CORNELL AGRICULTURAL ECONOMICS STAFF PAPER

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June 1987 A.E. Staff 87-12

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Abstract

I argue that much of what researchers have been measuring as technical inefficiency may be allocative inefficiency. When inputs are aggregated it is shown that any allocative inefficiency is manifested as technical inefficiency. Greater degrees of aggregation may produce lower measures of technical efficiency.

*Paper presented at the American Agricultural Economics Association Meetings, Michigan State University, East Lansing, Michigan, August 2-5, 1987.

**Associate Professor, Department of Agricultural Economics, Cornell University currently on leave with the USDA/ERS/RTD. Bernard F. Stanton and William Tomek provided helpful comments and suggestions.

A Note on Measuring Technical Efficiency

Renewed interest has surfaced in measuring technical and allocative efficiency in agriculture production, as evidenced by recent articles in the AJAE (Hall and LeVeen; Taylor, Drummond and Gomes), the regional agricultural economic journals (Bagi; Grisley and Mascarenhas), as well as other journals (Bravo-Ureta; Huang and Bagi; Russell and Young). These approaches have typically estimated a frontier function by econometric or mathematical programming techniques and measured deviations from that frontier. Research outside of agriculture has been even more extensive, resulting in an issue of the Journal of Econometrics discussing estimation, and the publication of a textbook whose goal is to define the measurement of efficiency (Fare, Grosskopf, and Lovell).

Many of the articles emphasize overcoming the econometric barriers that existed in estimating frontier functions. The initial and simplest approach, called corrected ordinary least squares (COLS), entails estimating an ordinary least squares equation (OLS) and shifting the intercept term by the largest residual. The total error term is attributed to inefficiency and has been shown to require assuming firm efficiency to be uniformly distributed. Other distributions of efficiency can be assumed by calculating and using the necessary moments of the error term that specifies those distributions. Although the estimates are consistent they are not necessarily efficient. Maximum Likelihood estimation has often been substituted (Greene). More recent advances involve defining both an error term and an efficiency component and using estimation procedures that allow individual firm measurement of efficiency (Jondrow, Lovell, Materov and Schmidt). Advances in defining mathematic programming methods to measure efficiency have also occurred (Fare, Grosskopf and Lovell).

The purpose of this note is to question the empirical measurement of technical inefficiency that is being reported in articles. I contend that much of what researchers have been measuring as technical inefficiency is really allocative inefficiency. Taken to the extreme it can be argued that all technical inefficiency is simply allocative inefficiency. This assertion is not new since Stigler has previously expressed the view that all perceived technical inefficiency is allocative inefficiency.[1]

Defining Efficiency

Following Farrell most researchers explain technical and allocative efficiency with a unit isoquant assuming constant returns to scale.[2] Any firm above the unit isoquant is technically inefficient. Any firm on the isoquant but not equating the MRTS with the input price ratio is technically efficient but allocatively inefficient. A firm both on the isoquant and equating the MRTS with the input price ratio is both technically and allocatively efficient, sometimes referred to as economically efficient. Allocative efficiency, as defined here and usually elsewhere, is based upon profit maximization (or cost minimization with constant returns to scale) as the behavior goal, but other objectives may be used.

The question that should be raised is why a firm is not technically efficient? Why should two firms that use identical input mixes produce two different amounts of output? One explanation is that the managerial ability

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[1] Stigler also goes on to say that even allocative inefficiency is perceived because of failure of the observer to measure all relevant inputs, or to correctly perceive what is being optimized or to account for all the constraints on the optimization process, etc.

[2] Fare et al. also define structural inefficiency as operating on the area of a backward bending isoquant. If constant returns to scale does not exist then scale efficiency is also an issue.

of the 2 farmers differ and that inclusion of a measurement of the management input would explain the difference in output. Incomplete knowledge of available techniques, motivation, learning, and psychological factors, all managerial factors, are potential reasons for deviations from the frontier (Leibenstein). Some have attempted to explain the degree of efficiency by measures of managerial attributes such as education, experience and age, but very little of the variation in efficiency is associated with such variables (Page; Pitt and Lee; Timmer). This conceivably could be in the inability to quantify management empirically. However, the role of management is to determine the combination of inputs to use to reach a goal. If 2 farmers use the same combination of inputs and have the same goal, how can it be argued that their management levels are different?

Perhaps farmers may waste inputs.[3] Yet, it is difficult to imagine a farmer being as careless as reported efficiency measures would suggest. It is also possible that a farmer may be applying or using the inputs at the wrong time during the production process leading to lower output. As Shapiro states "Whereas allocative efficiency is usually considered and measured in terms of the amounts of inputs combined in production, technical efficiency refers to the manner in which the inputs are used." This is not very plausible with an input like seed, but might be possible for labor. However, one can argue this is an allocative problem within the production process that would be apparent if the production process was separated into components. A farmer using more labor in planting than its marginal value warrants and less in cultivation would be considered allocatively inefficient if two separate production functions were estimated.

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[3] Stigler suggests that technical inefficiency is a useless concept until we have a theory of error (waste).

Hall and Winsten discussed the ambiguous notion of efficiency 30 years ago. They claim that what many call technical efficiency may really result from environmental variables. They state that, "Climatic variables might be considered as environmental for those problems in farm efficiency for which farmers cannot choose where they are to work." Or, "average seam thickness may be taken as an environmental variable for some problems concerned with mines." One could argue that those farms or mines are uneconomical to operate, but that is an economic (allocative) decision rather than a technical decision. Those marginal farms or mines may be technically efficient given their environment, but when compared to farms or mines operating under better environments they would be measured as technically inefficient. Pervasive statements such that, "the same volume of production could have been achieved with x percent fewer resources if all farms would have operated at 100 percent technical efficiency," may be meaningless. Of course, this limitation can be overcome if environmental variables are incorporated into the analysis. This is often either not done or cannot be done.

Hall and Winsten then discuss using linear programming as the paradigm, that technical inefficiency in the use of variable inputs may be the result of price (allocative) inefficiency in the selection of fixed inputs (techniques) given constant environmental factors (resources). They also comment on Farrell's earlier work as making a start -- though perhaps a false start since he did not analyze the concept of efficiency. Unfortunately, others may have continued from that false start, constructing elaborate measurement and estimation procedures without analyzing the concept of efficiency. An exception to that is Forsund et al. who discuss the nature of inefficiency.

Yet, they conclude by contending the discussion is philosophical and the real test of frontier models is likely to be an empirical one.

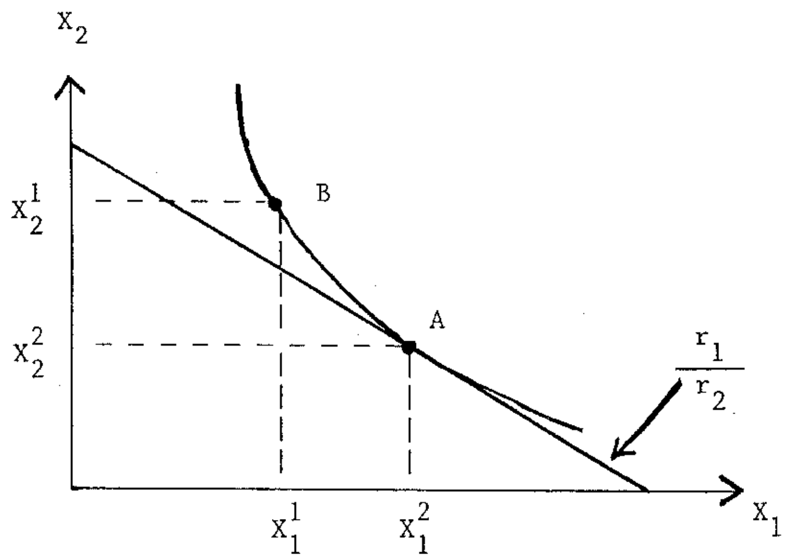
A limitation with estimating firm level production functions is that inputs must be aggregated in order to reduce the number of inputs for econometric or linear programming estimation of efficiency. In aggregating, the measured aggregated input may be in identical amounts for two firms although the disaggregated inputs may be quite different. In the following section it is shown that the disaggregated inputs may be technically efficient but allocatively inefficient, but when aggregated the result is manifested as technical inefficiency.

Aggregation and Allocative Inefficiency

Assume that two inputs will be aggregated into one input. Two different farmers both use the two inputs in different combinations but each produces the same amount of output. This is illustrated in Figure 1 as both farmers operating on the same isoquant and thus technically efficient. Assume however that only farmer A is allocatively efficient.

Now aggregate inputs X_1 and X_2 into a new input called X_A. Since X_1 and X_2 are different inputs it is necessary to weigh each by some value per unit of input measure. That value measure is typically price per unit of input. It would be logical to use r_1 and r_2 as the unit prices since these are the market prices. In fact, in empirical work it is common not to perform the actual aggregation, but to use expenditures based on sums of inputs which depend upon purchase prices. Farmers do purchase items at different prices but that problem is ignored or assumed away.

Figure 1.



Measuring the aggregate input for farmer B results in $r_1 X_1^1 + r_1 X_2^1$. Aggregation for farmer A results in $r_1 X_1^2 + r_2 X_2^2$. It can easily be shown that farmer A is using the least cost combination of inputs X_1 and X_2 to produce the output. Thus farmer A will have a lower expenditure "or use" of the aggregated input. When this lower amount of aggregated input is used to measure technical inefficiency it will be discovered that farmer B is technically inefficient when, in fact, he or she is simply allocatively inefficient.

An Example

To measure technical efficiency of dairy farms I constructed a unit isoquant using the procedure formulated by Boles. A sample of 112 dairy farms that participated in the 1983 New York Dairy Farm Business Summary Program was used.[4] Inputs were defined as the disaggregated expense categories on the itemized income statements used in the summary forms. Capital and labor

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[4] These 112 farms participated for 10 consecutive years from 1974 through 1983.

inputs were added. Output was milk and other receipts adjusted for inventory changes. In all, 28 inputs were defined. Analysis resulted in no farms being dominated by any linear combination of other firms using a step wise linear program (Boles). In this sense all farms were technically efficient. Then inputs were aggregated into nine categories including livestock expenses, crop expenses, etc. The analysis using these aggregated inputs suggest 56 of the 112 farms were technically inefficient to some degree.

Clearly each of the 112 farms are using slightly different techniques in producing milk. Some did not use a particular input which did not allow them to be dominated by any farm that used that input. A farmer may have errored in not using the input given it's price, but that is allocative rather than technical inefficiency. When inputs were aggregated that allocative inefficiency resulted in a higher cost of production which was manifested as technical inefficiency.

In measuring the technical efficiency of Brazilian farms Taylor et al. obtained a low average technical efficiency of 18%. They attribute it to some extent to the use of a full frontier specification where all unexplained variation in the sample, systematic or random, contributes to measured technical inefficiency. Maybe so, but the low technical efficiency may also be due to the fact that they only used three aggregated inputs in their production function specification -- land, labor, and materials.

These limitations not only question the validity of reported technical inefficiencies as entirely due to technical inefficiency, but more importantly the ability to compare technical efficiency measures between studies. Fare et al. criticism that economic theory textbooks should not assume that all firms operate on the production function may not be a valid criticism. By definition each firm does operate on a production function. French realizes this when he states that the firm "is said to be technically efficient if its production function yields the greatest output for any set of inputs, given its particular location and environment. If some other production function is used, the ratio of output obtained with this function to output obtained with the best function, given the input combinations, is a measure of the degree of technical efficiency." Since the production function representing the efficient boundary is a pervasive paradigm in our field, it is beneficial to look at input aggregation in more detail.

Aggregation of inputs

There are two situations under which inputs can be aggregated (Varian). These are Hicksian separability (composite commodity theorem) or functional separability (Blackorby, Primont and Russell). Under Hicksian separability it is necessary that the price vector q of the aggregated inputs always be proportional to some fixed vector q^o so that $q = tq^o$ for any scalor t. This implies that the isocost line for those aggregated inputs remain parallel for all observations. The expansion path may or may not be linear (homothetic). The result is that the production function can be written as $y = f(x,Z)$, where x is a vector of inputs (some may be aggregated) and Z is the aggregated input $Z = q^o z$. This aggregated or composite input is the individual inputs weighted by their input prices. This is the procedure used when expenditure data are used to arrive at aggregated inputs.

Appealing to Hicksian separability when estimating production functions requires two conditions. The first is that producers are faced with proportionate price vectors for the aggregated inputs. The market may or may not produce this proportionality (Gould and Segall) although it is typically assumed. The second is that each producer is equating the MRTSs of the aggregated inputs with the input price ratios, but that they are operating on different points on the expansion path because of different output prices.

The requirements for a frontier function estimate are not clear since it is assumed that a firm may not be operating on a production function. However, it would appear that proportional prices are required as well as equating the MRTSs with price ratios in order to consider the composite input as a valid aggregate input. Even if a producer is not operating on the frontier of a production surface, these conditions might be met. Yet, why a producer would be knowledgeable to equate MRTSs but not operate on the frontier is questionable. If a producer does not equate the MRTSs then the aggregated input incorporates allocative inefficiency as well as technical inefficiency.

A production function is (weakly) separable when the MRTSs for pairs of variables within the group of the aggregated input are unaffected by quantities of variables outside the group. This is a necessary but not sufficient condition for homotheticity within the group. Thus the expansion paths within the group may or not not be linear. However, more or less of the aggregated input may be used depending upon the output price and other inputs outside the aggregation. The production function can then be written as $y = (X_n, g(X_m))$ where g is the function of the aggregated inputs and X_n are the other inputs (some aggregated). The output from g can be viewed as an intermediate output that can be combined with x_n to produce y. This is often viewed as a two stage production process, not necessarily chronological.

There is no necessity to assume the same functional form for g as is used for f. In fact, empirically, the aggregator function normally used, at least implicitly, is the weakly additive form $g\ (X_m) = \sum_{i=1}^{m} r_i X_i$, which is the expenditure on the aggregated inputs. By definition this is the Leontief (fixed-proportions) value-added production function. The MRTSs are constant (linear isoquants) and the inputs are perfect substitutes. This may appear restrictive but is necessary at some level of aggregation unless the researcher collects data on all inputs and performs his own aggregation. If the same functional form applies to all inputs but a group of inputs are separable then the aggregation function depends on the functional form of the production function. This has been shown to be a geometric index with weights proportional to the elasticities of the respective inputs for the Cobb-Douglas (Solow). Of course, as Griliches succinctly states, "if we knew the elasticities, we would not be trying to estimate them." More recently Diewert derived aggregation indices for the most commonly used functional forms. Yet, simple sums are typically used by choice or necessity. As Griliches and others have shown, specification error has the tendency to reside in the error or constant term, concepts which are critical in empirically measuring technical efficiency.

Assuming the correct specification of the aggregation index, correctly measuring technical inefficiency from the frontier function requires no allocative inefficiency in the separable inputs aggregated or that allocative inefficiency would be manifested as technical inefficiency. Technical inefficiency in the separable inputs aggregated is permissible since it would be measured as technical inefficiency in the frontier production function. Again, if a farmer is allocatively inefficient in using aggregated inputs there is no reason not to expect allocative inefficiency within the aggregated inputs.

Conclusions

I have argued that what most authors have been reporting as technical inefficiency may also include elements of allocative inefficiency. This results because the aggregation of inputs to estimate the frontier production function incorporates allocative as well as technical inefficiency. The greater the amount of aggregation and the fewer inputs used in estimating the frontier function, the greater the amount of allocative inefficiency inherent in measures of technical inefficiency.

Additional research is needed to explain why a firm is technically inefficient. The selection of an incorrect technique that does not provide for a maximum output may simply be allocative inefficiency. By definition a separate technique must require at least one different input. The decision-maker may have errored in his selection of that technique given his goal objective (i.e., profit maximization). That error is manifested as technical inefficiency when inputs are aggregated explicitly or implicitly but it is really allocative inefficiency.

References

Bagi, Faqir S. "Stochastic Frontier Production Function and Farm-Level Technical Efficiency of Full-Time and Part-Time Farms in West Tennessee." North Central Journal of Agricultural Economics, 6(1984):48-55.

Blackorby, Charles, Daniel Primont and R. Robert Russell. "Duality, Separability, and Functional Structure: Theory and Economic Applications." New York: North-Holland, 1978.

Boles, J.N. "Efficiency Squared ... Efficient Computation of Efficiency Indexes." Western Farm Economic Association Proceedings, 1967.

Bravo-Ureta, Boris E. "Technical Efficiency Measures for Dairy Farms Based on a Probabilistic Frontier Function Model." Canadian Journal of Agricultural Economics, 34(1986):399-415.

Diewert, W. "Exact and Superlative Index Numbers." Journal of Econometrics, 4(1976):115-45.

Fare, Rolf, Shawna Grosskopf and C.A. Knox Lovell. The Measurement of Efficiency of Production. Boston-Dordrecht-Lancaster: Kluwer-Nijhoff Publishing, 1985.

Farrell, M.J. "The Measurement of Productive Efficiency." Journal of the Royal Statistical Society, 120(1957):253-281.

Forsund, Finn R., C.A. Knox Lovell and Peter Schmidt. "A Survey of Frontier Production Functions and of Their Relationship to Efficiency Measurement." Journal of Econometrics, 13(1980):5-25.

French, Ben C. The Analysis of Productive Efficiency in Agricultural Marketing: Models, Methods, and Progress in Martin, Lee R. A Survey of Agricultural Economics Literature, Vol. 1 Minneapolis: Univ. of Minnesota Press, 1977.

Greene, W.H. "Maximum Likelihood Estimates of Econometric Frontier Functions." Journal of Econometrics, 13(1980):27-56.

Griliches, Zvi. "Specification Bias in Estimates of Production Functions." Journal of Farm Economics, 39(1957):8-20.

Grisley, William and Juan Mascarenhas. "Operating Cost Efficiency on Pennsylvania Dairy Farms." Northeastern Journal of Agricultural and Resource Economics, 14(1985):88-95.

Gould, John and Joel Segall. "The Substitution Effects of Transportation Costs." Journal of Political Economy, (1968):130-137.

Hall, Bruce F. and E. Phillip LeVeen. "Farm Size and Economic Efficiency: The Case of California." American Journal of Agricultural Economics, 60(1978):589-600.

Hall, Margaret and Christopher Winsten. "The Ambiguous Notion of Efficiency." The Economic Journal, 69(1959):71-86.

Huang, Cliff J. and Faqir S. Bagi. "Technical Efficiency on Individual Farms in Northwest India." Southern Economic Journal, 51(1984):108-115.

Jondrow, J., C.A.K. Lovell, I.S. Materov, and P. Schmidt. "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model." Journal of Econometrics, 15(1982):233-38.

Journal of Econometrics, 13 (May, 1980)

Leibenstein, H. "Allocative Efficiency vs. 'X-Efficiency'." American Economic Review, 56(1966):392-415.

Page, John M. Jr. "Firm Size and Technical Efficiency." Journal of Development Economics, 16(1984):129-152.

Pitt, M. and L.F. Lee. "The Measurement and Sources of Technical Inefficiency in the Indonesian Weaving Industry." Journal of Development Economics, 9(1981):43-64.

Russell, N.P. and T. Young. "Frontier Production Functions and the Measurement of Technical Efficiency." Journal of Agricultural Economics, 34(1983):139-150.

Shapiro, Kenneth H. "Efficiency Differentials in Peasant Agriculture and their Implications for Development Policies." Journal of Development Studies, 19(1983):179-190.

Solow, Robert M. "The Production Function and the Theory of Capital." Review of Economic Studies, 61(1955):101-108.

Stigler, G.J. "The Xistence of X-Efficiency." American Economic Review, 66(1976):213-216.

Taylor, Timothy G., H. Evan Drummond and Aloisio T. Gomes. "Agricultural Credit Programs and Production Efficiency: An Analysis of Traditional Farming in Southeastern Minas Gerais, Brazil." AJAE, 68(1986):110-119.

Timmer, C. Peter. "On Measuring Technical Efficiency." Food Research Institute Studies in Agricultural Economics, Trade and Development, 9(1970):99-171.

Varian, Hal R. Microeconomic Analysis. New York: W.W. Norton, 1984.