Working Papers in
AGRICULTURAL ECONOMICS

April '88 **No. 88-2**

Pollution Control with Risk of Irreversible Accumulation

by

Jon M. Conrad

Department of Agricultural Economics
New York State College of Agriculture and Life Sciences
A Statutory College of the State University
Cornell University, Ithaca, New York, 14853-7801

Pollution Control with the Risk of Irreversible Accumulation

by

Jon M. Conrad
Associate Professor of Resource Economics
Department of Agricultural Economics
Cornell University
Ithaca, New York
14853

The author would like to acknowledge the helpful comments of Peter Berck, Oscar Burt, Richard Carson, Tony Fisher, Michael Hanemann, Richard Howitt, Larry Karp, Tim Mount, Lars Olson and seminar participants at Cornell University and the University of California at Berkeley, Davis, and San Diego. The usual academic absolution applies.

Pollution Control with the Risk of Irreversible Accumulation

Abstract

This paper is concerned with pollution control when excessive levels of a stock pollutant might result in a process of irreversible accumulation. Two dynamic models are constructed. The first is a deterministic model where a stock pollutant will increase according to the rate of residual discharge and decrease according to a known rate of degradation. The second model is stochastic, with the rate of degradation going to zero if the pollution stock exceeds some critical level. Risk arises because the critical value leading to irreversible accumulation is not known with certainty. Stochastic dynamic programming reveals a "probability effect" which slows the approach to a stochastic target which may be the same or less than the optimum in the deterministic model. If irreversible accumulation is triggered along the approach path one of two situations must occur. Either residual discharges continue until the pollution stock reaches a lower optimum (at which time residual discharge ceases and output is maintained at a lower level consistent with zero discharge) or discharges are ceased immediately. In the latter case the pollution stock is excessive and imposes a net social cost in perpetuity.

Pollution Control with the Risk of Irreversible Accumulation

I. Introduction

Residual wastes are sometimes classified as degradable or nondegradable (Kneese and Bower 1968). Degradable wastes are typically broken down into less complex, benign compounds by biological, chemical or physical processes. Organic residuals and thermal discharges might be examples of degradable residuals. In contrast, nondegradable wastes, while they may be dispersed throughout the environment, are not subject to decomposition and can only accumulate in a closed system. Mercury and other heavy metals would be examples of this type of waste.

While a disposal medium may have the ability to degrade a residual into constituent components, this ability may be impaired if high rates of discharge cause the pollution stock to accumulate. Under extreme conditions the degradation rate might become zero, at which time a process of irreversible accumulation ensues. We will see that the risk of irreversibility will slow the rate of approach to a stochastic target that may be the same or less than the deterministic optimum. The reduced rate of residual discharge along the approach path will be attributable to a "probability effect". It is possible that the process of irreversible accumulation might be triggered along the approach path. If it is then the economy will follow one of two courses. Either residual discharges will continue until the pollution stock reaches the optimum for a zero degradation rate (when they will cease) or they will cease immediately. In the latter case the pollution stock is excessive and imposes a net social cost in perpetuity.

The remainder of this paper is organized as follows. In the next section we present a dynamic model of pollution control with a known rate of degradation. This is followed by a stochastic model with a risk of irreversible accumulation. The section final summarizes the major results.

II. Optimal Pollution with Known Degradation

Why might a society choose some positive level of pollution? One possible answer is that a positive level of pollution might allow a society to produce more of some positively-valued commodity than if it devoted scarce resources to eliminating all pollution. Suppose Q_t is the level of a positively-valued commodity in period t while R_t is the level of a jointly produced residual. We will assume a fixed factor can be used to produce Q_t or *reduce* R_t according to an implicit transformation function $\phi(Q_t, R_t) \equiv 0$, with partials $\phi_Q > 0$ and $\phi_R < 0$. By devoting a significant portion of the fixed resource to pollution control it is possible to produce $Q_t = Q_0$ with no residual discharge ($R_t = 0$). To increase Q_t above Q_0 the fixed resource must be reallocated from pollution prevention toward commodity production. With no pollution control the available fixed resource would allow $Q_t = Q_{max}$ but residual discharge would be $R_t = R_{max}$. A plausible commodity-residual transformation curve is drawn in Figure 1.

Residual discharge is assumed to contribute to a stock pollutant according to the difference equation

$$X_{t+1} - X_t = -\gamma X_t + R_t \qquad (1)$$

where X_t is the pollution stock in period t and $0 \leq \gamma < 1$ is the rate of degradation. In each period the benefits of Q_t and the cost of X_t are assumed to be given by $W(Q_t, X_t)$. Maximization of the present value of net benefits subject to the transformation function and the dynamics of the pollution stock leads to the optimization problem

$$\text{Maximize} \sum_{t=0}^{\infty} \rho^t W(Q_t, X_t)$$

$$X_{t+1} - X_t = -\gamma X_t + R_t$$

$$\phi(Q_t, R_t) \equiv 0$$

$$Q_0 \leq Q_t \leq Q_{max}, \; 0 \leq R_t \leq R_{max}$$

where $\rho = 1/(1+\delta)$ is a discount factor and δ is the periodic discount rate. The current value Hamiltonian may be written as

$$\tilde{H} = W(Q_t,X_t) - \mu_t\phi(Q_t,R_t) + \rho\lambda_{t+1}[-\gamma X_t + R_t] \quad (2)$$

and with W(·) concave in Q_t the necessary and sufficient conditions for an interior maximum include

$$\frac{\partial\tilde{H}}{\partial Q_t} = W_Q - \mu_t\phi_Q = 0 \quad (3)$$

$$\frac{\partial\tilde{H}}{\partial R_t} = -\mu_t\phi_R + \rho\lambda_{t+1} = 0 \quad (4)$$

$$\rho\lambda_{t+1} - \lambda_t = -\frac{\partial\tilde{H}}{\partial X_t} = -W_X + \gamma\rho\lambda_{t+1} \quad (5)$$

Given the partials for $\phi(\cdot)$ and assuming the partials for W(·) are $W_Q > 0$ while $W_X < 0$, it will be the case that $\mu_t > 0$ and $\lambda_t < 0$. In steady state equations (1) and (3) - (5) can be shown to imply

$$\frac{W_Q\,\phi_R}{\phi_Q} = \frac{W_X}{(\delta+\gamma)} \quad (6)$$

and

$$R = \gamma X \quad (7)$$

Equations (6)-(7) along with the transformation function constitute a three-equation system defining the steady state optimum (X^*,R^*,Q^*). Equation (6) can be interpreted as requiring the marginal value of an incremental increase in R, which allows for an increase in Q according to ϕ_R/ϕ_Q, to be equated to the present value of marginal pollution cost.

The approach to steady state will, in general, be asymptotic.

There are, however, plausible instances where the approach to steady state will be "most rapid" utilizing "bang-bang" controls (Spence and Starrett 1975). Because it will give some benchmarks by which to evaluate the transition dynamics of the stochastic model we will consider such a model in more detail.

Suppose that $W(\cdot) = pQ_t - cX_t^2$ and $\phi(\cdot) = Q_t - b - nR_t = 0$. This specification assumes (1) the economy's welfare is linear in Q_t (Q_t might be exported at a constant price), (2) the costs of pollution are quadratic in X_t, and (3) the transformation function is linear permitting us to solve for $Q_t = b + nR_t$.

The current-value Hamiltonian becomes

$$\tilde{H} = p(b + nR_t) - cX_t^2 + \rho\lambda_{t+1}(-\gamma X_t + R_t) \qquad (8)$$

Equation (6) implies the optimal pollution stock is given by

$$X^* = \frac{np(\delta + \gamma)}{2c} \qquad (9)$$

Inspection reveals that the optimal pollution stock increases with n, p, δ, or γ and decreases with c. For this problem the most rapid approach path (MRAP) to X^* is optimal and there exists a switching function for R_t given by

$$\sigma_R = pn + \rho\lambda_{t+1} \qquad (10)$$

If $\sigma_R > 0$ then $R_t = R_{max}$ whereas if $\sigma_R < 0$ then $R_t = 0$. The first situation occurs when the pollution stock is below the optimum ($X_t<X^*$) while the second occurs when the stock is above the optimum ($X_t > X^*$). When $X_t = X^*$, $pn = -\rho\lambda_{t+1}$, $\sigma_R = 0$ and $R_t = R^* = \gamma X^*$.

To make things even more concrete, consider a numerical example where $b = 3.5$, $c = 0.01$, $\delta = 0.1$, $\gamma = 0.2$, $n = 0.5$ and $p = 1$. In this case $X^* = 7.5$, $R^* = 1.5$ and $Q^* = 4.25$. If $\gamma = 0$, and all other parameters are the same, then $X^* = 2.5$, $R^* = 0$ and $Q^* = 3.5$. With $X_0 = 0$ and $R_{max} = 2.5$, Figure 2 shows the solutions $X^*_{\gamma=0.2} = 7.5$ and $X^*_{\gamma=0} = 2.5$ and the MRAP approach to each.

III. The Risk of Irreversible Accumulation

Now suppose that the rate of degradation depends on the pollution stock. Specifically, suppose that $\gamma_t = \gamma$ if $X_t \leq X_c$, but $\gamma_t = 0$ if $X_t > X_c$ where X_c is an unknown critical value. Once the critical pollution stock is exceeded the process of accumulation is irreversible since the stock will never again degrade below X_c. Define Θ_t to be a binary variable indicating the following state of information: $\Theta_t = 1$ if $X_{t-1} \leq X_c$ while $\Theta_t = 0$ if $X_{t-1} > X_c$. Thus, it is only from the perspective of period t that one can determine if X_{t-1} exceeded X_c.

Let $F(X_t \mid \Theta_t) = \Pr(X_c \leq X_t \mid \Theta_t)$ denote the probability that the critical value is less than or equal to X_t given Θ_t. This distribution is shown in Figure 3 for $\Theta_t = 1$ and $\Theta_t = 0$.

If the critical value had not been exceeded before period t (Θ_t=1) the pollution stock in period t+1 would assume one of two values

$$X'_{t+1} = (1-\gamma)X_t + R_t \text{ with probability } 1 - F(X_t \mid \Theta_t)$$
$$X''_{t+1} = X_t + R_t \text{ with probability } F(X_t \mid \Theta_t)$$

We will proceed by solving a finite horizon optimization problem (t = 0,1,2,...,T) using stochastic dynamic programming and then, as $T \to \infty$, deduce the existence of a stochastic target which will permit us to make a comparison to the deterministic model of the previous section.

We wish to determine the optimal discharge policy R_t^* which will

$$\text{Maximize } E\left\{\sum_{t=0}^{T} \rho^t W(Q_t, X_t)\right\}$$

$$\begin{aligned} \text{Subject to } \; & X'_{t+1} = (1-\gamma)X_t + R_t \text{ with probability } 1 - F(X_t \mid \Theta_t) \\ & X''_{t+1} = X_t + R_t \text{ with probability } F(X_t \mid \Theta_t) \\ & \phi(Q_t, R_t) \equiv 0 \\ & X_0 \text{ and } F(X_t \mid \Theta_t) \text{ given} \end{aligned}$$

where $E\{\cdot\}$ denotes the expectation operator.

When t=T; that is, with one period to go we wish to maximize $W(Q_T, X_T)$ subject to $\phi(Q_T, R_T)=0$. Since $W_Q > 0$, and there is no future cost from pollution, the optimal decision is $Q_T = Q_{max}$ via $R_T = R_{max}$.

When t=T-1 maximizing $[W(Q_{T-1}, X_{T-1}) + \rho E\{W(Q_{max}, X_T)\}]$ subject to X_{T-1} (given), $\Theta_{T-1} = 1$, and the transformation function leads to the interior necessary condition

$$\frac{W_Q \phi_R}{\phi_Q} = \rho \, [W_{X_T'} (1-F(X_{T-1} \mid \Theta_{T-1})) + W_{X_T''} F(X_{T-1} \mid \Theta_{T-1})] \quad (11)$$

The left-hand-side (LHS) of equation (11) is evaluated at T-1. The RHS is the expected marginal damage of an additional unit of R_{T-1} when $X_{T-1} \leq X_c$ and when $X_{T-1} > X_c$. (Note that the partials of $W(\cdot)$ are evaluated at X_T' and X_T''). With only two periods to go R_{T-1} will influence pollution costs when t=T but it will *not* influence the transition probabilities (since the horizon ends at t=T).

When t=T-2 things become more complex. If $\Theta_{T-2} = 1$ then we face the following transition possibilities

$X_{T-2} \rightarrow X_{T-1}' \rightarrow X_T'$ with probability $[1-F(X_{T-2} \mid \Theta_{T-2})] \cdot [1-F(X_{T-1} \mid \Theta_{T-1})]$
$X_{T-2} \rightarrow X_{T-1}' \rightarrow X_T''$ with probability $[1-F(X_{T-2} \mid \Theta_{T-2})] \cdot [F(X_{T-1} \mid \Theta_{T-1})]$
$X_{T-2} \rightarrow X_{T-1}'' \rightarrow X_T''$ with probability $F(X_{T-2} \mid \Theta_{T-2})$

Note that irreversible accumulation eliminates the possibility of going from an X" state back to an X' state and thus there are only three transitions. Since R_{T-2} affects X_{T-1} it *will* influence the likelihood of the transitions $X_{T-1}' \rightarrow X_T'$ or $X_{T-1}' \rightarrow X_T''$. The effect of an incremental increase in R_{T-2} in period T is captured in the following term

$$\rho^2\{W_{X_T'} (1-\gamma)(1-F(X_{T-1} \mid \Theta_{T-1})) + W_{X_T''} F(X_{T-1} \mid \Theta_{T-1}) -$$
$$[W(Q_{max}, X_T') - W(Q_{max}, X_T'')] \, F'(X_{T-1} \mid \Theta_{T-1})\}$$

The first line inside {·} is the expected marginal pollution damage. If state X' obtains in period T the marginal damage from an increment in R_{T-2} will have been reduced by one period of degradation thus reducing its effect by $(1-\gamma)$. If state X'' obtains in period T, having been preceded by state X' in period T-1 (ie, the second transition listed above) then the full (undegraded) increment of R_{T-2} will carry over into period T.

The second line inside {·} is the "probability effect". Given a state of X' in period T-1, an increase in R_{T-2} will raise the likelihood of receiving $W(Q_{max},X''_T)$ and reduce the probability of receiving $W(Q_{max},X'_T)$, where $F'(X_{T-1} | \Theta_{T-1}) > 0$ is the increased likelihood of state X'' in period T. If $X_{T-2} > X_c$ then an incremental change in R_{T-2} would have no effect on the probability of X' or X'' in period T since X'' would occur with certainty. The combined effects in period T and T-1 from an incremental increase in R_{T-2} are given by

$$\frac{W_Q \phi_R}{\phi_Q} = \rho\{W_{X'_{T-1}}(1-F(X_{T-2} | \Theta_{T-2})) + W_{X''_{T-1}}F(X_{T-2} | \Theta_{T-2})\} +$$
$$\rho^2\{W_{X'_T}(1-\gamma)(1-F(X_{T-1} | \Theta_{T-1})) + W_{X''_T}F(X_{T-1} | \Theta_{T-1}) -$$
$$[W(Q_{max},X'_T) - W(Q_{max},X''_T)]F'(X_{T-1} | \Theta_{T-1})\} \qquad (12)$$

All the terms in equation (12) are negative. We are again seeking to equate the marginal value of transformation to the expected marginal cost of pollution *plus* the probability effect.

Working backwards in time the interior necessary condition becomes more complex as marginal pollution costs and probability effects extend over a longer and ultimately infinite future as $T \rightarrow \infty$. The probability effect will reduce the rate of discharge along an approach path from an initially small pollution stock. At $t = 0$, and depending on $F(X \mid \Theta)$, there will exist a stochastic target X_F^* where $X_{\gamma=0}^* \leq X_F^* \leq X_{\gamma>0}^*$, where $X_{\gamma=0}^*$ and $X_{\gamma>0}^*$ are the deterministic benchmarks of the preceding section. The logic behind the bounds for the stochastic target is as follows.

Suppose that the subjective distribution $F(X \mid \Theta)$ places a very high probability that $X_{\gamma>0}^* < X_c$ (ie, $F(X_{\gamma>0}^* \mid \Theta_0=1) = \Pr(X_c \leq X_{\gamma>0}^*) \approx 0$). With the same γ as in the deterministic model the stochastic target would be $X_F^* \approx X_{\gamma>0}^*$. The approach to X_F^* will be *less rapid* due to the lower (more cautious) discharge policy induced by the probability effect.

Alternatively, $F(X \mid \Theta)$ may imply that $X_{\gamma>0}^*$ is likely to exceed X_c. Then the stochastic target will be $X_F^* < X_{\gamma>0}^*$. The lower the prior on X_c the closer X_F^* will be to $X_{\gamma=0}^*$.

If it turns out that $X_c > X_F^*$ then the stochastic target will be reachable, given $R_t > \gamma X_t$ for all t along the approach. It is possible, however, that along the approach to X_F^* that X_c may be unexpectedly exceeded. Suppose this occurs at $t = \tau_1$ in Figure 4. At this point

$X_{\tau_1} < X^*_{\gamma=0}$ (the optimal pollution stock in the deterministic model when $\gamma = 0$) In this case X_t is allowed to increase (more rapidly since $\gamma = 0$) until $X_t = X^*_{\gamma=0}$ at which point $R_t = 0$ and $Q_t = Q_0$ thereafter. The economy is obviously at a lower level of welfare than had the critical pollution level not been exceeded at all. They might be better off, however, than if the irreversibility had been triggered later when $X^*_{\gamma=0} < X_{\tau_2} < X^*_F$.

If the irreversibility is triggered at $t = \tau_2$ then residual discharge stops immediately and $Q_t = Q_0$. In this case the pollution stock is excessive and the economy incurs a net social cost in perpetuity. At X_{τ_2} equation (6) does not hold, rather $W_Q \phi_R/\phi_Q < W_X/\delta$. To the extent that the probability effect slows the rate of residual discharge along an approach path it may reduce the magnitude of excessive pollution costs if a transition to irreversible accumulation takes place.

IV. Conclusions

The effect of the risk of irreversible accumulation was to slow the approach to a stochastic target X_F^*, where X_F^* is bounded by the deterministic optima when $\gamma > 0$ and when $\gamma = 0$ (ie, $X_{\gamma=0}^* \leq X_F^* \leq X_{\gamma>0}^*$). The stochastic target might be reachable and sustainable if the unknown $X_c > X_{\gamma>0}^*$. Since $F(X|\Theta)$ is a subjective distribution a process of irreversible accumulation may be triggered before reaching X_F^*. If this occurs one of two trajectories will be followed. If the pollution stock is less than $X_{\gamma=0}^*$, residual discharges will continue until $X_t = X_{\gamma=0}^*$, at which time they cease forever. If the pollution stock exceeds $X_{\gamma=0}^*$ residual discharges cease immediately and because the pollution stock is excessive for a world where $\gamma = 0$ a net social cost persists in perpetuity. The slower approach, induced by the probability effect, may help reduce the magnitude of such social costs if irreversibility is triggered.

References

Kneese, A. V. and B. T. Bower. 1968. *Managing Water Quality: Economics, Technology, Institutions* Johns Hopkins Press, Baltimore.

Spence. A. M. and D. Starrett. 1975. "Most Rapid Approach Paths in Accumulation Problems" *International Economic Review* 16 (June):388-403.

Figure 1: A Commodity-Residual transformation Curve Implied by $\phi(Q_t, R_t) \equiv 0$

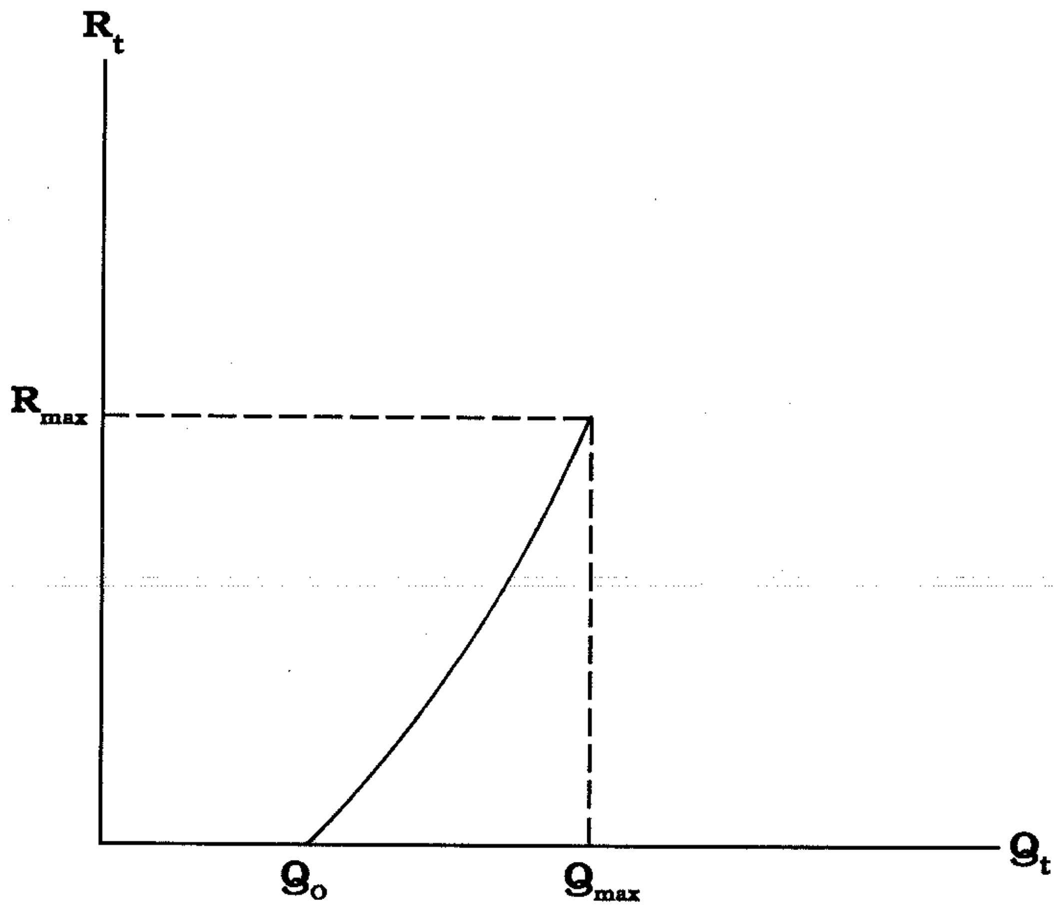

Figure 2: The Most Rapid Approach to $X^*_{\gamma=0}$=2.5 and $X^*_{\gamma=0.2}$=7.5 from X_0=0 for the Case when b=3.5, c=0.01, δ=0.1, n=0.5, p=1 and R_{max} =2.5

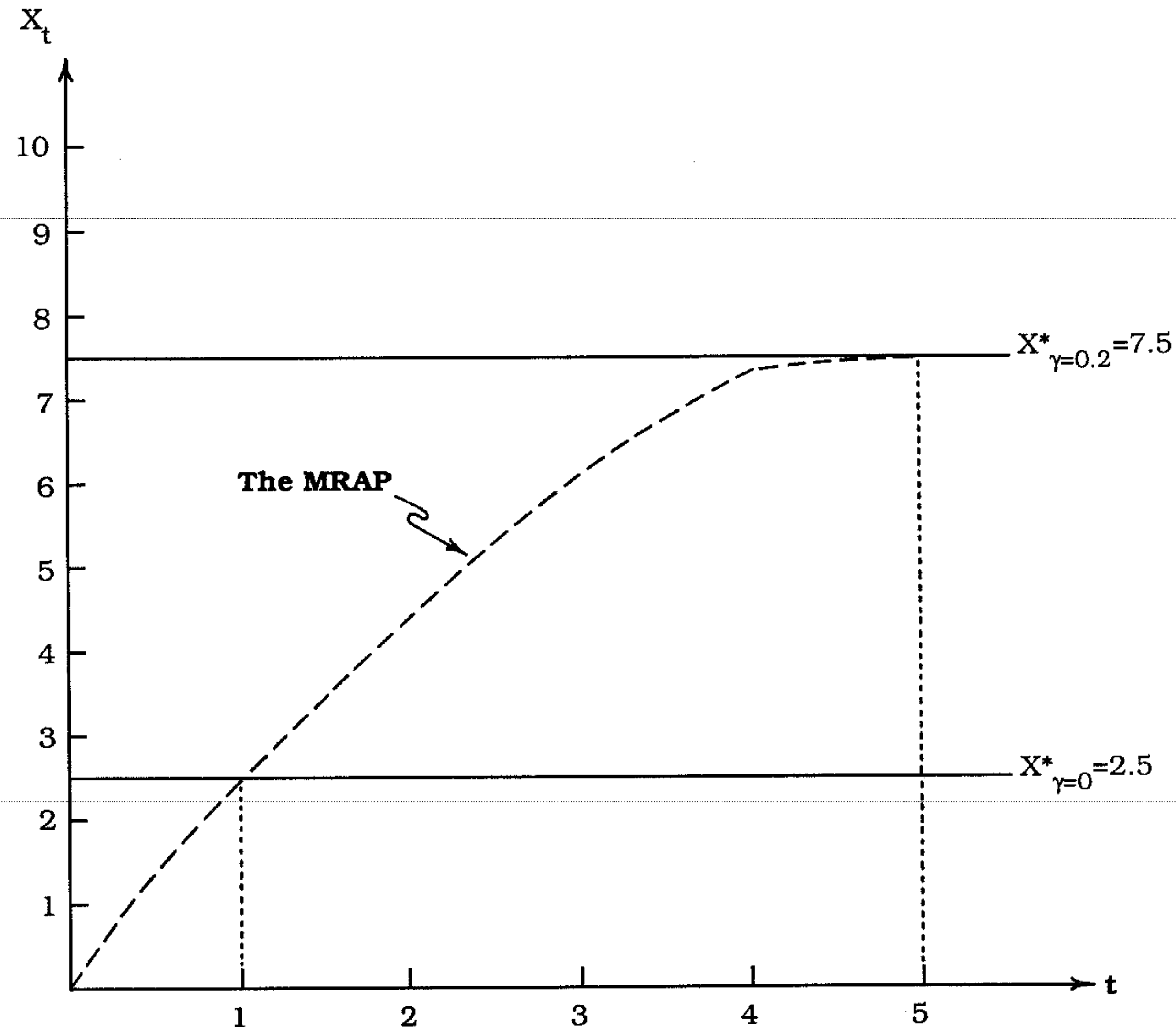

Figure 3: The Conditional Distribution $F(X_t \mid \Theta_t) = Pr\ (Xc \leq X_t \mid \Theta_t)$ When (a) $\Theta_t = 1\ (X_{t-1} \leq Xc)$ and (b) $\Theta_t = O\ (X_{t-1} > Xc)$.

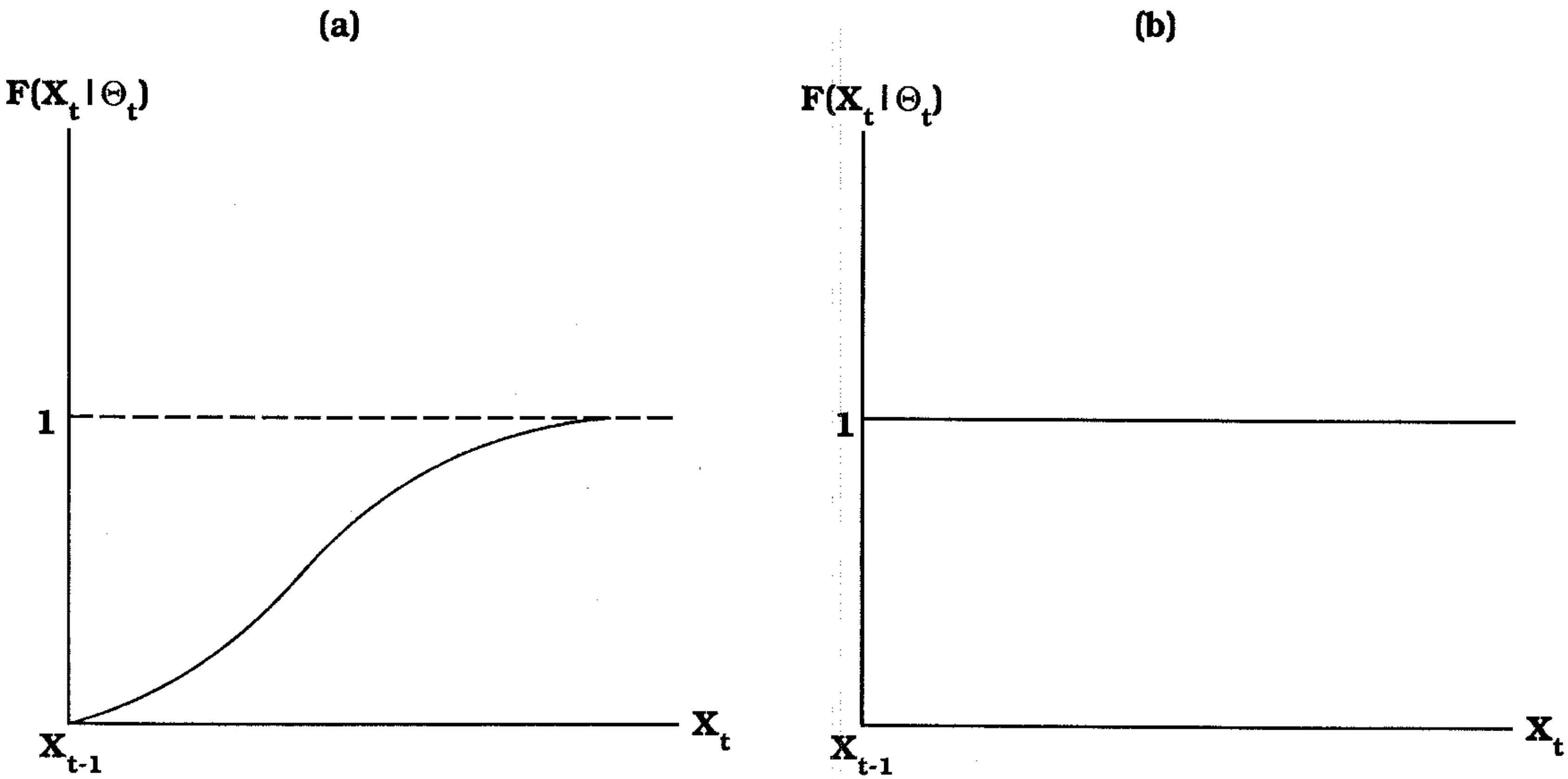

Figure 4: Approach Paths and Equilibria with the Risk of Irreversible Accumulation