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OPTIMAL FLUID MILK ADVERTISING IN NEW YORK STATE:

A Control Model

by
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ABSTRACT

Generic dairy promotion is big business. The 1983 Dairy and Tobacco Adjustment Act requires that all dairy farmers pay a promotion assessment of 15 cents per hundredweight on all milk sold commercially. Of the total assessment, up to 10 cents may be retained locally to fund regional or state dairy product advertising. The funding for national and state programs combined totals over $200 million annually. Thus, the program involves high stakes and, if not well conducted, can result in substantial losses in opportunity costs to dairy farmers. The size of the potential losses emphasizes the importance of understanding the economics of dairy promotion and the need to increase the efficiency of promotional efforts.

The purpose of this study is to use a comprehensive optimization framework to identify the optimal time path of advertising expenditures for the New York State fluid milk promotion program. New York State is the third largest milk producing state and the size of its consuming population is second only to the State of California. Currently, New York dairy farmers invest $15 million annually in dairy promotion efforts. The problem is cast in a deterministic optimal control framework with fluid sales equations for major New York cities and the farm milk supply equation for the entire state as the time-evolving equations. The objective is to choose the optimal advertising spending level for each city with the goal of maximizing the discounted net farm revenue stream. The model can be extended to optimization of national advertising expenditures across states or regions. Analytical insights

into the solution structure as well as empirical results based on alternative functional specifications are presented in the paper.

The empirical results indicate that advertising expenditure levels have been too high in the markets of New York City and Albany, although the spending level for Syracuse is found to be nearly optimal. The magnitude of the overspending, however, depends critically on the functional form chosen. For example, the result for New York City based on a semi-logarithmic specification indicates that the historical spending level is about 4.9 times that of the optimal level while the rate of overspending is 2.5 times when a double-logarithmic model is used. The overspending rate of 2.5 for New York City is consistent with that found in Liu and Forker, which also used a double-logarithmic specification. The analysis also shows that it is optimal to follow a seasonal pattern in allocating advertising funds: advertising should be intensified in the winter and at a lower level during the late spring and early summer. Further, the optimal seasonal pattern found is not sensitive to alternative functional form specifications. A casual examination of the expenditure data indicates that historically the seasonal spending pattern is far from optimal.

This study represents the first attempt to deal with commodity dairy promotion in a comprehensive optimization framework while taking into account the complexity of endogenous supply response and government price intervention. The advantage of the current approach over previous ad hoc simulation procedures is that both the short-term seasonal advertising pattern and the long-term spending time path can be identified in a more realistic setting. While the present model provides a more detailed dynamic picture of fluid milk sales, farm supply and advertising, it does suffer from some limitations. The optimal solutions are highly dependent on the functional form specified. This is a disturbing result, which confirms the finding of a previous study. The dilemma supports Kinnucan's call for devoting greater attention to theoretical underpinnings of the sales-advertising response relation in order to gain some insight into the appropriate *a priori* restriction to place on the functional form. The results also point out the need to develop a better and more comprehensive commodity promotion data set as argued by Forker et. al. Such a data set would enable researchers to narrow the choices of functional form empirically through appropriate specification tests.

In addition to resolving the functional form problem, the model could be improved to better reflect the characteristics of the dairy market environment. For example, the model does not account for the fact that political goodwill may accrue when advertising efforts increase demand and thereby reduce government expenditures on the dairy support program. In light of the 1985 Food Security Act, which gives the Secretary of Agriculture the power to adjust dairy support prices in response to surplus levels, the potential for political goodwill is of increasing importance to dairy farmers. If the possibility for political goodwill were incorporated, optimal advertising expenditure levels might be higher than those found in this study. To adapt the model to reflect the political economy of the entire dairy industry, researchers would need to specify the behavior of government in setting support prices. With an endogenized government support price which is a function of the dairy surplus, the adapted model should also allow for manufactured dairy product advertising, since the effect of such expenditures would no longer simply be to replace government purchases with private consumption, but rather would result in a farm price impact.

Optimal Fluid Milk Advertising in New York State: A Control Model

Donald J. Liu and Olan D. Forker

INTRODUCTION

Generic dairy promotion is big business. The 1983 Dairy and Tobacco Adjustment Act requires that all dairy farmers pay a promotion assessment of 15 cents per hundredweight on all milk sold commercially. Of the total assessment, up to 10 cents may be retained locally to fund regional or state dairy product advertising. The funding for national and state programs combined totals over $200 million annually. Thus, the program involves high stakes and, if not well conducted, can result in substantial losses in opportunity costs to dairy farmers. The size of the potential losses emphasizes the importance of understanding the economics of dairy promotion and the need to increase the efficiency of promotional efforts.

The continuing effects of advertising on sales after the original period of expenditure is a well-recognized phenomenon which is aptly summarized by Waugh's statement that "old advertisements never die -- they just fade away". Advertising's lingering impact has led analysts to seek a dynamic setting in which to explore promotion issues. Thus far, most of the attention has been focused on quantifying the sales-advertising relationship within the context of distributed-lag econometric models (Kinnucan, 1982; Liu and Forker, 1988a). Using such models, an ex-post evaluation of the costs and benefits of promotion programs is made by comparing actual sales during a given period with a sales level simulated under the assumption of no advertising effort.

Another important application of the sales-advertising models is the simulation of sales under various levels of advertising expenditures with the goal of identifying the optimal spending policy for the promotion agency. A serious drawback of this approach, however, is that the truly optimal solution may be missed since it is impractical to exhaust all possible policy scenarios in the simulation; a situation which is especially true when there exists an optimal seasonal allocation pattern. Additional complications are introduced if the interest is in long-term policy, in which case a time path for advertising must also be selected. In light of these drawbacks, the identification of a more comprehensive optimization framework for the study of optimal dairy promotion policy remains an important gap in the existing literature.

The purpose of this study is to use a comprehensive optimization framework to identify the optimal time path of advertising expenditures for the New York State fluid milk promotion program. New York State is the third largest milk producing state and the size of its consuming population is second only to the State of California. Currently, New York dairy farmers invest $15 million annually in dairy promotion efforts. The optimization problem is cast in a deterministic optimal control framework with the goal of choosing the optimal advertising spending level for major cities in the state. Though the analysis focuses on markets at the state level, the model can be extended to determine the optimal level for national advertising expenditures across states or regions. In the paper, both analytical insights into the solution structure and empirical results based on alternative functional form specifications are presented.

THE MODEL

Within the past 30 years several authors have examined advertising expenditure as a problem of optimal control. For example, Nerlove and Arrow's capital theoretic approach treats advertising as investment in the firm's goodwill, which in turn affects current and future sales. In Vidale and Wolfe's sales response model, advertising is viewed as a means to acquire (up to a saturation point) the uncaptured portion of a market's potential. Gould's diffusion approach to advertising explicitly admits the interaction between the uncaptured and the captured portions of the market either through inanimate media advertising or through word-of-mouth.[1] Each of the various theoretical models has yielded useful analytical insights into the structure of optimal advertising policy and has provided a framework for empirical studies by other researchers. For example, Rausser and Hochman used an adapted version of Vidale and Wolfe's sales response model to study the optimal orange juice advertising policy for the Florida Department of Citrus.

However, the above models are monopolistic in the sense that, in addition to being able to affect demand through advertising, the firm in question is assumed to have control over the price or quantity supplied of the good. Obviously, this is not the case for generic dairy promotion. In order to reflect more accurately the market structure of the dairy sector, the model developed in this paper includes an endogenously determined farm milk price and the subsequent supply

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1 For a more detailed review on optimal advertising control models, see Liu and Forker (1988b), and Sethi.

response arises from an advertising-induced farm price change, while taking into account the dairy price support program and the federal milk marketing order program.

Our analysis is greatly simplified by the assumption that the government support price is always binding. Given the huge dairy surpluses during the past decade, the assumption seems reasonable. As a result, the farm milk price becomes a function of fluid sales (Class 1 utilization) and the farm milk supply, given the exogenous Class 1 differential and the Class 2 price.[2] An additional implication of assuming a binding support price is that it is not essential to conduct manufactured dairy product advertising because the effect would be simply to replace government purchases of the dairy surplus by the increased private consumption, while leaving the farm milk price unchanged. Thus, the fluid-only advertising model to be constructed includes the evolution of retail fluid sales for major consumer markets in the state, the evolution of farm milk supply for the entire state, and the government equation for the average farm milk price. The objective of the promotion agency is to maximize the discounted net revenue stream from farm milk sales with the control variable being the level of fluid milk advertising expenditures in each of the consumer markets.

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2 Under the rules of the federal milk marketing order program, processors buy raw milk from dairy farmers paying a base price called Class 2 price for all the milk sold plus a premium called Class 1 differential for that milk sold to the fluid market. Since the government price support program sets a floor price for the Class 2 price and since the support price is assumed to be binding, the Class 2 price equals the government price. The Class 1 differential is exogenous, set by formula by the federal milk marketing order administrator.

Retail Fluid Sales Equation

The demand for fluid milk is specified as a function of advertising and other factors such as prices and income. Denote time t fluid milk sales in market i ($i = 1, 2, \cdots, I$) as $A_{i,t}$ and advertising expenditures as $U_{i,t}$. Since consumers need to hear or see, absorb, and act on the advertising message, it is assumed that there is a one-period time lag between the exposure of message and the action of purchasing. Further, since consumers will eventually forget, but only gradually, the advertising messages, sales are assumed to decay at a constant proportional rate ϕ_i ($0 \leq \phi_i \leq 1$). Then, the evolution of fluid milk sales over time can be specified as:

(1) $$A_{i,t+1} - A_{i,t} = \Phi_i(U_{i,t}) - \phi_i A_{i,t} + Z_{i,t+1}$$

where $\Phi_i(U_{i,t})$ captures the delayed impact of time t expenditures on t+1 fluid sales and $Z_{i,t+1}$ accounts for the contemporaneous effect of all other variables on $A_{i,t+1}$. It is assumed that advertising increases sales but at a decreasing rate (i.e., $\partial\Phi_i/\partial U_{i,t} \equiv \Phi_i' > 0$ and $\partial^2\Phi_i/\partial U_{i,t}^2 \equiv \Phi_i'' < 0$). Denote the sum of advertising expenditures across all markets as U_t and the sum of fluid sales as A_t:

(1.1) $$\sum_i U_{i,t} \equiv U_t$$

(1.2) $$\sum_i A_{i,t} \equiv A_t$$

Farm Milk Supply Equation

The supply of raw milk is specified as a function of the expected farm milk price and other factors such as production capacity and variable production costs. It is assumed that farmers have naive price expectations, so the expected next period price equals the current price. Thus, the time t+1 supply of milk (S_{t+1}) is in part a function of the farm milk price from the previous period (p_t^f). Denote the part of S_{t+1} contributed by p_t^f as $f(p_t^f)$. We assume that the price-induced farm supply curve is upwardly sloped (i.e., $\partial f/\partial p_t^f \equiv f' > 0$) and the supply response is somewhat contained by $\partial^2 f/\partial (p_t^f)^2 \equiv f''$ being negative. The evolution of farm milk supply can be specified as:

$$(2.1)\quad S_{t+1} - S_t = f(p_t^f) - \psi S_t + W_t$$

where ψ $(0 \le \psi \le 1)$ captures the depreciation in the farm production capacity and the cost of adjustment, while W_t accounts for the lag impact on S_{t+1} of other variables such as variable production costs at period t.

The farm milk price p_t^f is endogenous. Under the rules of the federal milk marketing order program, processors buy raw milk from dairy farmers paying a base price called Class 2 price (P_t) for all the milk sold plus a premium called Class 1 differential (δ_t) for milk sold in the fluid market. As such, the average farm milk price is:

$$(2.2)\quad p_t^f = \delta_t (A_t/S_t) + P_t$$

Given (2.2), the farm supply transition in (2.1) can be written as:

$$(2)\quad S_{t+1} - S_t = \Psi(A_t, S_t | \delta_t, P_t) - \psi S_t + W_t$$

where $\Psi(A_t, S_t)$ is conditional on the exogenous variables δ_t and P_t. Denoting $\partial\Psi/\partial A_{i,t}$ as Ψ_A (since A_t is linear in $A_{i,t}$), $\partial\Psi/\partial S_t$ as Ψ_S, and the corresponding second derivatives as Ψ_{AA}, Ψ_{SS}, and Ψ_{AS}, the following holds:

$$(2.3a)\quad \Psi_A = f' (\delta_t/S_t) \quad > 0$$

$$(2.3b)\quad \Psi_S = - f' (\delta_t A_t/S_t^2) \quad < 0$$

$$(2.3c)\quad \Psi_{AA} = f'' (\delta_t/S_t)^2 \quad < 0$$

$$(2.3d)\quad \Psi_{AS} = - (\delta_t/S_t^2) \{f'' (\delta_t A_t/S_t) + f'\}$$

$$(2.3e)\quad \Psi_{SS} = (\delta_t A_t/S_t^3) \{f'' (\delta_t A_t/S_t) + 2f'\}$$

The signs associated with the first derivatives of Ψ with respect to S and A are intuitively appealing. An increase in the current milk supply depresses the current average farm milk price and, hence, reduces the supply of milk in the next period. On the other hand, an increase in the current fluid sales causes the current average farm milk price to increase and, hence, the supply of milk in the next period to increase. Further, due to the assumption that the farm milk supply reacts to the price change at a decreasing rate, the second partial Ψ_{AA} is negative. Notice that the signs associated with the second derivatives Ψ_{AS} and Ψ_{SS} cannot be determined *a priori* because the farm milk price in (2.2) is not a linear function of S.

Inequality Constraints

In order for the solution to make sense, an additional restriction is needed: the sum of the fluid sales across all markets cannot be greater than the supply of milk:

(3.1) $A_t \leq S_t$

Also, the sum of advertising expenditures across all markets can be no greater than the available budget which, under the current dairy promotion program, equals a fixed assessment rate (τ) times the quantity of milk sold:[3]

(3.2) $U_t \leq \tau S_t$

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3 Since the carryover of funds has not been significant in practice, it is assumed that if the budget constraint is not binding at the optimal solution, the remaining money will go to manufactured dairy product advertising.

Finally, the following non-negativity constraints are imposed:

(3.3) $A_{i,t} \geq 0$

(3.4) $U_{i,t} \geq 0$

(3.5) $S_t \geq 0$

The Objective Function

For given initial state conditions $A_{i,0}$ and S_0, the agency's problem is to choose the time path for the control $\{U_{i,t};\ t = 0, 1, \cdots, T-1\}$ so as to drive the states $\{A_{i,t};\ t = 1, 2, \cdots, T\}$ and $\{S_t;\ t = 1, 2, \cdots, T\}$ over time in an optimal path which maximizes the discounted revenue stream from farm milk sales, net of advertising cost:[4]

$$Z = \sum_{t=0}^{T-1} \rho^t \left\{ p_t^f S_t - U_t \right\} + \rho^T V(A_T, S_T)$$

where $\rho \equiv (1 + r)^{-1}$ and r is the interest rate; and $V(A_T, S_T)$ is a salvage term including terminal cash flow and terminal value of the states $A_{i,T}$ and S_T. Making use of (2.2), the above objective can be expressed as a function of the exogenous government prices $\{\delta_t\}$ and $\{P_t\}$:

(4) $$Z = \sum_{t=0}^{T-1} \rho^t \left\{ \delta_t A_t + P_t S_t - U_t \right\} + \rho^T V(A_T, S_T)$$

SOLUTION INSIGHT

The framework presented in the previous section can be characterized as a dynamic nonlinear-nonautonomous optimization problem with multiple state variables. The nonlinearity is due to Φ_i and Ψ while the nonautonomy arises from the time-varying nature of $\{\delta_t\}$, $\{P_t\}$, $\{Z_{i,t+1}\}$ and $\{W_t\}$. As such, a complete analytical solution for the

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4 Actually, given $A_{i,0}$ and S_0, p_0^f is determined by (2.2) and, hence, S_1 is determined by (2).

problem is not readily available, leaving the alternative of numerical analysis. Before carrying out the empirical analyses, however, insight into the nature of the solution can be gained by examining the set of necessary conditions for optimality, deriving the steady state, and examining comparative statics results.[5]

The Necessary Conditions

To simplify the exposition, we assume an interior solution and, hence, ignore the inequality constraints in (3.1) to (3.5). We also suppress the exogenous variables $Z_{i,t+1}$ and W_t in state equations (1) and (2). Then, the problem is to maximize the objective in (4) by choosing $\{U_{i,t}\}$, $\{A_{i,t}\}$ and $\{S_t\}$, subject to the modified version of (1) and (2). The Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \textstyle\sum_{t=0}^{T-1} \rho^t \{ & \delta_t A_t + P_t S_t - U_t + \\
& \rho \textstyle\sum_i \lambda_{i,t+1} [\Phi_i(U_{i,t}) + (1 - \phi_i) A_{i,t} - A_{i,t+1}] + \\
& \rho \mu_{t+1} [\Psi(A_t, S_t) + (1 - \psi) S_t - S_{t+1}] \} \\
+ \rho^T V(A_T, S_T) &
\end{aligned}
$$

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5 It is of note that the above model can be made more general by respecifying the state equations in (1) and (2.1) as:

(1′) $A_{i,t+1} - A_{i,t} = \Phi_i(U_{i,t}, A_{i,t} | Z_{i,t+1})$

(2.1′) $S_{t+1} - S_t = f(p^f_t, S_t | W_t)$,

with the advantage that the resulting solution insight can be applied to a larger class of empirical functional specifications. For example, the interactions between $U_{i,t}$ and $A_{i,t}$ and between p^f_t and S_t are allowed in (1′) and (2.1′), respectively. However, in conjunction with the nonlinear farm price equation (2.2), this general approach tremendously complicates the derivation of steady-state comparative statics and, hence, it is not pursued here. Instead, the possibility of alternative functional specifications other than those admitted by (1) and (2.1) will be entertained in the empirical part of the analysis.

where λ_i and μ are the current-value adjoint variables for the state variables A_i and S, respectively, and they can be interpreted as the shadow prices of their corresponding states.

In accordance with Pontryagin's maximum principle (e.g. see Clark; Kamien and Schwartz), the necessary conditions include:

(i) the optimality conditions $\partial\mathcal{L}/\partial U_{i,t} = 0$ (t = 0 to T-1),

(ii) the adjoint equations $\partial\mathcal{L}/\partial A_{i,t} = 0$ and $\partial\mathcal{L}/\partial S_t = 0$ (t = 1 to T-1),

(iii) the transversality conditions $\partial\mathcal{L}/\partial A_{i,T} = 0$ and $\partial\mathcal{L}/\partial S_T = 0$ and,

(iv) the modified version of state equations (2) and (3) which can be recovered as $\partial\mathcal{L}/\partial(\rho\lambda_{i,t+1}) = 0$ and $\partial\mathcal{L}/\partial(\rho\mu_{t+1}) = 0$ (t = 0 to T-1).

Now making use of $U_t \equiv \sum_i U_{i,t}$ and $A_t \equiv \sum_i A_{i,t}$, the above conditions are:

(5.1) $\rho\,\lambda_{i,t+1}\,\Phi_i' = 1$

(5.2a) $\rho\,\lambda_{i,t+1} - \lambda_{i,t} = -\,\delta_t - \rho\,\mu_{t+1}\,\Psi_A + \rho\,\lambda_{i,t+1}\,\phi_i$

(5.2b) $\rho\,\mu_{t+1} - \mu_t = -\,P_t - \rho\,\mu_{t+1}\,\Psi_S + \rho\,\mu_{t+1}\,\psi$

(5.3a) $\lambda_{i,T} = \partial V/\partial A_{i,T}$

(5.3b) $\mu_T = \partial V/\partial S_T$

(5.4a) $A_{i,t+1} - A_{i,t} = \Phi_i(U_{i,t}) - \phi_i\,A_{i,t}$

(5.4b) $S_{t+1} - S_t = \Psi(A_t, S_t) - \psi\,S_t$

Condition (5.1) dictates that the last dollar spent in advertising must equal the shadow value of the additional fluid sales. The appearance of the discount factor ρ is due to the delay effect assumption of advertising.

Condition (5.2a) reflects that the change in the shadow price of fluid sales over time ($\rho\,\lambda_{i,t+1} - \lambda_{i,t}$), plus the marginal contribution of the fluid sales to the cash flow (δ_t), plus the marginal contribution of the fluid sales to the shadow value of milk supply in the next period

($\rho\ \mu_{t+1}\ \Psi_A$) must equal the costs of goodwill depreciation in the fluid market ($\rho\ \lambda_{i,t+1}\ \phi_i$). The appearance of the discount factor ρ is due to both advertising and production delay effects.

Similarly, condition (5.2b) says that the change in the shadow price of farm milk supply over time ($\rho\ \mu_{t+1} - \mu_t$), plus the marginal contribution of the milk supply to the cash flow (P_t) must equal the negative marginal contribution of the milk supply to the shadow value of milk supply in the next period ($-\ \rho\ \mu_{t+1}\ \Psi_S$), plus the costs of capacity depreciation in the farm sector ($\rho\ \mu_{t+1}\ \psi$).

Condition (5.3a) states that the shadow price of state A at the terminal time must equal its marginal contribution to the salvage value. Similarly, (5.3b) is the terminal condition for state S. Finally, (5.4a) and (5.4b) reflects the need for the optimal solution to observe the physical motion of the state variables.

<u>The Steady-State Solution</u>

To gain insight into the long-term solution of the problem, it is useful to investigate the steady state. In so doing, let $\{\delta_t\}$ and $\{P_t\}$ take their respective long-term constants δ and P and let the time horizon T be infinity. By definition, in the steady state $U_{i,t+1} = U_{i,t}$, $A_{i,t+1} = A_{i,t}$, $S_{t+1} = S_t$, $\lambda_{i,t+1} = \lambda_{i,t}$, and $\mu_{t+1} = \mu_t$. Denote the above values in the steady state as U_i, A_i, S, λ_i, and μ, respectively.

Now, with the assumption that the terminal value function V(·) is finite, the terminal term in the Lagrangian vanishes as T goes to infinity and, hence, the transversality conditions become $\lim_{t\to\infty} \lambda_{i,t}\ A_{i,t} = 0$ and $\lim_{t\to\infty} \mu_t\ S_t = 0$. Replacing variables with their steady states and making use of $\rho \equiv (1 + r)^{-1}$, other necessary conditions require:

$$(5.1') \qquad \rho\ \lambda_i = 1/\Phi_i'$$

$$(5.2a') \qquad \rho\ \lambda_i\ (r + \phi_i) = \delta + \rho\ \mu\ \Psi_A$$

$$(5.2b') \qquad \rho\ \mu\ (r + \psi - \Psi_S) = P$$

$$(5.4a') \qquad \Phi_i(U_i) - \phi_i\ A_i = 0$$

$$(5.4b') \qquad \Psi(A,S) - \psi\ S = 0$$

Substituting (5.1') and (5.2b') into (5.2a'), one has:

$$(6) \qquad r + \phi_i = \Phi_i' \left\{ \delta + \frac{P\ \Psi_A}{r + \psi - \Psi_S} \right\}$$

The interpretation for (6) is that the optimal steady-state expenditure level is such that the marginal opportunity costs of advertising equal the marginal benefits of advertising. The marginal opportunity costs of advertising include time costs (r) and the depreciation costs in the fluid sector (ϕ_i). The marginal benefits of advertising include the Class 1 premium from the additional fluid sales ($\delta\ \Phi_i'$) and the base revenue from the subsequent additional raw milk supply ($P\ \Psi_A\ \Phi_i'$). However, the benefit from additional farm supply is discounted by the opportunity costs of that additional farm supply which includes time costs (r), the depreciation costs in the farm sector (ψ), and the costs from the negative impact of additional supply on subsequent supply ($-\ \Psi_S$).

To obtain the steady-state solution, (5.4a'), (5.4b') and (6) have to be solved simultaneously for the unknown U_i, A_i, and S. It is important to note that all the markets (i = 1, ..., I) have to be solved simultaneously even though the budget constraint in (3.2), which acknowledges that the total expenditures across markets cannot exceed available funding, has been assumed away. The need for a simultaneous optimization across markets now arises from the supply response equation in (5.4b') as $\Psi(A, S)$ is a function of all the A_i's.

Comparative Statics

The conditions for the steady state can be used to determine the impact of changes in exogenous policy parameters such as r, δ, and P on the optimal level of U_i, A_i, and S. As pointed out previously, implicit in the function $\Psi(A, S)$, and hence in its first derivatives with respect to A and S, are the exogenous arguments δ and P. Totally differentiating (5.4a'), (5.4b') and (6) with respect to U_i, A_i, S, r, δ, and P yields the equation system $B\ \zeta = b$ with the following:

$$B \equiv \begin{pmatrix} \Phi_i' & -\phi_i & 0 \\ 0 & \Psi_A & \Psi_S - \psi \\ \Phi_i''[\delta + P\Psi_A/\theta] & \Phi_i' P[\Psi_{AA}/\theta + \Psi_A\Psi_{AS}/\theta^2] & \Phi_i' P[\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \end{pmatrix}$$

$$\zeta \equiv \begin{pmatrix} dU_i \\ dA_i \\ dS \end{pmatrix}$$

$$b \equiv \begin{pmatrix} 0 \\ -\Psi_\delta\, d\delta - \Psi_P\, dP \\ -\Phi_i' \left\{ 1 + P\,[\Psi_{A\delta}/\theta + \Psi_A\Psi_{S\delta}/\theta^2] \right\} d\delta - \Phi_i' \left\{ [\Psi_A + P\Psi_{AP}]/\theta + P\Psi_A\Psi_{SP}/\theta^2 \right\} dP + dr \end{pmatrix}$$

where $\theta \equiv r + \psi - \Psi_S$. It was established in the previous section that $\Phi_i' > 0$, $\Phi_i'' < 0$, $\Psi_A > 0$, $\Psi_S < 0$, $\Psi_{AA} < 0$. Further, it was shown in (2.3d) and (2.3e) that the signs associated with Ψ_{AS} and Ψ_{SS} cannot be determined without further assumptions. For example, there are two components with opposite signs on the right-hand-side of (2.3d): a direct supply effect of the farm price change (i.e., f') and an indirect effect arising from a change in f′ (i.e., $f''\delta A/S$). For equation (2.3d), assume the direct effect outweighs the indirect effect. As such, Ψ_{AS} is negative. Furthermore, it follows from (2.3e) that Ψ_{SS} is positive.

Now, making use of the average farm milk price equation in (2.2), the following additional conditions hold:

$$\Psi_{\delta} = f' A/S \qquad > 0$$

$$\Psi_{P} = f' \qquad > 0$$

$$\Psi_{AP} = \Psi_{AA} (S/\delta) \qquad < 0$$

$$\Psi_{SP} = - \Psi_{AA} (A/\delta) \qquad > 0$$

$$\Psi_{A\delta} = - \Psi_{AS} (S/\delta) \qquad > 0$$

$$\Psi_{S\delta} = \Psi_{AS} (A/\delta) \qquad < 0$$

Upon solving the equation system $B\ \zeta = b$ by Cramer's rule and making use of the signs established, the impact of an infinitesimal change in the interest rate (r) on the optimal level of the three endogenous variables are:[6]

(9.1) $dU_i/dr = \qquad < 0$

(9.2) $dA_i/dr = \qquad < 0$

(9.3) $dS/dr = \qquad < 0$

The impact of an infinitesimal change in the Class 1 differential (δ) on the optimal level of the three endogenous variables are:

(10.1) $dU_i/d\delta = \qquad > 0$

(10.2) $dA_i/d\delta = \qquad > 0$

(10.3) $dS/d\delta = \qquad > 0$

The impact of an infinitesimal change in the Class 2 price (P) on the optimal level of the three endogenous variables are:

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6 For a detailed derivation of the result, see Appendix A.

(11.1) $dU_i/dP \quad = \qquad \gtrless 0$

(11.2) $dA_i/dP \quad = \qquad \gtrless 0$

(11.3) $dS/dP \quad = \qquad > 0$

An increase in the interest rate causes a reduction in the optimal level of each of the three endogenous variables while an increase in the Class 1 differential increases the optimal level of the variables. These results are intuitively appealing. An increase in the interest rate increases the opportunity cost of money and hence reduces the incentive for advertising. On the other hand, an increase in the Class 1 differential increases the value of fluid sales and hence the incentive for fluid advertising. A less straightforward result is that an increase in the Class 2 price could cause a reduction in the optimal level of advertising: the sign is indeterminate.

The rationale for the indeterminate sign in (11.1) is as follows. On the one hand, an increase in the Class 2 price increases the farm milk supply which in turn depresses the Class 1 utilization rate; leading to a reduction in the effectiveness of fluid advertising in enhancing the average farm milk price in (2.2) and hence a reduction in the optimal level of advertising. On the other hand, an increase in the Class 2 price provides an incentive for more milk production and one way to further stimulate this additional production is to increase the average farm milk price even more through more fluid advertising. Since the above two forces work in opposite directions, the effect of a change in the Class 2 price on the optimal level of advertising is indeterminate. It follows that the corresponding effect on the optimal level of fluid sales is also indeterminate. In either case, however, the optimal farm milk supply reacts positively to an increase in the Class 2 price as indicated by (11.3).

THE ECONOMETRIC MODEL

The econometric model consists of retail fluid sales equations for three major cities in New York State and a farm milk supply equation for the entire state. The markets included in the analysis are New York City, Syracuse and Albany.[7] The estimation is based on monthly data from January 1983 to September 1987. The sales data are derived from fluid plant surveys conducted by the New York State Department of Agriculture and Markets, while advertising data are based on audits of the invoices of the New York State promotion unit. Other data are from public sources. A detailed listing of the data and their sources can be found in Appendix B.

Retail Fluid Sales Equations

In accordance with equation (1), a semi-logarithmic retail fluid demand equation is specified for each market. The dependent variable is the change in sales ($A_{t+1} - A_t$). The independent variables are lag fluid advertising expenditures deflated by the consumer price index for

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[7] Other major cities in the state such as Binghamton, Buffalo and Rochester are not included. Fluid sales data for Binghamton are not available while Buffalo and Rochester have independent regional promotion units that are not part of the New York-New Jersey federal marketing order region covering most of New York State. Notice that the combination of the two states in a federal order program means that the average farm milk price is a function of the fluid utilization and the total milk supply of the two states combined. This creates a problem for the analysis because the New York State promotion unit (ADA&DC) controls advertising funds for only the eastern part of New York. Also, sales data for New Jersey are not available. To deal with the problems, the analysis treats New York State as if it has its own individual federal order. Accordingly, the estimated supply equation pertains to New York State, rather than the area of New York and New Jersey in the order.

all items (U_t/CPI_t),[8] lag fluid sales (A_t), and other factors which include the price ratio (PR_{t+1}) between retail fluid milk price and consumer price index for food and beverage, average weekly earnings of production workers deflated by the consumer price index for all items ($DINC_{t+1}$) and a set of seasonal harmonic variables (SIN and COS).[9] In the case of New York City, a time trend (TIME) is also included.

The price for food and beverage is used as a proxy for prices of fluid milk substitutes, while the average weekly earnings for income. The harmonic variables account for seasonal patterns of the fluid sales. Due to the phenomenon that consumers tend to buy less milk in favor of soft drinks during the summer season, the retail fluid sales data often possess a "summer low" characteristic. Finally, the trend variable for the New York City equation captures the sales impact of the gradual change in the ethnic composition of population over time. It is generally observed that nonwhites tend to consume less milk and the nonwhite population is increasing faster than the white population in that city.

The equations are estimated by ordinary least squares and the estimation results are presented in Table 1 with the absolute values of the estimated student t ratios appearing in parentheses. All the coefficients in each of the three equations have their expected signs.

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8 In the New York City equation, the variable pertaining to deflated advertising expenditures is lagged two months; a specification consistent with Liu and Forker (1988a). Note that NDB expenditures were not included.

9 The variables COS1 to COS6 in Table 1 are the first to the sixth wave of the cosine term while SIN1 to SIN5 are the first to the fifth wave of the sine term. Doran and Quilkey argue that there is no theoretical grounds as to which wave will be more significant, hence, all the eleven terms should be empirically entertained.

For the New York City equation, all the included variables are significant, the adjusted R-Squared is reasonably high and the Durbin-Watson statistic does not indicate the existence of serial correlation. For the Syracuse equation, the income variable is not significant and the adjusted R-Squared is not as high as it was for the New York City equation. However, other variables are significant and the Durbin-Watson statistic is good. On the other hand, the Albany equation does not appear to be satisfactory at all. The insignificant variables include advertising, price and income. Given the limited availability of individual city data on a monthly basis, however, an alternative specification does not appear feasible at this time.

<u>Farm Milk Supply Equation</u>

In accordance with equation (2.1), a semi-logarithmic farm milk supply equation is specified for New York State. The dependent variable is the change in supply (S_{t+1} - S_t). Since the relevant quantity in calculating the average farm price in (2.2) and the advertising budget in (3.2) is the quantity "marketed" rather than "produced" in the state, the supply variable is the quantity of milk received by the plants in New York State (excluding Buffalo and Rochester). The independent variables are lag farm milk price (p_t^f) over lag feed cost index (FCI_t), lag fluid sales (S_t), and other factors which include lag slaughter cow price deflated by the index of price paid by dairy farmers ($DPCOW_t$), a dummy variable with January 1984 to June 1985 and April 1986 to September 1987 being one and zero otherwise (DUM), and a set of seasonal harmonic variables.

The feed cost index captures the effect of variable production cost while the slaughter cow price is the opportunity cost of keeping the dairy cow on farm. The dummy variable accounts for the supply effect of the 1984-85 Milk Diversion Program and 1986-87 Dairy Termination Program. Finally, the harmonic variables capture the seasonal pattern of farm milk production. Due to superior feed qualities and weather conditions in the spring, the farm milk supply data are often characterized by a "spring flush".

The equation is estimated by ordinary least squares and the estimation result is in Table 1. All the coefficients are significant and have the expected signs. The adjusted R-Squared indicates the model explains about 93% of the variation in the dependent variable. Finally, the Durbin-Watson statistic does not indicate the existence of serial correlation.

Alternative Functional Form Specification

The estimated retail fluid demand and farm milk supply equations presented in Table 1 are semi-logarithmic. These equations are consistent with the state equations in (1) and (2.1) and they will be used in the numerical computation of the optimal level of advertising expenditures. However, as found by Kinnucan (1983), there is potential for the functional form to condition the empirical findings. For example, in estimating a fluid sales equation for the Buffalo market under various functional form specifications, Kinnucan found that the estimated advertising elasticities differed by as much as 220% and the resulting simulated optimal level of advertising expenditures varied by 149%. In light of the divergence, Kinnucan argued that research results based on different functional forms should be presented.

Accordingly, the alternative of double-logarithmic specification is also considered. The double-logarithmic specification is consistent

Table 1: Estimation Results: A Semi-Logarithmic Specification*

New York City Retail Fluid Sales:

$$A_{+1} - A = \underset{(3.9)}{0.1696} \ln (U/CPI)_{-1} - \underset{(7.4)}{0.7932} A - \underset{(7.1)}{2.8499} \ln PR_{+1} + \underset{(2.2)}{15.7498} \ln DINC_{+1} + \underset{(3.1)}{0.4265} COS1_{+1} - \underset{(1.7)}{0.1414} COS6_{+1} + \underset{(1.9)}{0.2231} SIN3{+1} + \underset{(4.9)}{0.6230} SIN5{+1} - \underset{(2.5)}{0.4417} \ln TIME{+1}$$

Adjusted R-squared: 0.75 Durbin-Watson: 1.81

Syracuse City Retail Fluid Sales:

$$A_{+1} - A = \underset{(2.5)}{0.0273} \ln (U/CPI) - \underset{(5.2)}{0.6570} A - \underset{(2.2)}{0.2038} \ln PR_{+1} + \underset{(0.7)}{0.3369} \ln DINC_{+1} + \underset{(1.7)}{0.0385} COS1_{+1} - \underset{(2.6)}{0.0348} COS6_{+1} - \underset{(1.6)}{0.0296} SIN3_{+1} + \underset{(3.5)}{0.0660} SIN5_{+1}$$

Adjusted R-squared: 0.50 Durbin-Watson: 1.91

Albany City Retail Fluid Sales:

$$A_{+1} - A = \underset{(0.6)}{0.0060} \ln (U/CPI) - \underset{(4.4)}{0.5010} A - \underset{(1.0)}{0.1160} \ln PR_{+1} + \underset{(0.6)}{0.3926} \ln DINC_{+1} + \underset{(2.0)}{0.0458} COS1_{+1} - \underset{(2.1)}{0.0322} COS6_{+1} + \underset{(3.0)}{0.0630} SIN4_{+1} + \underset{(2.8)}{0.0600} SIN5_{+1}$$

Adjusted R-squared: 0.47 Durbin-Watson: 1.80

New York State Farm Milk Supply:

$$S_{+1} - S = \underset{(1.8)}{2.4139} \ln (p^{f}/FCI) - \underset{(8.1)}{0.2094} S - \underset{(7.8)}{8.1126} \ln DPCOW - \underset{(2.6)}{0.7063} DUM - \underset{(6.0)}{1.0806} COS2 + \underset{(6.4)}{0.7861} COS6 + \underset{(11.6)}{2.3488} SIN1 - \underset{(3.1)}{0.5350} SIN2 - \underset{(6.5)}{1.1404} SIN3 - \underset{(8.1)}{1.3955} SIN4 - \underset{(16.0)}{2.9086} SIN5$$

Adjusted R-squared: 0.93 Durbin-Watson: 1.98

* The measurements of data are: retail fluid sales and farm milk supply in ten million pounds, advertising expenditures in thousand dollars, retail fluid milk price in dollars per half gallon, average weekly earnings in dollars, farm milk price and slaughter cow price in dollars per hundredweight.

Table 2: Estimation Results: A Double-Logarithmic Specification*

New York City Retail Fluid Sales:

$$
\begin{aligned}
\ln A_{+1} - \ln A = {} & \underset{(4.4)}{0.0104} \ln (U/CPI)_{-1} - \underset{(6.8)}{0.6974} \ln A - \underset{(6.7)}{0.3855} \ln PR_{+1} \\
& + \underset{(1.8)}{0.6769} \ln DINC_{+1} + \underset{(3.3)}{0.0248} COS1_{+1} - \underset{(1.7)}{0.0075} COS6_{+1} \\
& + \underset{(1.7)}{0.0105} SIN3_{+1} + \underset{(4.9)}{0.0328} SIN5_{+1} - \underset{(3.6)}{0.0366} \ln TIME_{+1}
\end{aligned}
$$

Adjusted R-squared: 0.73 Durbin-Watson: 1.81

Syracuse City Retail Fluid Sales:

$$
\begin{aligned}
\ln A_{+1} - \ln A = {} & \underset{(2.4)}{0.0135} \ln (U/CPI) - \underset{(5.3)}{0.6732} \ln A - \underset{(1.5)}{0.0700} \ln PR_{+1} \\
& + \underset{(0.5)}{0.1355} \ln DINC_{+1} + \underset{(1.8)}{0.0208} COS1_{+1} - \underset{(2.7)}{0.0184} COS6_{+1} \\
& + \underset{(1.5)}{0.0149} SIN3_{+1} + \underset{(3.5)}{0.0343} SIN5_{+1}
\end{aligned}
$$

Adjusted R-squared: 0.51 Durbin-Watson: 1.91

Albany City Retail Fluid Sales:

$$
\begin{aligned}
\ln A_{+1} - \ln A = {} & \underset{(0.6)}{0.0036} \ln (U/CPI) - \underset{(4.3)}{0.4965} \ln A - \underset{(0.4)}{0.0234} \ln PR_{+1} \\
& + \underset{(0.6)}{0.2092} \ln DINC_{+1} + \underset{(1.9)}{0.0251} COS1_{+1} - \underset{(2.2)}{0.0186} COS6_{+1} \\
& + \underset{(2.9)}{0.0349} SIN4_{+1} + \underset{(2.8)}{0.0347} SIN5_{+1}
\end{aligned}
$$

Adjusted R-squared: 0.47 Durbin-Watson: 1.83

New York State Farm Milk Supply:

$$
\begin{aligned}
\ln S_{+1} - \ln S = {} & \underset{(2.5)}{0.0744} \ln (p^{f}/FCI) - \underset{(5.7)}{0.0613} \ln S - \underset{(5.6)}{0.1552} \ln DPCOW \\
& - \underset{(2.6)}{0.0139} DUM - \underset{(5.6)}{0.0199} COS2 + \underset{(5.4)}{0.0134} COS6 + \underset{(9.1)}{0.0360} SIN1 \\
& - \underset{(1.8)}{0.0061} SIN2 - \underset{(6.0)}{0.0209} SIN3 - \underset{(8.1)}{0.0280} SIN4 - \underset{(15.2)}{0.0536} SIN5
\end{aligned}
$$

Adjusted R-squared: 0.91 Durbin-Watson: 1.56

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* Measurements of data: See Table 1.

with equation (1') and (2.1') in footnote 5, which represent a generalization of (1) and (2.1) and admits more complicated interactions between variables in the state equation. The estimation results for double-logarithmic specification are in Table 2. In comparing the result with the semi-logarithmic equations in Table 1, it is evident that the statistical qualities such as the goodness of fit, the significance of variables, and the extent of serial correlation are similar for both specifications. The magnitudes of the estimated coefficients from the two specifications are not directly comparable as the dependent variables are expressed differently. Table 3 presents short-run and long-run advertising elasticities based on each specification. Overall, the elasticity estimates fall within the range of previous results (eg., see USDA). However, in all but the Syracuse market, the double-logarithmic specification results in slightly larger advertising elasticities.

Table 3: Sales Elasticities of Advertising

	NYC	Syracuse	Albany
Semi-Log Specification:*			
Short-Run	0.00858	0.01413	0.00338
Long-Run	0.01081	0.02151	0.00674
Double-Log Specification:			
Short-Run	0.01036	0.01347	0.00357
Long-Run	0.01486	0.02000	0.00719

* Evaluated at the historical mean quantities.

THE OPTIMIZATION

The estimated retail sales equations can be transformed readily into the form specified in (1) by collapsing all the terms as $Z_{i,t+1}$, except advertising expenditures (U) and lag sales (A). Similarly, the estimated supply equation can be transformed into that specified in (2.1) or by collapsing all the terms as W_t, except farm milk price (p_t^f) and lag supply (S). The remaining problem is to maximize the objective function in (4) subject to the state equations (1) and (2.1), the farm price formula (2.2), and the inequality constraints from (3.1) to (3.5).[10] Since the state promotion unit retains two-thirds of the total dairy promotion funds, the assessment rate τ in (3.2) is specified as 10 cents per hundredweight of milk sold. The interest rate is specified as 7% per annum which is the average rate of the 3-month Treasury Bills during the time period considered in this study. The terminal value function V($\cdot$) in (4) includes cash flow in the last period ($\delta_T A_T + P_T S_T$) and the values of the state variable $A_{i,T}$ and S_T which are computed as the future income stream from those two states, discounted by the interest rate (ρ) and the decay rate (ϕ_i for A_i and ψ for S). To make the computation of the future income stream possible, δ_T and P_T are assumed to prevail indefinitely into the future. The optimization problem is solved for the time period from January 1984 to September 1987 using GAMS/MINOS (Brooke et. al.).

The Optimal Advertising Policy

The optimal fluid advertising expenditure levels, based on both semi- and double-logarithmic specifications, along with the historical

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10 In the case of the double-logarithmic specification, equations (1) and (2.1) are replaced by (1′) and (2.1′).

Table 4: Optimal and Observed Advertising Expenditures (thousand dollars)

	New York City			Syracuse			Albany		
	Optimal		Observed	Optimal		Observed	Optimal		Observed
Time	Semi	Double		Semi	Double		Semi	Double	
84- 1	56.08	133.58	199.08	11.02	15.63	9.694	3.15	4.73	11.66
84- 2	57.11	123.93	183.86	10.92	16.03	10.71	3.12	4.84	12.64
84- 3	57.03	123.09	163.76	11.05	15.14	16.24	3.13	4.72	18.51
84- 4	55.47	116.56	259.05	10.96	16.19	12.59	3.07	4.83	17.23
84- 5	50.47	103.58	210.83	10.56	14.69	11.67	2.95	4.59	16.41
84- 6	46.07	97.680	330.52	9.653	13.50	17.09	2.72	4.40	22.57
84- 7	42.33	92.750	58.904	8.895	13.20	5.868	2.57	4.33	2.520
84- 8	45.58	104.88	321.37	8.421	12.88	24.85	2.52	4.27	27.43
84- 9	51.12	114.49	604.52	9.109	14.44	21.81	2.72	4.73	27.61
84-10	55.07	119.37	1017.8	10.13	15.51	40.51	2.99	5.26	47.65
84-11	59.16	137.33	542.02	10.86	16.57	19.91	3.18	5.56	29.30
84-12	60.18	126.63	426.55	11.57	17.90	13.16	3.35	5.79	19.09
85- 1	63.37	138.27	142.42	11.81	17.49	8.995	3.42	5.58	8.081
85- 2	65.97	146.29	243.90	12.36	18.49	12.22	3.53	5.83	15.87
85- 3	63.53	132.11	457.59	12.68	18.38	23.33	3.56	5.79	22.92
85- 4	61.18	122.16	465.61	12.18	17.11	16.57	3.41	5.52	14.77
85- 5	56.41	112.82	367.16	11.66	15.27	12.54	3.24	5.50	26.89
85- 6	50.35	101.86	347.19	10.72	14.71	13.96	2.99	5.16	39.01
85- 7	48.12	105.82	312.49	9.688	13.10	12.47	2.75	4.36	19.58
85- 8	45.96	106.24	276.26	9.303	12.76	13.19	2.66	4.30	20.13
85- 9	47.31	106.14	226.29	9.008	12.70	14.42	2.62	4.47	18.89
85-10	48.72	105.96	199.26	9.274	12.95	11.15	2.69	4.73	15.59
85-11	49.89	104.06	170.16	9.539	13.39	7.840	2.76	4.81	10.78
85-12	51.53	99.775	129.00	9.760	14.41	8.000	2.82	4.91	12.00
86- 1	51.25	97.280	87.014	10.03	14.23	7.585	2.88	4.99	12.36
86- 2	51.95	100.95	246.56	10.02	14.13	11.18	2.88	4.94	10.94
86- 3	56.08	111.01	229.11	10.18	14.07	10.43	2.92	4.61	11.81
86- 4	54.18	98.221	84.586	10.76	14.34	10.33	3.01	4.18	14.61
86- 5	49.60	96.285	256.96	10.31	13.31	8.790	2.87	3.89	18.67
86- 6	43.76	84.008	188.47	9.439	12.28	10.75	2.65	3.70	14.45
86- 7	42.07	78.924	309.81	8.485	11.51	14.51	2.45	3.40	16.23
86- 8	43.99	82.882	416.12	8.276	11.30	21.54	2.43	3.45	26.90
86- 9	44.57	81.686	225.73	8.648	11.05	13.71	2.54	3.56	39.78
86-10	48.14	89.702	463.46	8.889	10.93	15.07	2.65	3.60	19.72
86-11	55.74	101.02	146.59	9.669	11.94	6.280	2.87	3.87	7.685
86-12	62.10	108.24	116.00	11.00	13.25	7.000	3.17	4.22	9.000
87- 1	61.40	103.10	84.295	11.94	12.78	7.055	3.33	3.95	8.625
87- 2	54.44	85.782	138.91	11.63	12.74	6.865	3.20	3.87	11.86
87- 3	49.85	76.304	125.73	10.38	10.62	8.015	2.87	3.39	10.78
87- 4	47.43	67.953	139.29	9.570	9.899	5.000	2.63	3.09	8.885
87- 5	46.17	61.791	290.22	9.103	8.555	6.600	2.45	2.99	9.315
87- 6	60.75	70.591	152.37	8.767	8.459	5.550	2.25	2.43	8.380
87- 7	142.9*	115.51*	120.77	9.777	9.627	4.995	2.14	1.00	5.835
87- 8	NUC#	NUC#	292.46	13.87*	14.22*	17.86	2.09	1.00	24.07
87- 9	NUC#	NUC#	293.00	NUC#	NUC#	18.00	NUC#	NUC#	25.00

* Note that there is a jump in the optimal solution at the end of the control period. This is due to the specification of the terminal value function V(.).

\# NUC means "Not Under Control" which is due to the lag specification of the advertising variable in the retail sales equation.

Table 5: Optimal Retail Fluid Sales and Farm Milk Supply (ten million pounds)

	Retail Fluid Sales						Farm Milk Supply	
	New York City		Syracuse		Albany			
Time	Semi	Double	Semi	Double	Semi	Double	Semi	Double
84- 1	20.29	20.29	1.83	1.83	1.66	1.66	61.82	61.82
84- 2	19.28	19.28	1.75	1.76	1.46	1.46	58.61	58.61
84- 3	21.25	21.37	1.96	1.98	1.66	1.66	63.58	63.58
84- 4	18.96	19.10	1.70	1.71	1.49	1.50	62.86	62.87
84- 5	20.04	20.17	2.00	2.01	1.68	1.68	65.79	65.79
84- 6	18.89	19.01	1.81	1.82	1.58	1.59	63.02	63.02
84- 7	17.76	17.88	1.76	1.78	1.61	1.62	60.08	60.08
84- 8	18.68	18.78	1.89	1.90	1.73	1.74	57.32	57.32
84- 9	19.13	19.27	1.94	1.95	1.70	1.70	55.54	55.53
84-10	20.64	20.76	2.10	2.12	1.78	1.79	56.51	56.50
84-11	19.84	19.92	1.99	2.01	1.84	1.84	53.96	53.95
84-12	19.73	19.75	2.05	2.07	1.87	1.87	57.90	57.90
85- 1	20.03	20.06	2.08	2.10	1.94	1.94	57.90	57.89
85- 2	18.23	18.30	1.88	1.90	1.65	1.66	53.24	53.22
85- 3	20.38	20.48	2.02	2.04	1.79	1.79	60.74	60.72
85- 4	19.40	19.52	1.94	1.96	1.73	1.74	60.97	60.95
85- 5	19.78	19.87	1.96	1.97	1.69	1.69	65.96	65.93
85- 6	18.55	18.63	1.71	1.72	1.80	1.80	63.18	63.16
85- 7	18.83	18.89	1.84	1.85	1.98	1.99	61.44	61.42
85- 8	18.84	18.91	1.79	1.80	1.66	1.67	60.01	59.99
85- 9	18.71	18.80	1.79	1.80	1.61	1.62	57.63	57.60
85-10	20.46	20.54	1.91	1.92	1.79	1.79	59.45	59.42
85-11	19.63	19.73	1.86	1.87	1.89	1.90	56.59	56.56
85-12	19.88	19.98	1.87	1.88	1.91	1.91	60.14	60.10
86- 1	20.04	20.15	2.10	2.11	1.95	1.96	62.45	62.42
86- 2	18.40	18.66	1.95	1.96	1.91	1.92	57.23	57.20
86- 3	18.80	19.15	2.04	2.05	2.12	2.13	65.20	65.18
86- 4	17.92	18.14	2.04	2.05	2.06	2.07	65.34	65.33
86- 5	20.30	20.43	2.02	2.03	1.69	1.70	69.61	69.61
86- 6	19.02	19.14	1.92	1.93	1.60	1.60	64.74	64.74
86- 7	19.24	19.35	1.92	1.93	1.68	1.69	61.87	61.87
86- 8	20.10	20.18	2.06	2.08	1.70	1.70	59.59	59.58
86- 9	19.59	19.64	2.18	2.19	1.73	1.74	57.15	57.14
86-10	20.62	20.63	2.05	2.06	1.82	1.82	56.53	56.52
86-11	19.29	19.32	1.95	1.96	1.74	1.74	53.68	53.67
86-12	20.77	20.76	2.07	2.08	1.79	1.79	57.12	57.11
87- 1	20.95	20.99	2.14	2.16	2.00	2.01	57.59	57.58
87- 2	20.43	20.86	1.79	1.78	1.55	1.55	52.98	52.96
87- 3	21.03	21.42	1.97	1.97	1.76	1.77	60.84	60.84
87- 4	19.93	20.12	1.82	1.82	1.67	1.68	60.87	60.89
87- 5	20.37	20.47	2.00	2.00	1.76	1.76	65.44	65.46
87- 6	19.59	19.64	1.78	1.77	1.64	1.64	61.59	61.61
87- 7	18.66	18.66	1.89	1.89	1.72	1.72	58.90	58.92
87- 8	18.45	18.43	1.94	1.93	1.78	1.78	58.27	58.29
87- 9	19.36	19.29	2.00	2.00	1.78	1.77	54.80	54.81

spending are presented in Table 4.[11] In comparing the optimal with the observed level, it is implied that significant overspending of advertising expenditures has occurred in the markets of New York City and Albany. In addition, the overspending pattern persists regardless of the functional specification of the model. However, the magnitudes of the misallocation differ significantly between models, with the double-logarithmic specification generating a higher level of optimal spending for both markets. On the other hand, the result for the Syracuse market seems to indicate that the historical spending pattern is close to optimal regardless of the functional form chosen.

Table 6 presents the average ratio of historical spending to the optimal level, along with similar ratios pertaining to fluid sales and farm milk supply. Based on the semi-logarithmic specification, actual advertising expenditures have been about 4.9 times too high in the New York City market, 1.2 times too high in the Syracuse market, and 5.8 times too high in the Albany market. With the double-logarithmic specification, however, the result indicates that the overspending is only 2.5 times for New York City and 3.8 times for Albany, whereas for the Syracuse market a slight underspending of 10% is found. If the "outlier" expenditures of 1984-9 to 1984-11 were omitted from the calculation, the rate of overspending for New York city under the

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11 The corresponding optimal retail fluid sales and farm milk supply are in Table 5 with the observed values in Appendix B. It is of note that relatively high level of "outlier" expenditures occur at the periods of 1984-9, 1984-10, and 1984-11 for the New York city market; 1984-10 for the Syracuse market; and 1984-10, 1985-6, and 1986-9 for the Albany market.

Table 6: Ratios between the Actual and Optimal Levels of Endogenous Variables (actual/optimal)

	New York City		Syracuse		Albany		New York State	
Case	Semi	Double	Semi	Double	Semi	Double	Semi	Double
Ad. Expenditures:*								
1984	6.7	3.0	1.7	1.1	7.2	4.3		
1985	5.1	2.4	1.2	0.9	6.2	3.7		
1986	4.8	2.5	1.2	0.9	6.3	4.4		
1987	3.0	2.1	0.6	0.7	3.5	2.9		
Average	4.9	2.5	1.2	0.9	5.8	3.8		
Retail Fluid Sales:								
1984	1.014	1.009	1.007	1.000	1.010	1.006		
1985	1.019	1.015	1.005	0.997	1.012	1.009		
1986	1.006	1.000	0.992	0.986	1.008	1.005		
1984	0.993	0.983	0.968	0.969	1.001	0.999		
Average	1.008	1.002	0.993	0.988	1.008	1.005		
Farm Milk Supply:								
1984							1.0001	1.0001
1985							1.0002	1.0006
1986							1.0001	1.0003
1987							1.0000	1.0000
Average							1.0001	1.0003

double-logarithmic specification would have been 2.1; a result consistent with that found in Liu and Forker (1988a) where a double-logarithmic functional form was also used.

The significant differences in the optimal advertising level resulted from different functional specifications for New York City and Albany corroborate Kinnucan's functional form finding and the implications will be discussed later. At any rate, the high spending levels in the two markets have not yielded significant increases in fluid sales and farm milk supply, as indicated by the result that all the ratios between observed and optimal fluid sales and milk supply in Table 6 are close to unity.

Another observation that can be drawn from the optimal solution in Table 4 is the seasonal pattern of the advertising spending level. To demonstrate this more clearly, the optimal expenditure path and the observed path are plotted in Figures 1a and 1b, respectively, for the New York City market. In comparing the two figures, it is clear that the historical seasonal spending pattern is far from optimal. It is also evident that the optimal seasonal pattern generated by the semi-logarithmic model is consistent with that by the double-logarithmic model, except the former appears to be smoother. To investigate further the optimal seasonal spending pattern, Figure 2a shows the average monthly optimal expenditures for New York city under the double-logarithmic specification. For comparison, Figure 2b shows the corresponding average monthly observed expenditures. The figure indicates that it is optimal to advertise more during the winter season and less during the late spring and early summer. This result appears to be consistent with the *ad hoc* simulation result obtained by Kinnucan and Forker. Finally, the wide disparity between the actual and the optimal expenditure seasonal pattern found suggests that the economic effectiveness of advertising could have been enhanced had the actual seasonal pattern more nearly approximated the optimal pattern.

Figure 1a. Optimal Advertising Expenditures, New York City (Jan. 1984 - June 1987)

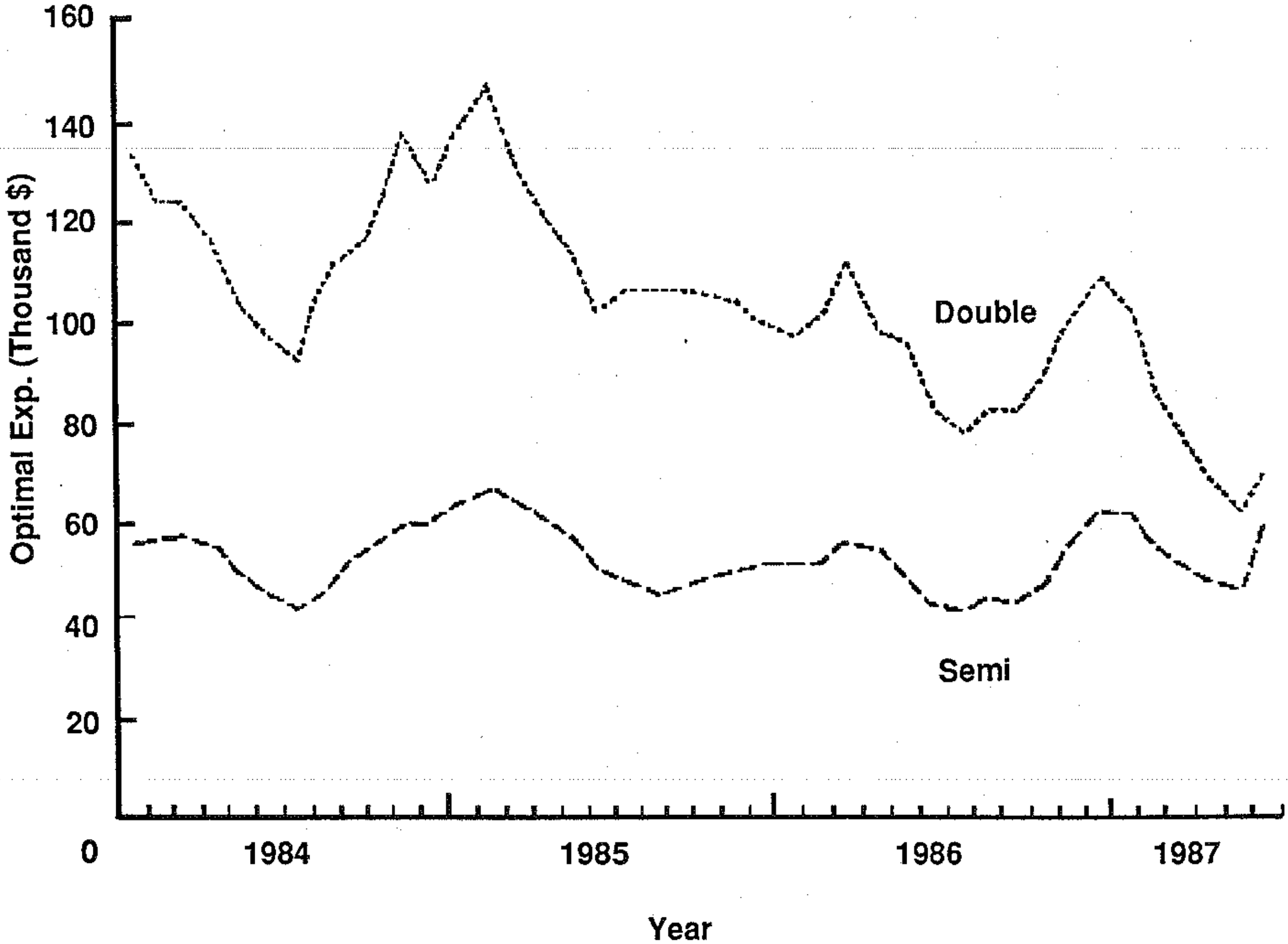


Figure 1b. Observed Advertising Expenditures, New York City (Jan. 1984 - June 1987)

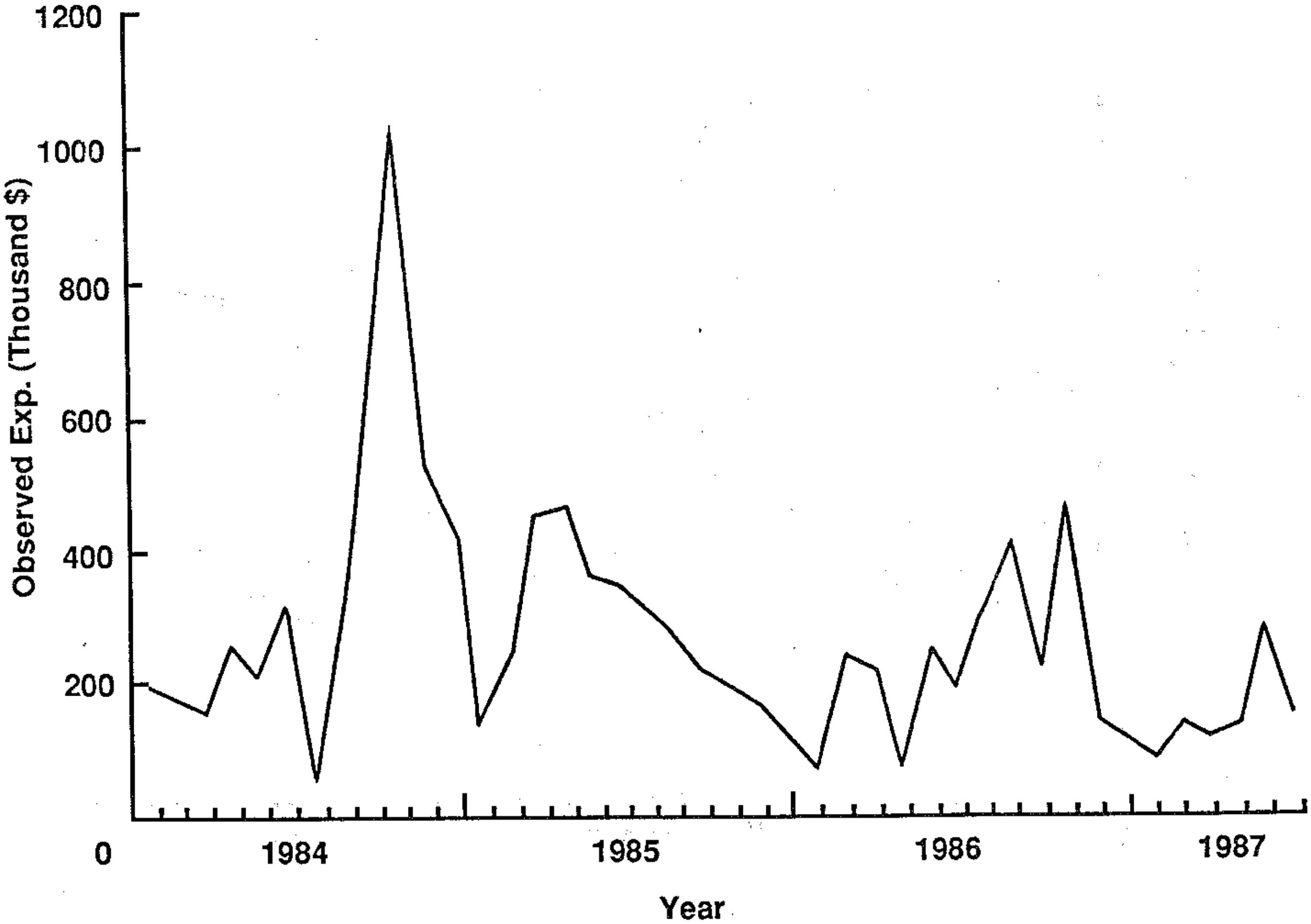


Figure 2a. Optimal Monthly Advertising Expenditures

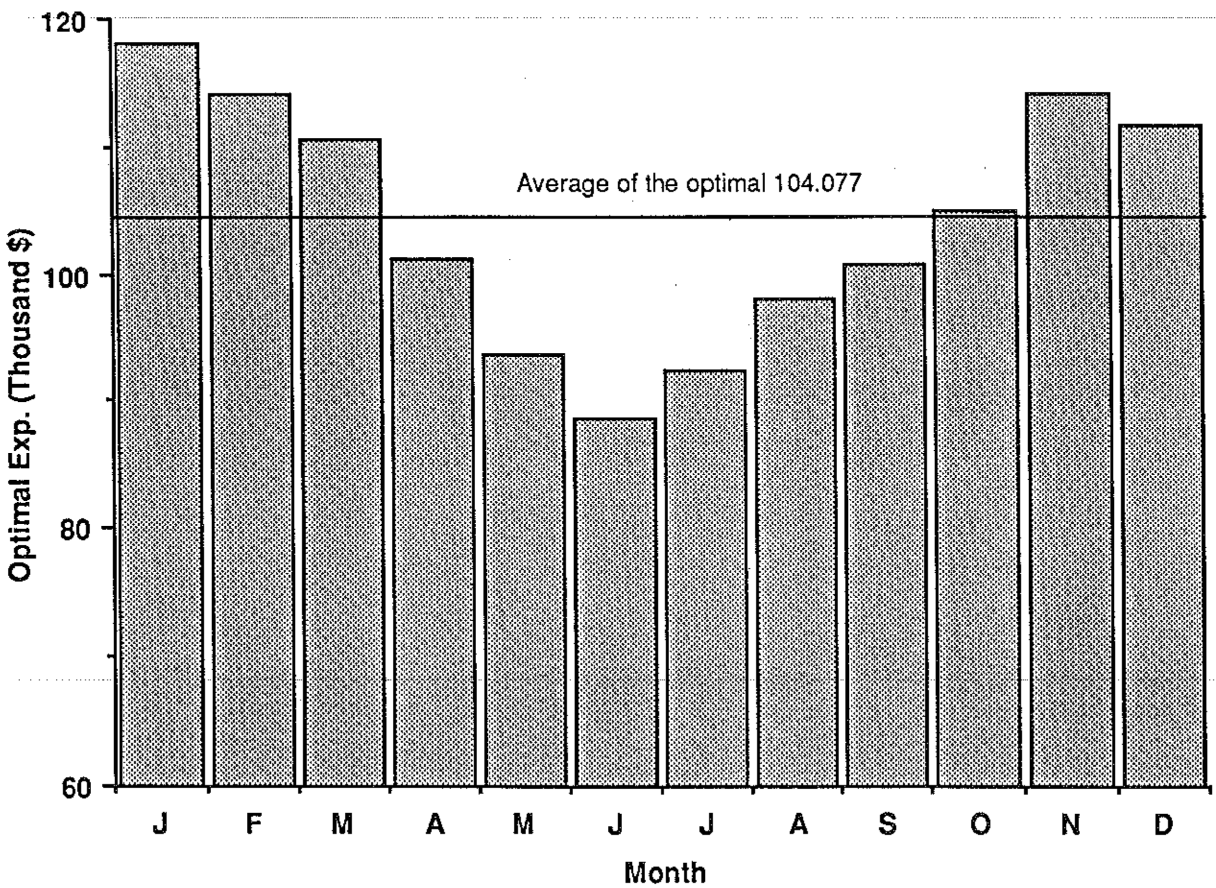
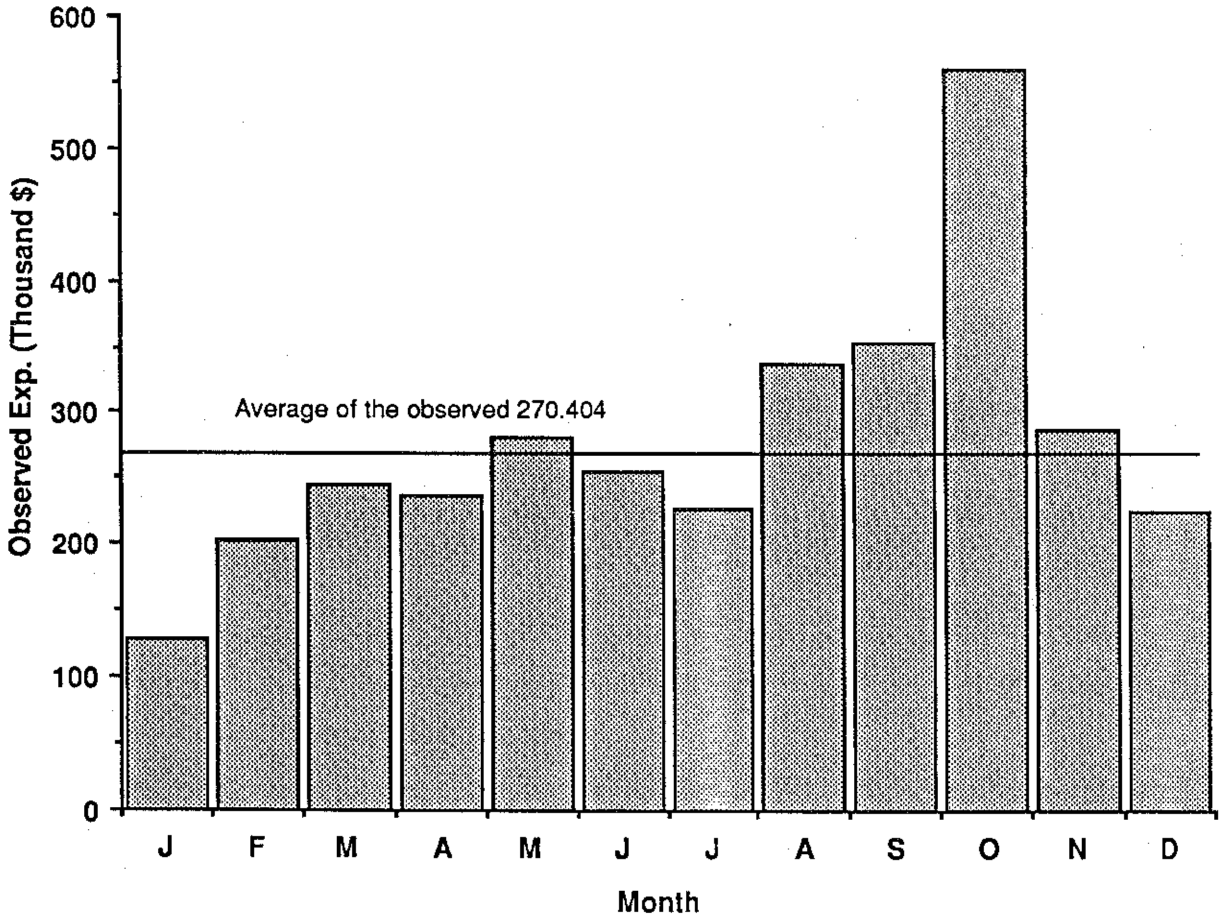


Figure 2b. Observed Monthly Advertising Expenditures



SUMMARY AND CONCLUSIONS

The purpose of this study is to identify the optimal time path of advertising expenditures for the New York State fluid promotion program. The problem is cast in a deterministic optimal control framework with fluid sales equations for major cities in the state and the farm milk supply equation for the entire state as the time-evolving equations. In the model, the endogenous variables include the farm milk price, which is determined in accordance with government dairy price regulation. The objective of the model is to choose the optimal spending level for each city with the goal of maximizing the discounted future net revenue stream. The model can be extended to optimization of national advertising expenditures across states or regions. Using the model, analytical insights into the nature of optimal solution are discussed and the steady-state conditions derived. Further, the model is implemented empirically and numerical solutions are obtained under different functional specifications.

The empirical results indicate that advertising expenditure levels have been higher than optimal in the markets of New York City and Albany, although the spending level for Syracuse is found to be nearly optimal. The magnitude of the overspending, however, depends critically on the functional form chosen. For example, the result for New York City based on a semi-logarithmic specification indicats that the historical spending level is about 4.9 times that of the optimal level while the rate of overspending is 2.5 times when a double-logarithmic model is used. The analysis also shows that it is optimal to follow a seasonal pattern in allocating advertising funds: advertising should be intensified in the winter and at a lower level during the late spring and early summer. Further, the optimal seasonal pattern found is not sensitive to alternative functional form specifications. A casual examination of the data indicates that the historical seasonal spending pattern is far from optimal.

This study represents the first attempt to deal with commodity dairy promotion in a comprehensive optimization framework while taking into account the complexity of endogenous supply response and government price intervention. The advantage of the current approach over previous ad hoc simulation procedures is that both the short-term seasonal advertising pattern and the long-term spending time path can be identified in a more realistic setting. While the present model provides a more detailed dynamic picture of fluid milk sales, farm supply and advertising, it does suffer from some limitations. The optimal solutions are highly dependent on the functional form specified. This is a disturbing result, which confirms the finding of a previous study. The dilemma supports Kinnucan's call for devoting greater attention to theoretical underpinnings of the sales-advertising response relation in order to gain some insight into the appropriate *a priori* restriction to place on the functional form. The results also point out the need to develop a better and more comprehensive commodity promotion data set as argued by Forker et. al. Such a data set would enable researchers to narrow the choices of functional form empirically through appropriate specification tests.

In addition to resolving the functional form problem, the model could be improved to better reflect the characteristics of the dairy market environment. For example, the model does not account for the fact that political goodwill may accrue when advertising efforts increase demand and thereby reduce government expenditures on the dairy support program. In light of the 1985 Food Security Act, which gives the Secretary of Agriculture the power to adjust dairy support prices in response to surplus levels, the potential for political goodwill is of increasing importance to dairy farmers. If the possibility for political goodwill were incorporated, optimal advertising expenditure levels might be higher than those found in this study. To adapt the model to reflect the political economy of the entire dairy industry, researchers would need to specify the behavior of government in setting support prices. With an endogenized government support price which is a function of the dairy surplus, the adapted model should also allow for manufactured dairy product advertising, since the effect of such expenditures would no longer simply be to replace government purchases with private consumption, but rather would result in a farm price impact.

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APPENDIX A: Derivation of Comparative Statics

The equation system to be solved is B ζ = b where the definitions of B, ζ, and b are:

$$B \equiv \begin{pmatrix} \Phi_i' & -\phi_i & 0 \\ 0 & \Psi_A & \Psi_S - \psi \\ \Phi_i''[\delta + P\Psi_A/\theta] & \Phi_i' P[\Psi_{AA}/\theta + \Psi_A\Psi_{AS}/\theta^2] & \Phi_i' P[\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \end{pmatrix}$$

$$\zeta \equiv \begin{pmatrix} dU_i \\ dA_i \\ dS \end{pmatrix}$$

$$b \equiv \begin{pmatrix} 0 \\ -\Psi_\delta\, d\delta - \Psi_P\, dP \\ -\Phi_i' \left\{ 1 + P\,[\Psi_{A\delta}/\theta + \Psi_A\Psi_{S\delta}/\theta^2] \right\} d\delta \;-\; \Phi_i' \left\{ [\Psi_A + P\Psi_{AP}]/\theta + P\Psi_A\Psi_{SP}/\theta^2 \right\} dP \;+\; dr \end{pmatrix}$$

In addition, the following holds:

θ	$\equiv$	$r + \psi - \Psi_S$	> 0	$(\because \Psi_S < 0)$
Ψ_A	$=$	$f'\delta/S$	> 0	$(\because f' > 0)$
Ψ_S	$=$	$-f'\delta A/S^2$	< 0	$(\because f' > 0)$
Ψ_δ	$=$	$f'A/S$	> 0	$(\because f' > 0)$
Ψ_P	$=$	f'	> 0	$(\because f' > 0)$
Ψ_{AA}	$=$	$f''\delta^2/S^2$	< 0	$(\because f'' < 0)$
Ψ_{AP}	$=$	$\Psi_{AA}S/\delta$	< 0	$(\because \Psi_{AA} < 0)$
Ψ_{SP}	$=$	$-\Psi_{AA}A/\delta$	> 0	$(\because \Psi_{AA} < 0)$
Ψ_{AS}	$=$	$-(\delta/S^2)\,\{f''\delta A/S + f'\}$		
Ψ_{SS}	$=$	$(\delta A/S^3)\,\{f''\delta A/S + 2f'\}$	> 0	(if $\Psi_{AS} < 0$)
$\Psi_{A\delta}$	$=$	$-\Psi_{AS}S/\delta$	> 0	(if $\Psi_{AS} < 0$)

$$\Psi_{S\delta} = \Psi_{AS}A/\delta \qquad < 0 \qquad (\text{if } \Psi_{AS} < 0)$$

Substituting $f'A/S$ for Ψ_δ, f' for Ψ_P, $\Psi_{AA}S/\delta$ for Ψ_{AP}, $-\Psi_{AA}A/\delta$ for Ψ_{SP}, $-\Psi_{AS}S/\delta$ for $\Psi_{A\delta}$, and $\Psi_{AS}A/\delta$ for $\Psi_{S\delta}$, the vector b can be written as:

$$b \equiv \begin{pmatrix} 0 \\ - f'A/S\, d\delta - f'\, dP \\ - \Phi_i' \left\{ 1 + P\, [(-S/\delta)\Psi_{AS}/\theta + \Psi_A(A/\delta)\Psi_{AS}/\theta^2] \right\} d\delta \\ \qquad - \Phi_i' \left\{ [\Psi_A + P(S/\delta)\Psi_{AA}]/\theta + P\Psi_A(-A/\delta)\Psi_{AA}/\theta^2 \right\} dP + dr \end{pmatrix}$$

<u>Comparative Statics with respect to r</u>:

Set dP and $d\delta$ equal to zero and express the resulting equation system as $B\,\xi = c$ where the definitions for ξ and c are:

$$\xi \equiv \begin{pmatrix} dU_i/dr \\ dA_i/dr \\ dS\ /dr \end{pmatrix}$$

$$c \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Using Cramer's rule, solution for the kth component of ξ (k = dU_i/dr, dA_i/dr, and dS/dr) can be expressed as:

$$\xi_k = |B_k| \ / \ |B|$$

where $|B|$ is the determinant of B and $|B_k|$ is obtained by replacing the kth column of the determinant $|B|$ by the column vector ξ. Specifically,

$$|B_1| = \phi_i\ (\psi - \Psi_S) \qquad > 0$$

$$|B_2| = \Phi_i'\ (\psi - \Psi_S) \qquad > 0$$

$$|B_3| = \Phi_i'\ \Psi_A \qquad > 0$$

In the above, Ψ_A is positive and Ψ_S is negative because we assume an upwardly slopped farm supply curve (i.e., $f' > 0$). Also, Φ_i' is positive as it is the marginal impact of advertising on fluid sales. With further assumptions that $\Phi_i'' < 0$ and $f'' < 0$, $|B|$ is negative:

$$\begin{aligned}
|B| &= \phi_i \Phi_i''(\psi - \Psi_S)\ [\delta + P\Psi_A/\theta] \ + \ (\Phi_i')^2 \Psi_A P\ [\Psi_{AS}/\theta + \Psi_A \Psi_{SS}/\theta^2] \\
&\quad + (\Phi_i')^2 (\psi - \Psi_S) P\ [\Psi_{AA}/\theta + \Psi_A \Psi_{AS}/\theta^2] \\
&= \phi_i \Phi_i''(\psi - \Psi_S)\ [\delta + P\Psi_A/\theta] \ + \ (\Phi_i')^2 \Psi_A P/\theta^2\ [\Psi_{AS}(r+\psi) + (f')^2 \delta^2 A/S^4] \\
&\quad + (\Phi_i')^2 (\psi - \Psi_S) P/\theta^2\ [\Psi_{AA}(r+\psi) - (f')^2 \delta^2/S^3] \\
&= \phi_i \Phi_i''(\psi - \Psi_S)\ [\delta + P\Psi_A/\theta] \ + \ (\Phi_i')^2 P(r+\psi)/\theta^2\ [\Psi_A \Psi_{AS} + (\psi - \Psi_S)\Psi_{AA}] \\
&\quad + (\Phi_i')^2 P/\theta^2\ [\Psi_A (f')^2 \delta^2 A/S^4 - (\psi - \Psi_S)(f')^2 \delta^2/S^3] \\
&= \phi_i \Phi_i''(\psi - \Psi_S)\ [\delta + P\Psi_A/\theta] \ + \ (\Phi_i')^2 P(r+\psi)\delta^2/(S^2\theta^2)\ [\psi f'' - (f')^2/S] \\
&\quad - (\Phi_i')^2 P/\theta^2\ [\psi (f')^2 \delta^2/S^3] \qquad < \ 0
\end{aligned}$$

Given the signs established above, it is clear that $dU_i/dr < 0$, $dA_i/dr < 0$, and $dS/dr < 0$.

<u>Comparative Statics with respect to δ:</u>

Set dr and dP equal to zero and express the resulting equation system as $B\ \xi = c$ where the definitions for ξ and c are:

$$\xi \equiv \begin{pmatrix} dU_i/d\delta \\ dA_i/d\delta \\ dS\ /d\delta \end{pmatrix}$$

$$c \equiv \begin{pmatrix} 0 \\ -\ f'A/S \\ -\ \Phi_i' \left\{ 1 + P\ [(-S/\delta)\Psi_{AS}/\theta + \Psi_A (A/\delta)\Psi_{AS}/\theta^2] \right\} \end{pmatrix}$$

We need to determine the signs of $|B_1|$, $|B_2|$, and $|B_3|$:

$$
\begin{aligned}
|B_1| &= -\ \phi_i(\psi-\Psi_S)\Phi_i' \left\{ 1 + (P/\delta)\ [-S\Psi_{AS}/\theta + A\Psi_A\Psi_{AS}/\theta^2] \right\} \\
&\quad -\ \phi_i\Phi_i' Pf'A/S\ [\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \\
&= -\ \phi_i(\psi-\Psi_S)\Phi_i' \ +\ \phi_i(\psi-\Psi_S)\Phi_i' P/(\delta\theta^2)\ [S\Psi_{AS}(r+\psi)] \\
&\quad -\ \phi_i\Phi_i' Pf'A/(S\theta^2)\ [\Psi_{AS}(r+\psi) + (f')^2\delta^2 A/S^4] \\
&= -\ \phi_i(\psi-\Psi_S)\Phi_i' \ -\ \phi_i\Phi_i' P(f')^3A^2\delta^2/(S^5\theta^2) \\
&\quad +\ \phi_i\Phi_i' P(r+\psi)/\theta^2 \left\{ [(\psi-\Psi_S)/\delta]\ S\Psi_{AS} - [f'A/S]\ \Psi_{AS} \right\} \\
&= -\ \phi_i(\psi-\Psi_S)\Phi_i' \ -\ \phi_i\Phi_i' P(f')^3A^2\delta^2/(S^5\theta^2) \\
&\quad +\ \phi_i\Phi_i' P(r+\psi)/\theta^2 \left\{ (\psi S/\delta)\Psi_{AS} \right\} \qquad < 0 \ \text{ if } \Psi_{AS} < 0
\end{aligned}
$$

$$
\begin{aligned}
|B_2| &= -\ (\Phi_i')^2 Pf'A/S\ [\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \ -\ (\Phi_i')^2(\psi-\Psi_S) \\
&\quad -\ (\Phi_i')^2 P(\psi-\Psi_S)/\delta\ [-S\Psi_{AS}/\theta + A\Psi_A\Psi_{AS}/\theta^2] \\
&= -\ (\Phi_i')^2 Pf'A/(S\theta^2)\ [\Psi_{AS}(r+\psi) + (f')^2\delta^2A/S^4] \ -\ (\Phi_i')^2(\psi-\Psi_S) \\
&\quad +\ (\Phi_i')^2 P(\psi-\Psi_S)/(\delta\theta^2)\ [S\Psi_{AS}(r+\psi)] \\
&= -\ (\Phi_i')^2 P(f')^3A^2\delta^2/(S^5\theta^2) \ -\ (\Phi_i')^2(\psi-\Psi_S) \\
&\quad -\ (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ [f'A/S]\ \Psi_{AS} - [(\psi-\Psi_S)/\delta]\ S\Psi_{AS} \right\} \\
&= -\ (\Phi_i')^2 P(f')^3A^2\delta^2/(S^5\theta^2) \ -\ (\Phi_i')^2(\psi-\Psi_S) \\
&\quad +\ (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ (\psi S/\delta)\Psi_{AS} \right\} \qquad < 0 \ \text{ if } \Psi_{AS} < 0
\end{aligned}
$$

$$
\begin{aligned}
|B_3| &= -\ (\Phi_i')^2\Psi_A \ -\ (\Phi_i')^2 P\Psi_A/\delta\ [-S\Psi_{AS}/\theta + A\Psi_A\Psi_{AS}/\theta^2] \\
&\quad +\ \phi_i\Phi_i'' f'A/S\ [\delta+P\Psi_A/\theta] \ +\ (\Phi_i')^2 Pf'A/S\ [\Psi_{AA}/\theta + \Psi_A\Psi_{AS}/\theta^2] \\
&= -\ (\Phi_i')^2\Psi_A \ -\ (\Phi_i')^2 P\Psi_A/(\delta\theta^2)\ [-S\Psi_{AS}(r+\psi)] \\
&\quad +\ \phi_i\Phi_i'' f'A/S\ [\delta+P\Psi_A/\theta] + (\Phi_i')^2 Pf'A/(S\theta^2)\ [\Psi_{AA}(r+\psi) - (f')^2\ (\delta^2/S^3)] \\
&= -\ (\Phi_i')^2\Psi_A \ +\ \phi_i\Phi_i'' f'A/S\ [\delta+P\Psi_A/\theta] \ -\ (\Phi_i')^2 P(f')^3A\delta^2/(S^4\theta^2) \\
&\quad +\ (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ [\Psi_A/\delta]\ S\Psi_{AS} + [f'A/S]\ \Psi_{AA} \right\} \\
&= -\ (\Phi_i')^2\Psi_A \ +\ \phi_i\Phi_i'' f'A/S\ [\delta+P\Psi_A/\theta] \ -\ (\Phi_i')^2 P(f')^3A\delta^2/(S^4\theta^2) \\
&\quad -\ (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ [(f')^2\delta/S^2] \right\} \qquad < 0
\end{aligned}
$$

Given the signs established above and $|B|$ being negative, $dU_i/d\delta > 0$, $dA_i/d\delta > 0$, and $dS/d\delta > 0$.

Comparative Statics with respect to P:

Set dr and $d\delta$ equal to zero and express the resulting equation system as $B\ \xi = c$ where the definitions for ξ and c are:

$$\xi \equiv \begin{pmatrix} dU_i/dP \\ dA_i/dP \\ dS\ /dP \end{pmatrix}$$

$$c \equiv \begin{pmatrix} 0 \\ -\ f' \\ -\ \Phi_i' \left\{ [\Psi_A + P(S/\delta)\Psi_{AA}]/\theta + P\Psi_A(-A/\delta)\Psi_{AA}/\theta^2 \right\} \end{pmatrix}$$

As before, we need to determine the signs associated with $|B_1|$, $|B_2|$, and $|B_3|$:

$$\begin{aligned} |B_1| &= -\ \phi_i(\psi-\Psi_S)\Phi_i'\Psi_A/\theta \ -\ \phi_i(\psi-\Psi_S)\Phi_i'P/\delta\ [S\Psi_{AA}/\theta - A\Psi_A\Psi_{AA}/\theta^2] \\ &\quad -\ \phi_i\Phi_i'Pf'\ [\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \\ &= -\ \phi_i(\psi-\Psi_S)\Phi_i'\Psi_A/\theta \ -\ \phi_i(\psi-\Psi_S)\Phi_i'P/(\delta\theta^2)\ [S\Psi_{AA}(r+\psi)] \\ &\quad -\ \phi_i\Phi_i'Pf'/\theta^2\ [\Psi_{AS}(r+\psi) + (f')^2\delta^2A/S^4] \\ &= -\ \phi_i(\psi-\Psi_S)\Phi_i'\Psi_A/\theta \ -\ \phi_i\Phi_i'P(f')^3\delta^2A/(S^4\theta^2) \\ &\quad -\ \phi_i\Phi_i'P(r+\psi)/\theta^2 \left\{ [(\psi-\Psi_S)/\delta]\ S\Psi_{AA} + f'\Psi_{AS} \right\} \\ &= -\ \phi_i(\psi-\Psi_S)\Phi_i'\Psi_A/\theta \ -\ \phi_i\Phi_i'P(f')^3\delta^2A/(S^4\theta^2) \\ &\quad +\ \phi_i\Phi_i'P(r+\psi)/\theta^2 \left\{ -\ \Psi_{AA}\psi S/\delta + (f')^2\delta/S^2 \right\} \\ &= -\ \phi_i(\psi-\Psi_S)\Phi_i'\Psi_A/\theta \ -\ \phi_i\Phi_i'P(f')^3\delta^2A/(S^4\theta^2) \\ &\quad +\ \phi_i\Phi_i'P(r+\psi)/\theta^2 \left\{ -\ f''\psi\delta/S + (f')^2\delta/S^2 \right\} \quad \gtrless 0 \end{aligned}$$

$$
\begin{aligned}
|B_2| &= - (\Phi_i')^2 Pf' \; [\Psi_{AS}/\theta + \Psi_A\Psi_{SS}/\theta^2] \;-\; (\Phi_i')^2(\psi-\Psi_S)\Psi_A/\theta \\
&\quad - (\Phi_i')^2 P(\psi-\Psi_S)/\delta \; [S\Psi_{AA}/\theta - A\Psi_A\Psi_{AA}/\theta^2] \\
&= - (\Phi_i')^2 Pf'/\theta^2 \; [\Psi_{AS}(r+\psi) + (f')^2\delta^2 A/S^4] \;-\; (\Phi_i')^2(\psi-\Psi_S)\Psi_A/\theta \\
&\quad - (\Phi_i')^2 P(\psi-\Psi_S)/(\delta\theta^2) \; [S\Psi_{AA}(r+\psi)] \\
&= - (\Phi_i')^2 P(f')^3\delta^2 A/(S^4\theta^2) \;-\; (\Phi_i')^2(\psi-\Psi_S)\Psi_A/\theta \\
&\quad - (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ f'\Psi_{AS} + [(\psi-\Psi_S)/\delta] \; S\Psi_{AA} \right\} \\
&= - (\Phi_i')^2 P(f')^3\delta^2 A/(S^4\theta^2) \;-\; (\Phi_i')^2(\psi-\Psi_S)\Psi_A/\theta \\
&\quad - (\Phi_i')^2 P(r+\psi)/\theta^2 \left\{ (\delta/S) \; [f''\psi - (f')^2/S] \right\} \qquad \gtrless 0
\end{aligned}
$$

$$
\begin{aligned}
|B_3| &= - (\Phi_i')^2\Psi_A \; [(\Psi_A/\theta + PS\Psi_{AA}/(\delta\theta) - PA\Psi_A\Psi_{AA}/(\delta\theta^2)] \\
&\quad + \phi_i\Phi_i''f' \; [\delta+P\Psi_A/\theta] \;+\; (\Phi_i')^2 Pf' \; [\Psi_{AA}/\theta + \Psi_A\Psi_{AS}/\theta^2] \\
&= - (\Phi_i')^2(\Psi_A)^2/\theta \;-\; [(\Phi_i')^2\Psi_A P/\delta] \; [S\Psi_{AA}/\theta - A\Psi_A\Psi_{AA}/\theta^2] \\
&\quad + \phi_i\Phi_i''f' \; [\delta+P\Psi_A/\theta] \;+\; (\Phi_i')^2 Pf' \; [\Psi_{AA}/\theta + \Psi_A\Psi_{AS}/\theta^2] \\
&= - (\Phi_i')^2(\Psi_A)^2/\theta \;-\; [(\Phi_i')^2\Psi_A P/(\delta\theta^2)] \; S\Psi_{AA} \; (r+\psi) \\
&\quad + \phi_i\Phi_i''f' \; [\delta+P\Psi_A/\theta] \;+\; [(\Phi_i')^2 Pf'/\theta^2] \; [\Psi_{AA}(r+\psi) - (f')^2\delta^2/S^3] \\
&= - (\Phi_i')^2(\Psi_A)^2/\theta \;+\; \phi_i\Phi_i''f' \; [\delta+P\Psi_A/\theta] \\
&\quad - (\Phi_i')^2 P(f')^3\delta^2/(S^3\theta^2) \qquad < 0
\end{aligned}
$$

Given the above and $|B| < 0$, one finds $dU_i/dP \gtrless 0$, $dA_i/dP \gtrless 0$, and $dS/dP > 0$.

APPENDIX B: Data and Sources

The data used in the estimation are presented in Table B. The sources for the data are listed below. In the table, the number in parentheses corresponds to the sources that the data were collected from.

(1) The New York State Department of Agriculture and Markets, Division of Dairy Industry Services. Contact person: Edward J. Johnston, Jr. TEL: (518) 457-5888.

(2) The American Dairy Association and Dairy Council. Contact person: Brian Ward. TEL: (315) 472-9143.

(3) "Employment Review". New York State Department of Labor.

(4) "Prevailing Retail Whole Milk Price in 1/2 Gallons in Supermarkets and Food Stores". The New York State Department of Agriculture and Markets, Division of Dairy Industry Services.

(5) "CPI Detailed Report". Bureau of Labor Statistics.

(6) "New York State Dairy Statistics". The New York State Department of Agriculture and Markets, Division of Dairy Industry Services.

(7) "New York Agricultural Statistics". The New York State Department of Agriculture and Markets, Division of Statistics.

Table B: Data Used in the Econometric Estimation

YEAR AND MONTH	RETAIL FLUID SALES (10 MILLION LBS)			GENERIC FLUID ADVERTISING (THOUSAND DOLLARS)		
	NYC	SYR	ALB	NYC	SYR	ALB
1983.01	20.202	1.9903	1.9676	131.95	9.9260	3.6950
1983.02	18.867	1.8156	1.7786	210.40	11.015	11.085
1983.03	20.452	2.0214	2.0065	178.30	9.3580	8.6850
1983.04	20.044	1.9578	1.9417	182.02	7.4170	7.6120
1983.05	19.446	1.9448	1.6115	218.83	8.8560	11.188
1983.06	18.714	1.9511	1.5693	193.49	9.9130	8.0690
1983.07	18.531	1.7408	1.6379	234.90	8.3540	7.9190
1983.08	19.134	1.7468	1.5872	201.94	8.2790	7.8690
1983.09	19.924	1.8969	1.6293	326.01	16.274	8.8310
1983.10	20.656	2.0487	1.7165	325.50	13.600	11.880
1983.11	20.229	2.0189	1.7137	230.91	9.9310	16.183
1983.12	21.376	2.0564	1.8194	164.52	7.9560	13.358
1984.01	20.291	1.8356	1.6642	199.09	9.6940	11.665
1984.02	19.282	1.7525	1.4715	183.86	10.710	12.640
1984.03	21.467	1.9664	1.6738	163.76	16.240	18.510
1984.04	19.207	1.7132	1.5122	259.05	12.590	17.238
1984.05	20.270	2.0072	1.7001	210.83	11.670	16.414
1984.06	19.204	1.8178	1.6044	330.53	17.093	22.571
1984.07	18.071	1.7834	1.6407	58.904	5.8680	2.5200
1984.08	19.083	1.8846	1.7458	321.37	24.858	27.439
1984.09	19.275	1.9688	1.7233	604.52	21.810	27.612
1984.10	21.007	2.1374	1.8091	1017.8	40.510	47.650
1984.11	20.343	2.0459	1.8710	542.03	19.915	29.303
1984.12	20.328	2.0894	1.9023	426.56	13.165	19.090
1985.01	20.530	2.1043	1.9680	142.42	8.9950	8.0810
1985.02	18.668	1.8856	1.6733	243.90	12.220	15.871
1985.03	20.610	2.0238	1.8116	457.59	23.330	22.929
1985.04	19.669	1.9631	1.7588	465.61	16.570	14.776
1985.05	20.175	1.9793	1.7097	367.16	12.540	26.893
1985.06	18.979	1.7225	1.8240	347.19	13.960	39.010
1985.07	19.237	1.8526	2.0143	312.49	12.470	19.581
1985.08	19.256	1.8007	1.6932	276.26	13.195	20.131
1985.09	19.120	1.8038	1.6387	226.29	14.420	18.894
1985.10	20.854	1.9303	1.8159	199.26	11.150	15.596
1985.11	19.979	1.8725	1.9218	170.16	7.8400	10.786
1985.12	20.195	1.8737	1.9331	0.0100	0.0100	0.0100
1986.01	20.319	1.9147	1.9288	87.014	7.5850	12.365
1986.02	17.008	1.8814	1.9154	246.57	11.180	10.945
1986.03	18.607	2.0218	2.1312	229.12	10.430	11.815
1986.04	18.148	2.0346	2.0782	84.586	10.330	14.615
1986.05	20.588	2.0212	1.7126	256.96	8.7900	18.675
1986.06	19.160	1.9170	1.6204	188.48	10.750	14.455

Table B (Continued): Data Used in the Econometric Estimation

YEAR AND MONTH	RETAIL FLUID SALES (10 MILLION LBS)			GENERIC FLUID ADVERTISING (THOUSAND DOLLARS)		
	NYC	SYR	ALB	NYC	SYR	ALB
1986.07	19.554	1.9280	1.7064	309.82	14.516	16.235
1986.08	20.418	2.0852	1.7250	416.13	21.545	26.900
1986.09	19.994	2.2189	1.7630	225.73	13.715	39.785
1986.10	21.092	2.0776	1.8517	463.46	15.075	19.725
1986.11	19.663	1.9810	1.7727	146.59	6.2800	7.6850
1986.12	21.232	2.0666	1.8162	0.0100	0.0100	0.0100
1987.01	21.215	1.9567	1.9790	84.295	7.0550	8.6250
1987.02	19.009	1.7147	1.5494	138.91	6.8650	11.860
1987.03	20.790	1.9283	1.7737	125.73	8.0150	10.780
1987.04	20.044	1.8059	1.6896	139.29	5.0000	8.8850
1987.05	20.557	1.9800	1.7752	290.22	6.6000	9.3150
1987.06	19.813	1.7656	1.6615	152.37	5.5500	8.3800
1987.07	19.019	1.8791	1.7367	120.78	4.9950	5.8350
1987.08	18.684	1.9177	1.7994	292.46	17.865	24.070
1987.09	19.387	2.0069	1.8043	12.850	0.6500	0.0100
SOURCES:	(1)	(1)	(1)	(2)	(2)	(2)

(1) The **retail fluid sales** data were provided by the New York State Department of Agriculture and Markets, Division of Dairy Industry Services. For detailed information, contact Edward J. Johnston, Jr. at (518) 457-5888.

(2) The **advertising expenditures** data were provided by the American Dairy Association and Dairy Council. For detailed information, contact Brian Ward at (315) 472-9143.

Table B (Continued): Data Used in the Econometric Estimation

YEAR AND MONTH	AVERAGE WEEKLY EARNINGS (DOLLARS PER WEEK)			RETAIL MILK PRICE ($ / 0.5 GALLON)			FOOD & BEVERAGE PRICE INDEX (1967 =100)	(1977 =100)
	NYC	SYR	ALB	NYC	SYR	ALB	NYC	NORTHEAST
1983.01	298.28	373.33	353.63	1.18	0.99	1.10	281.10	146.10
1983.02	296.24	376.67	362.58	1.18	0.99	1.13	282.70	146.10
1983.03	301.88	377.34	363.20	1.17	0.99	1.13	283.90	146.70
1983.04	305.75	375.47	365.22	1.17	0.99	1.13	284.60	147.30
1983.05	305.37	380.15	358.09	1.17	0.99	1.13	284.90	147.45
1983.06	309.32	381.05	362.79	1.17	0.99	1.13	285.00	147.60
1983.07	306.68	389.20	366.28	1.17	0.99	1.13	285.10	147.85
1983.08	303.62	394.54	367.83	1.17	0.99	1.13	285.40	148.10
1983.09	311.47	402.11	377.92	1.17	0.99	1.13	288.10	148.20
1983.10	316.99	401.23	377.92	1.17	0.99	1.13	287.80	148.30
1983.11	318.90	428.16	383.78	1.18	0.99	1.13	286.90	148.55
1983.12	324.10	431.96	382.75	1.18	0.99	1.13	289.00	148.80
1984.01	317.25	428.40	366.28	1.18	0.99	1.13	294.80	150.70
1984.02	323.47	430.31	369.36	1.18	0.99	1.13	296.90	152.60
1984.03	320.17	425.15	366.40	1.18	0.99	1.13	298.30	152.70
1984.04	323.47	426.63	380.55	1.18	0.99	1.13	297.70	152.80
1984.05	322.71	425.39	380.42	1.18	0.99	1.13	296.50	152.90
1984.06	324.23	428.48	383.13	1.18	0.99	1.13	298.20	153.00
1984.07	321.20	431.14	390.86	1.18	0.99	1.13	298.80	153.60
1984.08	321.39	427.03	386.72	1.18	0.99	1.13	300.90	154.20
1984.09	327.66	421.70	395.24	1.17	0.99	1.13	300.60	154.00
1984.10	330.81	441.83	396.58	1.18	0.99	1.13	300.90	153.80
1984.11	337.08	438.43	389.03	1.19	0.99	1.13	300.10	153.85
1984.12	345.32	450.66	393.73	1.20	1.03	1.16	302.20	153.90
1985.01	331.88	448.03	375.80	1.21	1.03	1.16	305.10	155.35
1985.02	342.27	444.39	371.99	1.21	1.03	1.16	307.00	156.80
1985.03	340.87	443.72	374.37	1.21	1.03	1.16	308.00	157.10
1985.04	337.55	439.55	363.08	1.21	1.03	1.16	308.40	157.40
1985.05	341.00	434.43	376.66	1.21	1.03	1.16	307.90	157.40
1985.06	343.04	443.50	387.25	1.21	1.03	1.16	308.20	157.40
1985.07	343.60	445.94	393.22	1.20	1.03	1.16	308.40	157.55
1985.08	339.96	442.26	382.64	1.21	1.03	1.16	309.50	157.70
1985.09	345.71	448.92	399.27	1.21	1.03	1.16	311.30	157.90
1985.10	350.10	444.44	401.23	1.21	1.03	1.16	311.50	158.10
1985.11	353.96	451.25	405.98	1.20	1.03	1.16	311.80	158.70
1985.12	360.45	460.51	418.16	1.20	1.03	1.16	314.00	159.30
1986.01	350.58	451.82	407.54	1.21	1.03	1.16	317.50	160.05
1986.02	351.94	442.09	413.24	1.21	1.03	1.08	316.90	160.80
1986.03	358.87	442.38	414.70	1.21	1.03	1.08	317.40	161.25
1986.04	357.34	453.70	415.33	1.20	1.03	1.08	319.70	161.70
1986.05	356.44	450.96	416.34	1.20	1.03	1.08	320.60	161.85
1986.06	357.20	462.79	417.99	1.20	1.03	1.08	319.70	162.00

Table B (Continued): Data Used in the Econometric Estimation

YEAR AND MONTH	AVERAGE WEEKLY EARNINGS (DOLLARS PER WEEK)			RETAIL MILK PRICE ($ / 0.5 GALLON)			FOOD & BEVERAGE PRICE INDEX (1967 =100)	(1977 =100)
	NYC	SYR	ALB	NYC	SYR	ALB	NYC	NORTHEAST
1986.07	359.29	440.89	403.90	1.20	1.03	1.08	325.20	163.60
1986.08	356.44	472.16	406.36	1.19	1.03	1.08	327.40	165.20
1986.09	361.34	476.57	418.80	1.20	1.03	1.08	327.10	165.55
1986.10	364.43	471.33	416.52	1.21	1.03	1.08	329.10	165.90
1986.11	369.02	470.12	420.86	1.22	0.93	1.16	329.20	166.25
1986.12	373.26	478.92	429.30	1.23	0.93	1.16	330.80	166.60
1987.01	370.56	464.53	416.96	1.23	0.93	1.16	335.20	168.20
1987.02	373.63	456.85	427.99	1.14	0.93	1.16	335.90	168.50
1987.03	373.84	465.22	418.40	1.13	0.93	1.19	336.70	169.00
1987.04	369.90	449.12	414.88	1.12	0.93	1.19	337.90	169.40
1987.05	375.55	444.51	415.51	1.12	0.93	1.19	339.70	170.30
1987.06	379.26	459.68	420.73	1.12	0.93	1.19	343.60	171.70
1987.07	378.62	447.73	416.85	1.12	0.93	1.19	342.70	171.50
1987.08	373.32	457.32	420.03	1.12	0.95	1.16	343.70	171.80
1987.09	375.38	471.17	421.40	1.16	1.00	1.16	345.00	172.30
SOURCES:	(3)	(3)	(3)	(4)	(4)	(4)	(5)	(5)

(3) The **Average Weekly Earnings** data pertain to "Production Worker of Manufacturing Sector" and were collected from Employment Review, published by the New York State Department of Labor. Data for New York City cover the area of Bronx, King, Nassau, New York, Queens, Richmond, Putnam, Rockland, Suffolk, and Westchester. Data for Syracuse cover the area of Madison, Onondaga, and Oswego. Data for Albany cover the area of Albany, Montgomery, Rensselarer, Saratoga, and Schenectady.

(4) The **Retail Milk Price** data pertain to "Prevailing Retail Whole Milk Price in 1/2 Gallons in Supermarkets and Food Stores" and were published by the New York State Department of Agriculture and Markets, Division of Dairy Industry Services.

(5) The Food and Beverage Price Index data were collected from CPI Detailed Report, published by the Bureau of Labor Statistics.

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