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Estimating Parameters in a Dynamic Open-Access Model with Application to the Flemish Cap Groundfishery

by

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Abstract

This paper estimates stock levels of groundfish, and other parameters of interest, in an area of the NorthWest Atlantic known as the Flemish Cap. To do so, it makes use of an algorithm designed to circumvent problems usually found in estimating the standard dynamic open-access model.

Empirical tests of the standard model have been relatively few. One difficulty is that fish stocks, the level of which affects the rate of catch, are not directly observable. Simplifying assumptions are generally required, such as the assumption that catchability does not change over time.

This paper describes an algorithm that imposes a less restrictive pattern (than constancy) on the catchability coefficient, and it reports on an application of the algorithm to the Flemish Cap groundfishery for the period from 1971 to 1985. Estimates are generated for the level of fish stocks, vessel productivity, biological growth rate and environmental carrying capacity. Given a minimum of *a priori* assumptions (e.g., that the biological growth rate is positive), a solution is obtained which appears to be unique.

As a secondary result, the hypothesis that catch per unit effort is proportional to stock levels is accepted at the vessel level.

Contents

1. Introduction and Overview

This paper describes an estimating technique for dealing with a problem commonly found in fisheries economics: catch rates are hypothesised to depend on the levels of fish stocks, but these stock levels are not only unobservable—they also fluctuate over time in a manner that itself needs to be modelled and estimated.

Once described, this estimating technique is applied to a fishery in the Northwest Atlantic known as the Flemish Cap. The resulting estimates are shown to be consistent with *a priori* expectations, both as to the actual parameter values and as to the interrelated behaviour of fishing effort and stock levels over time.

The paper is organised as follows: the rest of Section 1 briefly outlines the widely accepted theory of "open-access" regimes, and the characteristics of the Flemish Cap; Section 2 then describes the algorithm used to estimate the parameters of interest in this fishery; Section 3 reports on the empirical results, which are discussed in the concluding Section 4.

1.1 The Theory of "Open Access" Fisheries

Since the late 1970s, many of the world's richest fishing grounds have been regulated more stringently by the national governments within whose 200-mile Exclusive Economic Zone (EEZ) they now lie. While fisheries of the "open access" type, largely free from regulation, are now the exception rather than the rule, they remain of much theoretical and practical interest.

In the case of open access fisheries, there has been a large measure of agreement on the descriptive theoretical model. As a result, the following paragraphs set out such a model in an almost cursory fashion, with the main object of establishing the notation to be used later. Further detail can readily be found in such texts as Clark (1976).

Schaefer (1957) described the relationship between fishing effort E and the level of yield Y that could be maintained over time at various levels of effort. ("Fishing effort" is a somewhat slippery concept. In the current study, it is represented by the number of days a vessel spends fishing.) The general relationship is often represented as logistic.

Gordon (1954) had previously introduced economic considerations. Costs c per unit of effort E was assumed to be constant, as was the selling price p for each unit of catch Y. A point of equilibrium could thus be established, at which total costs and revenues would equal one another. (Gordon referred to this as a bionomic equilibrium.) Moreover, given the convexity of the yield/effort curve, and the constant level of costs per unit of effort, this equilibrium would be a stable one. The standard "Gordon-Schaefer model" is illustrated in Figure 1.

This Gordon-Schaefer equilibrium may also be seen as the steady state or fixed point in a dynamic process. Let X represent the stock, or biomass, of a given fishery at a certain point in time. Changes in stock levels are affected by two principal means: natural growth, and harvesting by fishermen. If $r > 0$ is the intrinsic growth rate, and $K > 0$ the environmental carrying capacity (both assumed to be constant), then biological growth is often represented by the logistic expression:

$$F(X) = rX\,(1 - (X/K))$$

If the catch rate, Y, fishing effort expended, E, and the resource stock, X, are linked by the production function:

$$Y = qEX,$$

where $q > 0$ is known as the catchability coefficient, then the overall rate of change in stock levels is given by the differential equation:

$$dX/dt = rX\,(1 - (X/K)) - qEX. \qquad (1)$$

(By setting the left hand side of this equation to zero and using a little algebra, an expression may be obtained for the yield-effort curve illustrated in Figure 1.)

Suppose that profit, π, is defined as follows:

$$\pi = pY - cE = (pqX - c)E. \qquad (2)$$

(It is assumed that yield does not affect price, and that costs are constant.) The next step, due to Smith (1969), views changes in fishing effort as determined by the level of profits. Specifically, the following differential equation is often used:

$$dE/dt = n\pi, \qquad (3)$$

where n is an adjustment parameter, scaled in effort per \$.

Equations (1) and (3) constitute a dynamical system. A point of equilibrium (X^*, E^*) is found where the two variables, X and E, show no tendency to change. Thus, making use also of Equation (2):

$$X^* = c/pq$$

$$E^* = r(1 - X^*/K)\,/\,q = r(1 - (c/pqK))\,/\,q$$

The equilibrium point is either a stable node or a stable focus, at least in the neighborhood of the equilibrium. A stable spiral is shown in Figure 2.

These theoretical expectations have been subjected to a limited amount of empirical testing. Wilen (1976) examined the North Pacific fur seal fishery over the period from 1882 to 1900; his results were consistent with the theoretical expectations of Figure 2, although the data series was too short to tell if an approach was being made to a point of equilibrium. The North Sea herring fishery for the period from 1963 to 1977 was studied by Bjørndal and Conrad (1987); their results do show a counterclockwise movement, consistent with the pattern found in Figure 2, that might or might not have developed into a inward spiral. Conrad (1987) examined the Western Arctic bowhead whale fishery for the period from 1848 to 1914; his findings were less easily reconciled to the counterclockwise loop called for by the model, though he suggested a number of reasons for this disparity.

1.2 The Flemish Cap

This fishing ground lies in the Northwest Atlantic, as shown in Figure 3[1]. It is the only fishery in that region to lie entirely outside the 200-mile Exclusive Economic Zones (EEZs) established in the late 1970s, and thus to remain suitable as an application for the open-access model.

Konstantinov (1970), writing before the introduction of EEZs, gives other reasons for using the Flemish Cap as a valuable test case. (His remarks deal with the cod stock alone, whereas the application described later in this paper does not distinguish among the various types of groundfish—of which cod are, nevertheless, the largest single species.) Konstantinov points out (i) the physical isolation of the area, which prevents (cod) stocks from mingling with those of other grounds; (ii) the "extremely stable oceanological conditions" on the Flemish Cap, which suggest a stable biological growth rate, *r*, and carrying capacity, *K*, as called for by

1 Figure 3 is taken from documents of the Northwest Atlantic Fisheries Organisation (NAFO). References to the Flemish Cap in this paper apply, strictly speaking, to NAFO Subarea 3M.

the model; and (iii) the fact that the area is not of such importance to any country that its fishermen have no alternative but to exploit it.

The Flemish Cap has been fished commercially only since 1957, and data on catch and effort are therefore available for the entire period of exploitation. The current study was limited to the period from 1971 to 1985 since data for that period were available in a particularly convenient form[2].

2. An Algorithm for Estimating the Dynamic Open-Access Model

Although data on catch and effort are available on a monthly basis from the NorthWest Atlantic Fisheries Organisation, this study uses annual aggregates. The open-access model described in the previous Section must therefore be transposed into discrete terms, so that changes in the stock level, *X*, are expressed as follows:

$$X_{t+1} - X_t = F(X_t) - Y_t + \varepsilon_{t+1}$$
$$= \text{(by supposition) } rX_t - rX_t^2/K - Y_t + e_{t+1}, \qquad (4)$$

where e_{t+1} picks up specification error in the growth function $F(X_t)$ as well as unexpected causes of mortality. It is assumed that e is i.i.d. $N(0; \sigma^2)$.

Suppose that yields at the fleet level may be represented by:

$$Y_t = q_t X_t E_t.$$

Now $X_t = Y_t / q_t E_t$; substituting for X_{t+1} and X_t in Equation (4):

$$Y_{t+1}/q_{t+1}E_{t+1} = (1+r)(Y_t/q_tE_t) - (r/K)(Y_t/q_tE_t)^2 - Y_t + e_{t+1}$$

2 I am very grateful to Joan Palmer and Patricia Hersey of the National Marine Fisheries Service, Woods Hole, MA, for providing me, on disk, with data from the Northwest Atlantic Fisheries Organisation and its predecessor.

or:

$$Y_{t+1}/E_{t+1} = (1+r)(q_{t+1}/q_t)(Y_t/E_t) - (r/q_tK)(q_{t+1}/q_t)(Y_t/E_t)^2 - q_{t+1}Y_t + q_{t+1}e_{t+1}. \quad (5)$$

2.1 The Nature of the Estimating Problem

Equation (5) exemplifies a difficulty that arises when estimating dynamic open-access models.

Y and *E* are observable, and there are three regressors. On the other hand, the parameters to be estimated include not only *r* and *K*, but also the various *q*s of which, in this application, there are 14.

This difficulty has usually been dealt with by assuming that *q* is constant over time. In that case, however, the yield relationship $Y_t = q_tX_tE_t$ can no longer be regarded as deterministic and now, when substituting for X_t and X_{t+1} in Equation (4), an error term associated with the yield function must be introduced, thus: $X_t = (Y_t / q_tE_t) + v_t$. Once these substitutions are made and an expression obtained that is analogous to Equation (5), the composite error term is found to be correlated with the set of regressors.

This paper makes use of an algorithm that economises on the number of parameters in Equation (5), and yet allows for a deterministic yield function. It therefore permits the estimation of that equation to remain tractable.

2.2 The Algorithm

The Flemish Cap has been fished, over the years, by vessels of widely varying tonnage classes, using gear of many different kinds and registered in several different countries. In the following discussion, a particular combination of class, gear and country is known as a vessel type. For any one vessel type *i* in time *t*:

$$q_{it} \equiv Y_{it}/X_tE_{it}$$

can always be defined. For this single observation, the "production function" $Y_{it} = q_{it}X_tE_{it}$ holds without an error term. (Weather effects and the like are assumed to act through q_{it}, and are thus captured by this relationship rather than through the error term in Equation (4).)

The meaning of q need not be precisely defined at this point. After its magnitude has been determined (for all *i*s and *t*s), its behaviour over time for any given type *i* can be examined to see if $Y_i = q_iXE_i$ is plausible. In the meantime q_{it} is only a construct, but it is not for that reason arbitrary; in the absence of other information, it would still be of interest as a crude measure of productivity.

Summing across *i*s:

$$Y_t = \Sigma Y_{it} = \Sigma q_{it}X_tE_{it} = X_t\Sigma q_{it}E_{it}.$$

Defining q_t as $\Sigma q_{it}E_{it} / E_t$:

$$Y_t = q_tX_tE_t.$$

At this stage, the model remains deterministic.

q_t is a weighted average of vessel-type catchability coefficients. There is no need to look on $Y_t = q_tX_tE_t$ as a "fleet production function"; according to Doll (1988), doing so in any short-run context makes "such formulations as [Y] = qEX ... misleading or at the least difficult to interpret" (p.119). Doll also requires that any fleet production function "should at least approximate the long-run engineering-type production function for the typical vessel in the fleet" (op.cit., p.122); he cites Nerlove (1965) to the effect that "in the absence of [such an assumption], the estimated coefficients ... have no readily interpreted meanings."

q_t, for the purposes of this discussion, may be looked upon as a measure of technology mix at the fleet level or, if preferred, as a purely arithmetical construct.

The problem identified in Section 2.1 is to economise on the number of parameters in Equation (5)—in particular, to impose a pattern on the ratio q_{t+1}/q_t without creating an error term that is correlated with the set of regressors. This can be achieved by hypothesising that the ratio q_{t+1}/q_t is distributed normally about a mean G.

Substituting $q_{t+1}/q_t = G + \eta_{t+1}$ into Equation (5):

$$\begin{aligned} Y_{t+1}/E_{t+1} = {} & G(1+r)(Y_t/E_t) - (Gr/q_tK)(Y_t/E_t)^2 - Gq_tY_t \\ & + q_{t+1}e_{t+1} + (1+r)(Y_t/E_t)\eta_{t+1} \\ & - (r/q_tK)(Y_t/E_t)^2\eta_{t+1} + q_tY_t\eta_{t+1}. \end{aligned} \tag{6}$$

The error term retains the usual desirable properties since η example, are not otherwise associated.

Before going further, an adjustment will be made which, though not essential to the analysis, focuses the discussion on the unknown stock level, X, rather than on the construct, q. Consider the first time period, with stock X_0. Now q_0 can be expressed in terms of X_0; say, $q_0 = Y_0/X_0E_0 = m_0/X_0$, where m_0 is known. If $E(q_{t+1}/q_t) = G$, an estimate of future qs is given by $q_t = G^t m_0/X_0$. Substituting into Equation (6):

$$\begin{aligned} Y_{t+1}/E_{t+1} = {} & G(1+r)(Y_t/E_t) - (GrX_0/G^t m_0K)(Y_t/E_t)^2 \\ & - GY_t(G^t m_0/X_0) + q_{t+1}e_{t+1} + (1+r)(Y_t/E_t)\eta_{t+1} \\ & - (r/q_tK)(Y_t/E_t)^2\eta_{t+1} + q_tY_t\eta_{t+1}. \end{aligned} \tag{7}$$

The parameters to be estimated are now *G*, *r*, *K* and X_0.

To solve Equation (7), an initial estimate of *G*, namely *G**, is used. (Lacking other information, a plausible initial value might be G* = 1.) This estimate is used only in the terms G^t, not where *G* appears alone. Regressing (Y_{t+1}/E_{t+1}) on (Y_t/E_t), $(Y_t/E_t)^2/G^{*t}$ and $G^{*t}Y_t$ (using OLS and suppressing the intercept), the following estimates are obtained:

$$\beta_1 = \hat{G}(1+\hat{r})$$
$$\beta_2 = -\ \hat{G}\hat{r}\hat{X}_0/m_0\hat{K}$$
$$\beta_3 = -\ \hat{G}m_0/\hat{X}_0.$$

Then, successively:

$$\hat{r} = (\beta_1/\hat{G}) - 1 \quad (8)$$
$$\hat{X}_0 = -\ \hat{G}m_0/\beta_3 \quad (9)$$
$$\hat{K} = -\ \hat{G}\hat{r}\hat{X}_0/m_0\beta_2 \quad (10)$$

Since there are four parameters to be estimated and only three regressors, the process just described gives rise to a range of solutions. It is convenient to label them by the value of *G* with which they are associated; thus, by the "G = 0.95" solution, one refers to estimates of *r*, X_0 and *K* given by substituting "G = 0.95" in Equations (8) through (10) successively.

A range of solutions (*G*, *r*, X_0, *K*) is thereby established. Certain of these solutions, such as those with negative *r*, can be eliminated on *a priori* grounds. The remainder may be thought of as a set of feasible solutions. (Although this is in principle an infinite set, in practice—and in the discussion that follows—its elements are a limited number of representative solutions, each associated with its particular value of *G**.)

3. Results

When this procedure was applied to the Flemish Cap groundfishery from 1971 to 1985, the "G = 0.956" solution gave rise to the following parameter estimates:

$$r = 1.56735 \quad K = 292{,}208 \text{ (tonnes)}.$$

Figure 4 shows the associated time path for *X* (plotted against *E* to be consistent with Figure 2). The apparent series of anticlockwise loops is further discussed below.

These estimates not only met least-squares criteria and *a priori* expectations, but were shown to be self-consistent: when used to generate a time series for *q*, the implied *G* (i.e., the relationship between successive *q*s) was equal to the initial value of G^*. The "G = 0.956" solution appeared to be the only one of which this could be said.

Before this result could be determined, a range of possible solutions was looked at, each based on a particular value for G^*. These solutions were tested for conformity with *a priori* expectations.

First, the biological growth rate, *r*, could not be negative. No other limits were put on *r*, since the model was applied to total catches of groundfish rather than to any particular species, such limits would be hard to determine *a priori*.

Second, estimates of the environmental carrying capacity, *K*, were examined. Just as negative rates of biological growth, *r*, could be ruled out *a priori*, so one might require that estimated stock levels not exceed estimated carrying capacity, i.e., that $K/X_0 > 1$. However, it was not sufficient to compare the mean values of $\hat{K}$ and $\hat{X}_0$. Because of the standard error associated with the parameter estimates β_2 (see page 9), the probability of rejecting the hypothesis that $K/X_0 > 1$ had to be calculated for each candidate solution[3]. It was discovered that in no case could the hypothesis be rejected at any reasonable α. For the "G = 0.956" solution, eventually chosen, $p(K/X_0 > 1) = 0.70$.

Similar results were found with respect to the third *a priori* consideration: the standard errors of β_3 were such that a hypothesis that $X_0 > 0$ could not be rejected. On the other hand, at approximately G = 1.03, the relationship between X_0 and G appeared to be discontinuous[4]; as $G \rightarrow 1.03$, $\hat{X}_0 \rightarrow \infty$.

A final *a priori* restriction could have been placed on *G* itself. If looked upon as a measure of change in "fleet technology", it might be thought of as unlikely to change by more than 10% per year. On the other hand, since *G* could be seen principally as a means to an end—as a mathematical construct designed to help in obtaining estimates of *r*, *X* and *K*—it was left unconstrained.

The dynamic properties of the various "solutions" were examined. Specifically, values were generated for $\hat{X}$ throughout the 15-year period under study, rather than just for $\hat{X}_0$. The results for the "G = 0.956" solution are given in Figure 4 as a plot of *E* against $\hat{X}$[5].

3 By contrast, standard errors for β_1 were found to be small for all candidate solutions. As a result, there was no need to take a probabilistic approach to the *a priori* requirement that $r > 0$.

4 "appeared", since the relationship was examined only by means of representative values for *G*, rather than derived explicitly

5 For other "solutions" where, say, $0.90 < G < 0.98$, the patterns were similar to Figure 4. For lesser values of *G* the series of loops turned into a much flatter series of kinks, as

It is possible to look on the loop from "1977" to "1985" as the main feature (a loop that is almost completely regular, except for the spur in "1981"); perhaps the 1977 introduction of the 200-mile limit in other fishing areas brought about a switch into the Flemish Cap around that time. Viewed in this light, the observations before "1977" come to appear simply as a vestige of the *ancien regime*, cluttering up what would otherwise be a classic open access picture.

However, another possibility enables all observations to form part of the pattern. This view sees a series of connected anticlockwise loops: (A) from "1971" to "1974"; (B) from "1974" to "1979"; and (C) from "1977" to "1985". The last two in particular overlap, suggesting the form of a helix, running from northwest to southeast.

These results call for discussion of those factors, such as fuel costs, which may have brought them about. Such a discussion is postponed to the following Section.

Having determined the time path of $\hat{X}$, it was possible to analyse the "productivity coefficient" q at the vessel-type level. In arguing for the original model, these definitions were used:

- $q_{it} \equiv Y_{it}/X_t E_{it}$
- $q_t \equiv \Sigma q_{it} E_{it}/E_t$.

Looking now at the productivity coefficient q_i for each combination i of country, gear and tonnage class, an error term must be introduced, thus:

$$Y_i = q_i X E_i + e_i, \tag{14}$$

where the characteristics of e are hard to anticipate on *a priori* grounds.

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when a spring is stretched beyond its limits.

OLS was therefore used, *faute de mieux*, to estimate q_i, though a case could be made for Seemingly Unrelated Regression since the *e*s might be related for different *i*s.

All but one of the equations generated high t-ratios for $\hat{q}_i$. However, in those (few) instances where it was possible to control for different reporting methods by examining the individual *q*s associated with one particular country, the estimates of q_i did not relate to one another as might have been anticipated on *a priori* grounds. Since effort was measured in terms of the number of days fished it might, for example, have been expected that larger vessels would have higher *q*s than smaller vessels, and that stern trawlers would have higher *q*s than the side trawlers they replaced. There were only four instance in which these expectations could be tested, but in only one case were they borne out.

On the other hand, as noted above, in almost every case $Y_i = q_i XE_i$ was strongly supported by OLS estimation. Of the 22 major vessel types (accounting in total for at least 85% of fleet effort in every year), only one exhibited a t-ratio as low as 4.3, with all the rest being greater than 8.4. These results provided a further measure of support for the functional form used in the standard open-access model.

From the vessel-level *q*s, a weighted average could be used to obtain estimates of *q* at the fleet-level. The algorithm had assumed that successive *q*s were linked by the relationship:

$$q_t = G^t q_0$$

or:

$$\log(q_t) = \log(q_0) + \log(G).t$$

As a check on the results, estimated fleet-level *q*s were used to determine the strength of this relationship, and to see whether the value for *G* implicit in the *q*-series was consistent with the *G** from which those *q*s were generated.

The mean estimate of *G* obtained from the series of *q*s coincided with the initial value, *G** (from which the *q*s themselves were derived), only when G ≈ 0.956. When some other initial value of *G** was used, successive estimates of *G*[6] converged to 0.956. (For the "G = 0.956" solution, the degree of association between $\log(q_t)$ and *t* was reflected in an adjusted R^2 of 0.69.)

4. Summary and Discussion

1. Applied to the Flemish Cap groundfishery (NAFO subarea 3M) for the period 1971–85, the model performed rather well. Solutions from the initial run were convergent, and estimates of the parameters *r*, *K*, X_0 and *G* were obtained.

   These estimates initially took the form of a fairly narrow range, within which the parameters satisfied least-squares criteria and largely met *a priori* expectations. However, while testing the consistency of these results, it was found possible—indeed neccesary—to move from a range to a point estimate, as follows:

   r = 1.56735 K = 292,208 (tonnes).

2. When solutions were used to generate values for $\hat{X}$ over the whole

6 Successive estimates of *G* would be formed by (i) picking some value of *G**; (ii) calculating the resulting*q*-series; (iii) calculating the value of *G* which best fitted the *q*-series; and (iv) using that value in place of *G**.

15-year period, the resulting plots of *E* against $\hat{X}$ exhibited not only reassuring resemblances to textbook illustrations of the open-access paradigm, but also interesting divergences from it. Although the conventional anticlockwise loop was displayed, with this data set it was "stretched out" so as to look more like the helix suggested in Figure 4.

Figure 2 had illustrated a theoretically determined point of equilibrium (X^*, E^*), where:

$$X^* = c/pq$$

$$E^* = r(1 - (c/pqK)) / q$$

During the second half of the 1970s fuel costs, a major component of the cost of fishing, rose steadily and substantially. All else being equal, the rise in unit costs *c* would cause (X^*, E^*) to move in the direction of increased stock levels and lower levels of effort. It may be that the apparent helix in Figure 4 owes its origin to this effect. On the other hand, an additional influence (at least with respect to the increase in stock levels) may be the decline in *q*, to be discussed below.

3. The model was also used to test the hypothesis that $Y_i = q_iXE_i$ is a plausible production function for specific vessel types. In OLS regressions, q_is were estimated and found to have, for the most part, quite formidable t-ratios. On the other hand, in those few cases where it was possible to form *a priori* expectations as to the relative size of particular $\hat{q}_i$s, these expectations were largely unmet.

At the aggregate level, the weighted average of vessel productivity coefficients $\hat{q}_i$, which may be known for want of a better term as the "fleet technology coefficient" q, exhibited some interesting characteristics. To solve the initial model, a ratio of successive qs, known as G, had been constructed, and the validity of this device was confirmed by tests on the resulting estimates. On the other hand, these estimated qs appeared to follow a 4.4% annual rate of decline.

The interpretation of a declining q requires further analysis. It is not an unprecendented result: Agnello and Anderson (1981), studying NAFO Subareas 5Y and 5Z with a very different model, found the "catchability" of cod, haddock and redfish to be declining over the period from 1960 to 1974[7]. In the present study, however, q is not so much a measure of catchability as of technological "mix" within the fleet. A theoretical relationship between changes in this composite measure on the one hand, and the levels of X and E on the other, needs to be developed.

One approach might be to expand the dynamical system made up of Equations (1) and (3). It might be valuable to add an equation for dq/dt, and to explore the theoretical properties of such a three-dimensional system. By suitably grounding all three relationships in behavioral plausibility, the theoretical time path of q—to the extent that it resembles empirical results—could then shed light on the decline in q apparently found in the Flemish Cap groundfishery over the study period.

7 The authors associated this decline with the use of quotas, at least in the case of haddock after 1969.

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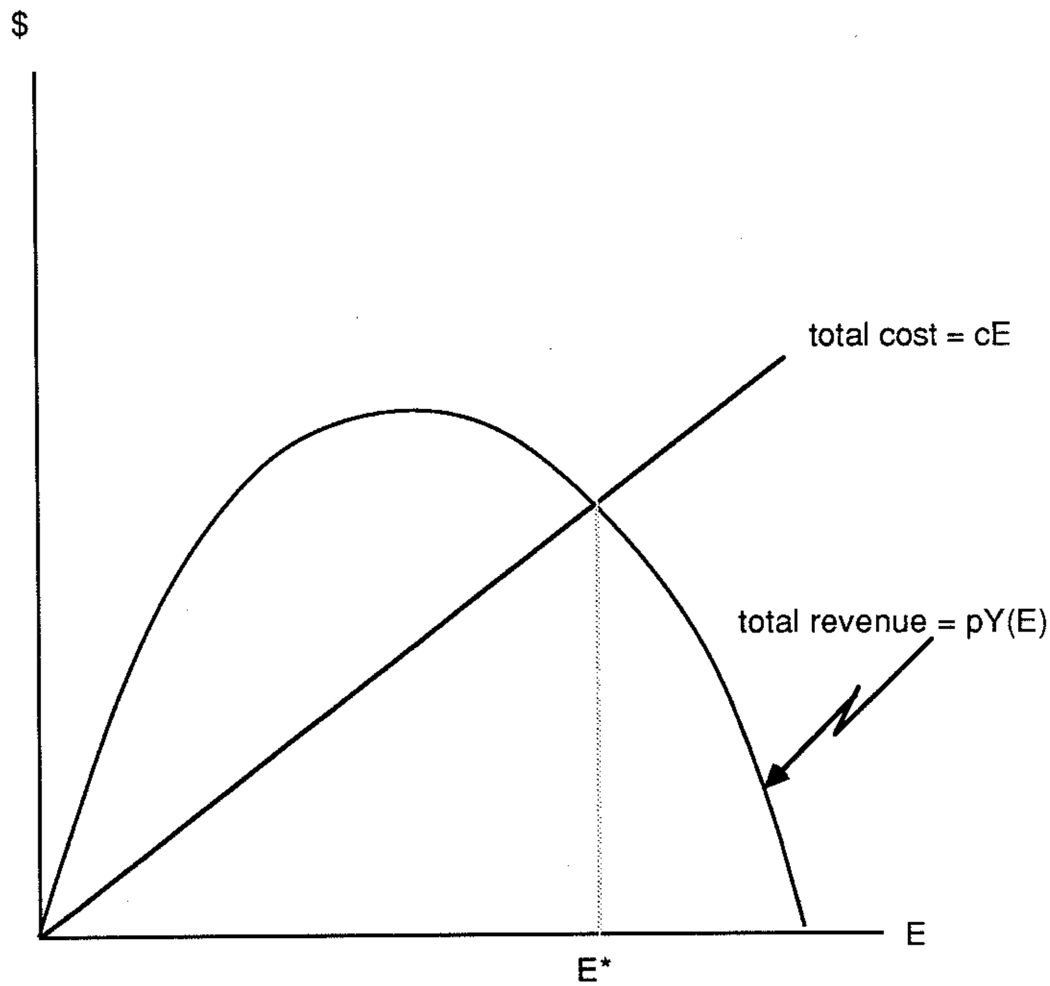

Figure 1: The Gordon-Schaefer "bionomic" equilibrium

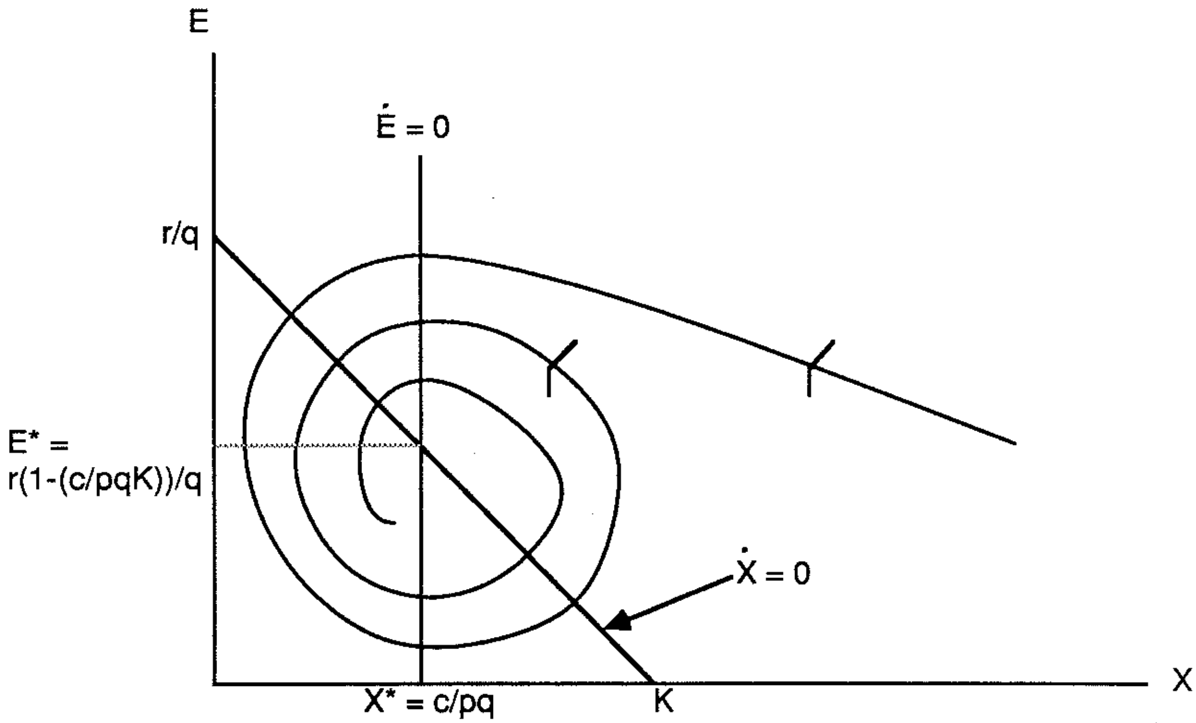

Figure 2: Theoretical approach of stocks and effort
to the equilibrium (X*, E*)

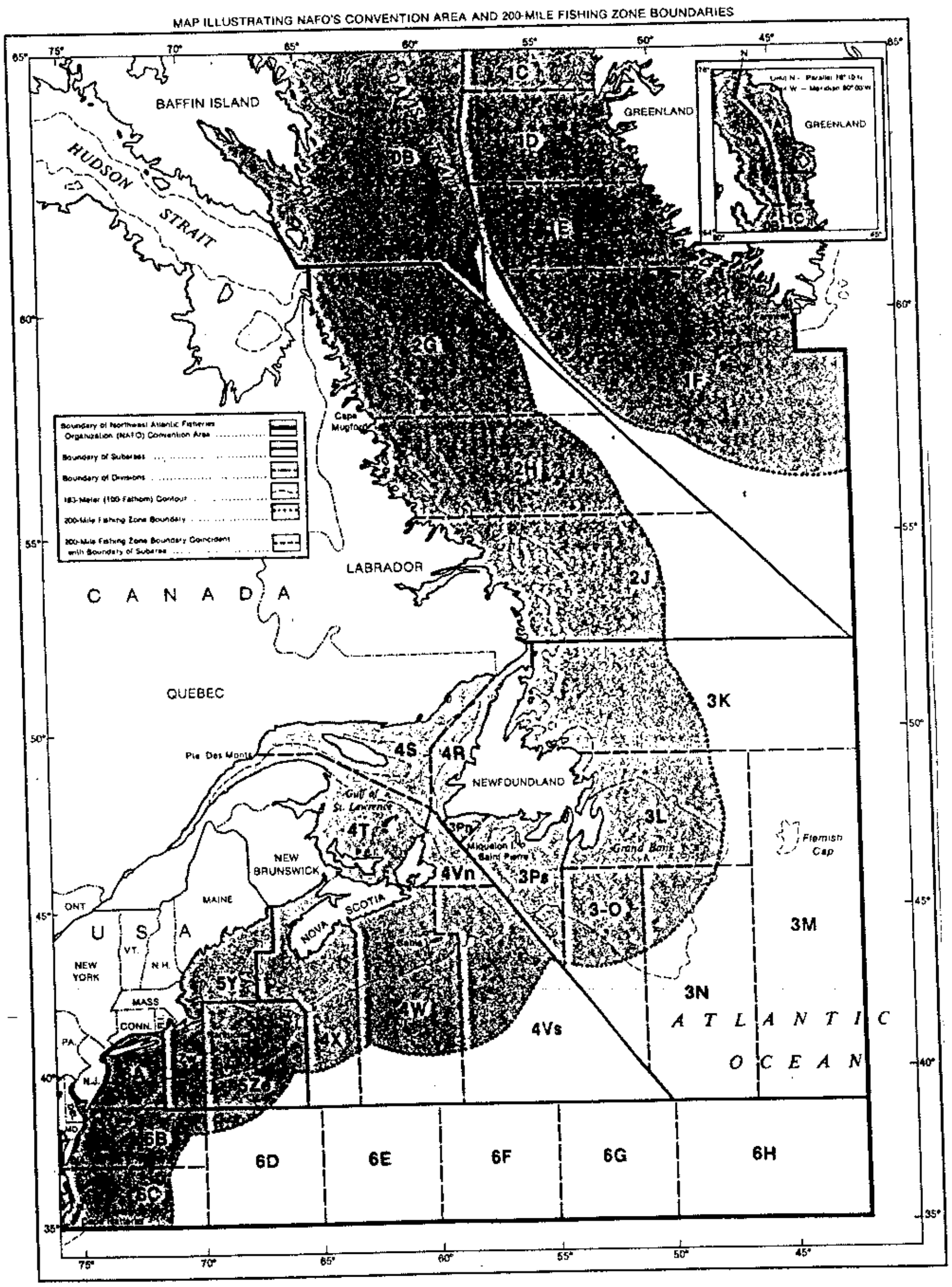

Figure 3: The Northwest Atlantic, showing major fishing areas

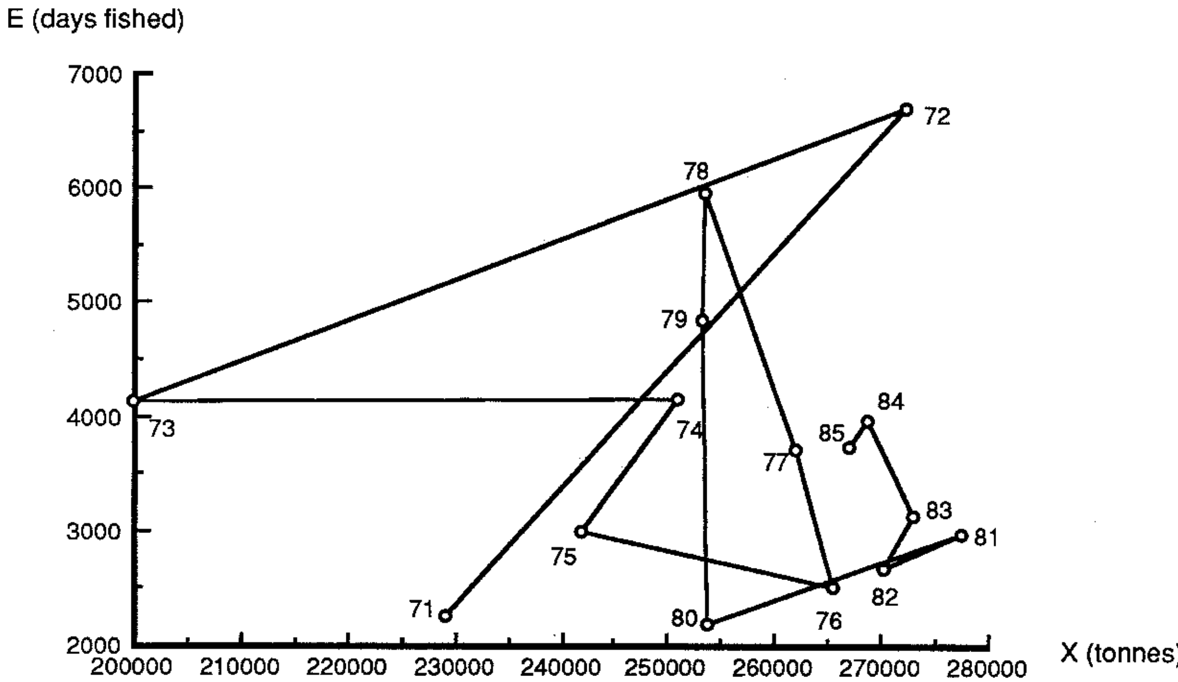

Figure 4: Relationship between observed fishing effort and estimated fish stocks (computed where G* = 0.956)

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