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**Economics, Mathematical Models,**  
**and**  
**Environmental Policy**

**by**  
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## **Economics, Mathematical Models and Environmental Policy**

### **ABSTRACT**

This paper briefly reviews several models of externality which provide the theoretical basis of environmental economics. An externality may be defined as a *situation* where the output or action of a firm or individual affects the production possibilities or welfare of another firm or individual who has no direct control over the initial level of the output or activity. Pollution, resulting from the disposal of residual wastes, is a classic example of externality.

Three static models examine the optimality conditions for (1) a two-person externality, (2) a many-person externality (where the externality takes the form of a "pure public bad"), and (3) a two-plant polluter. In the case of a two-person externality negotiation between the affected parties may lead to the optimal level for the externality *regardless* of the initial assignment of property rights. In the many-person case environmental policy, either direct controls or economic incentives, may be required to achieve an optimal allocation of resources. Economic incentives may take the form of per unit taxes on emissions or transferable discharge rights. In the third model it is shown how a tax can induce optimal (least cost) treatment from a two-plant polluter.

Two dynamic models examine the cases where (1) a pollution stock may accumulate or degrade according to rates of discharge and biodegradation and (2) a toxic residual must be transported from sites where it is generated to sites where it may be safely stored. The latter problem poses environmental risks from spills in transit or leakage at storage sites.

While radioactive and toxic wastes are likely to continue to be regulated by direct controls some of the more "benign" residuals are suitable for regulation by economics incentives. Effluent taxes in France and the Netherlands, transferable discharge permits on the Fox River in Wisconsin, transferable stove permits in Telluride, Colorado and the EPA's emission-offset policy are indications that economic incentives will play a greater role in the future management of environmental quality.



# **Economics, Mathematical Models and Environmental Policy**

## **I. Introduction**

The now extensive literature on environmental economics has its theoretical roots in the field of welfare economics and is specifically tied to the concept of externality (Mishan 1971). An externality might be defined as a situation where the output or action of a firm or individual directly affects the production possibilities or welfare of another firm or individual who has no direct control over the level of output or activity. Consider a brewery downstream from a pulp mill. The level of jointly produced pulp waste dumped into the river will directly influence the production process for beer by determining whether the brewery must treat water from the river or use water from an alternative source. As another example, suppose the volume (and type of music) on my stereo adversely affects the welfare of my neighbor. By lacking direct control over the amount of pulp waste or the volume and type of music we mean that the brewery or neighbor has no direct influence over the initial level for the externality. A unilateral decision designed to maximize the profit or welfare of a single individual may impose costs on others (the brewery

individuals. In contrast, a "purely public" externality occurs when a large number of individuals face the same (uniform) amount of some externality. The small-number externality may be amenable to private negotiation or bargaining of the sort envisioned by Coase (1960). It is possible to talk about an "optimal level" for an externality and examine the likely outcome of negotiations under different property rights or liability rules. An example of a purely public externality might be the current problems of acid rain or ozone depletion which affect a large number of individuals and where the level of externality may be more or less uniform across a large area or population. We will start by considering some static, two-party externality models.

Suppose Individual One would like to engage in an activity whose level is denoted by the continuous variable  $X$  and which results in net benefits according to the concave function  $N(X)$ . This activity, however, imposes costs on Individual Two according to the convex function  $C(X)$ . Then the welfare of a "community" comprised of Individuals One and Two might be calculated as

$$W = N(X) - C(X) \tag{1}$$

and the first order necessary condition for maximization of net social welfare requires  $dW/dX = 0$  implying  $N'(X) = C'(X)$ , and the optimal level of the externality is that which balances net marginal benefits to



Individual One with the marginal cost to Individual Two. Figure 1 shows a possible graph of  $N'(X)$  and  $C'(X)$  and the optimal externality  $X = X^*$  where  $N'(X)$  and  $C'(X)$  intersect.

Now consider the situation when Individual One has the property right to set  $X$  at a level which maximizes his or her net benefit. This would require  $N'(X) = 0$  which occurs at  $X = \bar{X}$ . At  $\bar{X}$ , however, significant marginal costs are being imposed on Individual Two. While Individual One has the right to set  $X$  at a level which maximizes his self-interest he or she might be amenable to a proposal to reduce  $X$  if a bribe or side payment were sufficient to compensate for any foregone net benefits. How much would it take and how much would Individual Two be willing to offer? At  $X = \bar{X}$  Individual Two would be willing to pay up to  $C'(X)$  for a marginal reduction in  $X$ , while Individual One would only require a small amount based on the very low net marginal benefits foregone for initial incremental reductions from  $\bar{X}$ . A comparison of  $C'(X)$  to  $N'(X)$  would indicate that Individual Two should be willing to offer up to marginal cost for an incremental reduction in  $X$ , while Individual One would require at least compensation of foregone net benefits. While the exact distribution of the vertical difference between  $C'(X)$  and  $N'(X)$  cannot be deduced it is positive for reductions in  $X$  all the way back to  $X^*$ . Thus, there is an incentive for negotiation to lead to a reduction in  $X$  from  $\bar{X}$  to  $X^*$ .

What if Individual Two has the property right to an environment free of cost imposed by  $X$ ? If Individual Two has the right to initially set the level for  $X$  he or she would obviously select  $X = 0$ , where marginal cost  $C'(X) = 0$ . However at  $X = 0$  Individual One is foregoing significant net benefits and has a strong incentive to approach Individual Two to see if he or she can be bribed into putting up with some costs associated with  $X > 0$ . With a similar logic as that governing the negotiated reduction from  $\bar{X}$ , Individual One would be willing to pay up to  $N'(X)$  while Individual Two would require at least a payment to cover the marginal costs,  $C'(X)$ . The vertical distance between  $N'(X)$  and  $C'(X)$  (and thus the incentive for negotiation) remains positive from  $X = 0$  up to  $X = X^*$ . Thus, there would a tendency for  $X$  to be negotiated upward from 0 toward  $X^*$ . What is surprising is that the level of externality after negotiation is the same regardless of the initial assignment of property rights. This is sometimes called the "Coase Theorem" in recognition of Ronald Coase's discussion of this subject in an article entitled "The Problem of Social Cost" (*Journal of Law and Economics*, October, 1960.).

Coase's Theorem hinges on some implicit assumptions, particularly that the transactions costs of negotiating are zero or symmetric when starting from  $X = \bar{X}$  or  $X = 0$  (ie, when property

rights are vested with Individual One or Individual Two), and that the net benefit and cost functions are invariant to changes in income. In other words, as side payments are made  $N'(X)$  and  $C'(X)$  do not shift. This latter assumption is referred to as a "zero income effect"

Coase concludes that there may be no need for government intervention in an externality situation if the social cost were generated and borne by a few individuals and property rights (liability rules) were well defined by common law. In such a case the concerned individuals could be expected to negotiate to a more or less optimal level of externality. These conditions are not likely to be met when an externality results from or adversely affects a large number of individuals. We now turn to this type of public externality. Baumol and Oates (1975) refer to it as an "undepleteable externality".

At a point in time suppose the productive resources for an economy are fixed and  $\phi(Q,S) \equiv 0$  is an implicit production possibilities or transformation curve identifying the feasible combinations of a positively valued commodity,  $Q$ , and a negatively valued residual,  $S$ . Thus, the fixed resources must be allocated between production of  $Q$  or reduction of  $S$ . By convention we assume  $\partial\phi(\cdot)/\partial Q > 0$  and  $\partial\phi(\cdot)/\partial S < 0$ . A graph of  $\phi(Q,S) \equiv 0$  is shown in Figure 2. There exists some implicit allocation of resources leading to  $S = 0$  and  $Q = Q_0$ . To

increase output above  $Q_0$  requires a reallocation of resources away from residual treatment. While  $Q$  can be increased,  $S$  increases at an increasing rate. The point  $(Q_{\max}, S_{\max})$  corresponds to an implicit allocation where all resources are devoted to production of  $Q$ .

Suppose there are  $I$  individuals with utility functions  $U_i = U_i(q_i, S)$  where  $q_i$  is the amount of  $Q$  going to the  $i^{\text{th}}$  individual. We assume that  $\partial U_i(\cdot)/\partial q_i > 0$  while  $\partial U_i(\cdot)/\partial S < 0$ . The residual is a "public bad" in the sense that the same level enters everyone's utility function and the "consumption" by one individual does not diminish the amount "consumed" by others. The commodity  $Q$  is a private good since an increase in the amount consumed by one individual reduces the amount available for others.

Conditions for a Pareto optimum  $(Q, S, q_i, i = 1, 2, \dots, I)$  can be derived by maximizing the utility of one individual subject to indifference curve constraints for all other individuals, a balancing equation for  $Q$  and the transformation function  $\phi(Q, S) \equiv 0$ . Let  $U_i^* > 0$  be the utility (indifference) constraint for the  $i^{\text{th}}$  individual. Then, the Lagrangian may be written as

$$L = \sum_{i=1}^I \lambda_i [U_i(\cdot) - U_i^*] + \omega \left( Q - \sum_{i=1}^I q_i \right) - \mu \phi(\cdot) \quad (2)$$

where  $\lambda_1 = 1$  and  $U_1^* = 0$ . Assuming a solution where  $Q > 0$ ,  $S > 0$  and

$q_i > 0$  (ie an interior solution), then first order conditions require

$$\frac{\partial L}{\partial Q} = \omega - \mu[\partial\phi(\cdot)/\partial Q] = 0 \quad (3)$$

$$\frac{\partial L}{\partial S} = \sum_{i=1}^I \lambda_i [\partial U_i(\cdot)/\partial S] - \mu[\partial\phi(\cdot)/\partial S] = 0 \quad (4)$$

$$\frac{\partial L}{\partial q_i} = \lambda_i [\partial U_i(\cdot)/\partial q_i] - \omega = 0 \quad (5)$$

$$\frac{\partial L}{\partial \omega} = Q - \sum_{i=1}^I q_i = 0 \quad (6)$$

and

$$\frac{\partial L}{\partial \mu} = -\phi(Q, S) = 0 \quad (7)$$

Given the signs for the partials of  $\phi(Q, S)$  and  $U(q_i, S)$  it can be shown that  $\omega > 0$  and  $\mu > 0$ . Some algebra will reveal that

$$\sum_{i=1}^I \frac{[\partial U_i(\cdot)/\partial S]}{[\partial U_i(\cdot)/\partial q_i]} = \frac{[\partial\phi(\cdot)/\partial S]}{[\partial\phi(\cdot)/\partial Q]} \quad (8)$$

Equation (8) requires that the sum of the marginal rates of substitution (of the residual for the commodity) over all individuals equal the marginal rate of transformation and is analogous to Samuelson's (1954, 1955) condition for optimal provision of a pure public good. In this case, however, we have the condition for the optimal level of an undepletable externality, or a pure public bad.

Equation (4) has an important interpretation. The term

$$-\sum_{i=1}^I \lambda_i [\partial U_i(\cdot) / \partial S] > 0$$

is the marginal social damage from an increase in the residual (which negatively affects all individuals). Some additional algebra will reveal that this term must be equated to

$$-\omega \frac{[\partial \phi(\cdot) / \partial S]}{[\partial \phi(\cdot) / \partial Q]} > 0$$

which is the marginal social value of an additional unit of S, which in turn allows the economy to produce more Q (which is what society positively values). Thus, equation (4) requires a balancing of the value of increased output with the marginal social cost resulting from the increase in pollution. The conditions for optimality raise an important practical question: How do you measure the marginal social cost of pollution? The losses are direct utility losses which creates a difficult evaluation problem. Suppose S was an air pollutant with a nonuniform affect over a large urban area. By a careful statistical analysis of property values it might be possible to estimate the disutility of living with alternative levels of S as reflected in reduced property values. Contingent valuation techniques, where an individual is asked to state his willingness to pay for a less polluted environment (or the required compensation for the individual to accept a more polluted environment) might also provide estimates of the marginal social cost

of pollution. There are difficulties with both of these approaches and it may be more practical to consider policies which do not require empirical estimates of marginal social cost. The following problem may suggest such a policy.

A corporation has two plants on a large lake. Under normal operating conditions, and without any environmental restrictions, the plant was discharging wastes at rates  $\bar{R}_1$  and  $\bar{R}_2$  from Plants #1 and #2, respectively. The EPA regards the combined discharge as excessive and requires that the total discharge from both plants not to exceed  $R$ . Denoting the untreated discharge from Plants #1 and #2 by  $R_1$  and  $R_2$ , respectively, the EPA discharge constraint implies  $R_1 + R_2 \leq R$ . The amount of waste treated at Plant #1 and #2 is  $(\bar{R}_1 - R_1)$  and  $(\bar{R}_2 - R_2)$ , respectively. Suppose the cost of treatment is a function of the amount treated and that the corporations combined treatment costs may be calculated according to

$$C = C_1(\bar{R}_1 - R_1) + C_2(\bar{R}_2 - R_2)$$

where  $C_1(\cdot)$  and  $C_2(\cdot)$  are treatment cost functions for Plants #1 and #2, respectively. The firm wishes to minimize the total cost of treatment subject to meeting the EPA discharge constraint. The Lagrangian for this problem may be written

$$L = C_1(\bar{R}_1 - R_1) + C_2(\bar{R}_2 - R_2) - \lambda(R - R_1 - R_2) \quad (9)$$

Assuming a positive level of treatment at both plants and that the EPA constraint is precisely met (ie, it holds as an equality), then the first order conditions for minimum treatment costs require

$$\frac{\partial L}{\partial R_1} = -C'_1(\bullet) + \lambda = 0 \quad (10)$$

$$\frac{\partial L}{\partial R_2} = -C'_2(\bullet) + \lambda = 0 \quad (11)$$

$$\frac{\partial L}{\partial \lambda} = -(R - R_1 - R_2) = 0 \quad (12)$$

Equations (10) and (11) imply that  $C'_1(\bullet) = C'_2(\bullet)$ , that is, the marginal cost of treating the last unit in each plant must be the same. Taken together, equations (10)-(12) constitute a three equation system which may be solved for the cost minimizing rates of discharge  $R_1$ ,  $R_2$  and the shadow price of the EPA constraint,  $\lambda$ . Note:  $\partial L / \partial R = -\lambda < 0$  is the marginal cost of the EPA constraint and as  $R$  decreases, treatment costs increase.

How does this problem relate to the difficulty of measuring marginal social damage and the formulation of environmental policy? The EPA specified an allowable discharge,  $R$ , hoping that this amount would result in acceptable ambient environmental quality. Alternatively, it could have specified a tax on each unit of  $R_1$  and  $R_2$



and let the corporation decide how much to treat from each plant and thus how much to pay in taxes. If the EPA had solved equations (10) - (12) and set the tax rate  $\tau = \lambda$ , then the corporation would presumably choose the same values for  $R_1$  and  $R_2$  that it originally chose when faced with discharge constraint,  $R$ . This is true since minimizing

$$TC = C_1(\bar{R}_1 - R_1) + C_2(\bar{R}_2 - R_2) + \tau(R_1 + R_2) \quad (13)$$

when  $\tau = \lambda$  will result in first order conditions requiring  $C'_1(\cdot) = C'_2(\cdot) = \tau$ , which was the same condition obtained for the original problem. The tax forces the corporation to determine if it can treat the marginal unit of waste for a cost less than the unit tax itself. If it can, it will do so, otherwise it will opt to pay the tax. The tax (called an effluent charge for water pollutants or an emission tax for air pollutants) has the desirable property of achieving a given emission reduction at least cost. Specifically, it causes firms to treat lower cost emissions first. If, after subsequent analysis, the EPA does not view the emission reduction as sufficient to achieve the desired level for ambient environmental quality, it can request a tax increase which will raise the cost of untreated emissions. The tax places the burden of finding the best way to reduce emissions on the corporation and the EPA does not have to spend time discovering the "best practical technology" nor inspecting firms to see that it has been installed and is being

maintained. It will need to monitor emissions to determine the level of untreated residuals in order to calculate the corporation's tax bill, but this should not increase transactions costs above the current costs of monitoring a system of emission standards

To summarize this section so far we have (1) considered a small-number externality (between two individuals) where negotiation between "polluter" and "pollutee" might lead to the same (optimal) level for the externality without government intervention, regardless of the initial assignment of property rights (liability). In the case of a large number externality, uniformly affecting a large number of individuals, we obtained an optimality condition equating the sum of all affected individuals' rate of commodity substitution (RCS) to the rate of transformation of the private good  $Q$ , for the residual  $S$ . The residual was a "pure public bad" and the optimality condition is analogous to Samuelson's condition for the optimal provision of a pure public good. The marginal social cost of pollution was the negative of the weighted sum of marginal disutility, which while being an interesting concept from a public policy perspective, presented difficult measurement problems. The model of the two-plant corporation, minimizing the cost of treatment subject to a total discharge constraint, revealed that the corporation would allocate

treatment so that marginal treatment costs were equal. The shadow price (Lagrange multiplier) on the discharge constraint, if used as a per unit tax on untreated discharges, would induce the same-least cost pattern of treatment. By taxing residual discharges an environmental agency has the ability, in theory, to bring about any desired reduction in emissions at least cost and with lower administration costs than policies that rely on direct regulation.

The three models presented thus far have not contained any detail about the physical characteristics of the residuals being jointly produced nor about the medium in which they might be disposed. A processing or manufacturing plant may often have some control over the chemical structure and form of a residual. It may then be able to dispose of the residual in one or more receiving media.

Environmental management becomes a complex problem of simultaneously determining not only the optimal allocation of resources and distribution of output, but the composition and disposition of the many forms of residual waste and the alternative media (air, water, land) into which these residuals may be disposed. To minimize the total cost of disposal one would like to transform and dispose of residuals in such a way that the marginal disposal plus marginal damage cost is the same for all media receiving waste (See Conrad and Clark 1987, Chapter 4, for a more complete static model

of multimedia residuals management). It may be optimal to *never* use some disposal options or some media if the marginal disposal cost or the marginal damage cost is always higher than the next best alternative. (For a discussion of this situation with regard to sludge disposal in the New York Bight see Conrad 1985).

In the late 1960s environmental economists became aware of the implications of the First Law of Thermodynamics which dictates a mass/energy balance in a closed system. In most economic systems this means that the mass of raw material inputs must ultimately equal the mass of waste (see Figure 3). Kenneth Boulding (1966) likened the earth to a spaceship with not only finite resources but a finite space to dispose of waste (residuals). Maximization of the rate of increase in Gross National Product may not be a desirable objective if the "newly produced goods and services" must ultimately become waste. The notion of a spaceship earth raises a fundamental problem: If we succeed in reducing the amount of Residual #1 being disposed via media #3, might we simply be creating another problem because we now must dispose of more of some other waste via an alternative medium. The management of residuals (which generate negative externalities) may ultimately require a large scale systems approach such as that envisioned by Kneese, Ayres and d'Arge (1970):

The Administrator of the World Environmental Control Authority sits at his desk. Along one wall of the huge room are real-time displays processed by computer from satellite data, of developing atmospheric and ocean patterns, as well as the flow and quality conditions of the world's great river systems. In an instant, the Administrator can shift from real-time mode to simulation to test the larger effects of changes in emissions of material residuals and heat to water and atmosphere at control points generally corresponding to the locations of the world's great cities and the transport movements among them. In a few seconds the computer displays information in color code for various time periods - hourly, daily, or yearly phases at the Administrator's option. ...Observing a dangerous reddish glow in the eastern Mediterranean, the Administrator dials sub-control station Athens and orders a step-up of removal by the liquid residuals handling plants there. Over northern Europe, the brown smudge of a projected air quality standards violation appears and sub-control point Essen is ordered to take the Ruhr area off sludge incineration for 24 hours but is advised that temporary storage followed by accelerated incineration - but with muffling - after 24 hours will be admissible. The CO<sub>2</sub> simulator now warns the Administrator that another upweller must be brought on line in the Murray Fracture Zone within two years if the internationally agreed balance of CO<sub>2</sub> and oxygen is to be maintained in the atmosphere.

It is unlikely that sovereign states would be willing to vest an international agency with such powers of intervention and regulation. However, the industrialized countries of western Europe have made some initial attempts to at least better coordinate environmental policies pertaining to transfrontier pollution (OECD 1974, 1976). The U. S. and Canada have a considerable history of diplomacy and cooperation in managing the quality of boundary waters and fishery

resources. Recent discussions on acid rain in North America have not led to any substantial agreement on policy, in part because of a reluctance by the current U.S. administration to commit itself to an expensive control program when they regard the consequences and costs of acid deposition as "poorly understood".

While multimedia residuals management makes sense from an economic point of view its feasibility from a political point of view must be questioned in light of the difficulty of siting sanitary landfills, nuclear waste repositories and other "nauseous" facilities. In other words, arranging for the transport and disposal of residuals in political jurisdictions different from where they were generated is a difficult and sensitive area of "intergovernmental relations" with very few communities willing to receive wastes from other areas. In a country with many many levels of local government (eg, state, county, city or town) the ability to transform residuals for disposal in a medium and at a location with the highest assimilative capacity or with the best storage characteristics may be blocked politically.

### **III. Dynamic Externality**

We now turn to a simple model of dynamic externality where residuals might accumulate as a pollution stock. It is this stock which

imposes a social cost and two important questions become (1) Is there an optimal, steady-state level for the pollution stock and the rate of residual discharge, and (2) If the current pollution stock is not optimal what is the optimal rate of residual discharge along an approach path?

Let  $X_t$  represent the pollution stock and  $R_t$  the rate of residual discharge in period  $t$ . We will assume that the pollution stock changes according to

$$X_{t+1} = X_t + R_t - D(X_t) \quad (14)$$

where  $D(X_t)$  is a degradation function specifying the rate at which the pollution stock degrades into its (harmless) organic constituents. If  $D(X_t) = 0$  then we have a model of pure accumulation.

Suppose  $Q_t$  is the output of some good or service sold at a constant per unit price,  $p$ . As before let  $\phi(Q_t, R_t) \equiv 0$  be an implicit production function expressing the maximum amount of  $Q_t$  attainable from the available fixed resources and a given level of  $R_t$ . Let  $S(X_t)$  denote the social cost from the pollution stock of  $X_t$  in period  $t$ . Then we are interested in the solution to the problem

$$\text{Maximize } \sum_{t=0}^{\infty} \rho^t \{pQ_t - S(X_t)\}$$

$$\begin{aligned} \text{Subject to: } & X_{t+1} = X_t + R_t - D(X_t) \\ & \phi(Q_t, R_t) \equiv 0 \text{ and } X_0 \text{ given} \end{aligned}$$

The Lagrangian expression for this problem may be written as

$$L = \sum_{t=0}^{\infty} \rho^t \{ pQ_t - S(X_t) + \rho\lambda_{t+1}(X_t + R_t - D(X_t) - X_{t+1}) - \mu_t\phi(Q_t, R_t) \} \quad (15)$$

The first order necessary conditions can be shown to imply

$$p - \mu_t \partial\phi(\cdot)/\partial Q_t \quad (16)$$

$$\rho\lambda_{t+1} - \mu_t \partial\phi(\cdot)/\partial R_t = 0 \quad (17)$$

$$\rho\lambda_{t+1} - \lambda_t = S'(X_t) + \rho\lambda_{t+1}D'(X_t) \quad (18)$$

With the conventional partials  $\partial\phi(\cdot)/\partial Q_t > 0$  and  $\partial\phi(\cdot)/\partial R_t < 0$  it will be the case that  $\mu_t > 0$  and  $\lambda_t < 0$ . In steady state  $\mu = p/[\partial\phi(\cdot)/\partial Q]$  and  $\lambda = (1+\delta)p[\partial\phi(\cdot)/\partial R]/[\partial\phi(\cdot)/\partial Q]$  and we obtain the following system defining the optimal pollution stock, residual discharge and output level

$$-p \frac{[\partial\phi(\cdot)/\partial R]}{[\partial\phi(\cdot)/\partial Q]} = \frac{S'(X)}{[\delta + D'(X)]} \quad (19)$$

$$R = D(X) \quad (20)$$

$$\phi(Q_t, R_t) = 0 \quad (21)$$

The last two equations simply require that the rate of residual discharge equal the rate of biodegradation and the implicit production function holds in steady state.

To see how equations (19)-(21) might define an optimal steady state consider the following numerical example. Suppose



$$\phi(Q,R) \equiv Q - \sqrt{100 + R} = 0$$

$$D(X) = 0.10X$$

$$S(X) = X^2$$

$$\delta = 0.10$$

Then equation (19) implies  $X = p(100+R)^{-1/2}/20$ , while equation (20) implies  $X=10R$ . Equating these two expressions and solving for an implicit expression in  $R$  yields:  $40,000R^2(100+R) - p^2 = 0$ . Given a value for  $p$ , the per unit price of  $Q$ , this last equation can be solved numerically for optimal  $R > 0$ . This was done for  $p = 10,000$  and (apparently) resulted in a unique optimum with  $R^* = 4.88$ ,  $X^* = 48.82$  and  $Q^* = 10.24$ . (The cubic in  $R$  was solved using Newton's method and starting from various positive initial guesses for  $R$  the algorithm always converged to  $R^* = 4.88$ ).

We have answered the first of our two questions in that equations (19)-(21) will, with appropriate convexity assumptions, define an optimal level for the pollution stock. Suppose this was solved for (as in the numerical example) and upon comparison with the initial condition we observe  $X_0 > X^*$ . What can we say about the optimal discharge policy along an approach path to  $X^*$ ? In general the optimal discharge policy will be for  $R_t < D(X_t)$ , and under certain circumstances it will be optimal for  $R_t = 0$ . What we're looking for is an optimal discharge rate, less than biodegradation, which will induce

the pollution stock to optimally decline to  $X^*$  (see Figure 4). If the objective function  $W_t = pQ_t - S(X_t)$  can be rewritten as an additively separable function in  $X_t$  and  $X_{t+1}$  and, via reindexing, that additively separable function leads to an objective that is the sum of quasi-concave functions, then a Most Rapid Approach Path (MRAP) is optimal (see Spence and Starrett 1975). In Figure 4 an MRAP from  $X_0 > X^*$  has  $R_t = 0$  for  $0 \leq t < \hat{t}$ . At  $\hat{t}$ ,  $X_t = X^*$  and  $R_t = R^*$  for  $t > \hat{t}$ . If the conditions for MRAP are *not* met the infinite horizon control problem must be solved for the asymptotic approach path along which  $0 < R_t < R^* = D(X^*)$ , and  $X_t$  asymptotically approaches  $X^*$  from above while  $R_t$  asymptotically approaches  $R^*$  from below. These two cases cover the optimal approach from  $X_0 > X^*$  provided  $R_t$  cannot be negative (ie, dredging or removal of  $X_t$  for disposal via an alternative media is not feasible).

This simple problem captures many aspects of a pollution stock that can accumulate or degrade. As noted earlier if  $D(X_t) = 0$  then the pollution stock can only accumulate. If the residual is highly toxic or radioactive then transport and storage so as to preclude leakage into the ambient environment is warranted. (The social cost of disposal via air, water or land are viewed as infinite). We now turn to a discussion of these types of residuals.

#### IV. Management of Toxic Residuals

By a toxic residual we will mean a residual which is not suitable for disposal into (onto) the traditional disposal media: air, water or land. The residual must be transported from the site where it is generated to a storage site. Uncertain social costs arise because the residual may be spilled in transit or it may leak while in storage.

Nuclear wastes would be a good example of the type of residual being considered. Other toxics, such as acids, pesticides or other chlorinated hydrocarbons might be amenable to treatment (eg, pyrolysis) at a specialized facility with nontoxic constituents discharged into the air or water. We will consider a model of toxics which must be stored.

Let  $R_{i,j,t}$  represent the amount of the toxic residual generated at site  $i$  which is transported for storage at site  $j$  during period  $t$ ,  $i = 1, 2, \dots, I$ ;  $j = 1, 2, \dots, J$ ; and  $t = 0, 1, 2, \dots$ . Assume that the cost of transport from  $i$  to  $j$  depends on the volume of waste according to  $C_{i,j}(R_{i,j,t})$ . Let  $X_{j,t}$  represent the amount of the toxic in storage at site  $j$  in period  $t$ . The change in the stocks of waste in storage will depend on the volume of residuals received from the various generating sites, the amount in storage in the previous period less any unintended leakage.

These dynamics are described by the difference equation

$$X_{j,t+1} = (1-\omega_{j,t})X_{j,t} + \sum_{i=1}^I (1-\omega_{i,j,t})R_{i,j,t} \quad (22)$$

where  $\omega_{j,t}$  and  $\omega_{i,j,t}$  are random variables indicating the fraction of  $X_{j,t}$  leaking from storage site  $j$  and the fraction of  $R_{i,j,t}$  spilled in transit during period  $t$ . The distributions for these random variables will depend on the characteristics (including age) of the storage site, the route and method of transport and other exogenous factors (eg storms, earthquakes, etc.). The formulation of subjective probability distributions for  $\omega_{j,t}$  and  $\omega_{i,j,t}$  presents our toxic waste manager with a difficult (impossible?) exercise in risk assessment.

The second difficult element is the assessment of the likely social costs in the event of a spill or a leak. Suppose these costs depend only on the size of the spill or leak in period  $t$  so that  $S_j(\omega_{j,t}X_{j,t})$  and  $S_{i,j}(\omega_{i,j,t}R_{i,j,t})$  represent the social cost of a leak at site  $j$  and a spill in transit from site  $i$  to site  $j$  during period  $t$ . In reality these costs may depend on the amount and number of previous leaks or spills and the exact location of the the spill on the route between  $i$  and  $j$ .

Finally, we will assume that the amount of waste generated at site  $i$  is given (exogenously) by  $R_{i,t}$  and that the storage capacity at site  $j$  is limited to a volume less than or equal to  $X_j$ . Then the toxic waste

manager may seek to minimize the expected disposal and social costs by solving the problem

$$\begin{aligned} \text{Minimize } & E \left\{ \sum_{t=0}^{\infty} \rho^t \left[ \sum_{i,j} [C_{i,j}(R_{i,j,t}) + S_{i,j}(\omega_{i,j,t} R_{i,j,t})] + \sum_{j=1}^J S_j(\omega_{j,t} X_{j,t}) \right] \right\} \\ \text{Subject to } & X_{j,t+1} = (1 - \omega_{j,t}) X_{j,t} + \sum_{i=1}^I (1 - \omega_{i,j,t}) R_{i,j,t} \\ & R_{i,t} - \sum_{j=1}^J R_{i,j,t} = 0 \\ & X_{j,0} \text{ given, } R_{i,j,t} \geq 0, 0 \leq X_{j,t} \leq X_j \end{aligned}$$

The above problem is a complex stochastic optimization problem and a numerical solution might be obtained using stochastic dynamic programming. In addition to the finite number of generating and disposal sites it may be necessary to assume that the random variables  $\omega_{j,t}$  and  $\omega_{i,j,t}$  are generated from a set of finite fractions with known, discrete probabilities. (Note, the summation over  $i,j$  is over all possible combinations of generation and disposal sites which is also a finite set). The solution will be in a feedback form since the optimal disposal pattern in period  $t$  will depend on the pollution stocks at the beginning of period  $t$  which in turn were determined by the random spills and leaks that occurred in period  $(t-1)$ .

The above model assumes the existence of  $J$  disposal sites and thus seeks to minimize the expected sum of transport and social costs.

If storage sites do not exist then the problem becomes more a complicated problem of siting and scale of investment in storage facilities. The objective of such a siting-storage problem might be the minimization of the expected present value of the sum of construction (capital), transport and social costs. The fact that alternative storage locations may influence spill or leakage probabilities from the different generating sites makes the problem especially difficult. The current controversy surrounding the location of nuclear waste repositories also points out the difficulty of finding local communities that would even consider the location of such a facility in their district (see Carter 1987). As opposed to transporting wastes from nuclear reactors used to generate electricity, Chapman (1987) estimates that on-site storage coordinated with decommissioning (shutdown and incasement of contaminated materials when the reactor is "retired") may be the least cost means of storing nuclear wastes.

## **V. Environmental Policy**

We noted earlier that the regulation of environmental externalities could basically take two forms: command and control (C&C) or economic incentives (EI) such as effluent taxes, pollution abatement subsidies or marketable pollution permits. Environmental policy in the U. S. has tended to rely on C&C type policies, specifically

emission standards and equipment (technology) standards. Subsidies have been provided to municipalities to construct wastewater treatment plants and firms have been given tax deductions or accelerated depreciation for equipment to reduce air pollution. (The plant construction subsidies are not the same as the per unit subsidy for emission reduction that economists tend to think of when considering economic incentives). In the area of oil spills and toxic wastes recent laws have employed the legal principle of *strict liability* for cleanup costs and damages from accidental spills, while a "cradle-to-grave" regulation has been used to control toxic wastes. These policies have produced some significant improvements in the quality of a few lakes, rivers, and "air sheds" but it is generally thought by economists that the accomplishments have been modest and probably achieved at an unnecessarily high cost. Problems which have not been adequately dealt with include transboundary pollution such as acid rain, groundwater contamination, and the disposal or storage of toxic and radioactive wastes. If the current C&C type policies are excessively costly would EI policies offer higher environmental quality at a lower cost to society? For which types of pollutants would EI policies be most appropriate?

We will consider two types of EI policies: a per unit emission

tax and a system of transferable pollution permits which allows the holder to discharge or emit a specified amount of some waste into a particular medium (stream, lake or airshed). As was noted in the third model of static externality, a per unit tax on residual discharge will force the cost-minimizing firm to determine if it can treat the marginal unit of residual at a cost less than the per unit tax. A higher tax rate would presumably induce more treatment, lower discharge and higher ambient quality.

Under a system of marketable pollution permits the environmental agency determines a total amount for some residual which might be discharged into a particular medium without resulting in unacceptable ambient quality. This total amount is then divided into smaller units corresponding to the amount or fraction of an amount which might be discharged by a typical firm generating this residual. Permits, entitling the holder to discharge this smaller unit might then be issued gratis to firms who had historically held emission permits (based on installed equipment) or they could be sold at auction. Once distributed the permits could be sold to another firm discharging into the same stream, lake or airshed. If a new firm wished to locate in the area it would have to purchase the necessary pollution permits from existing firms, thus keeping total discharge at



the desired level. The opportunity to sell some of their pollution permits to new or existing firms would create an opportunity cost for firms currently holding permits and make them sensitive to the possibility of selling if they could treat (or avoid generation in the first place) at lower marginal cost.

With either of these EI policies the environmental agency needs to know the relationship between total discharge and ambient quality within the receiving medium. With the pollution tax the environmental agency will also need to know (or subsequently learn by trial-and-error) the relationship between the tax rate and the total level of discharge. There are likely to be factors which cause these functional relationships to be stochastic or change over time. A prime consideration for the success of either policy is having a relatively stable relationship between total discharge and ambient environmental quality.

From our current understanding of the various processes whereby residuals are diffused, dispersed and altered within a disposal medium it would appear that the disposal of organic wastes in lakes, streams and estuaries and particulates in a local airshed are the best understood in terms of total discharge (loading) and the resultant ambient quality. While this understanding is far from perfect it is

better developed, for example, than our understanding of the movement of toxics in groundwater or the long range transport of oxides of sulfur or nitrogen. Thus, the use of either taxes or transferable pollution permits might be best suited to the more benign organic wastes disposed via water and certain residuals from combustion which are emitted through smoke stacks. In such cases there would exist a relatively long history on the characteristics of the residuals, their transport and behavior within the disposal medium, and the dimensions (metric) of ambient environmental quality.

Toxic and radioactive wastes, because of their more immediate and severe effects on human health, because of the complex way in which in which they can be transported through soils and groundwater and because of the inability of the individual to readily determine their concentration and the degree of risk to one's health would seem less suitable to control by EI policies and more appropriately controlled by C&C policies.

There have not been many instances of attempts to control pollution using taxes or marketable permits but the instances where EI policies have been tried would seem to support the above observation that they are better suited to nontoxic, degradable residuals. France and the Netherlands have used pollution fees to pay for treatment and to provide an incentive to reduce waterborne

residuals. Bower et al (1981) note that the fees (taxes) charged to date have been too low to create much of an incentive to reduce effluent loadings, but that with the system of fees in place the opportunity to increase fees to create a stronger incentive for reduced loadings does exist.

Experimentation with taxes or transferable permits in the U. S. is limited and restricted primarily to airborne pollutants. In Wisconsin, however the state Department of Natural Resources approved regulations whereby pulp mills along the lower Fox River were permitted to transfer permits allowing the discharge of a certain amount of waste into the river. The total amount had been reduced from earlier levels in an attempt to raise dissolved oxygen in the river. It was anticipated that firms that could treat wastes at lower cost might sell their permit to higher cost firms (see O'Neil et. al. 1983).

In 1979 amendments to the Clean Air Act allowed the EPA to institute the "bubble policy" where a firm in a particular airshed could transfer its pollution permit to another existing firm or to a new firm wishing to locate within the airshed. The number of transfers has not been large and thus there is not an organized market. Rather, transfer has been accomplished by negotiation between firms (see Hartwick and Olewiler 1986, pp. 443-444 for a discussion).

Finally in Telluride, Colorado city officials have put a moratorium on the installation of new wood-burning stoves.

Individuals wishing to install a new stove must persuade *two* other residents to give up theirs. In late 1986 the market price to get a resident to give up his permit for a stove was about \$1,000 (New York Times, November 30<sup>th</sup>, 1986, p.1).

While a C&C policy for toxic and nuclear wastes seems appropriate it is still possible for the private market to provide transport and storage where competing firms are subject to strict regulation, inspection, and liability in the event of a spill or leak. Under the cradle-to-grave management concept underlying the Comprehensive Environmental Response, Compensation, and Liability Act of 1980 (also known as CERCLA or the Superfund Act) a detailed accounting must be made of the volume of wastes generated and how they are disposed. The act also provides funds for the cleanup of toxic waste sites with subsequent compensation sought (often via litigation) from those responsible for the toxic site.

Scott (1986) believes that a system of marketable permits should be employed to control acid rain in North America. The U. S. and Canada would negotiate total emission rates for a set of regions in each country. Initially, the total emission rate for each region might

be similar to the amount currently emitted. Firms would receive marketable permits which could be transferred to existing or new firms within the region. Permits might be sold to firms outside the region but would be subject to an "environmental exchange rate". (A permit to emit X tons of SO<sub>2</sub> in region i might only permit a firm to emit X/2 tons of SO<sub>2</sub> if sold and transferred to region j).

Governments, individuals or environmental groups would be free to buy permits from firms in regions thought to contribute to acid deposition in their region and simply retire them. Scott feels that the initial total emission rate for each region should be subject to a negotiated schedule of decline which would be reflected in each permit issued to an individual firm. The individual permit would entitle the holder to a *declining* emission rate reflecting the overall regional decline. Firms wishing to maintain their emission rate into the future would have to acquire more permits at probably higher prices. By buying out the emission permits of existing firms interested parties would be compensating firms for an accelerated reduction in emissions.

In summary, environmental policy in the U. S. has relied primarily on direct regulation of firms and individuals generating externalities. At the encouragement of economists there has been a

few attempts at using economic incentives to reduce the level of untreated residuals. While the economic incentives created by taxes or marketable permits might be used to control any type of residual emission they are likely to be implemented in efforts to control the more familiar and benign residuals. Toxic and radioactive wastes are likely to continue to be regulated by command and control policies. Because of the limited success and higher cost of command and control policies it is important for environmental administrators to experiment with economic incentives that achieve the desired level of ambient environmental quality but allow firms to search for the least cost way of reducing emissions.

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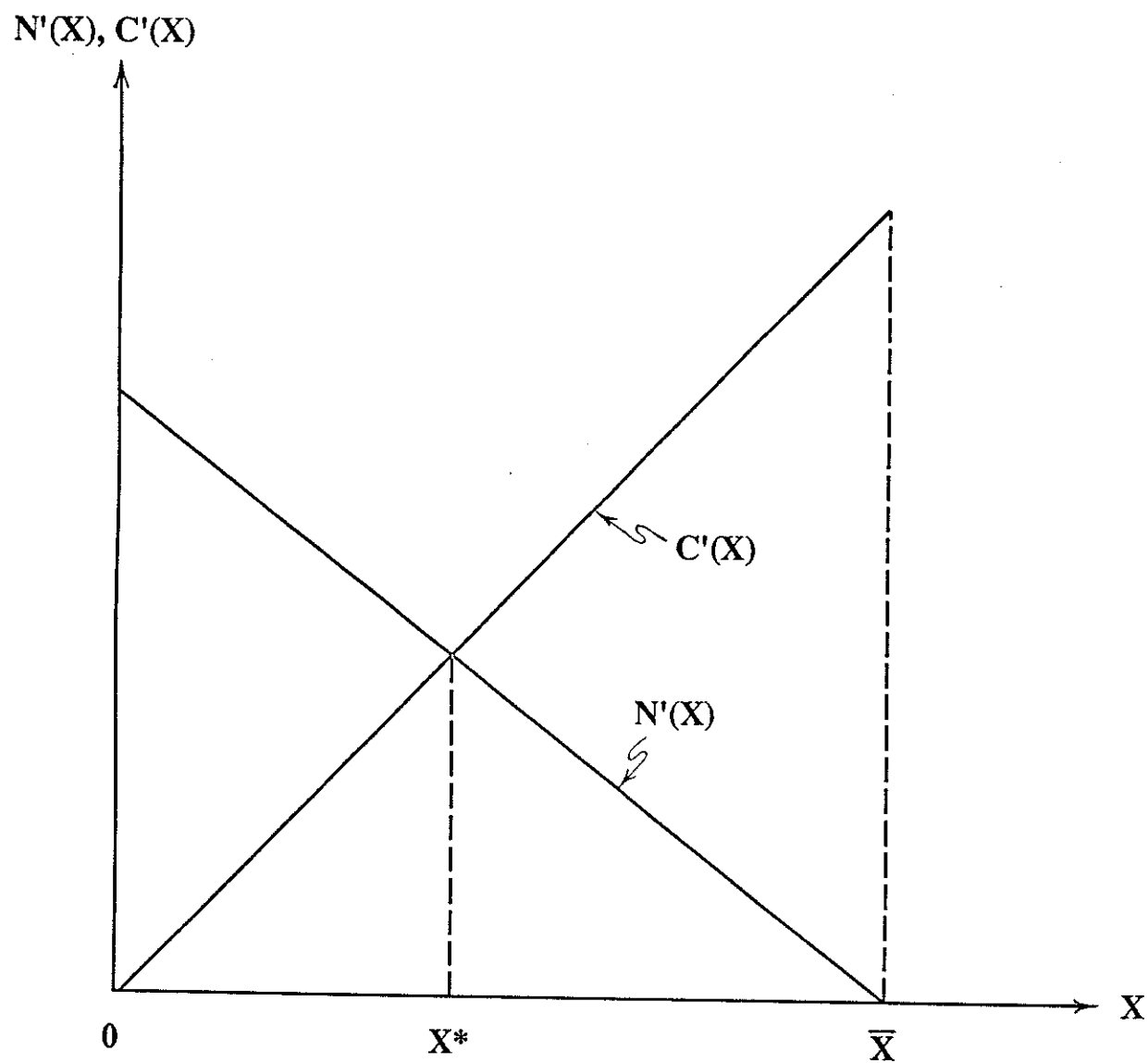
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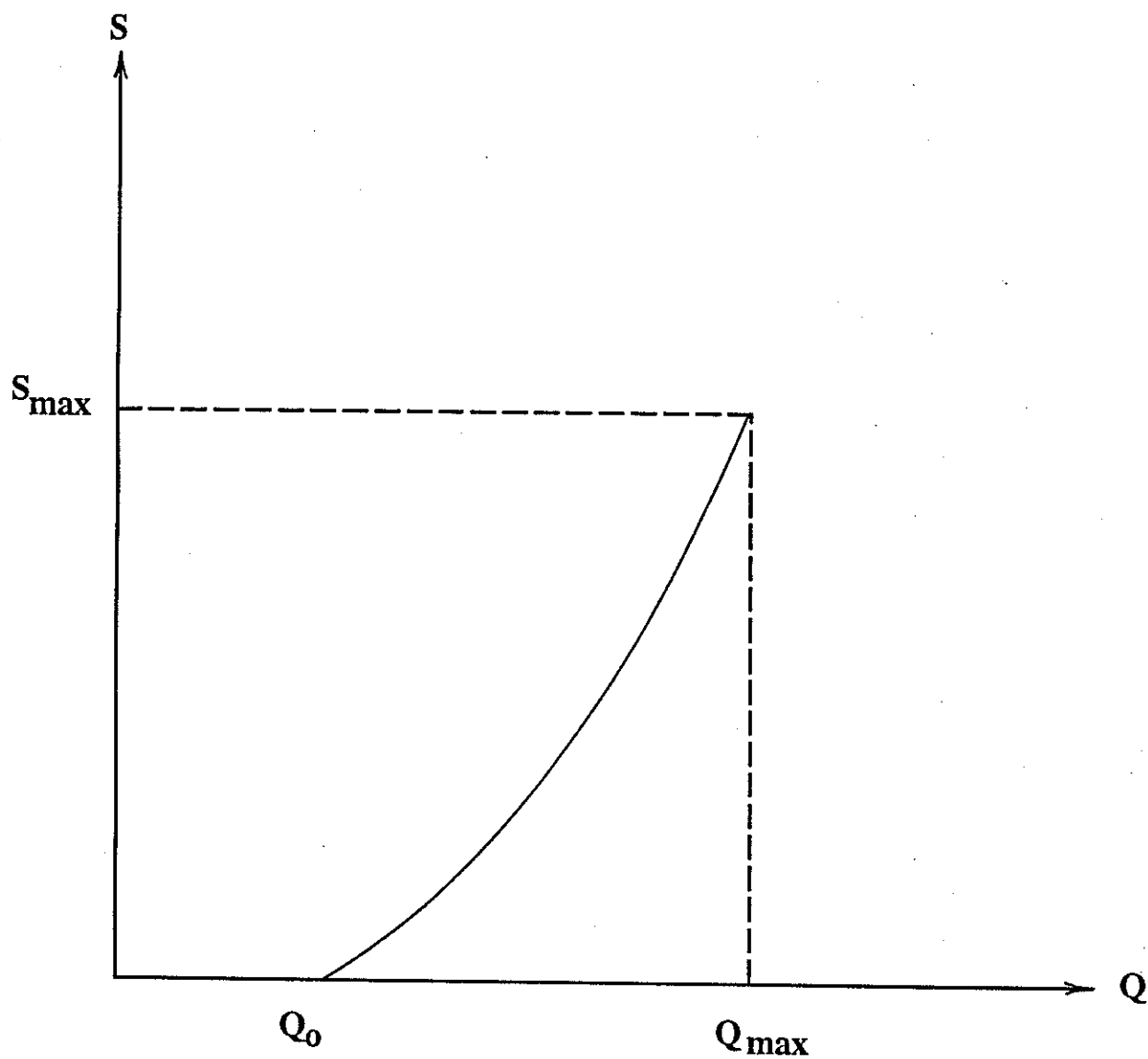
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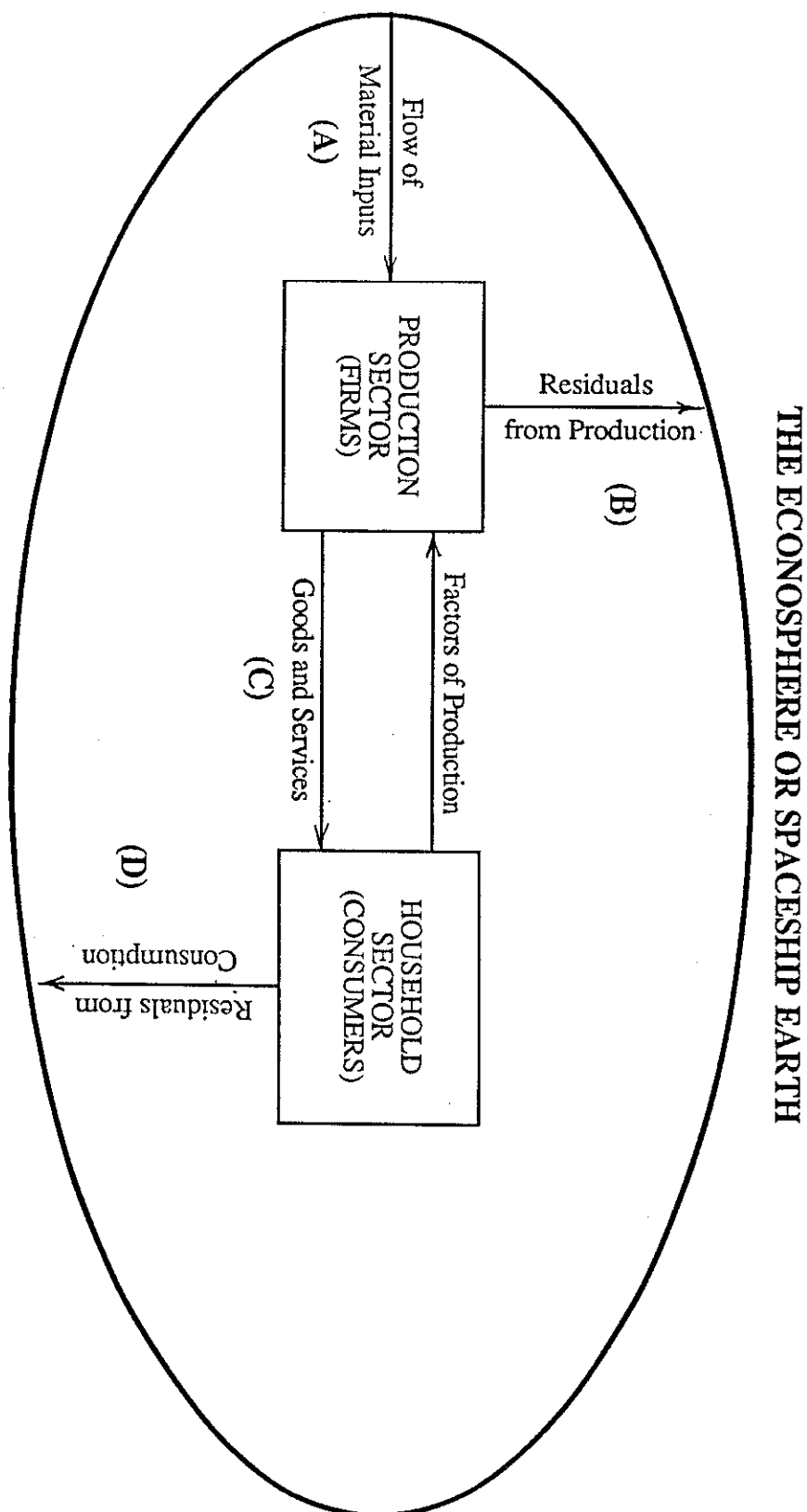
**Figure 1.** A Graph of Net Marginal Benefits,  $N'(X)$ , and Net Marginal Costs,  $C'(X)$ .



**Figure 2.** The Transformation Function  $\phi(Q, S) \equiv 0$  for Commodity  $Q$  and Residual  $S$ .



**Figure 3.** The Econosphere or Spaceship Earth, where materials balance ultimately requires  $A = B + C$ ,  $C = D$  and thus  $A = B + D$  (the mass of material inputs equals the sum of residuals).



**Figure 4.** Approach Paths for  $X_t$  and  $R_t$  when  $X_0 > X^*$ . Along the MRAP  $R_t^* = 0$  for  $0 \leq t \leq \hat{t}$  then  $R_t^* = R^*$ ,  $X_t^* = X^*$ , for  $t \geq \hat{t}$ .

