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Empirical Implications of Reconciling the Behavioral Assumptions in Dual Models of Production

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ABSTRACT

Duality theory states that all the economically relevant aspects of a production technology are summarized by both the production function and the cost function. Yet, in empirical applications where the primal and dual results have been compared, inferences based on the two models are markedly divergent (Burgess and Humphrey and Moroney). In this paper it is demonstrated that elasticity estimates, etc. are quite sensitive to maintained hypotheses concerning producer behavior and that these maintained hypotheses differ in the primal and dual models typically found in the literature. Several new production and cost models are developed which are consistent with regard to behavioral assumptions and from which it is possible to achieve more consistent empirical results.

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Empirical Implications of Reconciling the Behavioral Assumptions in Dual Models of Production

For at least the past 15 years, empirical studies in production economics have relied increasingly on the use of flexible functional forms and the application of duality relationships. Through the use of flexible production and cost functions, economists have been able to relax many of the maintained hypotheses embodied in non-flexible forms and to test a richer set of hypotheses about technical and behavioral relationships in production. In the early to mid-1970's, the literature contained a number of applications of both flexible production and cost functions (e.g. Berndt and Christensen; Vinod; Binswanger; Burgess and Humphrey and Moroney), but since that time, the vast majority of studies involving empirically estimated production systems have been based on dual cost or profit function formulations. An important exception by Capalbo and Denny (1986) uses a translog production function to develop the linkages between gross and net productivity indexes for Canadian and U.S. agriculture.

Despite the widespread application of duality, all economically relevant aspects of a production technology can in theory be derived by either the primal (or production) approach or a dual specification of the firm's optimization problem, and Theil, in comparing flexible functional forms, argues that:

> These approaches are not self-dual, but if (1) the translog production function is a close approximation of the true production function, and (2) the translog cost function is a close approximation of the true cost function, we might hope that the implications of the two competing translog approximations are not too far apart (p.154).

In empirical work, however, the power of this theoretical equivalence can be eroded by the inadequacy of the functional form, statistical problems such as simultaneity, omitted variables or errors in variables, or specifications based on different behavioral assumptions.[1] The actual cause or causes of the divergent results obtained from dual models have never been satisfactorily identified. In his text, Theil speculates that differences in elasticity estimates from the two systems result from the complicated matrix inversions required to generate elasticity estimates from production system parameters. Regardless of the sources for these disparate estimates, if they can be identified and if the equivalence of the two frameworks can be restored (or nearly so) at the empirical level, we will obtain an important check on estimated results.

The purpose of this paper is to explore one (possibly overlooked) explanation for the divergent empirical results: the behavioral assumptions implied in the systems of share equations which typically supplement the cost and production functions when they are estimated. It is generally believed that the cost and production system specifications in standard use embody the same behavioral assumptions, but except under fairly restrictive maintained hypotheses about the production technology they do not. In this paper we demonstrate how the dual models can be made behaviorally equivalent under a variety of assumptions about the technology and producer behavior and how the structure of production and cost models can be modified to test for profit maximizing behavior given a maintained hypothesis of cost minimization. These models are then estimated using data from the dairy processing sector (SIC 202) to illustrate the sensitivity of results to the various maintained hypotheses.

The remainder of the paper is organized as follows. First, the production and cost models involving equivalent behavioral assumptions are developed. For convenience, and in deference to its popularity, all models are based on the second order translog approximation to a three input technology. The extent to which different or unnecessarily restrictive maintained hypotheses may account for the different estimates is discussed, within the context of previous empirical studies where disparate elasticity estimates have been obtained from the 'dual' models. Second, hypothesis tests for the appropriate behavioral assumptions are developed. Third, the data from the dairy processing sector (SIC202) are described and estimated results are presented and discussed. The final section contains a summary and implications for empirical research.

The Models

As a point of departure, we choose the translog production system. The second order translog production function is

(1) $\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + e_0$,

where y = output; x_i = input i (i,j = 1...n); and e_0 is an error term, as are the e's and ϵ's in models below.

The production function describes a technological relationship between inputs and output and embodies no behavioral assumption. For estimation purposes, supplementing the production function with a set of 'share' equations requires an assumption about firm behavior. In the discussion that follows, four groups of models are formulated. Two groups of primal systems are developed, one with cost minimization and another with profit maximization as maintained hypotheses. Two groups of cost models are distinguished on the same basis.

Primal Systems with Maintained Hypothesis of Cost Minimization

Beginning with the less restrictive assumption of cost minimization, the first order conditions require that $p_i = \lambda f_i$, or $p_i x_i / y = \lambda f_i x_i / y = \lambda \, \partial \ln y / \partial \ln x_i$, where p_i = price of input i, $f_i = \partial y / \partial x_i$; λ is the Lagrangian multiplier. Continuing,

(2) $p_i x_i / \sum_j p_j x_j = (\partial \ln y / \partial \ln x_i) / (\sum_j \partial \ln y / \partial \ln x_j)$.

If cost shares are M_i, the cost share equations of the translog production system are[2]

(3) $M_i = (\alpha_i + \sum_j \beta_{ij} \ln x_j) / (\sum_i \alpha_i + \sum_i \sum_j \beta_{ij} \ln x_j) + e_i$ for i=1,n-1.

Denote the model consisting of equations (1) and (3) as model P1, a system of n equations. Only n-1 cost share equations may be included in the model since the n^{th} equation is not independent and a singular error covariance matrix would otherwise result.

If instead of imposing cost minimization (by equation (3)), the restriction $\sum_i \partial \ln y / \partial \ln x_i = 1$ is imposed, the more familiar share equations that are found most frequently in empirical applications of the translog production functions are obtained:

$$(4)\ M_i = (\alpha_i + \sum_j \beta_{ij} \ \ln x_j) + e_i.$$

For later reference, the model consisting of equations (1) and (4) is denoted as model P2. If symmetry and homogeneity are imposed the n^{th} share equation will be redundant and model P2 consists of n equations. If homogeneity is not imposed, the error covariance matrix will not be singular and a n+1 equation system may be estimated. In practice, however, such an error covariance matrix may be close to singular and one equation will have to be dropped.

The expression $\sum_i \partial \ln y/\partial \ln x_i$ is the sum of production elasticities and by definition equals returns to scale. By requiring that $\sum_i \partial \ln y/\partial \ln x_i = 1$, unit returns to scale are being imposed at each data point. This is not the same as imposing linear homogeneity globally which requires parametric restrictions on the α_i's and the β_{ij}'s; in practice, however, the estimated technology should be nearly linear homogeneous. It is clear that tests for linear homogeneity should not be conducted from an 'unrestricted' model which imposes unit returns to scale at each data point. It should also be recognized that share equations (4) are a special case of (3) and do not constitute the most general cost minimization specification available for the production system. In other words, although model P2 has been used frequently, it may be inappropriate for modeling many technologies.

<u>Primal Systems with Maintained Hypothesis of Profit Maximization</u>

For purposes of testing for profit maximizing behavior from a model with maintained hypothesis of cost minimizing behavior, it is convenient to develop model P1 further by adding a condition implied by profit maximizing behavior. That is,

$$(5)\ \ p_i = p_y f_i \text{ and}$$

$$\sum_i \partial \ln y/\partial \ln x_i = \sum_i f_i x_i/y = \sum_i p_i x_i/p_y y = C/p_y y,$$

where C is cost and p_y is the price of y. This equation relates the ratio of total production costs to the value of output to the sum of production elasticities or returns to scale. The supplemental equation in the primal translog system is then

$$(6)\quad C/p_y y = \sum_i \alpha_i + \sum_i \sum_j \beta_{ij} \ln x_j + e_n .$$

Equations (1), (3), and (6) then comprise the primal translog system with a behavioral assumption of profit maximization. This is model P3, a system of n+1 equations.

A primal translog system, which postulates profit maximization, also can be derived directly from the first order conditions for profit maximization, thus bypassing the need for the highly non-linear expressions contained in equation (3). Model P4 is behaviorally equivalent to model P3 and is derived as follows. Begin with the first order conditions for profit maximization given by equation (5)

$$f_i = p_i/p_y \quad \text{and } p_i x_i \,/\, p_y y = f_i x_i \,/y = \partial \ln y/\partial \ln x_i .$$

The share equations (now cost shares in terms of <u>value of output</u>) for the primal translog model are

$$(7)\quad p_i x_i/p_y y = \alpha_i + \sum_j \beta_{ij} \ln x_j + e_i \text{ for } i = 1,n.$$

Since the share equations in terms of value in output do not sum to a constant, no dependency will arise between the errors in the share equation no matter how the parameters of the system are restricted. The n+1 equation system for model P4 is comprised of the production function equation in (1) and the share equations in (7). Although model P4 is behaviorally equivalent to model P3, it is not possible to test for profit maximizing behavior from system P4, because profit maximizing behavior is not imposed on the system through parametric restrictions (as it is in model P3 through equation (6)),.

By making the assumption of constant returns to scale, model P4 reduces to model P2, because under these conditions, total costs exhaust the value of output. That is, $C = p_y y$, and the cost shares in value of output (equation (6)) are identical to input cost shares in total costs (equation (4)). Thus, model P2, derived from a maintained hypothesis of cost minimization, is observationally equivalent to one derived from a maintained hypothesis of profit maximization.

Of the four models described above and summarized in Table 1, the one found most often in empirical applications of flexible production systems is model P2. It is also the production system used by both Burgess and Humphrey and Moroney in making their comparisons of inferences obtained from dual production and cost models of production. Yet, of the four models discussed, model P2 is the most restrictive with respect to maintained hypothesis of the underlying technology. Model P1, on the other hand, has not been used in the empirical literature and is the least restrictive in terms of both behavioral assumptions and assumptions about the underlying technology. As discussed below, this model is the one to be used in testing the profit maximization hypothesis.

Dual Systems with Maintained Hypothesis of Cost Minimization

In order to facilitate a development of the cost system which parallels our treatment of the production system, it is convenient to begin with a translog system comprised of the cost function and its derivative relations given by Shephard's lemma. The second order translog cost function is

$$(8)\quad \ln C = \delta_0 + \delta_y \ln y + 1/2\, \delta_{yy} (\ln y)^2 + \sum_i \delta_i \ln p_i$$
$$+ 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln p_i \ln y + \epsilon_0$$

Since cost minimizing behavior was assumed in the derivation of the cost function, no additional behavioral assumption is required to generate the share equations of the cost system. Via Shephard's lemma, input demands can be related to the price derivatives of the cost function,

$$x_i = \partial C/\partial p_i \quad \text{and} \quad x_i p_i /C = (\partial C/\partial p_i)p_i/C = \partial \ln C/\partial \ln p_i.$$

The share equations which supplement (8) are

$$(9)\quad M_i = x_i p_i/C = \delta_i + \sum_j \gamma_{ij} \ln p_j + \gamma_{iy} \ln y + \epsilon_i \text{ for } i = 1,n-1.$$

The cost model composed of equations (8) and (9) is designated as model C1. In this model, as in all of the translog cost models below, linear homogeneity in prices and symmetry are imposed with the following parametric restrictions

Table 1. Alternative Translog Production Models

Production Models, cost minimization imposed

Model P1

$$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + e_0$$

$$\frac{p_i x_i}{C} = M_i = (\alpha_i + \sum_j \beta_{ij} \ln x_j) / (\sum_i \alpha_i + \sum_i \sum_j \beta_{ij} \ln x_j) + e_i$$

Model P2

$$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + e_0$$

$$M_i = \alpha_i + \sum_j \beta_{ij} \ln x_j + e_i$$

Production Models, profit maximization imposed

Model P3

$$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + e_0$$

$$\frac{p_i x_i}{C} = M_i = (\alpha_i + \sum_j \beta_{ij} \ln x_j) / (\sum_i \alpha_i + \sum_i \sum_j \beta_{ij} \ln x_j) + e_i$$

$$\frac{C}{p_y y} = \sum_i \alpha_i + \sum_i \sum_j \beta_{ij} \ln x_j + e_n$$

Model P4

$$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + e_0$$

$$\frac{p_i x_i}{p_y y} = (\alpha_i + \sum_j \beta_{ij} \ln x_j) + e_i$$

Note: C is cost, p_i and p_y are input and output prices, respectively, y is output; x_i's are inputs; and e's are error terms.

$\gamma_{ij} = \gamma_{ji}$, $\sum_i \delta_i = 1$, $\sum_j \gamma_{ij} = 0$ for all i, and $\sum_i \gamma_{iy} = 0$.

With symmetry and price homogeneity imposed, Model C1 is an n equation system which includes only n-1 cost shares, the n^{th} equation being redundant.

Regarding restrictions on the production technology, homotheticity requires that $\gamma_{iy} = 0$ for all i, homogeneity requires, in addition, that $\delta_{yy} = 0$ and linear homogeneity requires $\delta_y = 1$ as well. If the restrictions governing the technology are not imposed, then model C1 corresponds to primal model P1 with regard to maintained hypotheses about producer behavior and the technology. By imposing the linear homogeneity restrictions on technology, another model, call it C2, can be derived and is comprised of the unit cost function:

(10) $\ln C - \ln y = \delta_0 + \sum_i \delta_i \ln p_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \epsilon_0$

and share equations

(11) $M_i = \delta_i + \sum_j \gamma_{ij} \ln p_j + \epsilon_i$ for i = 1, n-1.

This model corresponds to production model P2 in terms of its assumptions regarding technology and producer behavior.

Dual Systems with Maintained Hypothesis of Profit Maximization

By adding the restriction that output price equals the marginal cost of production, it is easy to incorporate a maintained hypothesis of profit maximizing behavior within a cost system such as C1. To see this, set

$$p_y = MC = \partial C/\partial y \text{ or } p_y y / C = \partial \ln C/\partial \ln y.$$

In model C1, this restriction gives rise to an additional equation

(12) $p_y y/C = \delta_y + \delta_{yy} \ln y + \sum_i \delta_{iy} \ln y.$

Model C3, with maintained hypothesis of profit maximization, consists of equations (8), (9), and (12); it is an n+1 equation system and its assumptions regarding producer behavior and technology correspond to those of primal model P3. If linear homogeneity is imposed, equation (12) becomes a tautology, all of its parameters are restricted to zero save δ_y which is restricted to unity. In other words, if the production function is assumed to be linear homogeneous, then the cost system reduces to system C2.

These three cost systems are summarized in Table 2. Systems C1 and C2 have been used extensively in empirical studies. However, in studies that compare alternative production and cost specification, these cost models have not always been matched with the correct production specification. In one study of the "dual" translog models, for example, Humphrey and Moroney incorrectly chose model C1 as the cost system consistent with the behavioral and technological assumptions of primal model P2. Since model C1 is considerably more flexible with regard to the technology than model P2, it should come as no surprise that the inferences obtained from the two models regarding substitution possibilities for six two-digit manufacturing industries are at considerable variance. Capalbo made comparisons of models C1 and P4 for U.S. agriculture, and while the cost function seemed to work quite well, curvature conditions were violated in the production model and the demand for capital was positively sloped.

Furthermore, although in his study of "dual" models, Burgess correctly chose model C2 as having the same behavioral and technological assumptions as primal model P2, these models, which impose linear homogeneity in production, may not be well suited to many empirical exercises. As Burgess found in his study, it is quite possible for two behaviorally equivalent, but technologically misspecified models to produce different inferences about the factor substitution and demand elasticities.

Testing for Profit Maximizing Behavior

Of the systems discussed above, models C3 and P3 are used as the basis for conducting tests of the null hypothesis (H_0) that producers are profit maximizers. In general notation, the cost model under H_0 is:

(13) $C = F_0(y,p,\gamma_0) + \epsilon_0$

(14) $M_i = p_i x_i / C = F_i(y,p,\gamma_i) + \epsilon_i$ for $i=1,n-1$

(15) $p_y y / C = F_n(y,p,\gamma_n) + \epsilon_n$

Table 2. Alternative Translog Cost Models

Cost models, cost minimization imposed

Model C1

$$\ln C = \delta_0 + \delta_y \ln y + 1/2\ \delta_{yy} (\ln y)^2 + \sum_i \delta_i \ln p_i$$

$$+ 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln y \ln p_i + \epsilon_0$$

$$\frac{p_i x_i}{C} = M_i = \delta_i + \sum_j \gamma_{ij} \ln p_j + \gamma_{iy} \ln y + \epsilon_i$$

Model C2

$$\ln C - \ln y = \delta_0 + \sum_i \delta_i \ln p_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \epsilon_0$$

$$M_i = \delta_i + \sum_j \gamma_{ij} \ln p_j + \epsilon_i$$

Cost model, profit maximization imposed

Model C3

$$\ln C = \delta_0 + \delta_y \ln y + \delta_{yy} (\ln y)^2 + \sum_i \delta_i \ln p_i$$

$$+ 1/2 \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + \sum_i \gamma_{iy} \ln y \ln p_i + \epsilon_0$$

$$\frac{p_i x_i}{C} = \delta_i + \sum_j \gamma_{ij} \ln p_j + \gamma_{iy} \ln y + \epsilon_i$$

$$\frac{p_y y}{C} = \delta_y + \delta_{yy} \ln y + \sum_i \delta_{iy} \ln y$$

Note: C is cost, p_i and p_y are input and output prices, respectively, y is output; x_i's are inputs; and ϵ's are error terms.

where the ϵ's are stochastic disturbances and γ's are parameter vectors. Cost minimizing behavior is a maintained hypothesis. Therefore, Shephard's lemma applies and $\gamma_i \subset \gamma_0$. The implication of H_0 is that $\gamma_n \subset \gamma_0$ also. If γ_0 is partitioned as $\gamma_0 = (\gamma_0^0, \gamma_0^n)$ where the elements of γ_0^0 appear in (13) but not in (15) under H_0, then H_0 implies the restrictions $\gamma_n = \gamma_0^n$.

Similarly, in general notation, the production model under H_0 is

(16) $y = G(x,\beta_0) + e_0$

(17) $M_i = p_i x_i / C = G_i(x,\beta_i) + e_i$ for $i=1,n-1$

(18) $C/p_y y = G_n(x,\beta_n) + e_n$

where e's are stochastic disturbances and β's are parameter vectors. Again, under the maintained hypothesis of cost minimization, $\beta_i \subset \beta_0$. If β_0 is partitioned as $\beta_0 = (\beta_0^0,\beta_0^n)$ where elements of β_0^0 appear in (16) but not in (18) under H_0, then H_0 implies the restrictions $\beta_n = \beta_0^n$.

Among the translog cost models discussed above, tests for profit maximizing behavior may only be conducted using model C3. The Gallant-Jorgensen test statistic can be used to test H_0. It is obtained by estimating (using an iterative Zellner's estimator) an unrestricted model C3, where the parameters in equation (12) are not restricted to be equal to those in (8). The covariance matrix of residuals from the unrestricted model is retained. The restricted model C3 is then estimated (using a two-stage procedure with the retained error covariance matrix) where the parameters of equation (12) are restricted to equal those of (8). The test statistic is $T_0 = nS^{**} - nS^*$ where

(19) $S(\theta) = q'(\theta)\ (\sum^{-1} \otimes I)\ q(\theta)/n.$

S^{**} and S^* are $S(\theta)$ evaluated for the restricted and unrestricted models, respectively; $q(\theta)$ is the stacked error vector from the model evaluated at the converged parameter values, θ. Asymptotically, the statistic is distributed chi-squared with degrees of freedom equal to the number of extra parameters in the unrestricted model.

A similar test of profit maximization can be conducted using model P3 where the unrestricted model is such that the paramaters of equation (6) are not restricted to equal those in equation (1). The restricted model is where the parameters in equation (6) equal those in equation (1).

An Application to U.S. Dairy Processing

The models developed so far are now estimated using time series data from the U.S. dairy processing sector (SIC202) for the period 1957-81. This 3-digit SIC industry was chosen to isolate reasonably homogeneous production processes. Equally important, prices are supported in the industry and manufacturers have commitments to process milk from certain producers. Thus, following an argument similar to Klein's in studying the rail industry, it may be reasonable to hypothesize that manufacturers minimize the cost of producing a given output (e.g. processing all the milk from a certain set of producers) rather than maximize profits. Since this is an industry where the marginal productivity relations may not reflect profit maximization behavior, it was hoped that, for demonstration purposes, substantive differences would emerge in empirical results obtained from models embodying the different behavioral assumptions.

U.S. data for SIC202 are obtained from the Annual Survey of Manufacturers and from the Census of Manufacturers. Labor, capital, energy, and material inputs and expenditures, output, value of output, and appropriate deflators were collected. For a description of the way in which the variables for output, inputs, and prices were constructed, see Maier and Driscoll (1988). Value added in real terms is the difference between nominal value of production and material expenditures deflated by a value added price index. The value added price index is calculated as a Divisia index of labor, capital and energy prices. The capital service price is constructed using Berndt's modification of the Hall-Jorgensen method, while other input prices were obtained by dividing expenditures by input use.

Model Specifics

Because of the nature of the industry, materials (e.g. raw milk, containers, etc.) account for an overwhelming proportion of costs and, as demonstrated in numerous monte carlo studies (Driscoll 1985 and 1988), the extremely small size of the remaining input shares would have created problems in obtaining reliable estimates of elasticities involving these inputs. Partially in light of this problem, a value added approach was adopted. This approach may be justified if it can be argued that materials are separable from capital, labor, and energy and that SIC202 production processes are basically Leontief and have the following representation:

(20) $Y = \min(f(K,L,E), M)$.

That is, only so many quarts of milk may be bottled given a quantity of raw milk regardless of how large f is, but that given a quantity of raw milk, capital, labor, and energy must be sufficient to bottle it all.

If $f < M$, then the marginal products of capital, labor, and energy are strictly positive and their ratios are independent of M. If the case that $M < f$ is ruled out, then materials cannot influence the ratios of the marginal products and weak separability is established. There should be no objections to ruling out the $M < f$ case since it is typically assumed that the input set is a point on the boundary of the production possibilities set.

All seven of the models proposed in the first section are estimated using an iterative Zellner procedure in PROC SYSLIN (SAS). They are modified only to allow biased technological change. In reviewing the value added data, the possibility that technological change is embodied in one or more of the inputs is apparent. Value added in real terms declines about 20% over the period, yet labor and energy use decrease by 50% and capital use increases by only 15%.

Biased technological change is accomodated in the seven models by allowing nearly all parameters to vary with time. If the parameter vectors in each model are denoted generically by θ, then θ has the form

(21) $\theta = \alpha + \delta t$.

For instance, model P1 becomes

(22) $$\ln y = \alpha_0 + \sum_i \alpha_i \ln x_i + 1/2 \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j + \sum_i \alpha_{it}(\ln x_i)t + 1/2 \sum_i \sum_j \beta_{ijt} \ln x_i (\ln x_j)t + e_0$$

$$M_i = (\alpha_i + \alpha_{it}t + \sum_j \beta_{ij} \ln x_j + \sum_j \beta_{ijt}(\ln x_j)t) \;/\; (\sum_i \alpha_i + \sum_i \alpha_{it}t + \sum_i \sum_j \beta_{ij} \ln x_j + \sum_i \sum_j \beta_{ijt}(\ln x_j)t) + e_i$$

for all i = 1, n-1

where α_{it} and β_{ijt} have the t subscript to differentiate them from α_i and β_{ij}, respectively. Both α_{it} and β_{ijt} are time invariant.

This generalized, embodied technological change specification was tested in models P1 and C1 and some refinements resulted. First, model P1 required many iterations to attain convergence and sported a condition number of 13809.[3] Collinearity between the labor-time and energy-time terms was so severe that the energy-time terms were removed from model P1. As a result the condition number fell to 447. It is believed that these restrictions are not wholly unwarranted since it can be reasoned that the productivity of energy is unchangeable. Assuming energy saving improvements in capital design or the organization of production are embodied in capital and labor, parameters α_{3t} and β_{33t} are restricted to be zero in models P1-P4 (i = 3 refers to the energy input).

Subjective views relating to the way in which technological change is embodied in the inputs are easily adapted in the primal systems. This is not true of the dual systems and we have yet to specify the exact dual to the newly modified and slightly restricted model P1(it may not be possible to specify such a cost function). The fully general model C1 was estimated intact and the results pointed to slight alterations required of C1. Unlike model P1, the parameter estimates of model C1 converged quickly, and the condition number was reasonable. However, returns to scale and marginal product estimates were negative at several observations and concavity was violated at every data point. The output-time terms were found to be responsible for the monotonicity violations and by removing them from the C1 specification, the concavity violations vanished as well. In models C1-C3 then, parameters δ_{yt} and δ_{yyt} are restricted to be zero.

Although the models have been subjected to a certain amount of pretesting and the restricted models do not involve exactly the same technological assumption, the specifications are as consistent as possible regarding the technology and do impose the same behavioral assumptions. Let us now review the empirical results obtained from the seven models.

Empirical Results

The parameter estimates (and standard errors) of models P1-P4 are presented in table 3; similar information for models C1-C3 are in table 4. A comparison of the empirical estimates across the models indicates just how sensitive the estimates are to the maintained hypotheses. The hypothesis that profit maximization is the optimization criterion in the dairy processing sector is conducted using each of the tests described above. For the production system, the test statistic is 93.8.[4] For a chi-square distribution with 16 degrees of freedom, the null hypothesis is rejected at the 5% level for values exceeding 26.3. The cost system test rejects the profit maximization hypothesis more emphatically and produces a test statistic of 158.6, well above the critical value of 12.6 for a chi-square with 6 degrees of freedom. Thus, the profit maximization hypothesis is rejected in both cases.

Since the parameters of the functions are not directly comparable, the relative performances of the models are judged best by comparing estimates of returns to scale, marginal products, and Allen partial elasticities of substitution (Table 5).[5] Cost and production model estimates are grouped according to the underlying behavioral assumptions of the models, for instance, models C1 and P1 both maintain the weaker hypothesis of cost minimization and are thus grouped together. Factor demand elasticities are also calculated for models C1 and P1 (table 6).

Table 3. Parameter Estimates and Their Standard Errors Obtained from Four Production Models

	Model[a]							
	P1		P2		P3		P4	
Parameter[b]	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
α_0	-0.026	0.012	0.004	0.027	-0.005	0.011	-0.007	0.012
α_1	0.517	0.085	0.796	0.023	0.426	0.024	0.441	0.025
α_2	0.113	0.033	0.177	0.032	0.081	0.012	0.094	0.012
α_3	0.031	0.004			0.033	0.001	0.033	0.001
β_{11}	0.518	0.352	-0.103	0.051	-0.070	0.061	-0.115	0.064
β_{22}	-0.112	0.141	0.078	0.090	0.026	0.071	0.025	0.076
β_{33}	-0.010	0.006			-0.018	0.007	-0.009	0.006
β_{12}	0.054	0.091	0.069	0.057	0.022	0.014	0.004	0.031
β_{13}	0.105	0.030			0.014	0.002	0.006	0.007
β_{23}	0.070	0.031			0.012	0.007	-0.050	0.014
α_{t1}	-0.011	0.006	0.000	0.002	0.003	0.015	-0.004	0.002
α_{t2}	-0.004	0.003	-0.004	0.002	-0.001	0.002	-0.002	0.001
β_{t11}	-0.017	0.015	0.009	0.001	0.011	0.001	0.011	0.001
β_{t22}	0.005	0.007	-0.003	0.002	0.006	0.004	0.01	0.004
β_{t12}	-0.003	0.003	-0.008	0.001	0.001	0.001	0.001	0.001
β_{t13}	-0.006	0.001			0.002	0.001	-0.002	0.001
β_{t23}	-0.004	0.002			0.002	0.001	0.004	0.001

[a]See table 1 for a description of the models.

[b]Output is measured as value added, y and the subscripts are: 1 = labor input; 2 = capital input; 3 = energy input; t = time.

Table 4. Parameter Estimates and Their Standard Errors Obtained from Three Cost Models

Parameter[b]	Model[a] C1 Coefficient	C1 Standard Error	C2 Coefficient	C2 Standard Error	C3 Coefficient	C3 Standard Error
α_0	0.069	0.027	-0.001	0.026	0.018	0.025
δ_y	2.09	0.217			2.10	0.034
γ_{yy}	-11.46	2.93			-1.52	0.092
δ_1	0.849	0.007	0.849	0.007	0.936	0.011
δ_2	0.087	0.004	0.075	0.005	0.044	0.007
γ_{11}	0.126	0.019	0.134	0.018	0.230	0.032
γ_{12}	-0.112	0.007	-0.115	0.010	-0.168	0.016
γ_{22}	0.121	0.005	0.129	0.008	0.152	0.010
γ_{y1}	-0.037	0.025			-0.293	0.038
γ_{y2}	-0.022	0.021			0.087	0.018
δ_{t1}	-0.004	0.001	-0.004	0.001	-0.011	0.001
δ_{t2}	0.003	0.000	0.004	0.000	0.006	0.001
γ_{t11}	-0.001	0.001	-0.003	0.001	-0.003	0.002
γ_{t12}	0.002	0.000	0.003	0.000	0.001	0.001
γ_{t22}	-0.002	0.000	-0.003	0.000	-0.000	0.001
γ_{ty1}	0.009	0.003			0.014	0.004
γ_{ty2}	-0.002	0.001			-0.009	0.002

[a]See table 1 for a description of the models.

[b]Output is measured as value added, y and the subscripts are: 1 = labor input; 2 = capital input; 3 = energy input; t = time.

Table 5. Estimates of Returns to Scale, Marginal Products, and Allen Partial Elasticities of Substitution from Seven Production and Cost Models

	Returns to Scale and Marginal Products				Regularity Violations		
	RSCL	MPL	MPK	MPE	MV	CV	QCV
Cost Min.							
Model P1	0.48	0.37	0.07	0.04	0		0
Model C1	0.48	0.35	0.06	0.04	0	0	
Lin. Homo.							
Model P2	1.00	0.80	0.14	0.08	0		9
Model C2	1.00	0.78	0.14	0.09	0	0	
Profit Max.							
Model P3	0.48	0.38	0.07	0.04	0		19
Model P4	0.49	0.38	0.08	0.04	0		19
Model C3	0.47	0.36	0.06	0.04	0	24	

	Allen Partial Elasticities of Substitution					
	AESLL	AESKK	AESEE	AESLK	AESLE	AESKE
Cost Min.						
Model P1	-0.19	-3.85	-10.53	0.70	1.01	-0.94
Model C1	-0.07	-0.61	-5.28	0.14	0.57	-0.49
Lin. Homo.						
Model P2	-0.29	-12.45	-18.00	1.51	0.55	5.98
Model C2	-0.09	-0.79	-4.34	0.24	0.66	-1.15
Profit Max.						
Model P3	0.52	18.15	-8.66	-3.20	0.21	3.31
Model P4	0.11	7.17	-11.40	-1.18	1.06	-0.45
Model C3	0.05	2.56	-4.60	-0.51	0.25	1.24

Note: See tables 1 and 2 for model specifications. RSCL = returns to scale; MPI = marginal product of input I (I=L,K and E for labor, capital and energy, respectively). AESIJ = Allen partial elasticity of substitution for inputs I,J. MV, CV, and QCV give the number of data points where monotonicity, concavity and quasi-concavity conditions, respectively, are violated.

In terms of the primary objectives of this study and in light of the hypothesis tests, the most important comparisons are between the two cost minimization models. It is clear that by reconciling the behavioral and technological assumptions, the empirical performance of a production and cost model have been reconciled to a large degree. At the means of the data, estimates of returns to scale and marginal products are nearly identical; in a quantitative sense, the estimated Allen partial elasticities of substitution are not as comparable as the estimates of returns to scale or marginal products. The two estimates for AES_{LL} are the closest, but in general, the production system generates own-partial elasticity estimates which are smaller (larger in absolute value) than the cost system. Based on extensive monte carlo evidence from Driscoll (1985 and 1988), these remaining differences are probably due to errors in measurement of prices and inputs. The discrepancies are largest for those terms involving capital, the place where measurement error is most likely as well. Driscoll (1988) also points out that the sensitivity to measurement error is further exacerbated when input cost shares are relatively small as they are in the case of capital and energy (about 13% and 7%, respectively).

Despite these differences, however, the implications to be drawn from the factor price elasticities of demand are quite consistent (Table 6). Labor in both models is estimated to be quite own-price inelastic, while energy is only moderately so. In both models, labor and capital are substitutes in production, with the response of energy to the price of labor being much larger than the response of labor to energy prices. A similar relationship holds between labor and energy. Capital and energy, on the other hand, are mildly complementary.

To see that the reconciliation of the empirical estimates is in large part due to making the production model consistent with cost minimization behavior, one need only compare the results of model C1 with those of models P2, P3, P4 and C3. In model P4 (as in P3 and C3), the reguarlity conditions are violated at many data points and the own-Allen partial

Table 6. Factor Demand Price Elasticity Estimates for the U.S. Dairy Processing Industry[a]

Model[b]	$\partial \ln P_L$	$\partial \ln P_K$	$\partial \ln P_E$
Model C1			
$\partial \ln L$	-0.06	0.02	0.04
$\partial \ln K$	0.11	-0.07	-0.04
$\partial \ln E$	0.46	-0.06	-0.40
Model P1			
$\partial \ln L$	-0.16	0.09	0.07
$\partial \ln K$	0.56	-0.51	-0.06
$\partial \ln E$	0.81	-0.12	-0.69

[a]Price elasticity estimates are $\eta_{ij} = M_j \sigma_{ij}$, where M_j is the mean cost share of input j (Allen; Binswanger) over the 1959-82 period. L = labor; K = capital; E = energy.

[b]See tables 1 and 2 for model descriptions.

elasticities for both capital and labor are positive, rather than negative as they should be. More specifically, it is comparisons of models C1 with models P2 or P4 that have led to the disparate empirical results found elsewhere in the literature; similar comparisons here indicate disparate results as well. For example, by imposing the more restrictive behavioral assumption of Model P4 and C3, the regularity conditions are violated at many data points (Table 5). This is in stark contrast to either model C1 or model P1 where the concavity and quasi-concavity assumptions, respectively, are never violated. Capital and energy are shown to be substitutes in production, rather than complements as they are in both models C1 and P1.

Summary and Conclusions

This paper is in large part motivated by the belief that it would be useful to exploit fully the theoretical equivalence between primal (or production) and dual (or cost) models in empirical estimation of production economic systems using flexible functional forms. Unfortunately, in the few cases where elasticities of substitution and factor demand have been estimated using both primal and dual flexible-form models, the empirical estimates based on translog forms have been widely divergent. As is pointed out, the production and cost models in common use do not embody the same behavioral assumptions, but specifications can be developed which allow the profit maximization and cost minimization assumptions to be imposed in either the primal or dual models.

These extended models are then applied to aggregate time series data for the U.S. dairy processing industry. A test for profit maximization given cost minimization is developed and the profit maximization hypothesis is soundly rejected by both production and cost systems. The production system specification maintaining the weaker cost minimization assumption performs much better than the traditional production model when profit maximization is implied. The results from the respecified model are much more consistent with the traditional cost model. While it is quite premature to generalize on the basis of one empirical application, we believe that by reconciling the behavioral assumptions in primal-dual specifications one can provide an important internal check on the consistency on the empirical estimates of production relationships. This suggests that in future empirical work, the almost complete preoccupation with dual systems, as has been the case in the past, seems unjustified.

Footnotes

[1]In the few cases where the primal and dual results have been compared empirically, the implications regarding factor substitution and factor demand are widely divergent (e.g. Burgess; Humphrey and Moroney; Lessner; and Capalbo).

[2]In this model as well as the rest of the primal models discussed, symmetry requires $\beta_{ij} = \beta_{ji}$, homogeneity may be imposed by restricting $\sum_j \beta_{ij} = 0$ for all i, and linear homogeneity requires the additional restriction that $\sum_i \alpha_i = 1$.

[3]The condition number for a matrix (X'X) as defined by Belsley et al., is the ratio of the maximum to the minimum of its characteristic roots.

[4]Because of the addition of so many new parameters to the unrestricted, embodied technological change production model, the model was nearly singular and would not converge for SAS convergence criterion less than 0.17. Results presented are based on this convergence criterion. The estimated residual covariance matrix from this model looks reasonable, and parameter estimates are comparable to those obtained with model P1.

[5]The formulas for calculating these expressions for the translog models are easily found in the literature (e.g. Binswanger; Driscoll 1988; and Boisvert).

References

Allen, R. Mathematical Analysis for Economists. New York: St. Martin's Press, 1938.

Belsley, D., E. Kuh and R. Welsch. Regression Diagonostics. New York: John Wiley & Sons, 1980.

Berndt, E. "Reconciling Alternative Estimates of the Elasticity of Substitution". Review of Economics and Statistics. 58(1976):59-68.

Berndt, E. and L. Christensen. "The Translog Function and the Substitution of Equipment, Structures and Labor in U.S. Manufacturing 1929-68". Journal of Econometrics 1(1973):81-114.

Binswanger, H. "A Cost Function Approach to the Measurement of Elasticities of Substitution". American Journal of Agricultural Economics 56(1974):377-86.

Boisvert, R. N. "The Translog Production Function: Its Properties, Its Several Interpretations and Estimation Problems". A.E. Res. 82-28, Cornell University, September 1982.

Burgess, D. "Duality Theory and Pitfalls in the Specification of Technologies". Journal of Econometrics 3(1975):105-121.

Capalbo, S. "A Comparison of Econometric Models of U.S Agricultural Productivity and Aggregate Technology" in Susam Capalbo and John Antle (eds.) Agricultural Productivity. Washington, D.C.: Resources for the Future, 1988.

Capalbo, S. and M. Denny. "Testing Long-Run Productivity Models for the Canadian and U.S. Agricultural Sectors". American Journal of Agricultural Economics 68(1986):615-625.

Driscoll, P. "The Econometrics of Duality". Ph.D. Cornell University, 1988.

________________. "A Monte Carlo Study of Translog Production and Cost Systems". M.S. thesis, Cornell University, 1985.

Hall, R. and D. Jorgenson. "Tax Policy and Investment Behavior". American Economic Review (1967):391-414.

Humphrey, D. and J. Moroney. "Substitution Among Capital, Labor and Natural Resource Products in American Manufacturing". Journal of Political Economy 83(1975):57-82.

Klein, L. A Textbook of Econometrics. New York: Row, Peterson and Co., Inc. 1953.

Lessner, R. "The Translog Production Model: An Application to the Analysis of Small Farms in Two Villages of Nepal". M.S. thesis, Cornell University, 1982.

Maier, L. "Analysis of Multifactor Productivity Change in U.S. Food Manufacturing". M.S. thesis, Cornell University, 1987.

Theil, H. The System-Wide Approach to Microeconomics. Chicago: University of Chicago Press, 1980.

U.S. Bureau of the Census. Annual Survey of Manufacturers. Washington, D.C., various years.

________________. Census of Manufactures. Washington, D.C., various years.

Vinod, H. "Nonhomogeneous Production Functions and Applications to Telecommunications". The Bell Journal of Economics and Management Sciences, 3(1972):531-543.