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Pollution Control and Resource Management

by

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Pollution Control and Resource Management

Abstract

Certain levels of pollution, while not posing a direct or immediate threat to human health may directly affect the health or abundance of a species of value to man. These values might derive from commercial or recreational harvest, or from "nonconsumptive" activities such as observation or photography. A model is constructed where a stock pollutant adversely affects the growth of a renewable resource. Denoting resource biomass by X, the harvest rate by Y, the pollution stock by Z, the rate of commodity production by Q and the jointly produced residual by S, the equations of motion are given as $\dot{X} = F(X,Z) - Y$ and $\dot{Z} = -\gamma Z + S$, where $F(\cdot)$ is a growth function for the renewable resource and γ is the rate of biodegradation. Positive levels for S and Z may be optimal because $S > 0$ allows for an increase in Q according to the transformation function $\phi(Q,S) \equiv 0$. With Y and Q providing welfare flows according W(Y,Q) the general (nonlinear) problem becomes

$$\text{Maximize} \int_0^\infty W(Y,Q)e^{-\delta t}dt$$

$$\text{Subject to } \dot{X} = F(X,Z) - Y$$

$$\dot{Z} = -\gamma Z + S$$

$$\phi(Q,S) \equiv 0$$

where δ is the instantaneous rate of discount. The first order necessary conditions for this problem are obtained and evaluated in steady state. They provide insight into the optimal balance between a marginal increase in Q, S and Z and the reduction in Y through diminished growth. When $W(\cdot)$ and $\phi(\cdot)$ are separable and linear in Y,Q and Q,S, respectively, analytic solutions for X^* and Z^* are possible. The comparative statics yield different results than those obtained in single-state pollution or renewable resource models. For these models the current value Hamiltonian is linear in Y and S and the most rapid approach path is optimal. Two special cases, where pollution affects the intrinsic growth rate or the environmental carrying capacity, are analyzed in detail. A simple algorithm for driving (X,Z) to the optimum is developed and numerically applied to the case of rate-dependent pollution. A policy for coordinating pollution control and resource management is described.

Pollution Control and Resource Management

I. Introduction and Overview

Certain levels of pollution, while not posing a direct or immediate threat to human health, may directly affect the health or abundance of a species of value to man. These values might derive from commercial or recreational harvest of the species, "nonconsumptive" activities such as observation and photography, or simply knowing that the species and it environment are in a "healthy state". For example, at current levels the emission of the acid precursors SO_2 and NO_x, while not posing a direct threat to human health, are known to have eliminated certain species of fish from freshwater lakes in the U. S., Canada, and Northern Europe. In the Chesapeake Bay agricultural runoff of nutrients and herbicides along with industrial wastes are thought to have diminished the habitat for striped bass and oysters (see Kahn and Kemp 1985).

This paper is concerned with the optimal discharge of a residual waste and the simultaneous management of a renewable resource adversely affected by the stock of a pollutant. In the next section we introduce a general (nonlinear) model and derive conditions for optimal pollution control and resource management. In the third section we consider a specific form for the growth function for the renewable resource and examine two cases where the pollution stock may (1) adversely affect the intrinsic growth rate, or (2) adversely affect environmental carrying capacity. These cases are referred to as rate- and capacity-dependent pollution, respectively.

The fourth section imposes additional form by assuming that the net benefit function and the transformation function are separable and linear in their arguments. Analytic solutions for steady state are possible for both rate- and capacity-dependent pollution. The comparative statics of steady-state are presented for both models. The two-state models exhibit different comparative statics than typically found in single-state resource or pollution models. For the separable/linear models the optimal approach to steady state is the most rapid approach path (MRAP) using "bang-bang" controls While conceptually straightforward the MRAP approach proved difficult to identify numerically and an alternative algorithm is presented which permits rapid convergence to the steady-state optimum. The algorithm is illustrated for the case of rate-dependent pollution.

The final section discusses the implications of the models for pollution control and resource management.

II. The General Model

Let X be the population of some species at time t and Z the stock of a pollutant. Assume the species is harvested at rate Y and that the instantaneous change in population is given by

$$\dot{X} = F(X,Z) - Y \qquad (1)$$

where $F(\cdot)$ is the net growth function.

Let Q be the level of output of some positively-valued commodity and S the level of a residual waste, jointly produced according to the implicit function $\phi(Q;S) \equiv 0$. By convention we assume $\partial\phi(\cdot)/\partial Q = \phi_q > 0$ and $\partial\phi(\cdot)/\partial S = \phi_s < 0$ and that if it were desired, the fixed input implicit in $\phi(Q;S)$ could be allocated between commodity production and residual reduction so as to produce $Q = Q_0$ with $S = 0$. If the level of Q is to be increased beyond Q_0 then resources must be diverted from residual reduction toward commodity production. If all of the fixed resource is allocated to commodity production it is possible to produce $Q = Q_{max}$ and $S = S_{max}$ (see Figure 1).

The rate of residual discharge, if larger than the rate of degradation, can increase the pollution stock according to

$$\dot{Z} = -\gamma Z + S \qquad (2)$$

where γ is the rate of biodegradation.

Finally, we will assume that the social welfare of harvest Y and commodity Q are given by W(Y,Q). and thus neither the resource population nor the pollution stock directly affect welfare.

Maximization of the present value of welfare subject to the equations for resource and pollution dynamics and the implicit transformation function may be stated mathematically as

$$\text{Maximize } \int_0^\infty W(Y,Q)e^{-\delta t}dt$$

$$\text{Subject to } \dot{X} = F(X,Z) - Y$$

$$\dot{Z} = -\gamma Z + S$$

$$\phi(Q,S) \equiv 0$$

where δ is the instantaneous rate of discount. This leads to the current-value Hamiltonian

$$\tilde{H} = W(\cdot) + \mu_x[F(\cdot) - Y] + \mu_z[-\gamma Z + S] - \omega\phi(\cdot) \quad (3)$$

with first order necessary conditions that include

$$\tilde{H}_y = W_y - \mu_x = 0 \quad (4)$$

$$\tilde{H}_q = W_q - \omega\phi_q = 0 \quad (5)$$

$$\tilde{H}_s = \mu_z - \omega\phi_s = 0 \quad (6)$$

$$\dot{\mu}_x - \delta\mu_x = -\tilde{H}_x = -\mu_x F_x \quad (7)$$

$$\dot{\mu}_z - \delta\mu_z = -\tilde{H}_z = -\mu_x F_z + \gamma\mu_z \qquad (8)$$

where $\mu_x > 0$ and $\mu_z < 0$ are the current value shadow prices on the resource and pollution stocks, respectively.

In steady state equations (1)-(2), (4)-(8) and the transformation function can be shown to imply

$$F_x = \delta \qquad (9)$$

$$W_q = W_y F_z[\phi_q/\phi_s]/(\delta + \gamma) \qquad (10)$$

$$Y = F(X;Z) \qquad (11)$$

$$S = \gamma Z \qquad (12)$$

$$\phi(Q;S) \equiv 0 \qquad (13)$$

Equation (9) is a familiar expression in resource economics. In the nonlinear bioeconomic model (Clark 1976, p. 95) it requires that the optimal stock equate the biological growth rate to the rate of discount. In the present model the partial derivative F_x may involve both X *and* Z and so (9) may not uniquely determine the optimal population level.

Equation (10) may be interpreted as equating the instantaneous marginal social benefit of Q to the the discounted marginal social cost of an increase in S. Note that $[\phi_q/\phi_s]$ is the reciprocal of the marginal rate of transformation of residual S into commodity Q. An incremental increase in S, allowing Q to increase, will increase the steady-state stock of pollution. The increase in the pollution stock will reduce the net growth in X, which in turn will decrease Y, with a marginal loss of W_y in perpetuity. With W(·) concave in Y and Q equations (9)-(13) will define a unique steady-state optimum $(X^*, Z^*, Y^*, S^*, Q^*)$.

The local stability, and thus the approach to steady state proved difficult to ascertain for the general model. If W(·) and ϕ(·) are separable in Y,Q and Q,S respectively, then taking the time derivatives of (4) and (6) and noting from the transformation function that $\dot{Q} = -\phi_s \dot{S}/\phi_q$, it is possible to show that

$$\dot{Y} = \frac{W_y}{W_{yy}}[\delta - F_x] \tag{14}$$

$$\dot{S} = \frac{W_q \, \phi_s \, \phi_q^2 \, (\delta + \gamma) - \phi_q^3 \, W_y \, F_z}{W_q \, \phi_{ss} \, \phi_q^2 - \phi_s^2 \, W_{qq} \, \phi_q + W_q \, \phi_s^2 \, \phi_{qq}} \tag{15}$$

which along with equations (1) and (2) comprise a four dimensional dynamical system.

Even with the presumption of separability it was not possible to qualitatively deduce the dynamics in state (X,Z) space. We proceed by adding some additional structure.

III. Rate- and Capacity-Dependent Pollution

How might pollution affect the growth function of a renewable resource? We will consider two forms of growth suppression due to pollution. Recall that the logistic growth function takes the form

$$F(X) = rX(1 - X/K) \qquad (16)$$

where r is the intrinsic growth rate and K is the environmental carrying capacity. We will consider two cases where $r = r(Z)$ or $K = K(Z)$ with $r'(Z) < 0$ and $K'(Z) < 0$. In the first case pollution reduces the growth rate, while in the second case pollution reduces environmental carrying capacity. We refer to these cases as rate-dependent and capacity-dependent pollution, respectively.

Consider the rate-dependent case when $r(Z) = r/(1 + Z)$. The pollution-dependent growth function becomes

$$F(X,Z) = rX(1 - X/K)/(1 + Z) \qquad (17)$$

For $Z = \bar{Z}$ (a constant) equation (17) retains the "logistic shape" (symmetric, concave from below) with roots at $X = 0$ and $X = K$ and with a maximum sustainable yield (MSY) of $rK/[4(1 + \bar{Z})]$ occurring at $X = K/2$.

In the case of capacity-dependent pollution the form $K(Z) = K/(1 + Z)$ leads to the growth function

$$F(X,Z) = rX(1 - X(1 + Z)/K) \qquad (18)$$

With $Z = \bar{Z}$ equation (18) again retains the logistic shape with roots at $X = 0$ and $X = K/(1 + \bar{Z})$. Maximum sustainable yield again equals $rK/[4(1 + \bar{Z})]$ but occurs at $X = K/[2(1 + \bar{Z})]$. When $Z = 0$ equations (17) and (18) both collapse to (16).

IV. The Separable/Linear Model

To inquire further into the long run (equilibrium) and short run (dynamic) properties of these models we impose the additional assumption that W(·) and φ(·) are linear, taking the forms

$$W(Y,Q) = pY + Q \tag{19}$$

and

$$\phi(Q,S) = Q - (c + nS) = 0 \tag{20}$$

The parameter p is the relative price of a unit of the harvested resource, Y. Such a situation might arise for a small country or region facing constant prices for Y and Q.

The transformation function implies $Q = c + nS$, and the curve of Figure 1 becomes a straight line with $Q_0 = c$ and a slope $dS/dQ = 1/n$. Substituting the expression for Q directly into W(·) allows the current value Hamiltonian to be written as

$$\tilde{H} = (p - \mu_x)Y + (n + \mu_z)S + c + \mu_x F(X,Z) - \mu_z \gamma Z \tag{21}$$

which is linear in Y and S. The switching functions for Y and S are $\sigma_y = p - \mu_x$ and $\sigma_s = n + \mu_z$. When $\sigma_y < 0$, $Y = 0$, while if $\sigma_y > 0$, $Y = Y_{max}$.

Since the current value shadow price on the pollution stock is $\mu_z < 0$ we observe $S = 0$ when $\sigma_s < 0$ and $S = S_{max}$ when $\sigma_s > 0$. We also assume that Y_{max} is greater than maximum sustainable yield and that $S_{max} > \gamma Z$ for all Z. The shadow prices themselves can be shown to change according to

$$\dot{\mu}_x = \mu_x[\delta - F_x] \tag{22}$$

$$\dot{\mu}_z = -\mu_x F_z + (\delta + \gamma)\mu_z \tag{23}$$

Only in steady state, when $\sigma_y = \sigma_s = 0$, will $Y = Y^*$ and $S = S^*$. We will derive the steady-state solutions for rate- and capacity-dependent pollution and then discuss the approach dynamics.

For the case of rate-dependent pollution equation (17) implies $F_x = r(1 - 2X/K)/(1 + Z)$ and $F_z = -rX(1 - X/K)/(1 + Z)^2$. When $\dot{\mu}_x = 0$, $F_x = \delta$ implies $Z = [r(1 - 2X/K) - \delta]/\delta$. When $\dot{\mu}_z = 0$, $F_z = -(\delta + \gamma)n/p$ and we can solve for a second expression $Z = \sqrt{prX(1 - X/K)/[n(\delta + \gamma)]} - 1$. Equating these two expressions and solving for X yields

$$X^* = \frac{K}{2}\left\{1 - \sqrt{\frac{p\delta^2K}{4rn(\delta + \gamma) + p\delta^2K}}\right\} \tag{24}$$

The comparative statics of δ, γ, K, n, p and r on X^* and Z^* are summarized in Table 1. An increase in δ causes a decline in both X^*

and Z^*. While the decline in X^* is observed in most single-state bioeconomic models, the decline in Z^* is *not* typical. In the single-state pollution model briefly described in Appendix A an increase in the discount rate results in an *increase* in the pollution stock (see Conrad 1988 for additional details and a stochastic extension).

An increase in γ results in an increase in X^* and a decline in Z^*. This result seems jointly plausible, since a lower pollution stock would allow higher rates of growth, and "capital-theoretic" considerations would compel a larger resource stock. In the single-state model of Appendix A, however, an increase in γ leads to an *increase* in the steady-state pollution stock.

The remaining results in Table 1 are plausible, although it must be kept in mind that these are long-run equilibrium effects. For example, the decline in X^* resulting from an increase in price will be associated with a short-run increase in Y (ie, an increase in supply) as the stock is being reduced to its new long-run equilibrium level. Long run supply will decline if the initial equilibrium stock is less than X_{msy}.

For the case of capacity-dependent pollution, equation (18) implies $F_X = r(1 - 2X(1 + Z)/K)$ and $F_Z = -rX^2/K$. When $\dot{\mu}_X = 0$ we obtain

$$Z = [K(r - \delta) - 2rX]/(2rX) \tag{25}$$

while $\dot{\mu}_Z = 0$ uniquely determines the optimal resource stock as

$$X^* = \sqrt{K(\delta + \gamma)n/(pr)} \quad (26)$$

The comparative statics for this case are summarized in Table 2. In contrast to rate-dependent pollution, and most single-state bioeconomic models, an increase in the discount rate leads to an *increase* in the steady-state resource stock. Equally surprising is the result that an increase in the intrinsic growth rate leads to a *decrease* in the resource stock. The explanation may lie in the relative influence of the pollution stock in the rate- and capacity-dependent models. In the latter model the influence of Z on K is apparently strong enough to offset the usual comparative statics of the single-state model. For example, an increase in δ, results in a decline in the pollution stock which allows a net increase in the resource stock. An increase in r leads to an increase in Z which in turn reduces the carrying capacity to such an extent that the optimal biomass (X^*) actually *declines*.

It was initially thought that these results might be specific to the separable/linear forms. Relaxing either assumption makes the possibility of analytic solutions for either X^* or Z^* less likely. In a model where the transformation function took the form $Q = \sqrt{c + nS}$ and all other functions were the same, numerical analysis again produced $dX^*/d\delta > 0$ and $dX^*/dr < 0$. Thus, it would appear that the interaction between the stock pollutant and the renewable resource can produce comparative static results not typically encountered in single-state models.

In the separable/linear models, system dynamics are governed by four differential equations ($\dot{X}, \dot{Z}, \dot{\mu}_x, \dot{\mu}_z$) and the two switching functions σ_y and σ_s. Qualitative analysis of a point in state space, $\Re^2$, is made difficult because the complete dynamics takes place in $\Re^6$. A discrete-time analog was constructed and simulated for initial guesses for $\mu_{x,0}$ and $\mu_{z,0}$ and nonequilibrium values for X_0 and Z_0. Fairly intensive search failed to uncover the stable manifold which would lead to the known steady-state optimum. This is not too surprising and may be analogous to numerically trying to track a convergent separatrix to a saddle point equilibrium in a single-state model. It is more than a numerical inconvenience, however, since practical management might also be concerned with the optimal or at least "near-optimal" approach to a desired equilibrium.

Appendix B contains code (written in BASIC) for a simple algorithm which can assure convergence to a previously calculated equilibrium. It employs bang-bang controls to drive Z and Z^* and X to X^* as rapidly as possible and is thus similar "in spirit" to the MRAP solution. It does not employ the switching functions as the MRAP solution must, thus we cannot claim that the solutions obtained from the algorithm are optimal. The algorithm compares Z_t to Z^*, determines whether Z^* can be reached in t+1, and selects that S_t (subject to $0 \leq S_t \leq S_{max}$) which brings Z_{t+1} closest to Z^*. A similar set of calculations is made for Y_t based on a comparison of X_t and X^*.

Figure 2 shows four trajectories obtained from the initial conditions (1.0, 0), (0.7, 0.5), (0.1, 0.25) and (0.1, 0.15) when c=1, δ=0.1, γ=0.1, K=1, n=2, and r=0.5, for the case of rate-dependent pollution when Y_{max} = 0.20 and S_{max} =0.03. The numerical values for X_t, Z_t, Y_t, and S_t, for each are trajectory also given in Appendix B.

From the initial condition (1.0, 0) the point (X_t, Z_t) moves northwesterly reaching X^* first. At $X_t = X^*$ harvest is set equal to Y^* which, since the pollution stock is less than Z^*, will be less than growth, $F(X^*, Z_t)$. Thus, X_{t+1} will actually increase slightly above X^*, requiring $Y_{t+1} > Y^*$ so as to drive X_{t+2} back to X^*. All the while $S_t = S_{max}$ and Z_t is increasing toward Z^* as X_t undergoes damped oscillations along and slightly to the right of X^*.

The approach from (0.7, 0.5) proceeds southwesterly, again reaching X^* first. Since $Z_t > Z^*$ harvest at Y^* causes X_{t+1} to drop below X^*. The "zig-zag" down the left side of X^* is damped as Z_t approaches

Z^* with $S_t = 0$ if $Z_{t+1} = (1-\gamma)Z_t > Z^*$.

When a trajectory reaches Z^* first, $S_t = S^*$ and Z_t locks onto Z^* regardless of X_t. This is simply because the resource stock does not influence the dynamics of the pollution stock. This is revealed in the trajectories from (0.1, 0.25) and (0.1, 0.15).

The speed of convergence obviously depends on the relative growth rate, the rate of biodegradation, Y_{max} and S_{max}. The lower bounds of zero for Y_t and S_t presume that the species cannot be restocked from another location or hatchery, and that dredging of an excessive pollution stock is not feasible. In reality there are instances where both restocking and dredging have been employed to hasten recovery.

V. Policy

What are the implications, if any, of the above models for applied pollution control and resource management? The application of control theory to the management of environmental quality and renewable resources has been hampered by several factors. First, there are limited time-series data on which to generate estimates of the resource stock and even less data on the levels of pollution. These data limitations may make it difficult to estimate F(X,Z). Second, a single species is typically embedded in a larger system where food sources, predators, competitors and the physical environment are in a state of flux. These "external" factors frequently dominate the influence of biomass (X) on the change in biomass ($\dot{X}$), making the estimation of pollution-dependent growth functions statistically difficult *even if* sufficient time-series data can be assembled. Finally, while the preceding models presumed a commercial species with a marginal value equal to its per unit price, this need not always be the case. If a species' value derives from observation (as is the case for many marine mammals) then nonmarket valuation techniques must be employed. While recent advances in contingent valuation have increased the ability to estimate such values, significant problems remain and the validity of this approach is still controversial. How should one proceed?

Environmental economists when faced with the difficulty of estimating pollution damages have suggested that society should attempt to determine (via their elected representatives) an acceptable standard for ambient environmental quality. The objective of environmental policy might then be to achieve that standard at least cost. Suppose in an estuary that $\tilde{Z}$ is the maximum acceptable value for some stock pollutant and that the existing stock exceeds the target (ie, $Z_0 > \tilde{Z}$). If a pollution-dependent growth function had been estimated, optimal biomass could be calculated by finding the value of X which satisfies $\delta = F_x$ when $Z = \tilde{Z}$. Denote this value as $\tilde{X}$. If stock assessment reveals $X_0 < \tilde{X}$ then the separable/linear model of the preceding section would call for a moratorium on residual discharge *and* harvest. Such a policy is likely to have both commercial harvesters and the polluting firms "up-in-arms". A less rapid approach to $(\tilde{X},\tilde{Z})$ might be desirable.

Without some candidate F(X,Z) it is not clear what levels of pollution are consistent with the resource stock and sustainable yield. This does not mean that pollution control and resource management should proceed independently. One feasible approach, not totally inconsistent with the separable/linear models of the preceding section, is shown in Figure 3.

Suppose a team of scientists is empaneled to scrutinize all available historical data on harvest, discharge rates, estimates of the resource stock and the level of pollution. One of the panel's objectives is to provide interval estimates for the "optimal" resource and pollution stocks and point estimates of yield and residual discharge consistent with X and Z in those intervals. Suppose the optimal intervals are $X_1 \leq X^* \leq X_2$ and $Z_1 \leq Z^* \leq Z_2$, and $\tilde{Y}$ and $\tilde{S}$, are the estimates for sustainable yield and residual discharge in those intervals.

The second objective is to identify upper and lower bounds for Y and S which can "drive the system" (that is, cause X and Z to increase or decrease) and which are politically acceptable. These bounds are denoted as Y_{max}, Y_{min}, S_{max}, and S_{min}. Then, the initial condition (X_0, Z_0) must fall in one of the nine zones shown in Figure 3. The values for Y and S are uniquely determined by the initial condition and the subsequent evolution of the system. If the scientific committee has come up with a reasonably accurate assessment of the system's dynamics, the values for Y and S listed in the nine zones in Figure 3 might be thought of as "approximately optimal feedback controls" and they would guide the system to the "zone of optimality". There is no guarantee, however, given the initial state of ignorance and the political pressure which might influence the upper and lower bounds for Y and S, that the system will move toward this zone. Time may reveal the initial intervals to be inconsistent with the (true) underlying resource and pollution dynamics *or* less desirable than initially thought. It is again important for X and Z to be monitored as the initial feedback controls are applied. Such information may allow the estimation of a formal model, clarify the relative value (cost) of the resource (pollution) stock, and permit the identification of superior adaptive strategies.

Environmental economists have long realized that residuals management may require a multimedia approach. It is likely that pollution control and resource management will require greater synthesis and coordination in the future. This paper is an attempt at identifying the type of models and policies that may be useful in that synthesis.

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Table 1: Comparative Statics in the Case of Rate-Dependent Pollution

	δ	γ	K	n	p	r
X^*	-	+	+	+	-	+
Z^*	-	-	+	-	+	+

Table 2: Comparative Statics in the Case of Capacity-Dependent Pollution

	δ	γ	K	n	p	r
X^*	+	+	+	+	-	-
Z^*	-	-	+	-	+	+

Figure 1. A Graph of the Commodity-Residual Transformation Curve Implied by $\phi(Q,S) = O$

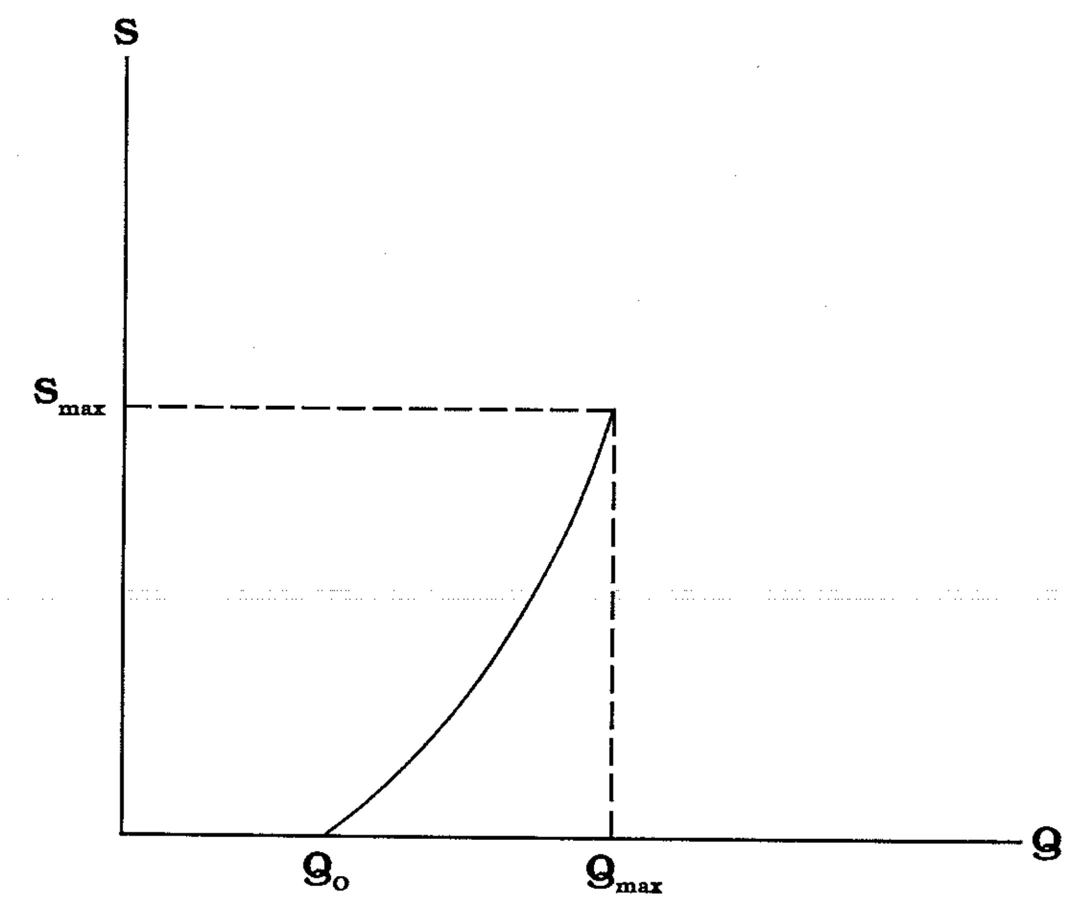


Figure 2: Convergence to the equilibrium (X^*, Z^*) for the case of Rate-Dependent Pollution when $c=1$, $\delta=0.1$, $\gamma=0.1$, $K=1$, $n=2$, $p=5$ and $r=0.5$ from initial conditions (1.0, 0), (0.7, 0.5), (0.1, 0.25) and (0.1, 0.15) when $Y_{max}=0.2$ and $S_{max}=0.03$ using the algorithm in Appendix B.

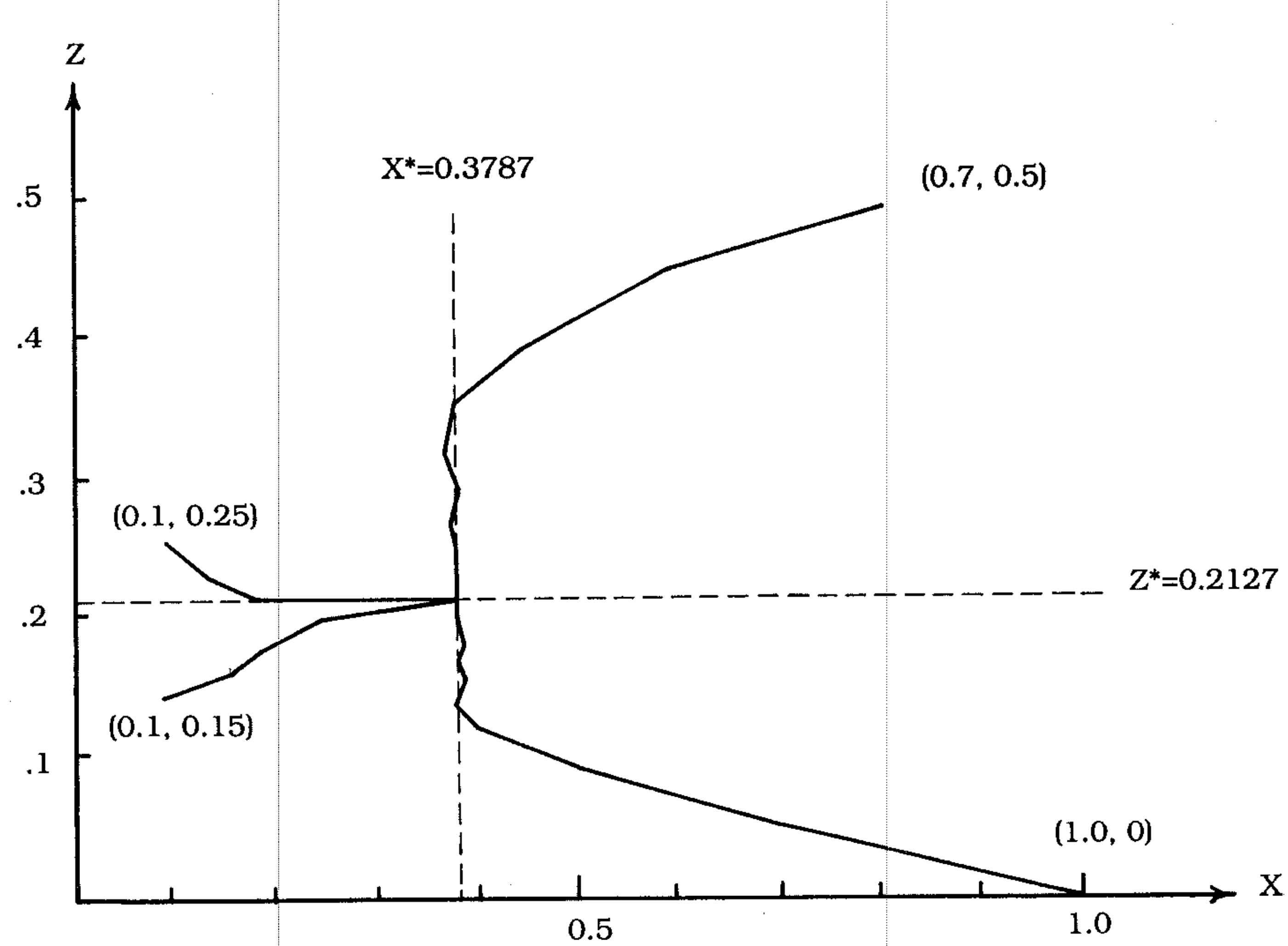
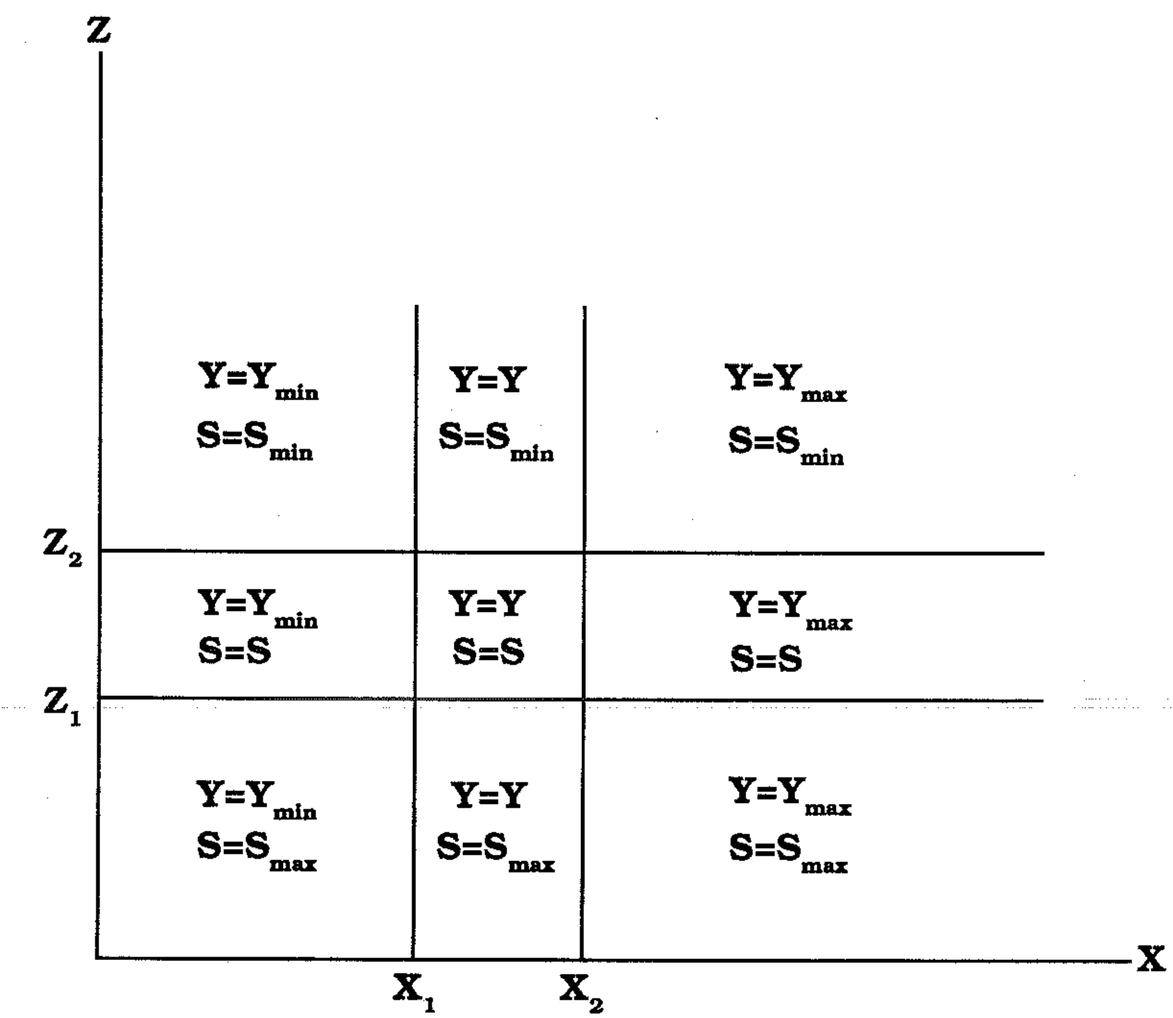


Figure 3. Harvest and Residual Discharge Based on Interval Guesses for Optimal Pollution and the Resource Stock



Appendix A

This appendix derives the steady-state comparative statics for a single-state pollution model. These results are useful in evaluating the comparative statics for the cases of rate- and capacity-dependent pollution.

Consider the problem

$$\text{Maximize} \int_0^\infty W(Q,Z)e^{-\delta t}dt$$

$$\text{Subject to } \dot{Z} = -\gamma Z + S$$
$$\phi(Q,S) = 0$$

where $W(\cdot)$ is a net benefit function with $W_q > 0$ and $W_z < 0$. The current value Hamiltonian may be written

$$\tilde{H} = W(Q,Z) + \mu[-\gamma Z + S] - \omega\phi(Q,S)$$

with first order conditions that include

$$\frac{\partial \tilde{H}}{\partial Q} = W_q - \omega\phi_q = 0$$

$$\frac{\partial \tilde{H}}{\partial S} = \mu - \omega\phi_s = 0$$

$$\dot{\mu} - \delta\mu = -\frac{\partial \tilde{H}}{\partial Z} = -[W_z - \mu\gamma]$$

In steady state these equations will imply

$$W_q = W_z[\phi_q/\phi_s]/(\delta + \gamma)$$

When $W(Q,Z) = pQ - aZ^2$, so that pollution damage is quadratic in the stock, and $Q = c + nS$, then the above equation may be solved for Z yielding

$$Z^* = \frac{np(\delta + \gamma)}{2a}$$

and the comparative statics are immediately apparent; namely that the optimal pollution stock increases with an increase in n, p, δ, or γ and decreases with an increase in a. The MRAP is optimal.

APPENDIX B

Program for Driving Z_t to Z^* and X_t to X^* for the Case of Rate-Dependent Pollution, $0 \leq Y_t \leq 0.2$, $0 \leq S_t \leq 0.03$

```
10 DATA 1,0.1,0.1,1,2,5,0.5
20 READ C,D,G,K,N,P,R
30 DIS=P*(D^2)*K/(4*R*N*(D+G)+P*(D^2)*K)
40 X=K*(1-SQR(DIS))/2
50 Z=R*(1-2*X/K)/D-1
60 Y=R*X*(1-X/K)/(1+Z)
70 S=G*Z
80 Q=C+N*S
90 PRINT "C=";C;"D=";D;"G=";G;"K=";K;"N=";N;"P=";P;"R=";R
100 PRINT:PRINT "When pollution affects the growth rate:"
110 PRINT USING "X=##.####";X
120 PRINT USING "Z=##.####";Z
130 PRINT USING "Y=##.####";Y
140 PRINT USING "S=##.####";S
150 PRINT USING "Q=##.####";Q
160 INPUT "HORIZON LENGTH = ";TT
170 DIM X(TT),Z(TT),Y(TT),S(TT)
180 X(0)=.7:YMAX=.2
190 Z(0)=.5:SMAX=.03
200 FOR T=0 TO TT-1
210 F=R*X(T)*(1-X(T)/K)/(1+Z(T))
220 IF Z(T)=Z THEN S(T)=S:GOTO 260
230 IF Z(T)*(1-G)>=Z THEN S(T)=0:GOTO 260
240 IF Z(T)*(1-G)+SMAX<=Z THEN S(T)=SMAX:GOTO 260
250 S(T)=Z-(1-G)*Z(T)
260 IF X(T)=X THEN Y(T)=Y:GOTO 300
270 IF X(T)+F<=X THEN Y(T)=0:GOTO 300
280 IF X(T)+F-YMAX>=X THEN Y(T)=YMAX:GOTO 300
290 Y(T)=X(T)+F-X
300 X(T+1)=X(T)+F-Y(T):Z(T+1)=(1-G)*Z(T)+S(T)
310 PRINT:PRINT "X(";T;")=";X(T)
320 PRINT "Z(";T;")=";Z(T)
330 PRINT "Y(";T;")=";Y(T)
340 PRINT "S(";T;")=";S(T)
350 IF ABS(X(T)-X)+ABS(Z(T)-Z)<.00001 GOTO 380
360 NEXT T
370 GOTO 390
380 PRINT "Steady State Attained at t=";T
390 INPUT "Do you want a print-out of X(t),Z(t),Y(t) and S(t)? Yes=1, No=0.";
400 IF W=0 GOTO 540
410 LPRINT "For rate-dependent pollution when:"
420 LPRINT:LPRINT "C=";C;"D=";D;"G=";G;"K=";K;"N=";N;"P=";P;"R=";R
430 LPRINT:LPRINT USING "X=##.####";X
440 LPRINT USING "Z=##.####";Z
450 LPRINT USING "Y=##.####";Y
460 LPRINT USING "S=##.####";S
470 LPRINT USING "Q=##.####";Q
480 LPRINT
490 LPRINT "          t","        X","      Z","   Y"," S"
500 LPRINT "____________________________________________________________"
510 FOR I=0 TO T
520 LPRINT USING "#######.####";I,X(I),Z(I),Y(I),S(I)
530 NEXT I
540 END
```

Actual Values of X_t, Z_t, Y_t and S_t for Trajectories in Figure 2.

From $(X_0,Z_0) = (1,0)$

t	X	Z	Y	S
0.0000	1.0000	0.0000	0.2000	0.0300
1.0000	0.8000	0.0300	0.2000	0.0300
2.0000	0.6777	0.0570	0.2000	0.0300
3.0000	0.5810	0.0813	0.2000	0.0300
4.0000	0.4936	0.1032	0.2000	0.0300
5.0000	0.4069	0.1229	0.1356	0.0300
6.0000	0.3787	0.1406	0.0970	0.0300
7.0000	0.3849	0.1565	0.1085	0.0300
8.0000	0.3787	0.1709	0.0970	0.0300
9.0000	0.3822	0.1838	0.1032	0.0300
10.0000	0.3787	0.1954	0.0970	0.0300
11.0000	0.3801	0.2059	0.0991	0.0274
12.0000	0.3787	0.2127	0.0970	0.0213

From $(X_0,Z_0) = (0.7,0.5)$

t	X	Z	Y	S
0.0000	0.7000	0.5000	0.2000	0.0000
1.0000	0.5700	0.4500	0.2000	0.0000
2.0000	0.4545	0.4050	0.1640	0.0000
3.0000	0.3787	0.3645	0.0970	0.0000
4.0000	0.3679	0.3280	0.0768	0.0000
5.0000	0.3787	0.2952	0.0970	0.0000
6.0000	0.3725	0.2657	0.0862	0.0000
7.0000	0.3787	0.2391	0.0970	0.0000
8.0000	0.3767	0.2152	0.0945	0.0190
9.0000	0.3787	0.2127	0.0970	0.0213

From $(X_0,Z_0) = (0.1,0.25)$

t	X	Z	Y	S
0.0000	0.1000	0.2500	0.0000	0.0000
1.0000	0.1360	0.2250	0.0000	0.0102
2.0000	0.1840	0.2127	0.0000	0.0213
3.0000	0.2459	0.2127	0.0000	0.0213
4.0000	0.3223	0.2127	0.0336	0.0213
5.0000	0.3787	0.2127	0.0970	0.0213

From $(X_0,Z_0) = (0.1,0.15)$

t	X	Z	Y	S
0.0000	0.1000	0.1500	0.0000	0.0300
1.0000	0.1391	0.1650	0.0000	0.0300
2.0000	0.1905	0.1785	0.0000	0.0300
3.0000	0.2560	0.1906	0.0000	0.0300
4.0000	0.3359	0.2016	0.0500	0.0300
5.0000	0.3787	0.2114	0.0970	0.0224
6.0000	0.3788	0.2127	0.0971	0.0213
7.0000	0.3787	0.2127	0.0970	0.0213