx -\frac{U_P\, Z'(C)}{[\delta + f\,'(P)]} + \frac{Z'(C)[U(C,P) - U(C_0,P + \Delta P)]\Gamma'(P)}{[\delta + f\,'(P)]} \tag{12}$$

and the second term on the RHS becomes recognizable as the "expected regret," in perpetuity, from the state $(P + \Delta P, C_0)$ when, in hindsight, one might have stayed at (P,C).

Expression (11) might be used to solve for a stochastic target; that is, a pollution level that the economy regards as worth the risk of trying to attain. Conrad (1988) has employed dynamic programming to show that the approach path to a stochastic target will be slower than the approach (from the same P_0) in a deterministic model having an optimal stock equal to the stochastic target. In each period there is a risk in allowing the pollution stock to increase further. The risk results in a probability effect that slows the rate of accumulation from what it would be in the deterministic model.

IV. An Example

To illustrate the above analysis consider the following functional forms: $U_t = C_t - \mu P_t^2$, $f(P_t) = \alpha P_t$ and $Z(C_t) = (C_t - C_0)/\eta$, where μ, α, and η are positive parameters. In this case equation (7) will imply an optimal level of pollution given by

$$P = \frac{\eta(\delta + \alpha)}{2\mu} \tag{13}$$

Figure 3 shows the optimal levels of the stock pollutant when $\alpha > 0$ and when $\alpha = 0$. They appear as horizontal lines in $(t\text{-}P_t)$ space and are denoted by $P_{\alpha>0}$ and $P_{\alpha=0}$. It can be shown that the functional forms specified above satisfy the conditions for the most rapid approach path (MRAP) to be optimal (see Spence and Starrett 1975). Thus, if P_0 is less than the optimal pollution stock, $C_t = C_{MAX}$ and $Z_t = Z_{MAX}$ until it is reached. In the case where $\alpha = 0$, once P_t reaches $P_{\alpha=0}$, $C_t = C_0$, while in the case where $\alpha > 0$, once P_t reaches $P_{\alpha>0}$, consumption is maintained at $C_t = C_0 + \eta\alpha P_{\alpha>0}$.

In the stochastic model, suppose we adopt the exponential distribution $\Gamma(P_{t+1}) = 1 - \exp[-\lambda(P_{t+1} - P_t)]$, where $\lambda > 0$. Note: As $(P_{t+1} - P_t) \rightarrow \infty$, $\Pr(P_{t+1} > P_c) \rightarrow 1$. Also, $\Gamma'(P_{t+1}) = \lambda\exp[-\lambda(P_{t+1} - P_t)]$, and when $P < P_c$ in steady state, $\Gamma'(P) = \lambda$. With ΔP small, approximation (12) may be solved for the stochastic target P_S, yielding

$$P_S = \frac{\eta[\delta + \alpha] - [C - C_0]\lambda}{2\mu} \tag{14}$$

Comparing (13) and (14) it is clear that $P_S < P_{\alpha>0}$. A logical lower bound on P_S would be $P_{\alpha=0}$, however it is possible that particular values for the parameters on the RHS of (14) may actually result in $P_S < P_{\alpha=0}$.

Figure 3 also shows the stochastic target P_S and a dashed line representing the optimal approach from $P_0 = 0$. The values of P_t along the approach path might be derived using stochastic dynamic programming. Because $\Gamma(P_{t+1})$ is subjective, there is a chance that P_c might be exceeded at any time along the approach path. If P_c were exceeded at $t = t_1$, then the problem reverts back to a deterministic problem with $\alpha = 0$ and an optimal stock of $P_{\alpha=0}$. Given the functional forms in this example the MRAP is now optimal and the approach from t_1 is shown as the more steeply sloped solid line segment.

If P_c were exceeded after the pollution stock was above $P_{\alpha=0}$ the economy would be unable to reduce the pollution stock. To prevent it from increasing (and lowering utility further) the best it can do is to set $C_t = C_0$ in perpetuity. In this example, if the economy is faced with the risk of irreversible accumulation it may be better to find out sooner, before $P_{\alpha=0}$ is exceeded, than later.

While this example is highly stylized, it may not be totally unrealistic. Consider a region or province extracting and processing a mineral resource which it exports at a constant price but which generates tailings and wastes from refining which accumulate locally. Net revenue might be linear in exports, C_t, while environmental costs might be approximated by a quadratic. At what level of pollution should stringent controls be introduced? If tailings were disposed in a local lake, dredging and alternative disposal may be possible, but have the dynamics of degradation been permanently altered?

V. Policy Implications

The preceding models, while simple, have some important implications for regulating stock pollutants that risk permanent environmental impairment. First, the setting of ambient standards should be more conservative when permanent impairment is possible. Not only do ambient standards need to reflect current damages associated with present or past concentrations of a stock pollutant, but they should also reflect the expected regret if a degradation or assimilative process is rendered less effective or if an ecosystem is threatened with a permanent loss in resiliency.

Second, ambient standards need to be set adaptively, so they can be changed if monitoring generates evidence that biological, chemical or physical processes have been altered, necessitating lower ambient concentrations and lower rates of emission.

Finally, when critical values or other potential bifurcations exist, monitoring is essential. Baseline studies on degradation and diffusion in relatively undisturbed, pristine environments need to be carried out as well as studies of these same processes in polluted environments. Such studies are essential for trying to identify both movements along as well as shifts of the degradation function.

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Figure 1. The Level of Residuals as a Function of Consumption

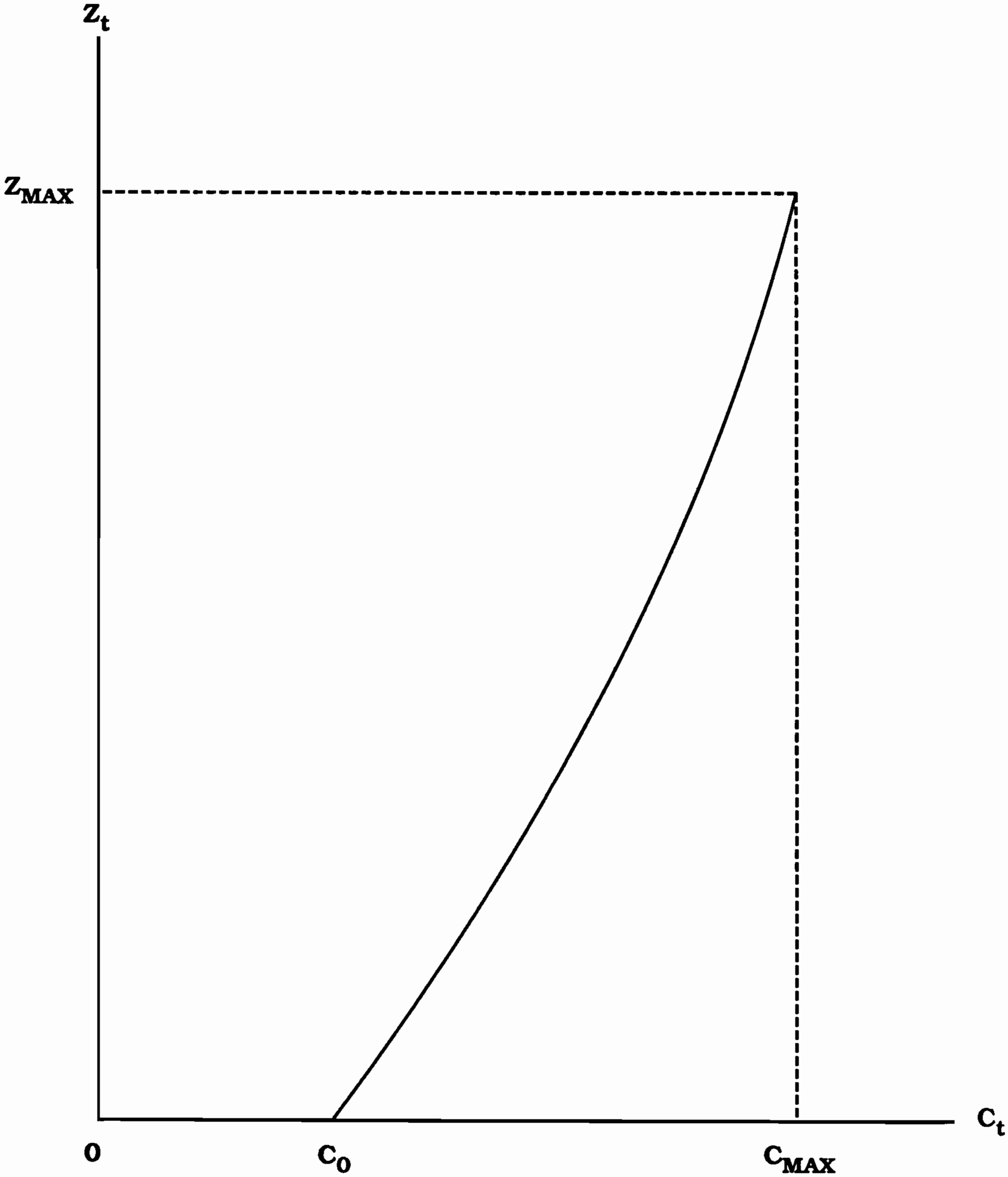


Figure 2. Degradation Functions

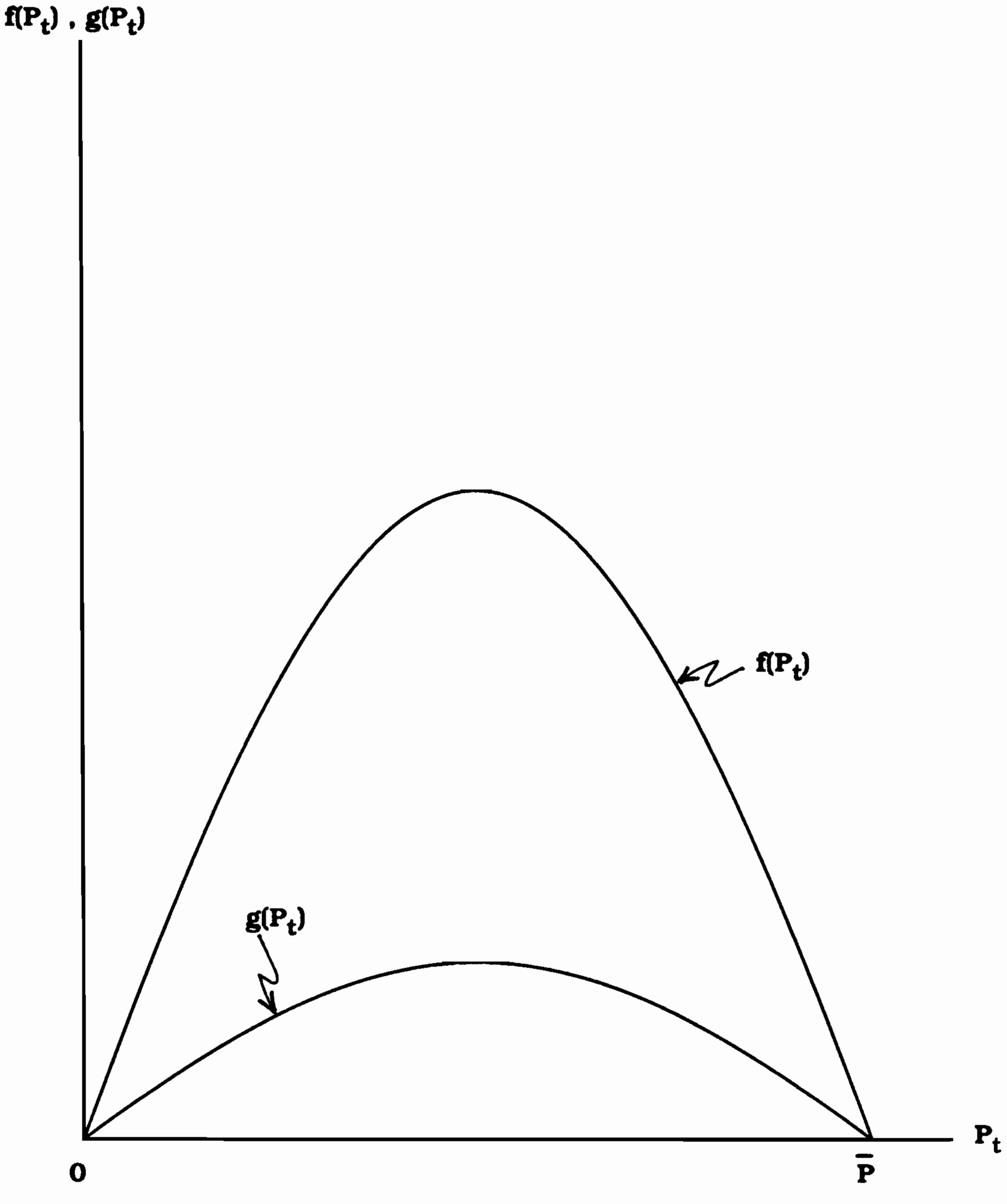
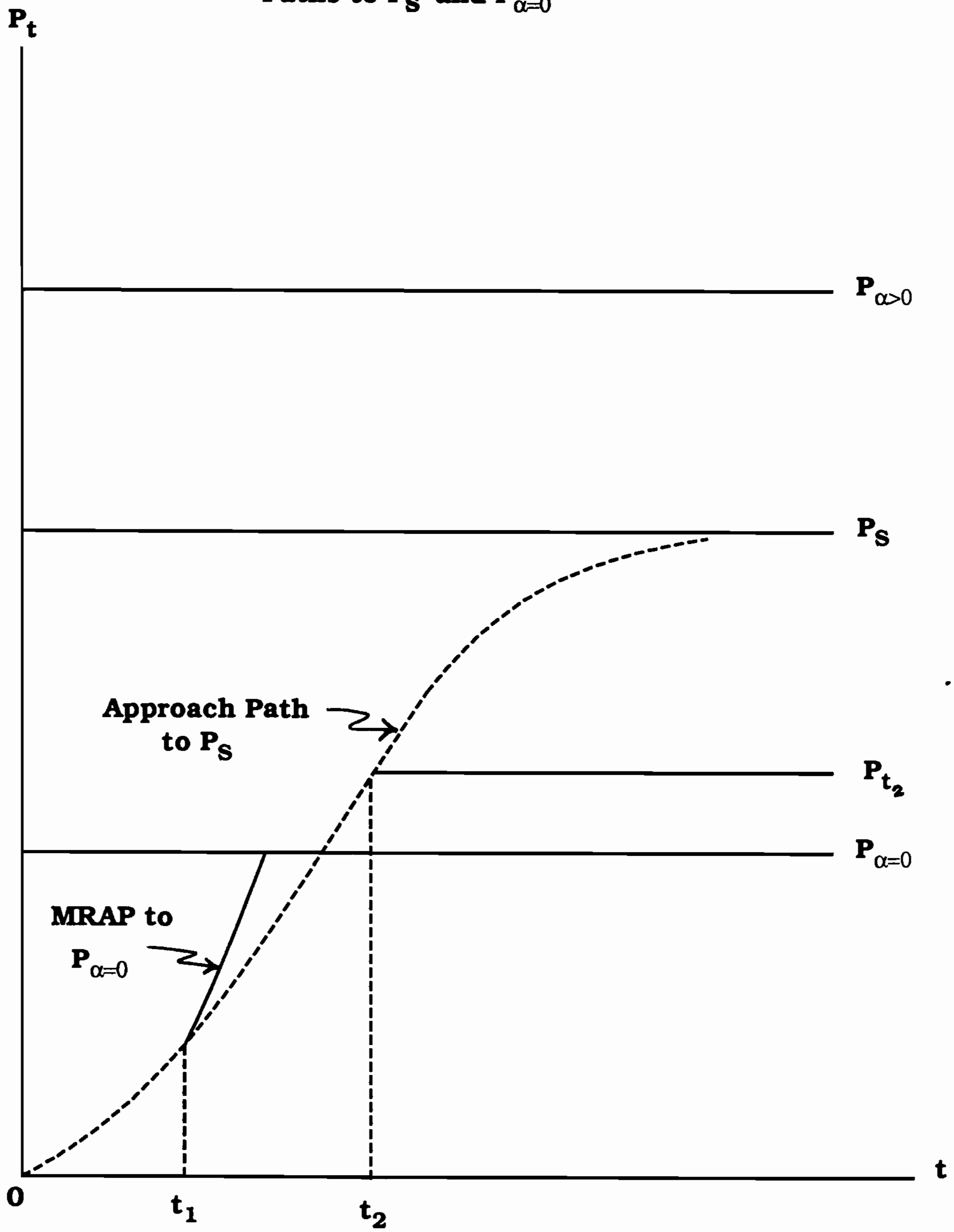


Figure 3. The Stochastic Target and Approach Paths to P_S and $P_{\alpha=0}$



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