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Resource Economics:
Five Easy Pieces

by

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Abstract

As in music and athletics, the development of technique and confidence in applied economics typically requires practice and exercise. The field of resource economics confronts both student and instructor with a difficult set of problems in static and dynamic optimization. This paper presents five numerical problems dealing with optimal forest rotation, management of a fishery, optimal depletion of an exhaustible resource, optimal static externality and control of a stock (dynamic) pollutant. By working through these problems the student should gain a better understanding of the economic theory underlying much of the recent literature in resource economics and see how the theory might be extended in new and important ways. A perverse few might regard these problems as fun.

Resource Economics:

Five Easy Pieces

Aspiring pianists frequently polish their skills by working through a set of compositions, often collected into a practice book and given a title like *Five Easy Pieces.*[1] These pieces are selected to develop technique and confidence; something sought in the arts, athletics and perhaps the applied fields of economics. This paper contains five numerical problems which students of resource economics might find helpful in strengthening their command of theory and enhancing their ability to "perform" applied research.

The problems presume an understanding of the maximum principle. No attempt is made to develop or review basic theory. Widely known results, from say forestry or bioeconomics, are simply stated, with references provided for those wishing to review the theory or the detailed derivation of specific formulae.

As in much of economics, the conditions for optimal resource allocation are often derived from the first-order conditions for an optimization problem. With appropriate concavity the first-order conditions become a system of equations whose solution yields the optimal allocation. For the simple (small-dimensioned) problems presented in this paper, elimination of variables might lead to a single equation in one unknown. For example, the Faustmann Formula in forestry or the "fundamental equation of bioeconomics" are single equations defining the optimal forest rotation and resource stock, respectively. Occasionally the equations will allow the derivation of an *explicit* expression for the optimal variable; that is, where the optimal variable is isolated on the left-hand-side (LHS) of an equation whose right-hand-side (RHS) is simply a function of constants and parameters. More often, when general optimality equations are evaluated for specific functional forms, they may only lead to an *implicit* expression for the optimal variable; that is, where the variable *cannot* be isolated and equated to a function of constants and parameters. In this case the unknown variable might be solved for using a numerical algorithm. The algorithm used for the problems dealing with forestry and fishery management is Newton's Method. It is briefly reviewed in the next section. In the remaining sections problems dealing with optimal forest rotation, fishery management, depletion of an exhaustible resource, static externality, and control of a stock pollutant will be posed and solved. Some extensions of these problems are briefly discussed in the final section.

Newton's Method

Suppose a set of first-order conditions may be reduced to a single equation written implicitly as

$$G(x,P) = 0 \tag{1}$$

where x is the remaining unknown and P is a vector of parameters. If the original optimization problem satisfied the appropriate concavity assumptions for existence and uniqueness, then the optimal value for x is the root (or zero) of G(x,P). Newton's method is one of several ways to numerically solve for the root of an implicit equation.[2] It requires that $G(\cdot)$ be continuous in x and have a continuous first derivative. The method is simple to code in any computer language and a 10 line program in BASIC is listed in Table 1. This program is used in the optimal forest rotation problem presented in the next section and here we will only discuss the logic behind the program's structure.

The parameters to the function $G(\cdot)$ are entered and read in lines 10 and 20. Line 30 requests a guess for the unknown variable, in this case the optimal rotation length, T^*. This guess serves as an initial condition for a process of adjustment which will hopefully converge to within some ε of the unknown optimal rotation.

After entering the guess for T^*, line 40 calculates values for two expressions (Q and E) which appear in $G(\cdot)$ and its derivative and their definition in line 40 shortens the amount of code to be written in lines 50 and 70. Line 50 calculates the value of $G(\cdot)$ based on the initial guess and line 60 determines if $G(\cdot)$ is sufficiently close to zero. In this example the root has been found if the absolute value of $G(\cdot)$ is less than $\varepsilon = 0.0001$.

In the unlikely case that an initial guess was sufficiently close to the root ($|G(\cdot)| < \varepsilon$), line 60 would instruct the computer to go to line 90 and print the result, calling it T^*. If it were not sufficiently close ($|G(\cdot)| \geq \varepsilon$) the computer proceeds to line 70 where it calculates the value of the derivative $G'(\cdot)$, defined as DG in the code. The derivative of $G(\cdot)$ is used in updating the guess for T^* according to line 80 where the new guess is equal to the old guess less the ratio $G(\cdot)/G'(\cdot)$. The computer is then directed back to line 40 where it recalculates Q and E, then to line 50 to calculate G, and so on. For a formal derivation of this updating formula and a graphical portrayal of the process of convergence see Conrad and Clark (1987).

Newton's method can be extended to the solution of simultaneous equations where it is referred to as the Newton-Raphson method. While these methods are useful, there is no guarantee that the zero(s) will be found. Convergence may depend on the initial guess and on ε. In Figure 1 we show a plot of G from line 50 for the parameter values in line 10. By inspection the zero appears at about 37. If an initial guess of 20 is entered the algorithm drives T toward zero where $G'(\cdot)$ is undefined and an overflow error is given. Further, if uniqueness is not guaranteed a priori there may be multiple zeros and the particular zero found by the algorithm will again depend on the initial guess. With these pitfalls in mind we proceed to our five numerical problems.

Optimal Forest Rotation

One of the oldest questions in forestry is concerned with the optimal time to cut an even-aged stand of trees. The interval between planting and cutting is called the rotation length. The Faustmann formula can be used to solve for the rotation length which maximizes the present value of net revenues from an infinite series of rotations.

Let Q(t) represent the volume of merchantable timber in the current stand. Assume the parcel of land was just planted with seedlings and will require no thinning. It will be cut at some unknown date T years into the future and immediately replanted. If p is the price per unit of timber and c is the cost of of cutting and replanting, then the net revenue at $t = T$ is $N(T) = pQ(T) - c$. The present value of this single rotation is $N = [pQ(T) - c]\, e^{-\delta T}$, where δ is the instantaneous rate of discount.

With price, cost, the discount rate and soil productivity unchanging the rotation length is unchanging and the present value of an infinite number of future rotations is given by

$$V = [pQ(T) - c]e^{-\delta T}\{1 + e^{-\delta T} + e^{-\delta 2T} + \dots\} = \frac{[pQ(T) - c]}{[e^{\delta T} - 1]} \qquad (2)$$

The Faustmann formula is derived from the first order condition $dV/dT = 0$ and requires

$$\frac{pQ'(T)}{[pQ(T) - c]} = \frac{\delta}{(1 - e^{-\delta T})} \qquad (3)$$

The above expression can be shown to imply

$$pQ'(T) = \delta[pQ(T) - c] + \delta V \qquad (4)$$

which has a nice economic interpretation. The optimal rotation equates the marginal *value* of waiting, $pQ'(T)$, to the marginal *cost* of waiting, which is in turn comprised of two components: the interest payment foregone on the current stand, $\delta[pQ(T) - c]$, and the interest payment foregone by a marginal delay of all *future* stands, δV. This latter cost term is also the rental value of the parcel in a competitive land market and is frequently referred to as "site value." [See Johansson and Lofgren (1985) and Clark (1976) for additional details].

A functional form for $Q(t)$ which provides a good fit for many commercially grown species is

$$Q(t) = e^{a - b/t} \tag{5}$$

A graph of the function is shown in Figure 2. The volume of timber will asymptotically approach e^a. The rotation that maximizes $Q(T)/T$, sometimes called mean annual increment, is $T = b$.

The Faustmann formula leads to the implicit equation

$$G = pQ'(T)(1 - e^{-\delta T}) - \delta[pQ(T) - c] = 0 \tag{6}$$

With the above form for $Q(T)$ we note that $Q'(T) = e^{a - b/T}(b/T^2) = bQ(T)/T^2$, yielding the specific form for G given in line 50 of Table 1. The derivative of G is a bit messy, but some perseverance will show it to be equal to DG defined in line 70.

The optimal rotation will thus depend on the parameters a, b, c, δ and p (see line 20). Gamponia and Mendelsohn (1983) estimate that a = 13.06 and b = 145.61 for Douglas fir grown on a high quality site. If c = 150, δ = 0.1 and p = 100, then the optimal rotation length, from an initial guess of 30, is T^* = 37.72 years. If the discount rate is changed to δ = 0.05 the optimal rotation increases to T^* = 51.91 years. The comparative statics of changes in a, b, c, and p can be numerically explored and conform to the analytical results in the literature. [See Johansson and Lofgren (1985), pp. 80-85].

Fishery Management

A fairly general problem in the management of a single species fishery is to

$$\text{maximize} \quad \int_0^\infty W(X,Y)\, e^{-\delta t}\, dt$$

$$\text{subject to} \quad \dot{X} = F(X) - Y$$

where W(X,Y) is a welfare or net benefit function, dependent on the fish population (or stock) X and harvest Y, and where F(X) is a growth function. The current-value Hamiltonian is

$$\tilde{H} = W(X,Y) + \mu[F(X) - Y] \tag{7}$$

First-order conditions include $W_Y = \mu$ and $\dot{\mu} - \delta\mu = -W_X - \mu F'(X)$, where W_X and W_Y denote partial derivatives. In steady state we obtain

$$F'(X) + \frac{W_X}{W_Y} = \delta \tag{8}$$

With Y = F(X), equation (8) will become a single equation in X^*, the optimal steady-state stock. The left-hand-side of (8) may be interpreted as the resource's own (internal) rate of return. X^* is determined so as to equate that internal rate to the rate of discount, δ. The resource's rate of return has two components; the biological rate, $F'(X)$, and what has been referred to as the "marginal stock effect", W_X/W_Y (Clark and Munro 1975).[3]

Let E denote fishing effort (an aggregate economic input) and assume the production function $Y = qXE$ and the cost equation $C = cE$, where $q > 0$ is called the "catchability coefficient" and $c > 0$ is the cost per unit effort. These forms will imply $c(X) = c/(qX)$ as the stock-dependent average cost function.

With logistic growth, $F(X) = rX(1 - X/K)$. The parameters r and K are referred to as the intrinsic growth rate and the environmental carrying capacity, respectively. Collectively the entire model is referred to as the Gordon-Schaefer model and in this case equation (8) permits an explicit solution for the steady-state optimal stock, X^*, as a function of the bioeconomic parameters c, δ, K, p, q and r. Specifically

$$X^* = \frac{K}{4}\left[\frac{c}{pqK} + 1 - \frac{\delta}{r} + \sqrt{\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\gamma}{pqrK}}\right] \qquad (9)$$

This fortuitous outcome (an explicit solution for X^*) is the exception rather than the rule. More often, equation (8) will result in an implicit equation that might be solved numerically for a given set of parameters. For example, the Gompertz growth function takes the form $F(X) = rX \ln(K/X)$ with the derivative $F'(X) = r[\ln (K/X) - 1]$. If we retain the same production function and cost equation, so that $c(X) = c/(qX)$ and $c'(X) = -c/(qX^2)$, then equation (8) leads to

$$G = [r \ln(K/X) - (r + \delta)]pqX + (r + \delta)c = 0 \tag{10}$$

with the derivative

$$DG = [r \ln(K/X) - (2r + \delta)]pq \tag{11}$$

Table 2 contains a BASIC program, again using Newton's method, to solve for the optimal population when $c = 1$, $\delta = 0.1$, $K = 1$, $p = 2$, $q = 1$, and $r = 1$. From an initial guess between zero and one the optimal stock is $X^* = 0.7169$. From numerical analysis we would infer $dX^*/dc > 0$, $dX^*/d\delta < 0$, $dX^*/dK > 0$, $dX^*/dp < 0$, $dX^*/dq < 0$, and $dX^*/dr > 0$.

Optimal Depletion of a Nonrenewable Resource

Let R denote the remaining reserves of a nonrenewable resource at instant t. With no exploration or discovery of new reserves, R would decline over time according to $\dot{R} = -q$, where q is the rate of production. Suppose the net benefits to society are strictly a function q and given by B(q) at instant t. With initial reserves of R(0) the problem of maximizing the present value of net benefits may be stated

$$\text{maximize} \quad \int_0^T B(q)\, e^{-\delta t}\, dt$$

$$\text{subject to} \quad \dot{R} = -q,\ R \geq 0,\ R(0) \text{ given}$$

The current-value Hamiltonian is simply

$$\tilde{H} = B(q) - \mu q \tag{12}$$

with first order necessary conditions that include $B_q = \mu$ and $\dot{\mu} - \delta\mu = 0$. The latter condition implies that the current-value shadow price on remaining reserves is growing at the rate of discount. The transversality condition is $\tilde{H}(T) = 0$, which with $\mu(T) > 0$ will imply $q(T) = R(T) = 0$ and the optimal date of exhaustion is determined so that cumulative production just equals initial reserves.

Suppose extraction costs are zero and that marginal net benefit equals price defined by the inverse demand curve

$$p = ae^{-bq} \tag{13}$$

where $a > 0$ and $b > 0$. Then $B_q = \mu = p$ implies $\dot{\mu} = ae^{-bq}(-b\dot{q})$ and $\dot{\mu} - \delta\mu = 0$ implies $\dot{q} = -\delta/b$ which can be integrated directly yielding $q = c - \delta t/b$, where c is a constant of integration. At $t = T$, $q(T) = 0$ implying $c = \delta T/b$. This leads to the particular solution

$$q^* = (T - t)\delta/b \tag{14}$$

and production is seen to decline linearly over time.

The exhaustion constraint requires

$$R(0) = \frac{\delta}{b}\int_0^T (T - t)\,dt = \frac{\delta T^2}{2b} \tag{15}$$

implying that the date of exhaustion is $T = \sqrt{2bR(0)/\delta}$.

For $R(0) = 1$, $b = 5$ and $\delta = 0.1$ we obtain $T = 10$ and

$q^* = 0.02\ (10 - t)$ as the optimal time path for production. For other inverse demand curves it is unlikely that the exhaustion constraint will lead to an explicit equation for T and one may need to solve for T as the zero of an implicit expression.

Bees, Honey and Almonds

One of the classic examples of a positive externality involves the pollination services provided by bees as they collect nectar for the production of honey (Meade 1952)[4]. Consider the grower whose production of almonds benefits from pollination by a neighbor's bees according to the production function

$$A = 10\sqrt{X}(1 + 0.4\sqrt{B}) \tag{16}$$

where A is the output of almonds, B is the number of hives put out by the neighboring beekeeper and X is an aggregate input representing other factors of production. Assume the bees can derive nectar from a variety of sources (for free) and the production function for honey, H, depends only on the number of hives so that

$$H = 40\sqrt{B} \tag{17}$$

Suppose the unit cost of X is $w = 0.5$, the unit cost of operating a hive is $c = 20$, the unit price of almonds is normalized to $p_a = 1$ and the unit price of honey is $p_h = 5$. In isolation the beekeeper would seek to maximize profit given by

$$\pi_h = 200\sqrt{B} - 20B \tag{18}$$

and $d\pi_h/dB = 0$ requires $B = 25$.

To have a well-defined problem we assume that the almond grower can observe his neighbor and *given* $B = 25$ will seek to maximize

$$\pi_a = 10\sqrt{X}(1 + 0.4\sqrt{25}) - 0.5X = 30\sqrt{X} - 0.5X \tag{19}$$

Then $d\pi_a/dX = 0$ implies $X = 900$. This solution is not optimal and from society's point of view the number of hives and the amount of honey and almonds is deficient.

The optimal levels for B and X are found by maximizing

$$\pi = 10\sqrt{X}(1 + 0.4\sqrt{B}) + 200\sqrt{B} - 0.5X - 20B \tag{20}$$

The equations which result from $\partial\pi/\partial X = 0$ and $\partial\pi/\partial B = 0$ can be solved simultaneously to yield $B^* = 100$ and $X^* = 2{,}500$. In isolation the beekeeper's profits are $\pi_h = 500$, while the almond grower nets $\pi_a = 450$. At (X^*, B^*) the almond grower earns profits of $\pi_a = 1{,}250$, while the beekeeper nets nothing ($\pi_h = 0$). Obviously compensation of at least 500 would have to be offered to the beekeeper for him to increase the number of hives to $B^* = 100$. The almond grower could offer up to 800 and still achieve a profit of 450, which was earned in isolation. Thus, there is an incentive for negotiation and (X^*, B^*) might be supported by a payment of between 500 to 800 from the almond grower to the beekeeper.

Control of a Stock Pollutant

Let X now denote a stock pollutant; that is, a pollutant which can accumulate or degrade over time depending on the rate of residual discharge, R, and the rate of decomposition. The dynamics of the stock pollutant are given by

$$\dot{X} = -\gamma X + R \tag{21}$$

where γ is the rate of biodegradation.

The residual is jointly produced with the positively-valued commodity Q according to the commodity-residual transformation function, written implicitly as

$$\phi(Q,R) = 0 \tag{22}$$

By convention we assume the partials $\phi_Q > 0$ and $\phi_R < 0$. A graph of the transformation curve implied by $\phi(Q,R) = 0$ is shown in Figure 3. It assumes the existence of a fixed resource that may be allocated to commodity production or residual reduction (pollution control). If desired it would be possible to produce $Q = Q_0$ with no residual discharge, $R = 0$. If additional units of Q are desired the fixed resource must be reallocated from residual reduction to commodity production, resulting in $R > 0$. If all of the resource is allocated to commodity production then $Q = Q_{MAX}$ and $R = R_{MAX}$.

Suppose the net benefits from Q and the social cost from X are evaluated according to the concave function

$$W = W(Q,X) \tag{23}$$

with partials $W_Q > 0$ and $W_X < 0$. The problem of maximizing the present value of net benefits subject to the dynamics of the stock pollutant and the transformation function may be stated as

$$\text{maximize} \quad \int_0^\infty W(Q,X)\, e^{-\delta t}\, dt$$

$$\text{subject to} \quad \dot{X} = -\gamma X + R$$

$$\phi(Q,R) = 0$$

The current-value Hamiltonian for this problem is

$$\tilde{H} = W(Q,X) + \mu(-\gamma X + R) - \omega\phi(Q,R) \tag{24}$$

where $\mu < 0$ is the shadow price on the stock pollutant and $\omega > 0$ is the multiplier associated with the transformation function.

The first order conditions include $W_Q = \omega\phi_Q$, $\mu = \omega\phi_R$ and $\dot{\mu} - \delta\mu = -(W_X - \mu\gamma)$. In steady state these conditions imply

$$\frac{W_Q\, \phi_R}{\phi_Q} = \frac{W_X}{(\delta + \gamma)} \tag{25}$$

The left-hand-side of (25) may be interpreted as the the marginal value product from an incremental increase in R which allows an incremental increase in Q. (ϕ_R/ϕ_Q is the marginal rate of transformation of R for Q). On the right-hand-side we have the marginal social cost in perpetuity for the increase X that must also accompany the increase in R according to the steady-state relationship $X = R/\gamma$.

Consider the case where $W = pQ - cX^2$ and $\phi = Q - b - nR = 0$. Such a situation might arise in a small country or region which exports Q at constant world price p, but faces local environmental costs which increase at an increasing rate according to the quadratic term cX^2. The transformation curve is linear with $Q_0 = b$ and $\phi_R/\phi_Q = -n$. For these forms equation (25) may be solved for the optimal steady-state pollution stock yielding

$$X^* = \frac{np(\delta + \gamma)}{2c} \tag{26}$$

The comparative statics are immediately apparent: an increase in n, p, δ or γ causes an increase in X^*, while an increase in the social cost parameter, c, causes a decrease in the optimal pollution stock.

For these forms the current-value Hamiltonian is linear and the most rapid approach path (MRAP) is optimal (Spence and Starrett 1975). In this case if $X > X^*$, $R = 0$, while if $X < X^*$, then $R = R_{MAX}$. In the first instance, where the current pollution stock exceeds the optimal stock, residual discharge is zero (and $Q = Q_0$) and the stock decays according to $X = X(0)e^{-\gamma t}$. In the second instance, where the pollution stock is below the optimal stock, $R = R_{MAX}$ and the stock accumulates from X(0) according to

$$X = \frac{R_{MAX}}{\gamma} + \left[X(0) - \frac{R_{MAX}}{\gamma}\right]e^{-\gamma t} \tag{27}$$

By way of a numerical example, if $b = 3.5$, $c = 0.01$, $\delta = 0.1$, $\gamma = 0.2$, $n = 0.5$ and $p = 1$ equation (26) implies $X^* = 7.5$ and the state equation and transformation function imply $R^* = 1.5$ and $Q^* = 4.25$. If $R_{MAX} = 2.5$ and $X(0) = 0$ then the pollution stock will accumulate toward X^* according to $X = 12.5(1 - e^{-0.2t})$. It will take until $t^* = 4.58$ for X to reach $X^* = 7.5$. The complete solution is shown in Figure 4.

Extensions

There are numerous deterministic and stochastic extensions to the preceding problems. We conclude with a brief discussion of some of the deterministic problems which have been examined.[5]

What if a stand of timber provides a flow of nontimber benefits such as wildlife habitat for small game or recreation benefits for hikers? The Faustmann formula can be modified so that the optimal rotation accounts for these continuous nonmarket benefit flows. For the derivation of such a formula, a discussion of the likely affect on rotation length and possible corner solutions see Hartman (1976).

As a practical matter fishery management is likely to involve conservation and employment objectives. Charles (1988a, 1988b) reviews a growing literature on the socioeconomics of fisheries management and derives a modified "golden rule" when giving positive weight to employment (effort). The inclusion of a "preservation value", as would be appropriate for many marine mammals, has been analyzed by Vousden (1973).

The literature on nonrenewable resources is now quite vast, containing a rich selection of refinements and extensions to the seminal work of Hotelling (1931). For an interesting piece on the price implications of exploration and discovery see Pindyck (1978).

For a "gallery" of static models of externality see Baumol and Oates. For some dynamic models see Keeler, Spence and Zeckhauser (1972), d'Arge and Kogiku (1973), and Conrad (1988a, 1988b).

The problems contained in this paper are meant to illustrate some of the basic theory in resource economics. Mastery of these concepts can provide an entree to a large and growing literature important to the managment of resources and environmental quality.

Endnotes

[1] Moviegoers may recall the 1970 film *Five Easy Pieces* starring Jack Nicholson and Karen Black. Nicholson plays the prodigal son of a famous musician. The film's title refers to a practice book from Nicholson's more innocent youth.

[2] Interval bisection is another method which is easily coded for the computer. For details on this method and others see any introductory text on numerical methods, for example Henrici (1982).

[3] Clark (1976) originally derived the equation

$$F'(X) + \frac{c'(X)\,F(X)}{[p - c(X)]} = \delta$$

as a singular solution for the special case where $W(X,Y) = [p - c(X)]Y$, p being the per unit price for fish and c(X) being stock-dependent average cost. Derivation of equation (8) was left as an exercise. (See Clark 1976, pp.176-177).

[4] For additional information on the type of technological interdependence that can occur between growers and beekeepers see Cheung (1973) and Johnson (1973). The role of contracting is

discussed and empirical evidence suggests that the market for pollination services may have "internalized" this static externality. These articles are summarized in Hartwick and Olewiler (1986, pp. 432-435).

[5] For a brief introduction to some stochastic models see Conrad and Clark (1987, Chapter 5). For a more thorough presentation see Mangel (1985).

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Table 1. A Program to Solve for the Optimal Forest Rotation when Volume is given by $Q(t) = e^{a-b/t}$

```
10 DATA 13.06,145.61,150,0.1,100
20 READ A,B,C,D,P
30 INPUT "GUESS FOR T*=";T
40 Q=EXP(A-B/T):E=1-EXP(-D*T)
50 G=P*B*Q*E/(T^2)-D*(P*Q-C)
60 IF ABS(G)<.0001 GOTO 90
70 DG=E*(P*B*Q*(B-2*T)/(T^4)-D*P*B*Q/(T^2))
80 T=T-G/DG:GOTO 40
90 PRINT "T*=";T
100 END
```

Table 2. A Program to Solve for the Optimal Fish Stock with Gompertz Growth, $F(X) = rX \ln(K/X)$

```
10 DATA 1,0.1,1,2,1,1
20 READ C,D,K,P,Q,R
30 INPUT"GUESS FOR X*=";X
40 G=(R*LOG(K/X)-(R+D))*P*Q*X+(R+D)*C
50 IF ABS(G)< .00001 GOTO 80
60 DG=(R*LOG(K/X)-(2*R+D))*P*Q
70 X=X-G/DG:GOTO 40
80 PRINT"X*=";X
90 END
```

Figure 1. A Plot of G from Line 50 of Table 1

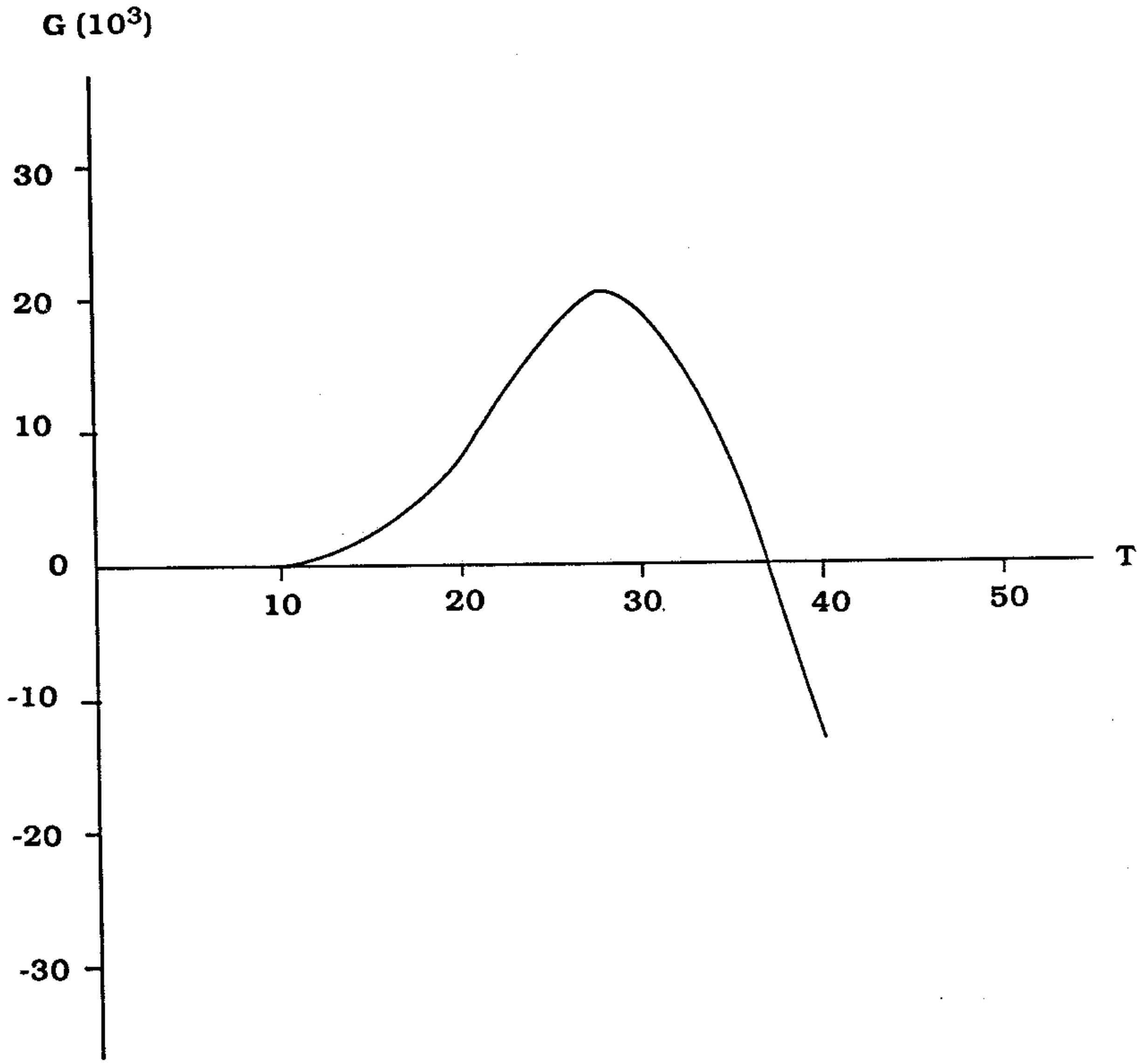


Figure 2. A Graph of the Volume Function
$Q(t) = e^{a - b/t}$

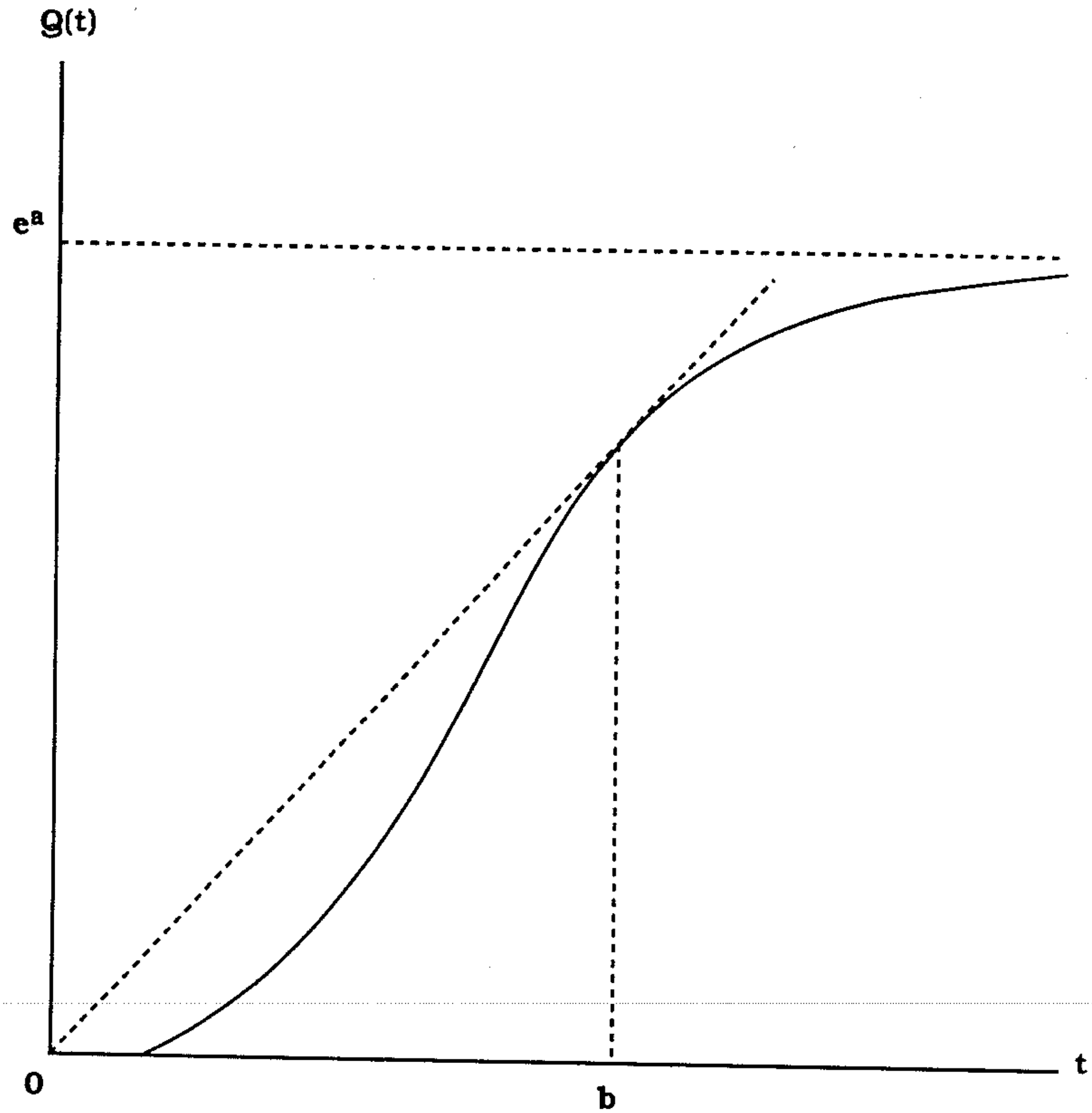


Figure 3. A Graph of the Commodity-Residual Transformation Curve

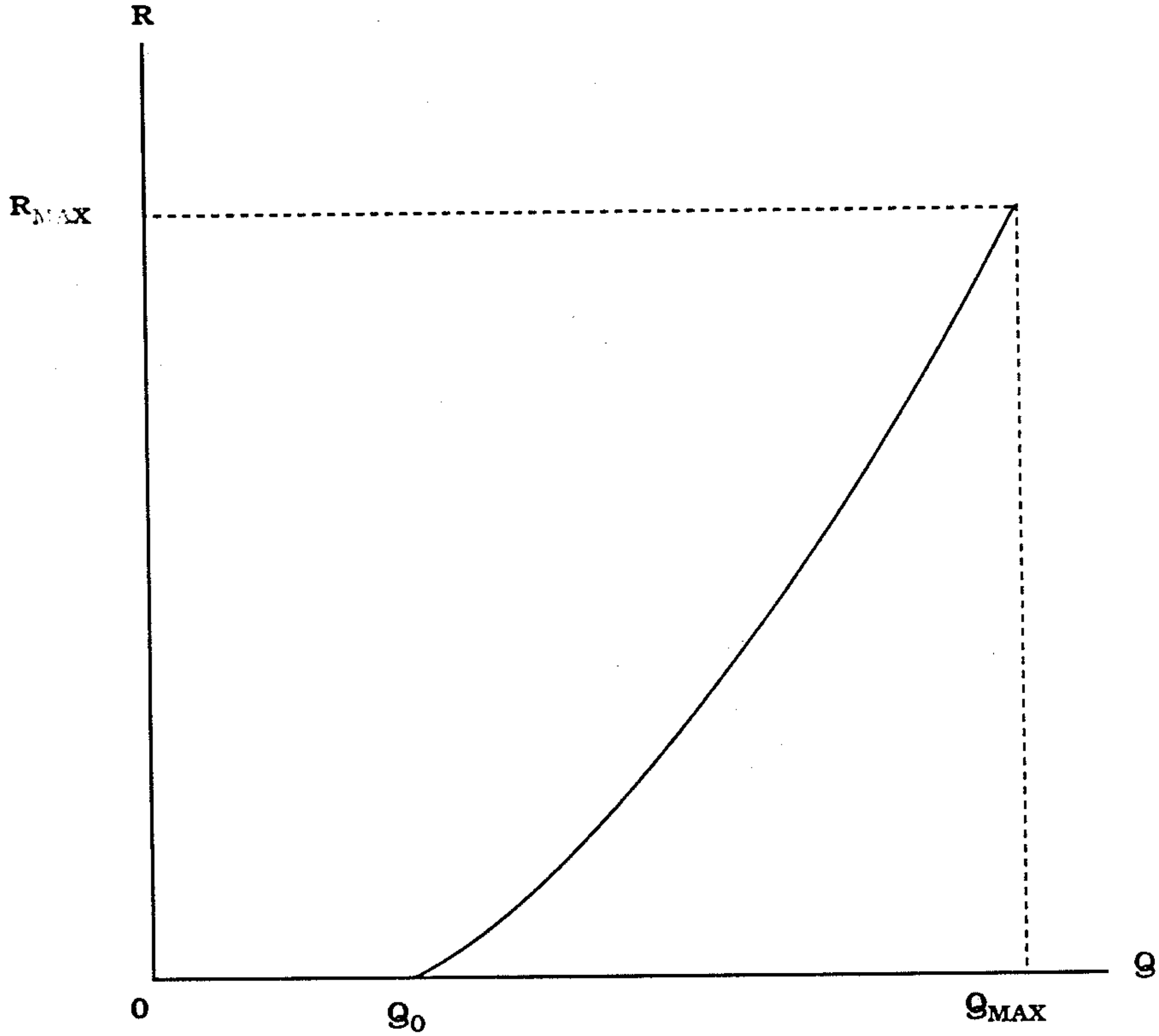
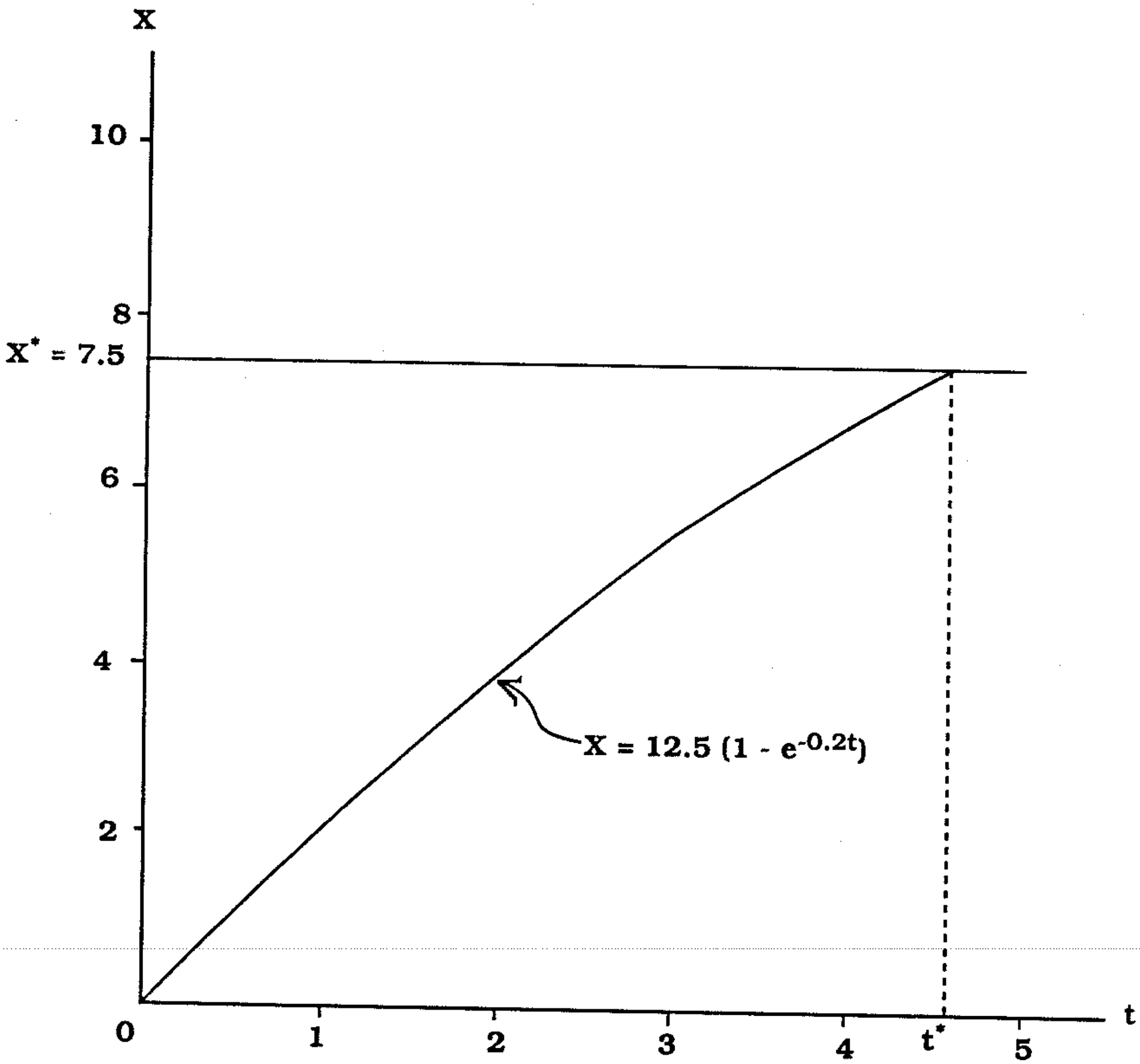


Figure 4. The Most Rapid Approach Path to the Optimal Pollution Stock $X^* = 7.5$ from $X(0) = 0$



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