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**A Bioeconomic Model of the Harp Seal in the
Northwest Atlantic**

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A Bioeconomic Model of the Harp Seal in the Northwest Atlantic

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ABSTRACT

The biology and recent economic history of the harp seal (*Phoca groenlandica*) in the Northwest Atlantic is briefly reviewed. A simple dynamic model of pup production and seals one year of age or older (1+ seals) is constructed. Explicit solutions for the long-run, zero-harvest equilibrium are derived and a sufficient condition for stability is identified. The model is calibrated based on estimates of annual mortality for pups and 1+ seals, and on estimates of the 1+ population supporting net maximum pup production. The model is simulated for the period 1952-1980 and the results conform well to more complex models with multiple cohorts and multiple density-dependent parameters. Economic factors are introduced (pelt prices, marginal cost and discounting) and a dynamic optimization problem is posed which seeks to maximize the present value of net revenues (producer surplus). If net revenue only depends on the harvest of pups and 1+ seals it is possible to derive an explicit expression for the optimal stock of 1+ seals. If net revenue is separable and quadratic in harvest it is possible to derive explicit expressions for the optimal harvest of pups and 1+ seals. When annual net revenues are estimated for the bioeconomic parameters which might have prevailed in 1976 we obtain values ranging from \$1,288,959 to \$1,381,257 for the optimally managed fishery. While these values may be sufficiently low that some could justify stopping the harp seal hunt, the importance of the hunt both culturally and economically to the residents of Newfoundland, along with the value of fish consumed by an expanded seal population, may warrant seal harvest and management at some future date.

Key Words: harp seal, population dynamics, bioeconomics

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The order *Pinnipedia* or "fin-feet" contains three families: walruses, "true seals" and eared seals. Nearly all species of pinnipeds have been hunted by man for their meat, fur, oil or ivory. Perhaps the most controversial hunt in the long history of sealing involved the harvest of the harp seal (*Phoca groenlandica*) in the Gulf of St. Lawrence and off the coast of Newfoundland. From the late 1960s until 1982 various environmental and animal rights groups sought to stop the hunt on the grounds that the harp seal population had been seriously depleted and that the hunt was inhumane and unethical. A flood of protest letters was directed at the Canadian government who regulated the hunt and the Norwegian government whose sealing industry, by the late 1970s, purchased most of the pelts.

While the Canadian government never stopped the hunt, environmental and animal rights groups achieved a *de facto* moratorium when the European Community (EC) banned the import of many seal products in 1982. This resulted in a dramatic drop in prices for seal pelts and since 1983 there has been a significant reduction in the harvest of the harp seal in the northwest Atlantic.

Subsistence harvest continues from various coastal communities in Labrador, Newfoundland and Quebec and by the Inuit of the Canadian Arctic and the Eskimo of western Greenland.

This paper is "academic" in at least two senses. First, the ban on the import of seal products into the EC has made the determination of harvest quotas for pups and seals unnecessary. In an economic sense the harp seal is no longer a scarce resource.¹ Second, interest in the harp seal might now be limited to academicians; biologists, historians and resource economists, who are intrigued by the population dynamics of this particular animal, the colorful history of its commercial harvest (dating back to at least 1593) and how the harp seal *might* have been harvested *if* one sought to maximize some measure of economic value.

This paper contains sections that may be of interest to all of the above academic groups. The biology of the harp seal is briefly reviewed in the next section. This is followed by a simple model of population dynamics for pups and seals one year of age or older (1+ seals). The third section briefly discusses the economic history of the hunt in the post-World War II era. The model of population dynamics is then used to determine (simulate) the impact of harvesting on the seal population from 1952 to 1981. In the fourth section an

optimization problem is posed and solved to determine the optimal herd size and harvest of pups and 1+ seals if one sought to maximize the present value of net revenues from the hunt. The fifth and final section summarizes the paper, discusses the growing conflict between the expanding populations of certain marine mammals and presents an argument for active management as opposed to a policy of strict protection.

I. Biology

Much of the recent biological research on the harp seal would not have been possible without the seminal work of D. E. Sergeant and the sampling and monitoring of the kill which began in the early 1950s. This brief review on the biology of the harp seal draws heavily from Sergeant (1976).

The harp seal reproduces in three spatially separated populations on the pack ice off Newfoundland, Jan Mayen Island (off the east coast of Greenland) and in the White Sea (in the northwest Soviet Arctic). Statistical analysis of skull and body dimensions reveals that the western Atlantic population is more distinct from the two eastern populations than the eastern populations are from each other.

The Newfoundland population is divided into two sub-

populations. Approximately one-third of the total population reproduces on the drifting pack ice near the Magdalen Islands in the Gulf of St. Lawrence (see Figure 1). Pregnant females in the "Gulf" herd give birth to (whelp) a single pup beginning in late February. Whelping may continue until the third week in March.

The remainder of the population, approximately two-thirds of the total, winter on the pack ice off southern Labrador. This group is referred to as the "Front" herd. Females in this herd begin to whelp around March 8th on ice which has drifted south to the northern coast of Newfoundland. Lett *et al.* (1979) note that younger females whelp first and that older females may feed for two more days before hauling out to give birth. In the interim, the pack ice may have drifted 50 km further south, thus giving the false impression that the Front herd is further divided into two distinct sub-herds.

Newborn pups are covered with a long white fur and are called "whitecoats." After whelping, the Front herd drifts further south into Notre Dame Bay on the northeast coast of Newfoundland. This was an important area for hunting by landsmen from Newfoundland.

The pups are nursed for about nine days and then abandoned by their mothers. After lactation, but before leaving the whelping area,

the females are courted by males. Mating appears to be promiscuous.

After about 18 days the pups begin to shed their white fetal hair replacing it with a spotted juvenile pelage. The mottled pups are called "beaters." In addition to whitecoats and beaters, the landsmen, reaching the ice in small motor boats and vessels up to 20 meters in length, would kill immature seals called "bedlamers."

In early April the bedlamers and mature adults begin to molt. The Front herd has now moved northward, reaching the pack ice just east of the Strait of Belle Isle. Molting takes three to four weeks. Adult males and bedlamers molt first, followed by females who begin to molt by the third week of April. After molting, the adults and bedlamers begin their northward migration. The beaters, leaving later in May, will rejoin the bedlamers in mid-June in waters off the west coast of Greenland. The mature adults will spend the summer and early fall on feeding grounds further west in the Canadian Arctic.

The migration pattern of the Gulf herd is initially different. From the Magdalen Islands the beaters, bedlamers and adults move eastward toward Cabot Strait. Here they begin a northward migration up the west coast of Newfoundland where beaters and bedlamers were once subject to significant kill by landsmen. Molting might take place in the vicinity of Esquiman Channel, although the exact location is

highly variable for the Gulf herd. Molting will again take three to four weeks and the seals will usually leave the Gulf through the Strait of Belle Isle by the first week in May. As in the Front herd, mature adults will migrate to the Canadian Arctic while the beaters and bedlamers will migrate to the west coast of Greenland.

The precise reason for the summer separation of adults from beaters and bedlamers is not known. Sergeant (1973) speculates that it may relate to food preferences, with the younger seals feeding on capelin (*Mallotus villosus*) and shrimp (*Pandalus borealis*) while the adults concentrate on larger fish, such as cod, in the Canadian Arctic.

In late September the adults and juveniles begin their southward migration, moving in advance of the fresh forming Arctic ice. By December the first migrants have reached northern Newfoundland where the herd splits and the cycle repeats itself.

The majority of the pups and seals killed during the commercial hunt would be taken at the whelping and molting patches in March and April. Longlining from various ports in northern Newfoundland as well as a net fishery along the north shore of Quebec accounted for most of the remaining commercial harvest in the post-war era.

II. A Model of Population Dynamics

Let P_t denote the number of pups born in year t , X_t the number of seals one year of age or older (1+ seals), H_t the harvest of pups in year t and Y_t the harvest of 1+ seals. We will assume that the dynamics of pups and 1+ seals are governed by the following first-order difference equations

$$P_{t+1} = rX_t(1 - X_t/K) \quad (1)$$

$$X_{t+1} = (1 - M_0)(P_t - H_t) + (1 - M)(X_t - Y_t) \quad (2)$$

where pup production is assumed logistic, r being the "intrinsic rate" of pup production, K a density-dependent parameter affecting the relative rate of pup production, M_0 the first-year mortality of pups and M the annual mortality of 1+ seals. This is obviously a simple model. We will demonstrate that this model has certain advantages because it allows us to obtain analytical estimates of the harp seal population with zero harvest ($H_t = Y_t = 0$). This approximates the situation in the harp seal fishery since 1983. Furthermore, we can analytically assess the stability of the long-run, zero-harvest equilibrium. When economic

considerations are introduced, such as pelt price, marginal cost and the rate of discount, it is also possible to construct a bioeconomic model which can be optimized.

But do equations (1)-(2) constitute an acceptable model to a biologist? The model can be thought of as a simplification of a multiple cohort model where all age classes one year of age or older have been collapsed into a single 1+ cohort. This precludes consideration of many, possibly important, biological details. For example, it is thought that pup mortality, female sexual maturity (age at first parturition) and female pregnancy rates are density dependent (Lett *et al.* 1979). If this is the case, and if the age structure of the population is changing over time, then r , K , M_0 and M are likely to be changing and the collapsing of all nonpups into a single cohort results in a serious misspecification. If the simple model is used to simulate the 1+ population over some time horizon (given initial conditions, estimates of r , K , M_0 and M and the harvest of pups and 1+ seals) erroneous population projections may result. It is, of course, very difficult to determine the direction or extent of such errors because we do not know what the population was actually doing over the interval of interest. Thus, the best we can do is to compare the results of our simple model with those obtained from more complex models

(multiple-cohort or multiple-dependency). We will do this in the next section. But first, let us demonstrate the analytical attributes that make this model convenient to work with and relatively easy to calibrate.

In the zero-harvest environment in the long run, the populations of pups and 1+ seals would presumably approach a stationary state where $P = rX(1 - X/K)$ and $X = (1 - M_0)P + (1 - M)X$. These two equations can be solved for the equilibrium stocks of 1+ seals and pups yielding

$$\tilde{X} = \frac{K[r(1 - M_0) - M]}{r(1 - M_0)} \quad (3)$$

$$\tilde{P} = \frac{KM[r(1 - M_0) - M]}{r(1 - M_0)^2} \quad (4)$$

This equilibrium is shown in Figure 2 where (\tilde{X}, \tilde{P}) occurs at the intersection of $P = rX(1 - X/K)$ and $P = MX/(1 - M_0)$. At \tilde{X} there would be no "surplus production" and recruitment from the pup population would precisely offset the mortality of 1+ seals. A sufficient condition for the local stability of (\tilde{X}, \tilde{P}) is

$$2 > 1 + (1 - M_0)r - 2M > (1 - M) \quad (5)$$

This condition guarantees that the eigenvalues of the linearized system, evaluated at (\tilde{X}, \tilde{P}) , will be less than one in absolute value and thus (\tilde{X}, \tilde{P}) will be stable (see Edelstein-Keshet 1988).

From Figure 2 we note that $K > \tilde{X}$. The maximum possible population of pups is $P_{MAX} = rK/4$, which will occur at $X = K/2$. For populations of 1+ seals between zero and \tilde{X} there is a harvestable surplus of pups defined by

$$S = rX(1 - X/K) - MX/(1 - M_0) \quad (6)$$

This surplus is maximized at $X = \tilde{X}/2$ and is denoted by S_{MAX} in Figure 2.

Research in the 1970s and 1980s provides us with direct estimates of some of the biological parameters and indirect estimates for others. We seek estimates of r , K , M_0 and M . With initial conditions $(X_0$ and $P_0)$ and data on harvest $(H_t$ and $Y_t)$ we are then able to simulate the population of pups and 1+ seals. Independent point estimates for pups or 1+ seals during the interval of simulation and

comparison of our results with simulations from other models will be used as a means of "validating" the estimates of r , K , M_0 and M .

Roff and Bowen (1983) estimate the annual natural mortality of 1+ seals to be $M = 0.075$ with a 95% confidence interval of approximately 0.06 to 0.09. They then consider a multiple cohort model for two cases; where $M_0 = M$ and where $M_0 = 3M$. We will report on simulations where M_0 is varied from 0.075 to 0.225. In the optimization analysis we have chosen to adopt the more conservative position that $M_0 = 3M$. It would seem plausible that pup mortality should be greater than the mortality of bedlamers and adults.

No direct estimates of r and K exist. There are, however, indirect estimates of \tilde{X} . Lett and Benjaminsen (1977) estimate that the population of 1+ seals that would support maximum (net) sustainable yield (MSY) of pups was 1.6×10^6 . Lett *et al.* (1979), in a model with three density-dependent parameters, revise the estimate of the population supporting MSY downward to 1.4×10^6 .

In our model the population of 1+ seals that sustains the highest net pup production corresponds to $\tilde{X}/2$. The above estimates would imply that \tilde{X} is between 2.8×10^6 and 3.2×10^6 . Suppose $\tilde{X} = 3.0 \times 10^6$. Since $\tilde{P} = M\tilde{X}/(1 - M_0)$ we can calculate $\tilde{P} = 290,322$ pups when $M = 0.075$ and $M_0 = 0.225$.

Finally, suppose $\tilde{X}/K = 0.75$. For $\tilde{X} = 3.0 \times 10^6$ this would imply $K = 4.0 \times 10^6$. The equation for pup production, evaluated at (\tilde{X}, \tilde{P}) , implies $r = \tilde{P}/[\tilde{X}(1 - \tilde{X}/K)] = 0.387$. After sensitivity analysis (via simulation) we opted for $r = 0.4$. Our basic parameter set thus becomes $r = 0.4$, $K = 4.0 \times 10^6$, $M_0 = 0.225$ and $M = 0.075$.

The value for r is similar to the value estimated by Wilen (1976) in his study of the North Pacific fur seal. This value is considerably larger than estimates of the intrinsic growth rate for baleen whales which frequently range from 0.05 to 0.13.

For these values of r , K , M_0 and M equations (3) and (4) imply that $\tilde{X} = 3,032,258$ and $\tilde{P} = 293,444$. This "pristine" equilibrium is locally stable since condition (5) is satisfied ($2 > 1.16 > 0.925$). One test of the predictive suitability of this model would be to see if the unharvested population of harp seals stabilizes in the vicinity of (\tilde{X}, \tilde{P}) .

III. Economics and Population Dynamics After World War II

The harp seal in the western north Atlantic has been harvested intensively for almost 200 years (Busch 1985). The history of the fishery in the eighteenth, nineteenth and early twentieth century is a colorful and culturally important legacy for the people of

Newfoundland, Quebec and Labrador. Space precludes a discussion here and we will restrict our economic history to the post-World War II period.

The Second World War afforded the harp seal population a respite from commercial harvest. In 1942 only two large vessels went to the ice and in 1943 no vessels participated in the fishery as the war took priority for both men and ships. In 1944 only one vessel, the 457 ton, wooden-hulled *Eagle* went to the ice. Most of the vessels in the pre-war fleet must have been converted to other uses or deteriorated beyond repair. With the end of the war, however, the industry was poised for an infusion of new capital.

In 1945 five diesel vessels with a total tonnage less than the *Eagle* entered the fishery. These vessels were around 65 feet in length and used longline techniques to harvest seals in open water. Crew size varied between 40 and 50 men, a far cry from the average of 150 men aboard the larger steam vessels of the early twentieth century.

In 1947 the steamer *Sable Island* joined the *Eagle* and a diesel fleet which had grown to 15 vessels; several making two or three trips to the ice. Crews totalling 628 men harvested 97,535 seals for an average share of \$186 per man (Busch 1985). With a

relatively abundant seal population the industry entered the post-war era with bright prospects.

Prior to 1938 Norwegian sealers had harvested harp and hooded seals (*Cystophora cristata*) on the "west ice" off Jan Mayen and on the "east ice" west of Novaja Semlja, south to the White Sea. Access to seals on the east ice had been restricted as the USSR sought to expand its sealing industry in its post-revolutionary period. In 1938 Norway was encouraging its sealing fleet to participate in the hunt in the Gulf of St. Lawrence and off Newfoundland. The German occupation of Norway in April of 1940 brought a halt to large scale commercial sealing and several vessels who were hunting off Newfoundland chose to remain in Canada.

It took Norway longer to rebuild its sealing fleet after the war. In 1946 and 1947 only one vessel participated in the Newfoundland hunt. In 1948 four vessels were equipped for sealing off Newfoundland and in 1950 the number had increased to 14.

Table 1 shows the Canadian and Norwegian harvest of harp seals (pups plus 1+ seals) for the period 1952-1980. Norway generally sent its larger steel-hulled ships of 400 to 600 gross registered tons, while smaller boats went to the west and east ice. Norwegian harvest exceeded Canadian harvest for the first time in 1959 and three times

during the 1960s. The maximum number of vessels sent to Newfoundland was 16 in 1960 and again in 1964.

Table 2 reports the average prices for whitecoats and adult seals for the same period. These are average prices because the pelts of whitecoats, "ragged-jackets" (a stage in the molt of a pup between the whitecoat and the beater), beaters, bedlamers and adults are graded by both quality and color.

We observe a reversal in the average prices. The whitecoats are worth almost twice as much during the 1950s, but were generally priced lower than adults after 1960. We will make use of the prices in 1976 in an optimization model in the next section.

In the 1970s, Norwegian sealing went into decline. To a large extent this was the result of catch quotas imposed by the Canadian government beginning in 1971. The Norwegian quota was initially set at 100,000 seals but it was gradually reduced during the 1970s as symptoms of over-exploitation became evident in the fishery. By 1982 the quota had been reduced to 24,000 seals (see Table 1).

With the rapid expansion by both Canada and Norway after the war a fortuitous decision was made to learn more about the harp seal and to actively monitor the hunt. In particular, samples were taken at various times and different locations during the hunting season to

learn something about the age composition of the harvest. This allowed researchers to develop time series data on the harvest of pups and 1+ seals, data which is appropriate to the model constructed in the previous section of this paper.

The fourth and fifth columns of Table 3 present estimates of the harvest of pups (H_t) and 1+ seals (Y_t) from 1952 to 1980 as reported in Bowen (1982). These data appear in lines 50-80 (H_t) and 120-150 (Y_t) of the program (written in BASIC) which appears in Table 4. The initial conditions for the stock of 1+ seals ($X_0 = 2,530,208$) and pups ($P_0 = 566,323$) in 1952 come from Lett *et al.* (1979). When equations (1)-(2) are simulated from (X_0, P_0) for our basic parameter set ($K = 4.0 \times 10^6$, $M = 0.075$, $M_0 = 0.225$, and $r = 0.4$) we obtain trajectories for X_t and P_t given in the second and third columns of Table 3. A plot of these values is shown in Figure 3. The stock of 1+ seals declines from $X_0 = 2,530,208$, reaching a low of 1,017,593 in 1972 and then increasing to 1,298,529 in 1981. The production of pups dips initially from $P_0 = 566,323$, increases slightly from 371,887 in 1953 to 399,389 in 1958, and then follows a nearly monotonic decline to 303,487 in 1973. From 1973 pup production slowly increases to 347,478 in 1981. Figure 3 shows a relatively

constant level for pup production after 1952. While this is a function of the scaling in Figure 3, pup production is remarkably stable, ranging between 303,487 and 399,389.

These trajectories do not display the rapid increase in the population of 1+ seals and pups which Roff and Bowen (1983) obtain after 1972 in their multiple cohort model with eight age groups.

There is no density dependence in their model and one would expect it to begin to grow exponentially if harvest were less than surplus production and the dominant eigenvalue were greater than one.

Lett *et al.* (1979), in a model with three density-dependent factors, simulate the population of pups and 1+ seals from the same initial conditions for the period 1952 to 1977 and then simulate their model (with hunting restricted to aboriginal harvest), in order to infer the stationary state of the "relatively pristine" system. In their sequential population analysis the population of 1+ seals declined from 2.5 million in 1952 to 1.0 million in 1972 and increased to 1.3 million in 1977. Our estimate of the 1+ seal population in 1977 is approximately 1.2 million. At the time of their research Lett *et al.* did not have access to the revised estimates of harvest by Bowen (1982) which slightly increased the harvests of pups and 1+ seals. We suspect that if Lett *et al.* were to rerun their sequential population analysis

with Bowen's revised harvest estimates our simulations would be nearly identical. Our analytical results for zero or very low aboriginal harvest imply $\bar{X} = 3,032,258$ which is lower than the apparent steady state of about 4.0 million 1+ seals when all three density-dependent factors are at work.

Table 5 contains additional simulation results. When K is increased to 5.0×10^6 and r is reduced to 0.35 we obtain similar results for the period 1952-1981, but the long-run, zero-harvest equilibrium is increased. Note, that the population of 1.18 million 1+ seals in 1972 grows to 1.33 million in 1977. With M and M_0 unchanged, equation (3) implies a long-run, zero-harvest population of $\bar{X} = 3,617,511$.

Reduction of M_0 has the obvious effect of increasing pup survival and therefore the number of pups or 1+ seals which could be harvested without reducing the population. The last two columns in Table 5 give the numerical results when M_0 is reduced from 0.225 to 0.075 ($= M$). In our model the reduction in M_0 would appear to have a more significant effect than in the multiple cohort model of Roff and Bowen (1983). When compared to our base case the minimum 1+ population is 1.45 million in 1971 (versus 1.01 million in 1972) rising to 1.88 million in 1977 (versus 1.19 million). The long-run, zero-

harvest steady state is $\tilde{X} = 3,189,189$.

We conclude from the above simulations that our model, while simple, is a reasonable description of harp seal dynamics. It was relatively easy to incorporate existing information on mortality rates (M and M_0) and to identify plausible values for r and K which resulted in simulations similar to those obtained by other researchers using more complex models. We make no claim that our basic parameter set is "correct" or unchanging. We would argue that the model is easy to recalibrate when new stock estimates become available, particularly new estimates for \tilde{X} .

Our model permitted analytic solutions for the long-run, zero-harvest equilibrium (\tilde{X}, \tilde{P}) , and a sufficient condition for its stability. The model's simplicity will permit use to introduce economic factors such as price, cost and the rate of discount in an attempt to maximize some measure of economic value from the resource. While there are many potential management objectives we will focus on the maximization of the present value of net revenue or producer surplus.

IV. Optimization

In an attempt to determine the importance of the harp seal

fishery, Dunn (1977) made some value-added calculations for both the primary (harvesting) and secondary (processing) sectors in Newfoundland and Nova Scotia. Total value-added for both sectors in 1976 was estimated to be about \$5.5 million. A Royal Commission (1986) report to the Canadian government estimated the net benefits in 1982 to be \$3.2 million. This included the value of oil and meat as well as skins.

In this section we will provide a rough estimate of net value for the harvesting sector in 1976 if the fishery had been "optimally" managed. In the process we will calculate the optimal stock of 1+ seals, X^* , pup production, P^* , pup harvest, H^* , and harvest of 1+ seals, Y^* . Our definition of an optimally managed fishery is one which maximizes the present value of net revenues (producer surplus) to the harvest sector. This might be regarded as a narrow criterion, but we will see that it contains, as a special case, the maximum sustainable yield policy frequently proposed by biologists. We will also comment on how the objective function might be modified if other factors are deemed to be relevant.

Suppose the net revenues to the harvest sector in year t are a function of the rate of harvest of both pups and 1+ seals and are calculated according to the function $N_t = N(H_t, Y_t)$. The problem of

maximizing the present value of net revenues may be stated mathematically as

$$\begin{aligned} &\text{maximize } \sum_{t=0}^{\infty} \rho^t N(H_t, Y_t) \\ &\text{subject to } P_{t+1} = rX_t(1 - X_t/K) \\ &\quad X_{t+1} = (1 - M_0)(P_t - H_t) + (1 - M)(X_t - Y_t) \end{aligned}$$

where $\rho = 1/(1 + \delta)$ is the discount factor and δ is the annual rate of discount. This is a dynamic optimization problem with two state variables (X_t and P_t) and two control variables (H_t and Y_t). The Lagrangian expression for this problem may be written

$$\begin{aligned} L = \sum_{t=0}^{\infty} \rho^t \{ &N(H_t, Y_t) + \rho \lambda_{t+1} [rX_t(1 - X_t/K) - P_{t+1}] \\ &+ \rho \mu_{t+1} [(1 - M_0)(P_t - H_t) + (1 - M)(X_t - Y_t) - X_{t+1}] \} \end{aligned}$$

where λ_{t+1} and μ_{t+1} are Lagrange multipliers equal to the "shadow price" or marginal value of an additional pup and 1+ seal in period $t+1$, respectively. First order necessary conditions for an interior solution include

$$N_H(\bullet) = (1 - M_0)\rho\mu_{t+1} \quad (7)$$

$$N_Y(\bullet) = (1 - M)\rho\mu_{t+1} \quad (8)$$

$$\lambda_t = (1 - M_0)\rho\mu_{t+1} \quad (9)$$

$$\mu_t = \rho\lambda_{t+1}r(1 - 2X_t/K) + (1 - M)\rho\mu_{t+1} \quad (10)$$

where $N_H(\bullet) = \partial N(\bullet)/\partial H_t$ and $N_Y(\bullet) = \partial N(\bullet)/\partial Y_t$. In addition to equations (7) - (10) the first order conditions also include equations (1) and (2). If the net revenue function is concave in H_t and Y_t , then the first order conditions are necessary and sufficient and there will exist a unique steady-state optimum. When $N(\bullet)$ is only a function of H_t and Y_t we can derive an explicit expression for the optimal stock of 1+ seals as follows.

In steady state equations (7) and (8) imply

$$\frac{N_H}{(1 - M_0)} = \frac{N_Y}{(1 - M)} \quad (11)$$

Equation (9) requires $\lambda = (1 - M_0)\rho\mu$ and equation (10) can be shown

to imply $\rho\mu[(1 - M) - (1 + \delta)] = -\rho\lambda r(1 - 2X/K)$. Substituting the expression for λ and canceling $\mu > 0$ from each side leaves a linear expression in X which can be solved for

$$X^* = \frac{K}{2} \left[\frac{r(1 - M_0) - (M + \delta)(1 + \delta)}{r(1 - M_0)} \right] \quad (12)$$

Note, the optimal population of 1+ seals depends on the biological parameters K , M , M_0 , and r and on the annual discount rate δ .

Inspection will reveal that as $\delta \rightarrow 0$, $X^* \rightarrow \bar{X}/2$, a result consistent with earlier bioeconomic models [for example, Clark (1976)]. Knowing X^* we can calculate pup production as to $P^* = rX^*(1 - X^*/K)$. Equation (2), evaluated at (X^*, P^*) becomes an equation in H and Y ; specifically

$$(1 - M_0)H + (1 - M)Y = (1 - M_0)P^* - MX^* \quad (13)$$

Given a functional form for $N(H, Y)$ equations (11) and (13) might be solved for the optimal harvest rates H^* , for pups, and Y^* , for 1+ seals.

When the net revenue function is a separable-quadratic, taking the form

$$N(H_t, Y_t) = aH_t - bH_t^2 + cY_t - dY_t^2 \quad (14)$$

equation (11) implies

$$2(1 - M)bH - 2(1 - M_0)dY = a(1 - M) - c(1 - M_0) \quad (15)$$

Equations (13) and (15) comprise a two-equation system in H and Y and may be solved yielding

$$H^* = \frac{[a(1 - M) - c(1 - M_0)](1 - M) + [(1 - M_0)P^* - MX^*]2(1 - M_0)d}{2(1 - M)^2 b + 2(1 - M_0)^2 d} \quad (16)$$

and

$$Y^* = \frac{[(1 - M_0)P^* - MX^*]2(1 - M)b - [a(1 - M) - c(1 - M_0)](1 - M_0)}{2(1 - M)^2 b + 2(1 - M_0)^2 d} \quad (17)$$

The separable-quadratic form might be appropriate if (a) the price for pelts were constant and (b) the marginal costs of harvesting an additional pup and 1+ seal were linear and independent. The first assumption seems plausible and the shape of the marginal cost

schedule for landmen and small vessels may well be linear, given the small amounts of fixed capital. For larger vessels, however, marginal cost is likely to have been nonlinear, exhibiting an initial range of decreasing average and marginal cost, as a result of significant fixed costs.

Figure 4 depicts producer surplus when demand and marginal cost are linear in harvest. If demand were perfectly elastic at P_H and marginal cost were given by $MC_H = m + nH$ then producer surplus is quadratic in H and the coefficient a in the separable-quadratic net revenue function becomes $a = P_H - m$, while $b = n/2$.

For the year 1976 we adopt an average price for whitecoats and beaters of $P_H = \$17$ (see Table 2). If we assume an intercept of $m_H = \$2$ then $a = \$15$. (All values are in 1976 Canadian dollars.) If the marginal cost line intercepted P_H at 137,000 (the approximate harvest in 1976), this would imply $b = n_H/2 = 5.47 \times 10^{-5}$.

For 1+ seals we adopt the values $P_Y = \$20$, $m_Y = \$1$ (implying $c = \$19$), and with marginal cost intercepting P_Y at 37,600 we calculate $d = 2.53 \times 10^{-4}$. With these values for a , b , c and d and the previous values for K , M , M_0 and r , Table 6 shows the values for X^* , P^* , H^* and Y^* when δ varies between 0.00 and 0.10.

When $\delta = 0$ we note that $X^* = \tilde{X}/2 = 1,516,129$ seals producing 376,586 pups of which 173,760 are harvested. Of the 202,826 pups that escape harvest another 45,635 die of natural causes leaving 157,191 to join the class of 1+ seals next year. This precisely offsets the 47,007 1+ seals which had been harvested the previous year and the 110,184 that died of natural causes. The net value to the harvest sector is $N^* = \$1,288,959$ (a rough estimate in 1976 dollars). This value increases to \$1,381,257 when $\delta = 0.1$. (The present value of the fishery declines, of course.) This is less than half of the \$3 million in value-added by the harvesting sector as estimated by Dunn (1977). In our analysis, however, total producer surplus is the sum of two triangular areas similar to that shown in Figure 4, as opposed to the rectangular areas implied by standard value-added calculations.

The net revenue values in Table 6 are also less than the 1982 estimate of \$3.2 million obtained by the Royal Commission (1986). The average price in 1982 was \$24 per pelt (as opposed to the 1976 prices of \$17 and \$20 for pups and 1+ seals, respectively) and the Commission had a more detailed accounting of costs by harvest technique (landsmen, longliner, or large vessel), as well as transport and processing costs.

As the discount rate increases to $\delta = 0.1$ we see a monotonic

decline in stock and harvest variables. Annual net revenue increases in this range, thus, total cost must be declining more rapidly than revenue. When $\delta = 0.1$ the optimal stock is 758,064 which is less than the low estimate obtained in the simulations of the previous section. Pup production declines to 245,759 of which 129,825 are harvested.

Because net revenues were only a function of harvest (H_t and Y_t), there was no "marginal stock effect" and the optimal population of 1+ seals had to be less than $\tilde{X}/2$ for $\delta > 0$. If harvest costs are stock dependent; that is, they depend on the number of pups and the size of the 1+ population, then the optimal stock may exceed $\tilde{X}/2$. In this case net revenue would be modified to $N_t = N(H_t, Y_t, P_t, X_t)$. It is frequently maintained that search costs are stock dependent. Prior to 1970 planes and helicopters had been used in locating the whelping and molting patches. Even without them the cost of search may not be large, thus, the marginal stock effect is likely to be small.²

Another way in which the stock of pups and 1+ seals might directly enter the net benefit function would be if society placed a direct value on their existence. Existence value will typically increase the size of the optimal stock.³ The controversy over the hunt in the 1970s and 1980s reflected, in part, a concern over the depletion of

the harp seal population as a result of the large harvests taken during the 1950s and 1960s. Based on our simulations (and in the opinion of most seal biologists) the harp seal was never threatened with extinction. The arguments by protesters who sought to ban the hunt outright must ultimately rest on ethical considerations that go beyond the scientific role of resource economics.

V. Conclusions

This paper has attempted to review the recent and controversial history of the harp seal fishery in the Northwest Atlantic. The biology of the harp seal was discussed and a model describing the dynamics of pup production and 1+ seals was constructed. While simple, this model was capable of calibration so as to produce results comparable to the more complex multiple cohort and multiple density-dependent models when simulated from 1952 to 1981.

The simplicity of the model might be viewed as an advantage. It permitted analytic expressions for the long-run, zero-harvest steady state that the current fishery appears headed toward, given the ban on the importation of various seal products to the EC. A sufficient condition for the stability of the long-run, zero-harvest equilibrium was

also identified. The model exhibited a robust stability for a plausible set of biological parameters and when simulating the population of 1+ seals using estimates of harvest from 1952 to 1980. This is consistent with the commercial history of a resource which has been intensively harvested for almost 200 years.

The equations of dynamics were then taken as constraints in a dynamic optimization problem which sought to maximize the present value of net revenue (producer surplus). When net revenue depended only on the harvest of pups and 1+ seals it was possible to derive an explicit expression for the optimal stock of 1+ seals. For a zero discount rate the optimal stock was equivalent to the population level that would maximize the net production of pups, a result consistent with single-state bioeconomic models.

With net revenues quadratic and separable in harvest, explicit expressions for the optimal harvest of pups and 1+ seals could also be derived. For very rough estimates of price and the marginal cost of harvest it was possible to estimate the annual net revenues in the harvest sector in 1976. These estimates were less than half of the earlier estimate of \$3 million obtained through value-added calculations.

Given the low estimates of annual net revenue from an

optimally managed fishery it is tempting to conclude that the cost of the economic moratorium is small. Several factors would caution against such a simple conclusion. First, the importance of the seal fishery, both culturally and economically, to the residents of Newfoundland is probably impossible to reduce to a dollar metric. Second, as the seal population moves toward a zero-harvest equilibrium (estimated for our basic parameter set at 3,032,258 seals) it will come into greater competition with man for various fish stocks in the Northwest Atlantic. Bowen (1985) identifies caplin (*Mallotus villosus*), Atlantic cod (*Gadus morhua*), herring (*Clupea harengus*) and shrimp (*Pandalus borealis*) as among the species of value to commercial fishermen.

A similar controversy has developed in the North Pacific where the northern fur seal (*Callorhinus ursinus*) has recovered to near carrying capacity on the Pribilofs Islands and is now thought to be consuming significant quantities of Alaskan pollock (Swartzman and Haar 1985). The health of the herd (in terms of parasitic infestation and outward appearance) also declines when food and space limits are reached.

The sea otter (*Enhydra lutris*), while nowhere near the numbers that were found off the California coast during the eighteenth

and nineteenth centuries, has raised the ire of abalone divers and other shellfishermen on the grounds that even a small population can significantly reduce local shellfish populations (Estes and VanBlarcom 1985).

It is currently difficult to evaluate the extent to which marine mammals such as the the sea otter and the various pinnipeds will ultimately compete for fish and shellfish resources that are valued by man. A larger model, with dynamic equations for cod and other species would need to be constructed to estimate the impact of a larger harp seal population. The protection afforded these marine mammals is undoubtedly the result of what many view to be the bloody and brutal history of their exploitation and the fact that some pinniped populations have yet to recover from the harvest by sealers during the eighteenth and nineteenth centuries. In an evolutionary sense, however, it is not possible to go back to the pre-exploitation state that existed in the eighteenth or nineteenth century. The marine environment is no longer the same, neither physically nor in the composition or size of species it supports. Man must now take responsibility for the environment which he has altered. While it would be indefensible to deliberately extinguish another species, the humane management of marine mammals that are valuable or whose expanded

populations impose significant opportunity costs, should not be dismissed as out of hand.

Endnotes

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¹Some view the harp seal population as a nuisance or pest. During the controversy in the 1970s, fishermen from Newfoundland formed a group that called themselves "Codpeace," defending the harvest of harp seals as a way of managing the predation (thus competition) for cod. We will return to this potential conflict between man and seal in the last section of this paper.

²As with schooling fish, the fact that seals establish rookeries or haul out to whelp or molt at certain locales in certain seasons makes them relatively easy to locate even as the overall population declines. For a discussion of this behavioral attribute and its implications for the marginal stock effect in the North Sea herring fishery, see Bjørndal (1988).

³For a discussion of existence value and its effect on the optimal level for escapement and harvest in the bowhead whale fishery see Conrad (1989).

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Figure 1. Location Map of the Harp Seal Fishery in the Northwest Atlantic

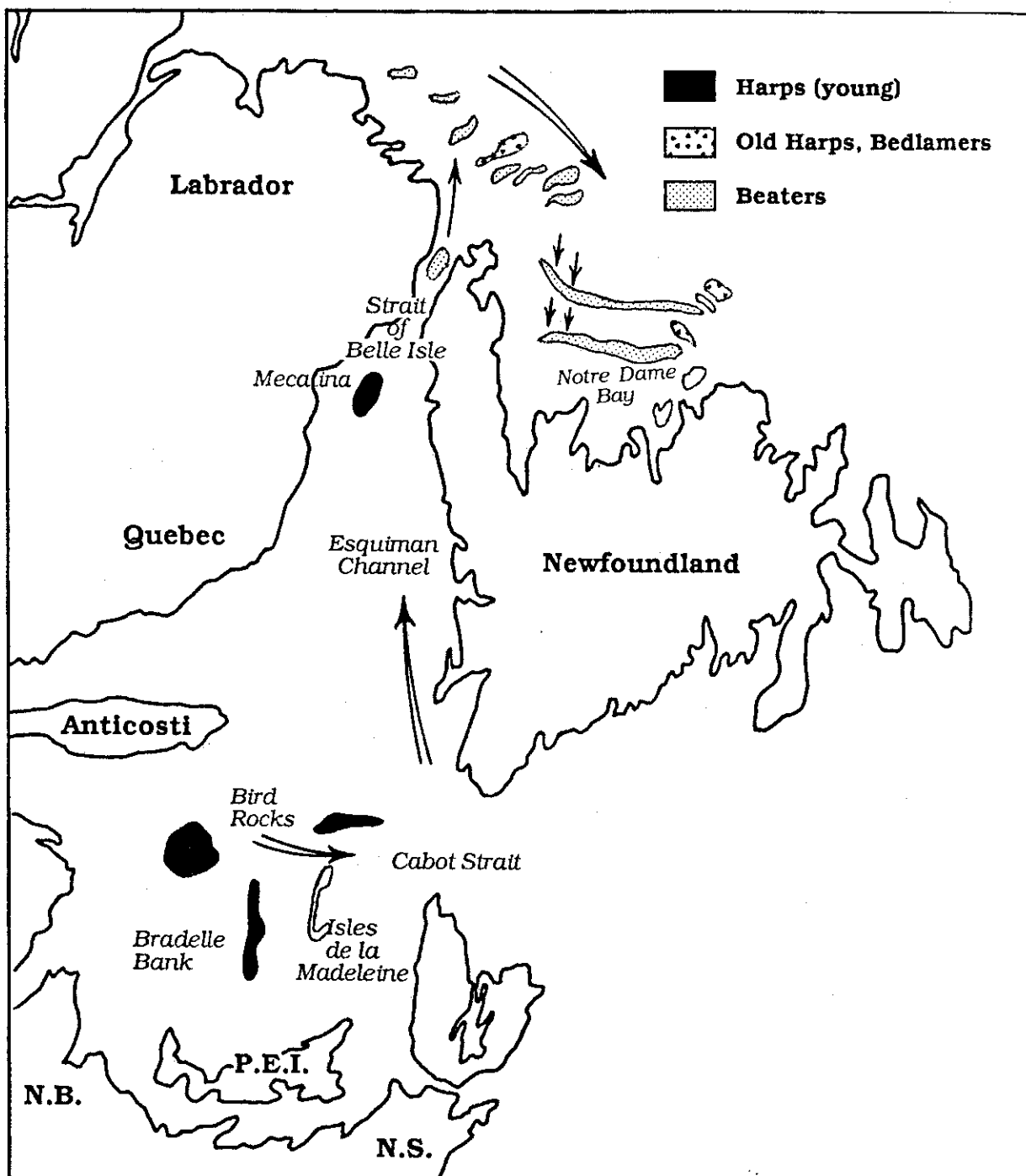


Figure 2. Equilibrium in the Harp Seal Model

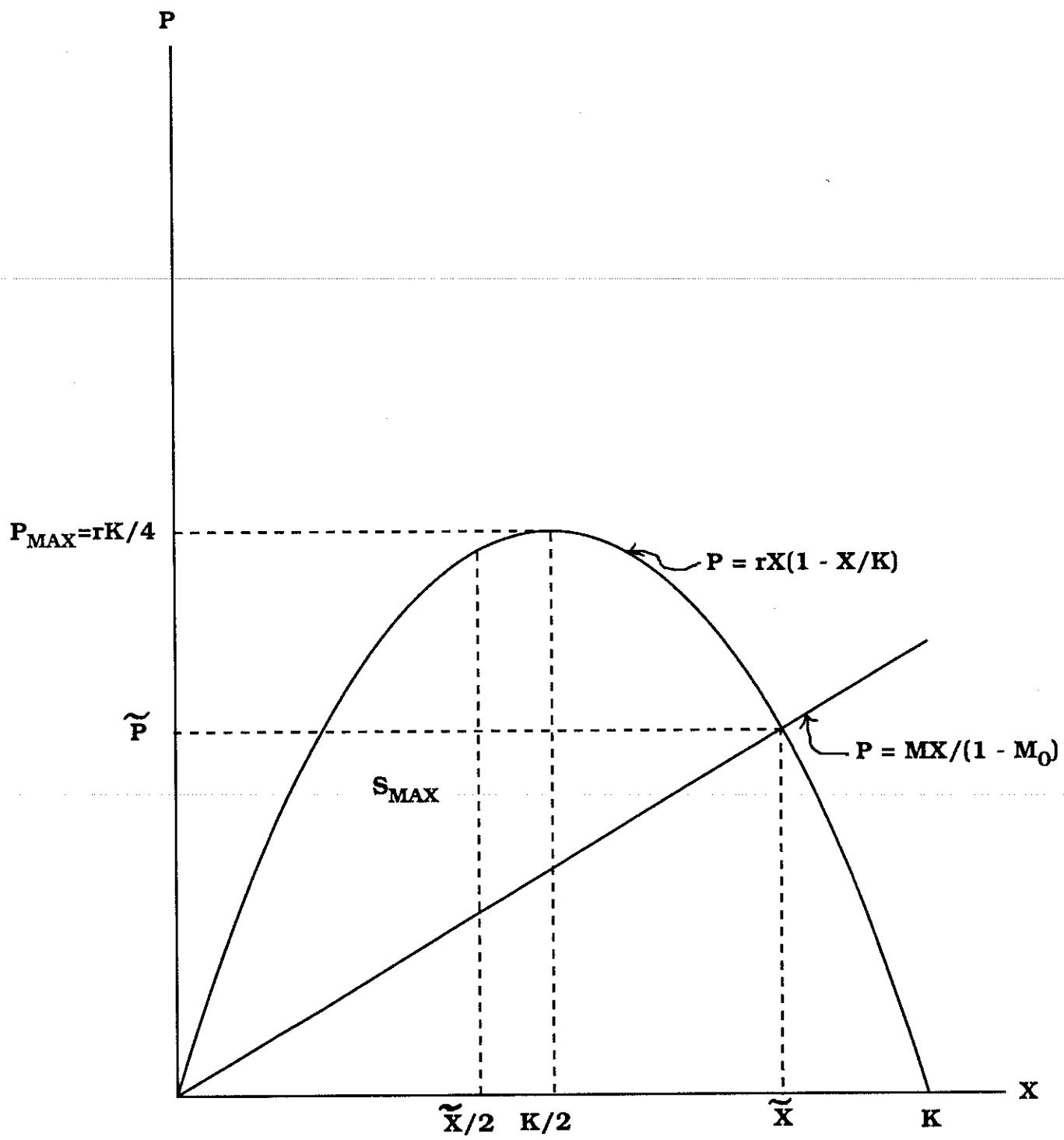


Figure 3. A Plot of Pups and 1+ Seals from Table 3

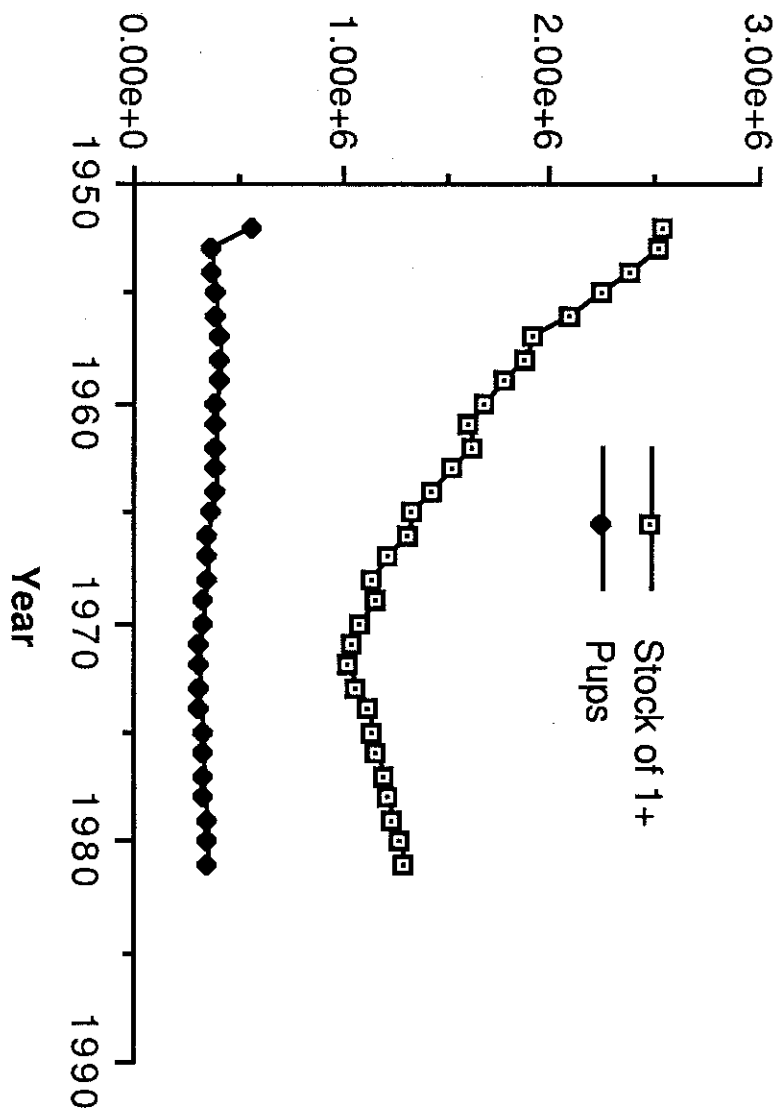
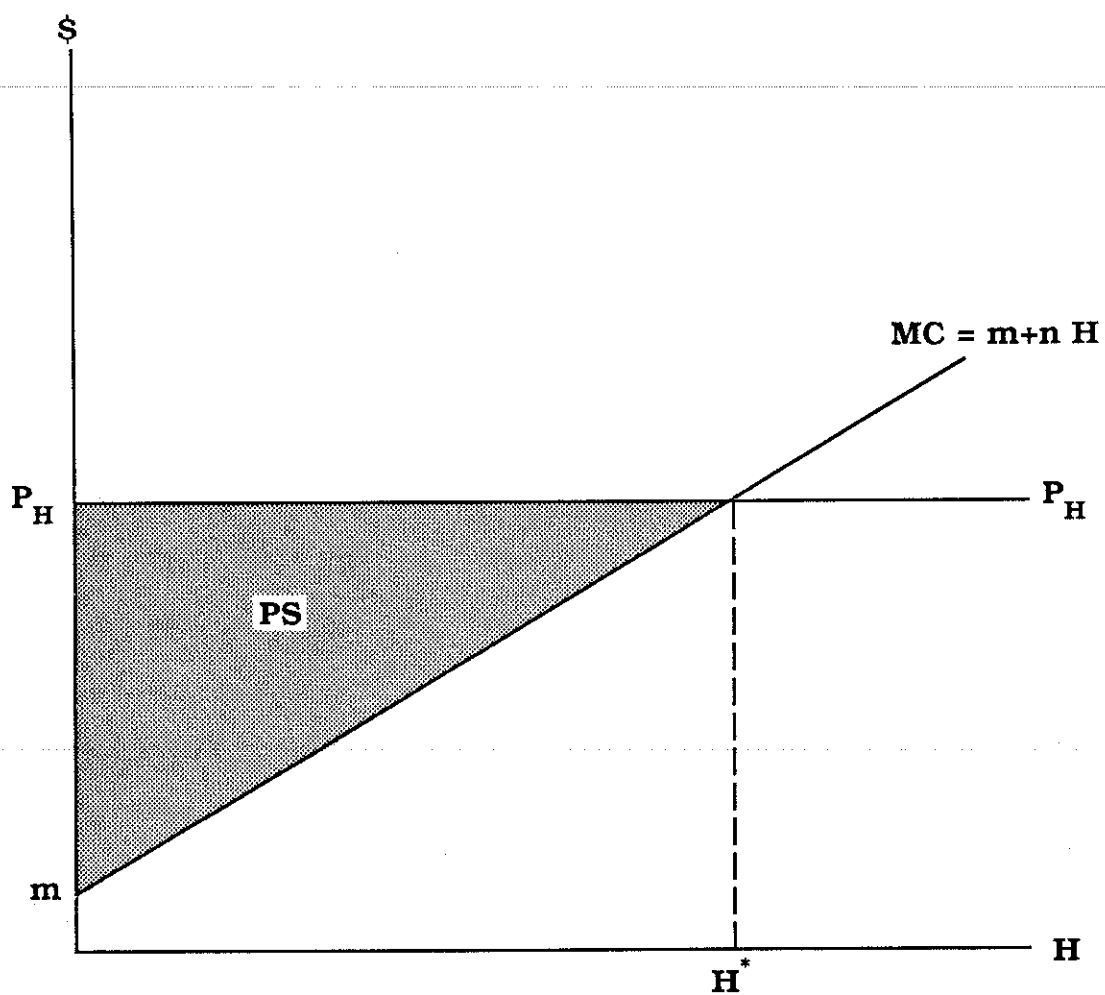


Figure 4. Producer Surplus in the Harvest of Harp Seal Pups:
 $PS = (P_H - m)H^* - (n/2)(H^*)^2 = aH^* - b(H^*)^2$



**Table 1. Canadian and Norwegian Harvest of Harp Seals
(Pups and 1+ Seals) and the Norwegian Large Vessel Fleet
1952 - 1980**

Year	Canadian Harvest	Norwegian Harvest	Norwegian Vessels
1952	195,255	130,037	11
1953	194,040	97,090	7
1954	157,417	127,696	9
1955	185,158	165,450	10
1956	212,149	189,943	10
1957	137,029	123,057	15
1958	184,387	131,891	13
1959	113,048	217,715	13
1960	165,756	129,357	16
1961	89,596	111,940	13
1962	175,660	154,507	13
1963	215,670	138,159	13
1964	163,920	188,667	16
1965	178,131	67,157	14
1966	181,275	150,677	13
1967	153,043	187,312	15
1968	113,246	88,260	10
1969	179,565	117,414	14
1970	150,259	115,198	13
1971	139,651	98,639	10
1972	84,319	53,300	11
1973	76,488	58,290	10
1974	100,833	55,585	8
1975	121,986	60,161	8
1976	129,903	44,683	6
1977	131,241	35,624	5
1978	159,753	16,254	4
1979	154,299	20,288	4
1980	160,928	20,213	3

Sources: Norwegian harvest from Director General of Fisheries (1952 - 1980). Canadian harvest based on Bowen (1982).

**Table 2. Average Prices for Whitecoats and Adult Seals
1952 - 1980 in Nominal Canadian Dollars**

Year	Price of Whitecoats	Price of Adult Harp Seals
1952	3.35	1.78
1953	3.85	2.54
1954	4.08	2.99
1955	5.50	4.26
1956	5.91	4.46
1957	6.83	4.69
1958	8.27	3.39
1959	10.07	4.03
1960	10.18	4.95
1961	2.83	5.31
1962	8.97	10.46
1963	15.06	20.33
1964	15.04	22.56
1965	15.06	18.83
1966	15.04	12.78
1967	9.04	9.04
1968	9.04	9.04
1969	10.54	11.30
1970	11.66	12.76
1971	12.21	14.37
1972	12.76	15.02
1973	14.78	17.39
1974	16.25	19.43
1975	18.41	21.31
1976	17.15	19.86
1977	18.85	23.81
1978	21.69	28.20
1979	24.19	32.26
1980	27.12	37.74

Source: Director General of Fisheries (1952 - 1980). Norwegian prices were converted to Canadian Dollars using the average exchange rate for that year.

Table 3. Harvest Estimates for Pups (H_t) and 1+ Seals (Y_t) and the Simulated Values for the Stock of 1+ Seals (X_t) and Pup Production (P_t) for the Basic Parameter Set $K = 4.0 \times 10^6$, $M = 0.075$, $M_0 = 0.225$, and $r = 0.40$ from $X_0 = 2,530,208$ and $P_0 = 566,323$

Year	X_t	P_t	H_t	Y_t
1952	2,530,208	566,323	207,799	117,493
1953	2,509,618	371,887	207,771	83,359
1954	2,371,526	374,029	186,254	98,859
1955	2,247,743	386,196	261,475	89,133
1956	2,093,373	393,862	347,887	54,205
1957	1,921,861	399,128	173,063	87,023
1958	1,872,426	399,389	150,915	165,363
1959	1,771,601	398,372	244,111	86,652
1960	1,678,131	394,783	165,656	129,457
1961	1,610,097	389,640	175,892	25,644
1962	1,631,273	384,797	212,100	118,067
1963	1,533,557	386,404	276,282	77,547
1964	1,432,153	378,243	271,744	80,843
1965	1,332,499	367,755	188,183	57,105
1966	1,318,908	355,444	255,273	76,679
1967	1,226,694	353,611	280,256	60,099
1968	1,135,951	340,199	160,594	40,912
1969	1,152,106	325,342	237,102	59,877
1970	1,078,698	328,107	221,074	44,383
1971	1,039,692	315,120	213,853	24,437
1972	1,017,593	307,780	120,262	17,357
1973	1,070,545	303,487	103,434	31,344
1974	1,116,303	313,611	119,364	37,054
1975	1,148,847	321,907	144,478	37,669
1976	1,165,348	327,553	136,981	37,605
1977	1,190,856	330,335	134,892	31,973
1978	1,223,436	334,528	121,750	54,257
1979	1,246,394	339,694	137,315	37,272
1980	1,275,282	343,207	136,417	44,724
1981	1,298,529	347,478	-----	-----

Source: The values for H_t and Y_t are from Bowen (1982).

Table 4. The Simulation Program

```

10 DATA 4.0E6,0.075,0.225,0.40,29
20 READ K,M,MO,R,TT
30 DIM P(TT),X(TT),H(TT),Y(TT)
40 X(0)=25302081:P(0)=566323!
50 DATA 207799,207711,186254,261475,347887,173063,150915,244111
60 DATA 165656,175892,212100,276282,271744,188183,255273,280256
70 DATA 160594,237102,221074,213853,120262,103434,119364,144478
80 DATA 136981,134892,121750,137315,136417
90 FOR T=0 TO TT-1
100 READ H(T)
110 NEXT T
120 DATA 117493,83359,98859,89133,54205,87023,165363,86652
130 DATA 129457,25644,118067,77547,80843,57105,76679,60099
140 DATA 40912,59877,44383,24437,17357,31344,37054,37669
150 DATA 37605,31973,54257,37272,44724
160 FOR T=0 TO TT-1
170 READ Y(T)
180 NEXT T
190 FOR T=0 TO TT-1
200 P(T+1)=R*X(T)*(1-X(T)/K)
210 X(T+1)=(1-MO)*(P(T)-H(T))+(1-M)*(X(T)-Y(T))
220 NEXT T
230 LPRINT:LPRINT "K=";K,"M=";M,"MO=";MO,"R=";R:LPRINT
240 LPRINT " T", " X(T)", " P(T)", " H(T)", " Y(T)"
250 LPRINT "
260 FOR T=0 TO TT
270 LPRINT 1952+T,X(T),P(T),H(T),Y(T):LPRINT
280 NEXT T
290 END

```

"

Table 5. Additional Simulation Results

<hr/>				
$K=5.0 \times 10^6, M=0.075, M_0=0.225, r=0.35$			$K=4.0 \times 10^6, M=0.075, M_0=0.075, r=0.40$	
Year	X_t	P_t	X_t	P_t
1952	2,530,208	566,323	2,530,208	566,323
1953	2,509,618	437,436	2,563,396	371,887
1954	2,422,326	437,493	2,445,898	368,258
1955	2,343,918	437,077	2,339,366	380,117
1956	2,221,768	435,794	2,191,210	388,483
1957	2,073,124	432,081	2,014,281	396,343
1958	2,037,883	424,744	1,989,248	399,979
1959	1,944,298	422,551	1,917,478	399,988
1960	1,856,614	415,883	1,837,701	399,319
1961	1,791,547	408,523	1,796,264	397,365
1962	1,813,750	402,366	1,842,687	395,849
1963	1,715,963	404,534	1,765,241	397,525
1964	1,614,930	394,470	1,673,267	394,488
1965	1,514,144	382,665	1,586,532	389,324
1966	1,498,485	369,466	1,600,775	382,904
1967	1,403,670	367,287	1,527,848	384,062
1968	1,310,253	353,364	1,453,689	377,707
1969	1,323,537	338,415	1,507,648	370,154
1970	1,247,403	340,615	1,462,262	375,759
1971	1,205,438	327,670	1,454,621	371,083
1972	1,180,634	320,187	1,468,359	370,256
1973	1,230,974	315,649	1,537,421	371,735
1974	1,274,124	324,770	1,674,601	381,803
1975	1,303,480	332,306	1,757,487	389,411
1976	1,316,442	337,283	1,817,395	394,118
1977	1,338,159	339,443	1,884,158	396,665
1978	1,366,749	343,008	1,955,412	398,658
1979	1,385,531	347,602	2,014,708	399,801
1980	1,410,112	350,557	2,071,928	399,978
1981	1,428,942	354,350	2,118,958	399,482
<hr/>				

Table 6. The Bioeconomic Optimum for $K = 4.0 \times 10^6$, $M = 0.075$, $M_0 = 0.225$, $r = 0.40$, $a = \$15$, $b = 5.47 \times 10^{-5}$, $c = \$19$, $d = 2.53 \times 10^{-4}$ and δ ranging from 0.00 to 0.10

		δ					
		0.00	0.02	0.04	0.06	0.08	0.10
X*	1,516,129	1,374,838	1,228,387	1,076,774	920,000	758,064	
P*	376,586	360,917	340,461	314,765	283,360	245,759	
H*	173,760	172,234	167,430	159,002	146,591	129,825	
Y*	47,007	46,612	45,373	43,198	39,995	35,669	
N*	\$1,288,959	\$1,296,796	\$1,319,288	\$1,350,770	\$1,378,626	\$1,381,257	