BIOECONOMICS AND THE BOWHEAD WHALE

By

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ABSTRACT
The bowhead whale (Balaena mysticetus) was once widely distributed across the oceans and polar seas of the northern hemisphere. It was harvested by the Eskimo and Thule Indians in present day Alaska, Canada, Russia and Greenland for many centuries before European exploration and settlement. Up until 1848 it is believed that the stock of bowheads in the western Arctic was in a relatively stationary state with the indigenous Eskimo population. In that year Captain Thomas Roys of Sag Harbor, New York navigated the whaling bark Superior through the Bering Strait and discovered the western Arctic bowhead stock which supported a commercial fishery until 1914. After the departure of the Yankee whalers, the remaining stock of bowheads was once again subject to subsistence harvest by the Eskimo. In the mid-1970s the International Whaling Commission (IWC) became concerned with the increased harvest of bowheads by the Alaskan Eskimo and the increased number of whales thought to have been "struck-but-lost." In 1977 the scientific committee of the IWC recommended a moratorium on Eskimo harvest. The ensuing controversy, which saw the U.S. Government side with the Eskimo's demand for continued harvest, led to a vast research effort to estimate (1) the bowhead population in 1848, (2) the current population, and (3) the likely effect of continued Eskimo harvest on the stock of bowheads. This paper reports new estimates of the bowhead stock during the open access period of commercial exploitation. The population of bowheads in 1848 is estimated to have been between 11 and 18 thousand. By 1914 the stock had probably been reduced to between 1,000 and 3,500 adult whales. A state-space diagram is constructed which shows the dynamic evolution of the resource and industry for the years 1848-1914. The current management controversy is formulated as a dynamic optimization problem where Eskimo welfare is assumed linear in stock and kill. Steady-state optimal escapement will depend on the relative weight assigned to the bowhead stock, the rate of time preference (discount) and parameters of the delayed-difference equation describing population dynamics. Optimal stock ranged from approximately 4,000 to 13,500 whales with kill ranging from 40 to 145 whales per year.
Bioeconomics and the Bowhead Whale

I. Introduction and Overview

The bowhead whale (Balaena mysticetus) was once widely distributed across the oceans and polar seas of the northern hemisphere (see Figure 1). It was harvested by the Eskimo and Thule Indians in present day Alaska, Canada, Greenland and Russia for many centuries before European exploration and settlement. Until the 19th century the bowhead and the indigenous Eskimo populations in the western Arctic are thought to have existed in relatively stable numbers in something akin to a stationary state.

In 1848, however, Captain Thomas Roys of Sag Harbor, New York navigated the whaling bark *Superior* through the Bering Strait into the Chukchi Sea and there discovered a vast population of bowhead (see Figure 2). Captain Roys and his crew found the bowhead to be slow and docile and so thickly layered with blubber that an average whale yielded over 11,923 liters of oil (about 100 barrels). In addition, the average bowhead produced 681 kg (1500 lbs) of baleen which, in the late 1800s, would be highly demanded for use in corsets, skirts and umbrellas (Bockstoce 1980).

Roys easily filled his hold and returned to Honolulu where
word of his success spread rapidly. In 1852 more than 200 whalers were operating in the Bering and Chukchi seas. Bockstoce and Botkin (1980, p.2) described the typical whaling voyage as follows.

Leaving New England in the autumn and rounding Cape Horn in the southern summer, they outfitted at Hawaiian ports or San Francisco, sailing for the Arctic in late March to reach the pack ice of the central Bering Sea a month later. Informal accounts suggest that they took a few whales as they worked their way north toward Bering Strait through the melting flows, but by early June most of the whales had passed them and gone deep into the safety of the ice on the migration to their summer feeding grounds in the Arctic Ocean. As the fishery progressed into its second decade the whalers generally would not see their quarry again until late July when the ice allowed the ships to approach the north coast of Alaska and intersect the whales traveling from the Beaufort Sea to their autumn feeding grounds near Hearld Island in the Chukchi Sea. The ships often cruised near Hearld Island until the violent weather and encroaching ice of early October drove them back to ports in the Pacific Ocean.1

While the price of whale oil generally declined after 1870, the rising price of baleen and the development of steam-auxiliary whaling vessels allowed the fleet to profitably pursue the bowhead to the least accessible reaches of the Arctic Ocean. In 1889 the steamers discovered the bowhead's summer feeding grounds off the MacKenzie River delta in Canada's Northwest Territories. For the rest of the 19th century, and until the end of the commercial fishery in 1914,
the whalers concentrated their efforts in these waters.

By the early 20th century the stock of bowheads in the western Arctic had been drastically reduced and the landings had declined to less than 10 percent of the annual average during the first decade of commercial exploitation. In 1908 the introduction of flexible form steel and a change in ladies' fashion caused the baleen market to collapse and in 1915 only one whaler (with no reported catch) made a voyage to the western Arctic (Bockstoce 1980, p.25).

With the closing of shore-based stations and the departure of the pelagic whalers the Eskimo once again became the sole harvester of the bowhead whale. While the bowhead and the hunt itself continued to play a traditional role in the Eskimo culture, the technology (equipment) used in harvesting the bowhead was altered after contact with the Yankee whalers. Before the turn of the century the Eskimo traded furs and baleen to acquire the darting gun and the bomb lance which greatly reduced the time to kill and retrieve a whale. Even with these technological changes the number of crews, their catch and the number of whales that were "struck- but-lost" did not seem to pose a threat to the recovery of the bowhead. It was not until the early 1970s when greater employment opportunities (many
associated with oil development on Alaska's north slope) allowed more Eskimos to assemble the financial resources to equip and supply a whaling crew. The estimate of total kill (landed plus struck-but-lost) for the period 1960-1967 was 153 whales, while for the period 1970-1977 it was 383. This increase caused the International Whaling Commission (IWC) in 1977 to recommend a zero catch for the bowhead whale in Alaska. A similar recommendation was made in each of the subsequent years through 1982.

The Eskimo opposed the IWC recommendations claiming that the hunt and harvest were critical to their cultural survival and that even the higher kill rates observed in the 1970s could not be shown to exceed recruitment. The U.S. government, long a champion of whale conservation and an advocate of a moratorium on commercial whaling, found itself supporting the Eskimo position and recommending (in 1980) that approximately 30 bowhead be allowed to be taken on the basis of subsistence and cultural need. The controversy lead to research attempting to answer three questions: (1) what was the population of bowhead in the western Arctic before the start of commercial exploitation, (2) what is the current bowhead population, and (3) how would Eskimo harvests at an annual rate of zero to 40
whales affect future population levels?

In the next section we will review the results of the biological research which attempted to answer the above questions. Previous estimates of the bowhead whale population for the pelagic fishery (1848-1914) will be compared to new estimates derived by a different procedure. A state-space diagram for this period will be constructed and compared to similar diagrams depicting the evolution of other open access resources.

In the third section a bioeconomic model is developed to consider optimal escapement and harvest for the current Eskimo fishery. Because the fishery is important to the economic subsistence and cultural preservation of the Eskimo villages, the traditional bioeconomic model might initially seem an inappropriate paradigm for considering resource management in a nonmarket setting. As it turns out, the case where Eskimo welfare depends linearly on stock and yield, provides a means of estimating optimal stock and harvest for various combinations of the weight assigned to the bowhead population, the discount rate and other biological parameters.

The fourth and final section of the paper summarizes the major conclusions and indicates the likely affect of factors not considered in the simple bioeconomic model of Section III. This section concludes
with some recommendations for resource management within a subsistence economy.

Section II. Population Dynamics and the Open Access Fishery

A population biologist, attempting to estimate the change in the stock of a renewable resource, will typically require (1) time series data on harvest, (2) a model of population dynamics and (3) an estimate of the population at some point in time. With time series data on harvest and some point estimate of stock, the model of population dynamics can be used to generate stock estimates for other periods (years).

In a report to the National Marine Fisheries Service, Bockstoce and Botkin (1980) provided estimates of the number of whales killed by the pelagic fleet in each year from 1849 through 1914. The number of whaling voyages to the western Arctic was obtained by an exhaustive analysis of the Whalemen's Shipping List and Merchants' Transcript as well as other accounts of whaling voyages found in newspapers, magazines, books and government documents. Over 25,000 reports were processed revealing more than 2,700 cruises to the western Arctic.
At the same time a list of approximately 4,000 logbooks and journals held in public collections in the United States, Canada and Australia was constructed. Of this number 800 described seasonal cruises to the western Arctic. Regrettably less than 550 were complete and legible. These accounts, however, were spread more or less uniformly over the entire period of the pelagic fishery with approximately 20 percent of the voyages documented in detail each year.

If $V_t$ denotes the estimate of the total number of voyages to the western Arctic in year $t$, $v_t$ the number of sample voyages for which legible, complete catch data exist, and $y_t$ the sample annual catch, then total catch, $Y_t$, was estimated as $Y_t = y_t / (v_t / V_t)$, for each year. From the detailed logs and journals it was also possible to estimate the number of whales struck-but-lost. These figures could be scaled up in a similar manner to estimate the total number of whales struck-but-lost in each year. Over the history of the pelagic fishery Bockstoce and Botkin estimate that 18,650 bowhead whales were killed.

As important as this research was in attempting to estimate the initial bowhead stock and its subsequent dynamics, it did not include estimates of the shore-based kill by Eskimos and nonnatives.
during the commercial period. Breiwick et. al. (1984) combine the pelagic estimates of catch and kill from Bockstoce and Botkin with published estimates of shore-based catch and kill to obtain the total man-induced mortality for the years 1848-1914. The number of vessels (voyages), pelagic catch, pelagic kill and total kill are given in Table 1.

The next step in estimating the population of bowhead in 1848 would be to determine a model that would describe the change in the population as a result of net natural growth and man-induced mortality. While detailed biological information on the "life history" of the bowhead whale is incompete, the available information, plus information on baleen whales in general, permits the construction of several plausible models. We will consider a single-state, delayed-difference equation model which has been used extensively in modelling baleen whale populations (Allen 1963, Clark 1976 a). In this model it is assumed that the bowhead stock can be adequately described by a single state variable, the number of adult whales. This model might be contrasted to a multiple cohort model where the resource is described by a vector indicating the number of individuals in each year class or age group.
A delay-difference equation implies that there is a lag between birth and "recruitment" into the fishery; when an individual is at risk from fishing. Let \( X_t \) denote the stock of adult bowhead whales in period \( t \). Then before commercial exploitation the stock of adults might change according to the equation

\[
X_{t+1} = (1-M)X_t + F(X_{t-\tau})
\]

where \( X_{t+1} \) is the population of adult bowhead in year \( t+1 \) determined by the number of adults surviving from the previous year plus new adults (recruits) determined by the adult population in period \( t-\tau \).

Thus, \( M \) is the adult annual mortality rate and \( F(\cdot) \) is a recruitment function. The population would be unchanging at a stationary point \( X \) where \( MX = F(X) \) and adult mortality is precisely offset by recruitment.

Clark (1976 a) notes that a sufficient (though not necessary) condition for the stationary point to be locally stable is \( F'(X) < MX \).

We will use a generalized logistic to characterize \( F(\cdot) \). This may be written as

\[
F(X_{t-\tau}) = r X_{t-\tau} [1-(X_{t-\tau}/K)^{\alpha}]
\]

where \( r \) is the maximum recruitment rate, \( K \) is a population size so large that recruitment is zero and \( \alpha \) is a density dependence parameter which will determine the population level at which
recruitment is a maximum \((X_{MR})^5\). Some algebra will show that
\[ X_{MR} = K \left[ 1/(\alpha + 1) \right]^{1/\alpha}, \]
while the stock level maximizing net recruitment is given by
\[ X_{MNR} = K \left[ (r-M)/(r(\alpha+1)) \right]^{1/\alpha} \]
and there is a unique, nonzero stationary state at
\[ X_0 = K \left[ (r-M)/r \right]^{1/\alpha} \tag{3} \]
(see Figure 3).

With commercial harvest the model of population dynamics must be modified to account for fishing mortality. Let \(K_t\) denote total kill and \(Z_t\) denote escapement, where \(Z_t = X_t - K_t\). Then, assuming that survival and recruitment are based on escapement in period \(t\) and \(t-\tau\) respectively, equation (1) becomes
\[ X_{t+1} = (1-M)Z_t + F(Z_{t-\tau}) \tag{4} \]
Given the time series data on commercial mortality (total kill in Table 1) and if (a) the population was in a stationary state prior to commercial exploitation and (b) values for \(\alpha, K, M, r,\) and \(\tau\) are known, then one can calculate \(X_0\) and simulate the system comprised \(Z_t = X_t - K_t\) and (4) forward in time over the interval of commercial exploitation to determine what happened to the bowhead population.

Unfortunately, while plausible ranges for \(\alpha, K, M, r,\) and \(\tau\) might be suggested, none of the parameters are known precisely. As
an alternative to step (b) above, if one had a point estimate of the population in some future year, one could search for alternative combinations of the parameters to find sets which simulate to that estimate. If the departing whalers had left us a note as to the remaining stock of bowheads in 1914, we could employ this procedure to reconstruct the time path of the population for the commercial period. No such note was left, and it was not until the 1970s, and the concern over increased Eskimo harvests that scientific stock estimates were attempted. To simulate to a date in the 1970s, however, one would need estimates of total kill for the period since 1914, when the fishery reverted back one which supported Eskimo subsistence.

Marquette and Bockstoce (1980) and others worked to assemble estimates of shore-based kill both during and after the commercial period. These data were much more difficult to assemble and are undoubtedly less precise than the data culled from the records kept by the industry and firms engaged in commercial whaling. Pelagic and shore-based kill are summarized in Breiwick et al. (1984, Table 1). They used a value of 4,000 as a point estimate for the bowhead stock in 1970 and employed a multiple cohort model to
determine the combinations of biological parameters and 1848 stock which would simulate to that point estimate. In our single-state model, where $F(.)$ is a generalized logistic, the parameters will imply a stationary stock for 1848, and using the estimates of total kill from 1848 to 1982 one can simulate the population to the beginning of 1983. A simple program, listed in Appendix A, was written to perform such a simulation and by trial-and-error one can quickly determine parameter combinations which simulate to approximately 4,000 whales in 1970. The search was restricted to a "plausible parameter space" defined by

$$\alpha = 1.00, 2.39, 4.80$$

$$19,000 \leq K \leq 50,000$$

$$0.03 \leq M \leq 0.09$$

$$0.01 \leq (r-M) \leq 0.04$$

$$\tau = 3, 5, 7$$

Table 2 summarizes nine simulations that led to a bowhead population of approximately 4,000 in 1970. The stationary state implied by equation (3) is listed as the population in 1848, and the population levels for 1914 and 1983 are also given. The estimates of the population in 1848 range from 11,753 to 17,290. This is reasonably consistent with the estimates of Breitwick et al. which
ranged from 14,000 to 20,000 based on simulations with their multiple cohort model. In 1914 the stock ranged from a low of 1,482 (simulation #1) to a high of 3,383 (simulation #6). The population increases from 1970 to 1983 for all simulations, but it may be a slight increase, as in simulations #6 and #8, or large a increase, as in simulations #1 and #4. There are undoubtedly other combinations of \( \alpha, K, M, r, \) and \( \tau \) which would also lead to a population level of approximately 4,000 in 1970. Figure 4 shows the entire time path for the bowhead population for simulation #5 during the commercial period 1848-1914.

A simple differential equation model of open access discussed by Clark (1976 b) and employed by Wilen (1976) in a study of the north Pacific fur seal takes the following form:

\[
\begin{align*}
\dot{X} &= rX(1-X/K) - qXV \\
\dot{V} &= \eta[pqXV-cV]
\end{align*}
\]

(5)

where \( rX(1-X/K) \) is the symmetric logistic growth function, \( qXV \) is a production function defining catch (\( q \) is called the catchability coefficient), \( \eta > 0 \) is a "stiffness parameter" determining the responsiveness of the number of vessels, \( V \), to profit \( pqXV-cV \) and where \( p \) is the price received per unit of harvested resource and \( c \) is
the unit cost of operating a vessel. The state-space diagram for this system is shown in Figure 5. The eigenvalues for the linearized system reveal the stationary state \((X_\infty, V_\infty)\) to be stable (a node or a spiral) and the equilibrium stock \(X_\infty\) is a breakeven stock level in the sense that for \(X > X_\infty\) profit is positive and \(\dot{V} > 0\) while for \(X < X_\infty\) profit is negative and \(\dot{V} < 0\). This assumes, of course, that price, cost and all other parameters are constant.

Wilens (1976) concludes that the first loop of a ragged but convergent spiral seems to have been completed during the later years of the open access period for the north Pacific fur seal. Apparently the exit of pelagic sealers was rapid enough to allow the stock of fur seals to start to increase even before the International Fur Seal Treaty of 1911. Bjorndal and Conrad (1987) did not see any indication of recovery for the North Sea herring, in spite of what appeared to be rapid exit by herring seiners. They conclude that Norway and the European Community were probably justified in closing the fishery in 1977.

For the bowhead whale the the collapse of the market for baleen in 1908 probably saved the species from open access extinction. Modelling open access with delayed-recruitment
necessitates a more complex model. One possible specification might be

\[ Z_t = X_t - (1 + \beta_t)H(X_t, V_t, \omega_t) \]

\[ X_{t+1} = (1 - M)Z_t + F(Z_{t-1}) \]

\[ V_{t+1} = (1 - s_t)V_t + \eta[H(X_t, V_t, \omega_t) - c_{1,t}V_t + c_{0,t}]/V_t \]  

(6)

where \( \beta_t \) is the struck-but-lost rate as a fraction of catch, \( \omega_t \) is a binary variable where \( \omega_t = 1 \) indicates good weather/ice conditions while \( \omega_t = 0 \) indicates bad weather/ice conditions, \( s_t \) is the fraction of vessels lost during year \( t \), \( \eta \) and \( p_t \) are as in the simple model, and \( c_{1,t} \) and \( c_{0,t} \) are the variable and opportunity costs for a vessel, respectively. The collection of data, estimation and simulation of this model is the focus of ongoing research. Given the estimates of the total number of vessels (from Table 1) and the estimates of the bowhead population from the simulations summarized in Table 2, it is possible to construct a state-space diagram for the evolution of \( (X_t, V_t) \). This has been done for the values of \( X_t \) from simulation #5. The result is Figure 6. One sees the start of a spiral for the points from 1848-1857, but from 1858 to 1914 there is nearly a monotonic decline in the bowhead stock and a ragged but overall decline in the number of vessels. This perhaps reflects the fluctuations in the prices for whale oil and baleen, the fact that ships were lost to the pack ice (and Confederate ships of
war), and increases in the the costs of operating and constructing whaling vessels. According to simulation #5, by 1911 the bowhead population had reached its low point of 1,960 adult whales and by 1914 the stock had only recovered slightly to the 2,022 reported in Table 2.

III. Bioeconomics and the Eskimo Fishery

In spite of the increased harvest and kill during the 1970s it would now appear that the bowhead population has continued to increase from its low just prior to the end of the commercial fishery. Simulation #5 led to an estimate of 4,543 adult whales in 1983. Current estimates, based on aerial survey, put the stock at around 5,000 (J. M. Breiwick, personal communication, April, 1987). Thus, while the population does not seem to be endangered by the Eskimo hunt, the allowable kill (catch plus struck-but-lost), and the desired (optimal) population size are still subject to debate. As the population increases the number of whales which could be taken by the Eskimo under the IWC aboriginal exemption could also increase. Should it? What is the "optimal" population size? What factors should be considered when specifying an Eskimo quota?
Bioeconomic models attempt to maximize some measure of economic performance subject to the population dynamics of the harvested species. For a commercial fishery, maximization of the present value of net revenue might be an appropriate objective. In a sport or subsistence fishery other factors may lead to alternative objectives or a multiobjective problem. As we shall see the most general optimization problem can yield some insight into the quantitative questions of optimal catch and population size, and will also permit us to infer the qualitative effect of factors that are not easily measured.

Consider the following problem.

Maximize \[ \sum_{t=0}^{\infty} \rho^t W(X_t, K_t) \]

Subject to \[ X_{t+1} = (1-M)Z_t + F(Z_{t-\tau}) \]  \[ Z_t = X_t - K_t \] (7)

where \( W(\cdot) \) is a utility or social welfare function indicating that Eskimo welfare may depend on the bowhead population \( X_t \), and kill, \( K_t \). The discount factor is \( \rho = 1 / (1 + \delta) \), where \( \delta \) is the periodic discount rate. It can be shown (Appendix B) that a steady-state optimum must satisfy

\[
\left[ \frac{W_X + W_K}{W_k} \right] [1 - M + \rho^\tau F'(Z)] = 1 + \delta
\] (8)
where $W_X$ and $W_K$ denote the partials of $W(\cdot)$ with respect to $X$ and $K$, respectively. A simple form for $W(\cdot)$ is

$$W(X, K) = \gamma X + K \quad (9)$$

where $\gamma$ is the marginal value of an additional adult bowhead, in the water, relative to an additional bowhead killed and caught. Define

$$\theta = r(\gamma + 1) - (1+\delta)\{M + \delta - (1-M)\gamma\} \quad (10)$$

Then, the optimal level for escapement implied by $F(Z) = rZ(1 - (Z/K)^\theta)$ and equations (8) and (9) is

$$Z^* = K\{\theta/[r(\gamma + 1)(\alpha + 1)]\}^{1/\alpha} \quad (11)$$

and knowing $Z^*$ one can calculate $X^* = (1-M)Z^* + F(Z^*)$ and $K^* = X^* - Z^*$. For small values of $r$ and $\gamma$ and relatively modest values of $M$, $\delta$, and $\tau$ it is possible that $\theta \leq 0$, implying that extinction is optimal. Table 3 gives the optimal values for stock, $X^*$, and kill, $K^*$, for $\gamma = 0.00$ to 0.05 and $\delta = 0.00$ to 0.05. For example, when $\gamma = 0$ (thus $W_X = 0$) and $\delta = 0$ the optimal stock is $X^* = 8,735$ and the optimal kill is $K^* = 145^7$. When the discount rate increases to 0.01, the optimal stock declines to $X^* = 6,549$ and kill to $K^* = 131$. Entries of zero for $X^*$ and $K^*$ indicate that extinction is optimal for those combinations of $\delta$ and $\gamma$ and the values for $\alpha$, $K$, $M$, $r$, and $\tau$ defining parameter set #5.

Analysis of the other parameter sets in Table 2 will reveal that
the likelihood of extinction increases with increases in $M$, $\delta$ or $\tau$, and decreases with increases in $r$ and $\gamma$. While increases in $\gamma$ will (ultimately) increase optimal stock, $X^*$, it is interesting to note that increases in $\gamma$ may cause an increase or decrease in $K^*$ depending on whether the stock is initially to the left or right of the stock supporting maximum net recruitment ($X_{MNR} = 8,590$). For example, moving down the column $\delta = 0$ the optimal stock monotonically increases while the optimal kill monotonically decreases. For $\delta = 0.01$, $K^*$ increases as $\gamma$ goes from 0.00 to 0.01 but decreases for subsequent increases in $\gamma$. If one precludes combinations of $\delta$ and $\gamma$ that result in extinction, the optimal stock ranges from 3,898 to 13,611 while the optimal kill rate ranges from 39 to 145. Again, as $X^*$ approaches $X_0$ the optimal kill rate must ultimately decline since net recruitment approaches zero (see Figure 3).

IV. Conclusions and Implications for Resource Management in Subsistence Economies

The bowhead whale fishery in the western Arctic presents resource economists with an interesting case study of open access dynamics as well as a contemporary problem in resource management.
The analysis reported in this paper would indicate that stock of adult bowhead whales, at the time of their discovery in 1848, was probably between 11,000 and 18,000. This compares to an estimate of 14,000 to 20,000 obtained by Brevik et al. (1984) using a multiple cohort model. The pelagic and shore-based whalers "mined" the population (kill rates in excess of recruitment) to a probable low of 1,000 to 3,500 adult whales just prior to the abandonment of commercial fishing in 1914. The population has slowly recovered, even with the increased Eskimo kill rates recorded in the 1970s. Of the nine simulations reported in this paper estimates of the stock in 1983 ranged from a low of 3,998 to 4,914. This is consistent with the aerial surveys that put the stock at about 5,000 in 1986.

The estimates of vessel numbers and population during the commercial period (1848-1914) permit a state-space plot which appears consistent with the received theory of open access dynamics. Estimation of a more detailed structural model is currently in progress and should afford greater insight into the precise effects of changes in the prices for oil and baleen, operating and opportunity costs. From 1880 to 1908 the relatively high price for baleen sustained the Arctic fishery in what was then an obviously moribund industry. Writing in 1906 W. S. Tower concluded
What the future of whaling is to be, is, of course, much in the nature of mere prophecy - yet the signs seem easy to interpret. It appears reasonable enough to say that the fishery for right whales will be carried on in the northern seas as long as the demand for whalebone continues and as long as the price remains at its present high figure.... The prospect for the Atlantic sperm whale fishery is not nearly so promising. The low price of oil is rather discouraging to the merchants, and only the good luck of the vessels in securing large catches in a short time has made it possible to continue the business with any profit.... Beyond these possibilities the future seems to hold nothing.... The economic conditions under which whaling prospered have ceased to exist, never to be revived.... (Tower pp. 114-115).

In 1910 the *Whalemen's Shipping List and Merchants' Transcript* reported no market for baleen, and Tower's prophecy with regard to the Arctic fishery was fulfilled. Tower, of course, did not foresee the development of the pelagic fishery in the Antarctic Ocean which from 1925 to 1965 constituted an even larger fishery in terms of physical yield and economic value. This fishery was ultimately based on large factory vessels being supplied with whales taken by smaller catcher vessels. Japan, Norway and the USSR would dominate in this fishery and New Bedford and the US never achieved a "prominence" in the modern whaling era. (For an economic history of this fishery see Clark and Lamberson 1982).

The current controversy over the bowhead whale resulted from
a lack of information on the size of the population and the impact of
increased Eskimo kill rates during the 1970s. It now appears that,
while the increased Eskimo harvest may have slowed the continued
recovery of the bowhead population, it did not cause a decline. The
nine simulations described in this paper show the bowhead population
increasing from 1970 to 1983, although the size of the increase
ranged from 4 to 908 depending on the set of parameters employed in
the simulation. It should also be noted that the delayed-difference
equation model on which the population simulations were based is
capable of oscillatory and perhaps more complex ("chaotic") behavior.

Determining the optimal stock and kill for the Eskimo fishery
is problematic. The traditional bioeconomic model, which seeks to
maximize the discounted sum of net revenues, would not seem
appropriate for resource management within the context of a
subsistence economy. The more general problem of maximizing the
present value of welfare, while capable of encompassing a variety of
welfare measures, raises questions as to the variables affecting welfare
and their relative importance. The simple approach taken in this
paper was to presume that welfare was a linear function of stock and
kill. This permitted us to solve for the optimal level for escapement
given certain biological parameters, a discount rate ($\delta$) and the weight
($\gamma$) of an increment to the bowhead stock relative to an incremental
increase in harvest today. Treating extinction as an inadmissible
outcome, assuming the biological parameters in set #5, and values of $\delta$
and $\gamma$ from 0.00 and 0.05 (inclusive), the optimal stock ranged from
approximately 4,000 to 13,600 adult whales. Total allowable kill
ranged from approximately 40 to 145 whales per year.

The simple model of the preceding section was deterministic.
Scientists and consultants for the IWC have noted that natural
mortality is stochastic and that periodic "mass groundings" will
frequently result in abnormally high yearly mortality rates. Further,
the long term effect of north slope oil activities (exploration, drilling
and production) may adversely affect the krill resource upon which
the bowhead feeds, or detrimentally alter the bowhead's migratory
route or summer feeding grounds. This might counsel for a more
conservative quota recommendation than that emerging from a
deterministic model of surplus production.

Finally, the current bowhead controversy is not dissimilar to
other resource management problems that arise in subsistence
economies or less developed countries. How should renewable
resources be managed when their value arises from direct consumption by a household and the resource itself is an integral part of the indigenous, typically non-western, culture? Resource economists have made advances in the theory and techniques to elicit nonmarket values. Contingent valuation methods may also provide a means to assess the relative values of increments to the standing stock ($y$) and the absence, presence and magnitude of the culture's rate of time preference ($\delta$). If successful, the attempts to elicit such weights from a non-western culture may permit the identification of a range of stocks and associated yields which would enhance, if not maximize, a notion of welfare meaningful to a society dependent on such resources.
Endnotes

1A detailed history of the bowhead whale fishery in the western Arctic may be found in Whales, Ice, and Men by John R. Bockstoce (University of Washington Press, Seattle, 1986).

2The darting gun fires an explosive bomb (on a delayed fuse) into the whale at the same time that an iron (harpoon) affixes a line and float. The harpooned whale is pursued and when it can be approached at close range a bomb lance is fired from a smooth-bored shoulder gun. If the bomb is well placed the delayed explosion will usually kill the whale outright.

3In 1978 the International Whaling Commission estimated that the cost of equipment and supplies to support a whaling crew amounted to about $10,000. Many of the equipment items could be used for several seasons.

4Shore-based catch and kill are estimated by Marquette and Bockstoce (1980) for the US, Canada and the USSR. Breiwick et al. (1984) supplement these data with additional kill data reported by Bogoslovskaya et al. (1982).

5It is generally believed by biologists that the recruitment curve for baleen whales is asymmetric with the stock supporting maximum recruitment $X_{MR} > 0.5K$. For $\alpha=2.39$, $X_{MR}=0.6K$, while for $\alpha=4.80$ the stock level which maximizes net recruitment is $X_{MNR}=0.6K$. These values of $\alpha$ will be used in the set of "plausible parameter values" to be discussed shortly.

6In the program in Appendix A, the DATA statements beginning at line 220 contain the estimates of total kill (pelagic and shore-based) from 1848-1982. Thus, in 1848 Captain Roys captured 15 bowhead, while 3 were thought to have been mortally struck-but-lost, for a total of 18 bowhead killed.

7If Eskimo welfare depends only on kill ($W_X=0$) then the equation defining optimal escapement becomes

$$Z^* = K \left\{ (r - (1 + \delta)^{5}(M + \delta))/[r(\alpha + 1)] \right\}^{1/\alpha}$$

for any concave function $W(K)$.
References


Figure 1. The Bowhead whale (Balaena mysticetus) is believed to have existed in five distinct stocks in the oceans and polar seas of the northern hemisphere. During the early years of the pelagic fishery in the western Arctic a large bowhead might run up to 65 ft. in length and weigh 65 tons. Such a whale could yield over 300 barrels of oil, although the average yield was about 100 barrels (1 barrel=31.5 gallons). In addition to oil the bowhead produced baleen (whale bone) used in the manufacture of corset stays, parasols, umbrellas and buggy whips. The average bowhead yielded about 1,500 pounds of baleen. In 1892, at 42.5 cents per gallon for oil and $5.35 per pound of baleen, the average bowhead was worth $9,364.
Figure 2. Location Map for the Pelagic Bowhead Whale Fishery in the Western Arctic (1848-1914).
Figure 3. A Graph of \( F(X) = rX(1- (X/K)\alpha) \) and \( MX \) for \( \alpha=2.39, K=1, M=0.03 \) and \( r=0.06 \). With \( \alpha=2.39 \) maximum recruitment is \( X_{MR} = K[1/(\alpha+1)]^{1/\alpha} = 0.6K \), while \( X_0 = K[(r-M)/r]^{1/\alpha} = 0.75K \), and \( X_{MNR} = K[(r-M)/(r(\alpha+1))]^{1/\alpha} = 0.45K \).
Figure 4. The Reduction in the Bowhead Whale Population, $X_t$, in the western Arctic Based on the Parameters in Simulation #5.
Figure 5. A State-Space Diagram for the Dynamical System

\[
\dot{X} = rX(1-X/K) - qXV \\
\dot{V} = \eta[pqXV - cV]
\]
Figure 6. A State-Space Diagram of the Population of Bowhead Whales, V, and Whaling Vessels, X, from 1848-1914 Based on Simulation #5.
Table 1. The Number of Vessels, Pelagic Catch, Pelagic Kill and Total Kill for the Western Arctic Bowhead Fishery (1848-1914)\textsuperscript{1}

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<tr>
<th>Year</th>
<th>Vessels</th>
<th>Pelagic Catch</th>
<th>Pelagic Kill\textsuperscript{2}</th>
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\(^1\)Source: Breitwick et. al. (1984), Table 1, pp.488-490.

\(^2\)Pelagic kill is the sum of pelagic catch plus struck-but-lost.

\(^3\)Total kill is the sum of pelagic kill plus shore-based catch plus struck-but-lost.
Table 2. Estimated bowhead population in 1848, 1914, 1970 and 1983 from simulations using the generalized logistic recruitment function.

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<th>$r$</th>
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Table 3. Optimal stock, $X^*$, and kill, $K^*$, for the parameter set #5 in Table 2, $\gamma = 0.00$ to 0.05 and $\delta = 0.00$ to 0.05.

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Appendix A

10 REM This program simulates the the population of bowhead
20 REM whales in the western Arctic assuming a recruitment function
30 REM F(Z)=R.Z(1-(Z/K)^A) where Z is escapement, R is the maximum
40 REM recruitment rate, A is a density dependent parameter, and
50 REM K is the positive population level so large that gross recruit-
60 REM ment is zero. The mortality rate is M and the values for total kill
70 REM come for Breiwick et al. (1984). The bowhead population in 1848
80 REM implied by the biological parameters (and assuming a stationary
90 REM state) is X(0)=((R-M)/R)^(1/A)).K. Stationarity of the unexploited
100 REM population implies X(t)=X(0) and kill, K(t)=0 for the years
110 REM (1848 - I) for i=0,1,...,T-1, where T is the recruitment lag. Of
120 REM interest are those combinations of parameters resulting in a
130 REM population of bowheads of approximately 4,000 in 1970.
140 DATA 2.39,20100,0.03,0.054,5
150 READ A,K,M,R,T
160 DIM K(135+T),X(135+T),Z(135+T)
170 X(0)=((R-M)/R)^(1/A)) K(0)=0; Z(0)=X(0)
180 FOR I=1 TO T-1
190 X(I)=X(0); K(I)=K(0); Z(I)=Z(0)
200 NEXT I
210 X(T)=X(0)
220 DATA 18,573,2067,898,2709,807,166,2,0,78,459,372
230 DATA 221,306,157,303,434,590,554,599,516,382
240 DATA 637,138,200,147,95,200,76,270,80,266
260 DATA 136,282,343,180,228,117,118,130,303,228
270 DATA 147,50,162,109,82,105,66,87,122,57
280 DATA 30,41,27,8,54,6,24,23,27,27
290 DATA 32,8,39,9,39,51,33,15,29,30
300 DATA 17,30,24,21,18,14,23,23,21,12
310 DATA 18,38,21,14,3,23,20,17,8,9
320 DATA 14,21,12,35,8,35,11,5,5,2
330 DATA 29,17,20,15,24,14,23,11,26,32
340 DATA 48,26,43,48,40,32,74,72,17,23,53,16
350 FOR I=T TO 134+T
360 READ K(I)
370 NEXT I
380 FOR I=T TO 134+T
390 Z(I)=X(I)-K(I)
400 X(I+1)=(1-M)*Z(I)+R*Z(I-1)*T*Z(I-1/T)/K)^A
410 IF X(I+1)<=0 GOTO 440
420 NEXT I
430 GOTO 450
440 PRINT "Extinction in year":1848+(I+1)-T
460 PRINT " X(1848)"," X(1914)"," X(1970)"," X(1983)
470 PRINT X(T),X(66+T),X(122+T),X(135+T):PRINT:PRINT
480 INPUT "Do you want a print-out of t, X(t), K(t),and Z(t)? Yes=1. No=0.";W
490 IF W=0 GOTO 570
510 LPRINT " YEAR"," X(t)"," K(t)"," Z(t)
520 LPRINT 
530 FOR I=T TO 135+T
540 YR=1848-T+1
550 LPRINT YR,X(I),K(I),Z(I)
560 NEXT I
570 END
Appendix B

Equation (8) in the text may be derived by a discrete-time extension of the method of Lagrange multipliers (Conrad and Clark 1987). The problem of maximizing the present value of welfare subject to escapement and the equation describing delayed-recruitment was stated as

Maximize  \[ \sum_{t=0}^{\infty} \rho^t W(X_t, K_t) \]

Subject to  \[ X_{t+1} = (1-M)Z_t + F(Z_{t-\tau}) \]
\[ Z_t = X_t - K_t \]

The Lagrangian expression associated with this problem may be written as

\[ L = \sum_{t=0}^{\infty} \rho^t \{ W(X_t, K_t) + \rho \lambda_{t+1} [(1-M)(X_t - K_t) + F(X_{t-\tau} - K_{t-\tau}) - X_{t+1}] \} \]

The first order necessary condition require

\[ \frac{\partial L}{\partial K_t} = \rho^t \frac{\partial W(\cdot)}{\partial K_t} - (1-M) \rho \lambda_{t+1} - \rho \lambda_{t+1} \lambda_{t+1} \lambda_{t+1} F'(\cdot) = 0 \]
\[ \frac{\partial L}{\partial X_t} = \rho^t \frac{\partial W(\cdot)}{\partial X_t} + (1-M) \rho \lambda_{t+1} - \rho \lambda_{t+1} + \rho \lambda_{t+1} \lambda_{t+1} F'(\cdot) = 0 \]
\[ \frac{\partial L}{\partial (\rho \lambda_{t+1})} = \rho^t [(1-M)(X_t - K_t) + F(X_{t-\tau} - K_{t-\tau}) - X_{t+1}] = 0 \]

which may be simplified to

\[ \frac{\partial W(\cdot)}{\partial K_t} = (1-M) \rho \lambda_{t+1} + \rho \lambda_{t+1} \lambda_{t+1} F'(\cdot) \]
\[ \lambda_t = \frac{\partial W(\cdot)}{\partial X_t} + (1-M) \rho \lambda_{t+1} + \rho \lambda_{t+1} \lambda_{t+1} F'(\cdot) \]
\[ X_{t+1} = (1-M)(X_t - K_t) + F(X_{t-\tau} - K_{t-\tau}) \]

When evaluated in steady state the first of these conditions may be
solved for $\rho \lambda$ yielding

$$\rho \lambda = \left[ \frac{\partial W(*)}{\partial K} \right] / [(1-M) + \rho^5F'(\cdot)]$$

The second of the first order conditions becomes

$$\rho \lambda [(1-M) + \rho^5F'(\cdot) - (1+\delta)] = -\frac{\partial W(*)}{\partial X}$$

Substituting the expression for $\rho \lambda$ into the above yields

$$\left[ \frac{\partial W(*)}{\partial K} \right] [(1-M) + \rho^5F'(\cdot) - (1+\delta)] = \left[ -\frac{\partial W(*)}{\partial X} \right] [(1-M) + \rho^5F'(\cdot)]$$

and isolating $(1 + \delta)$ on the right-hand-side results in

$$\left[ \frac{W_x + W_k}{W_k} \right] [1 - M + \rho^5F'(Z)] = 1 + \delta$$

as given in the text. For an alternative derivation see Clark (1976 a).