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BIOECONOMICS AND THE HARVEST OF TWO

COMPETING SPECIES

by

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I. Introduction and Overview

Much of the economic wisdom on how to manage fishery resources is

based on a single species model where the resource in question is described

by a single state variable, usually representing biomass. While single-

species biomass models have afforded important descriptive and prescrip-

tive insights into the causes of overfishing and the form that effective

policy might take, they are typically regarded as simplistic from both

an economic and ecological perspective. Recent research has sought to

determine the economic importance of stochastic environmental effects

(Sutinen 1981), price uncertainty (Andersen 1982), age structure

(Conrad 1982), and searching strategies to locate schools of fish

(Mangel and Clark 1982).

Yet another area of interest is the management of multispecies

systems. Clark (1976a) examines several two-species models where ecolo-

gical or technical interdependencies may exist. An ecological interde-

pendency would occur if the two species exhibited a prey-predator, competi-

tive, or symbiotic relationship. A technical interdependency is said to

occur if the two species are subject to joint (nonselective) harvest.

In his analysis, Clark assumes a pristine system attributed to

Gause (1935) and examines the bioeconomic consequences when only one of

the two species is harvested (Clark 1976a, pp. 311-317), and when both

species are harvested selectively (Clark 1976a, pp. 317-325).

Getz (1979) also examines the selective harvest of two competing

species and shows that when each production function is linear in

effort, (a) the maximum sustainable rent (MSR) solution is singular, (b) the optimal approach to MSR lies along partially singular curves, and (c) while it may not be optimal, a minimum-time solution exists which may be determined in a relatively straightforward fashion. The minimum-time or most rapid approach path (MRAP) will drive the system from its initial condition to the MSR in the least amount of time, subject to constraints on harvest. Spence and Starrett (1975) define conditions where MRAP is optimal for the single-species biomass model.

Neither Clark nor Getz, however, examines the problem of harvesting competing species when technology (fishing gear) is nonselective. There are at least two important fisheries where this situation is thought to exist. In the Eastern Tropical Atlantic (ETA), Yellowfin and Skipjack tuna, often mixed within a single school, are thought to be interspecific competitors and are harvested jointly by purse seiners. On Georges Bank, off the coast of Massachusetts, cod and haddock may compete for a common food source and are jointly havested by trawlers. This paper will focus on the nonselective harvest of competing species.

In the next section we will develop a relatively general statement of the problem, derive expressions for the steady state optimum and conditions for stability. The third section presents a particular specification employing a Gaussian system of interspecific competition and two transcendental production functions. A numerical example is provided to illustrate analytical techniques and the behavior which might result when competitive species are harvested under conditions of open access and central management. The final section summarizes the major conclusions and policy implications.

II. The Nonselective Harvest of Competing Species

To set the stage for the numerical analysis in the next section and in view of the data typically encountered in empirical research, we have chosen to partition time into discrete intervals. The system we wish to consider is comprised of three difference equations of the general form:

$$X_{1,t+1} = F_1(X_{1,t}, X_{2,t}; E_t) \quad,$$

$$X_{2,t+1} = F_2(X_{1,t}, X_{2,t}; E_t) \quad, \tag{1}$$

$$E_{t+1} = N(X_{1,t}, X_{2,t}; E_t) \quad.$$

The first two equations determine the population levels of species one and two in period t+1 as a function of stocks and fishing effort in period t. The third equation determines the level of fishing effort in period t+1, again as a function of stocks and effort in period t. The first two equations are bio-technical in nature, in the sense that they reflect biological interactions between species one and species two, and the technical relationship between effort and the yield or harvest of each species. By joint or nonselective harvest, we mean a situation where a positive level of effort results in a positive yield of both species (assuming neither extinct). Selective harvesting would be characterized by two types of effort (technology), say $E_{1,t}$ and $E_{2,t}$, where the yield of species one depends on only the level of $E_{1,t}$ and the yield of species two on $E_{2,t}$.

The third equation is a behavioral equation in the sense that it determines how fishermen adjust (or are allowed to adjust) effort as an implicit function of net value. Yield, gross revenue, cost, and user cost can be shown to be explicit functions of stocks and effort, and

thus _implicit_ in the third equation of our system is the behavioral response of fishermen or a central management authority to net private or net social value.

The first order partial derivatives of $F_1(\cdot)$, $F_2(\cdot)$ and $N(\cdot)$ may take the following signs:

$$\partial F_i(\cdot)/\partial X_i \gtreqless 0 \quad ,$$

$$\partial F_i(\cdot)/\partial X_j < 0 \quad ,$$

$$\partial F_i(\cdot)/\partial E < 0 \quad , \qquad\qquad i,j=1,2 \qquad\qquad (2)$$

$$\partial N(\cdot)/\partial X_i > 0 \quad , \qquad\qquad i \neq j$$

$$\partial N(\cdot)/\partial E \gtreqless 0 \quad .$$

A stationary or fixed point $(X_1, X_2; E)$ is one where

$$X_1 = F_1(X_1, X_2; E) \quad ,$$

$$X_2 = F_2(X_1, X_2; E) \quad , \qquad\qquad\qquad (3)$$

$$E = N(X_1, X_2; E) \quad .$$

The local stability of such a point may be determined from the Eigen values of the Jacobian Matrix

$$J = \begin{bmatrix} \partial F_1(\cdot)/\partial X_1 & \partial F_1(\cdot)/\partial X_2 & \partial F_1(\cdot)/\partial E \\ \partial F_2(\cdot)/\partial X_1 & \partial F_2(\cdot)/\partial X_2 & \partial F_2(\cdot)/\partial E \\ \partial N(\cdot)/\partial X_1 & \partial N(\cdot)/\partial X_2 & \partial N(\cdot)/\partial E \end{bmatrix} \qquad (4)$$

The characteristic equation of this matrix is a cubic with real coeffi-cients. The equation can have either three real roots or one real and two complex (conjugates). Many cases can occur depending on the arrange-ments of the roots λ_1, λ_2, and λ_3 in the plane of the complex variable λ.

Seven nondegenerative cases are described in Table 1. There are three types of invariant manifolds: (a) stable, which is tangent to the space E^s spanned by the Eigen vectors corresponding to Eigen values with $|\lambda_j| < 1$; (b) unstable, which is tangent to the space E^u spanned by the Eigen vectors corresponding to Eigen values with $|\lambda_j| > 1$; and (c) center, which is tangent to the space E^c spanned by Eigen vectors corresponding to Eigen values with $|\lambda_j| = 1$. In Table 1, points lying along the horizontal axis of the unit circle correspond to real roots, while those lying above or below are complex. In phase space triple arrows indicate the direction of most rapid movement.

We will consider a slightly less general system of the form:

$$X_{1,t+1} = G_1(X_{1,t}, X_{2,t}) - H_1(X_{1,t}, E_t) \quad,$$

$$X_{2,t+1} = G_2(X_{1,t}, X_{2,t}) - H_2(X_{2,t}, E_t) \quad, \tag{5}$$

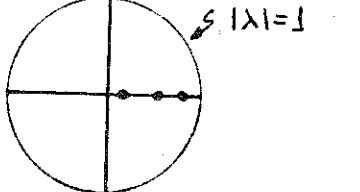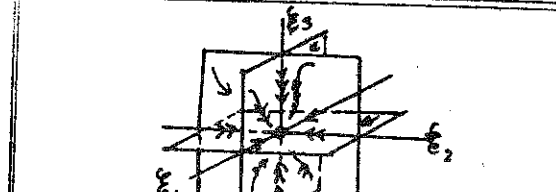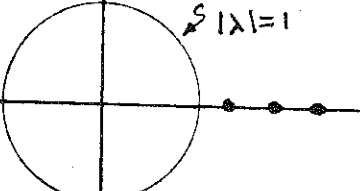$$E_{t+1} = E_t + \eta\pi(X_{1,t}, X_{2,t}; E_t) \quad.$$
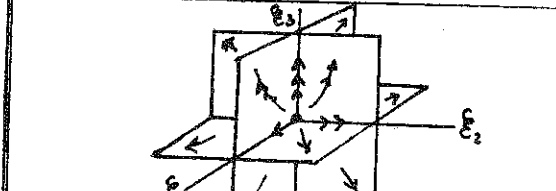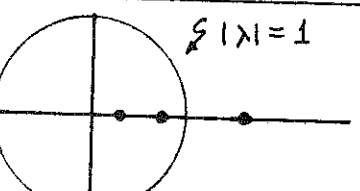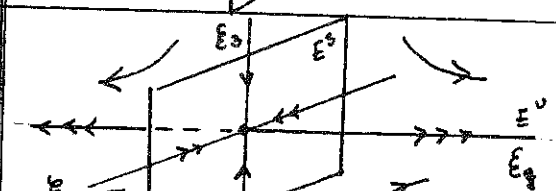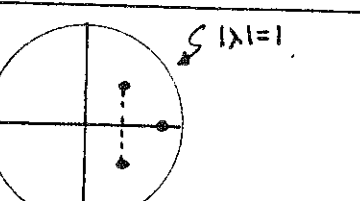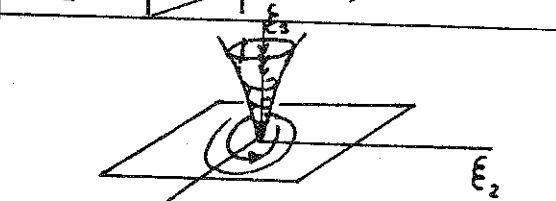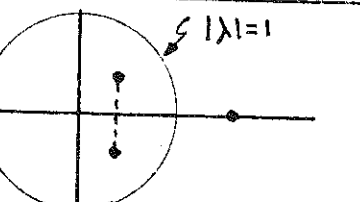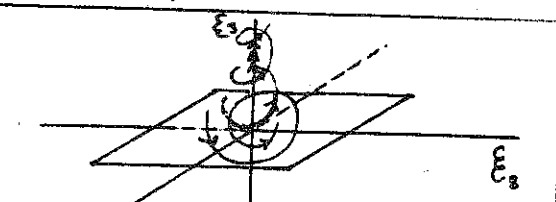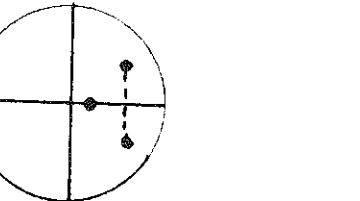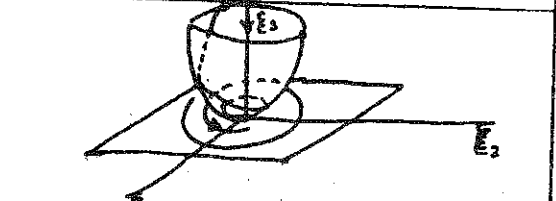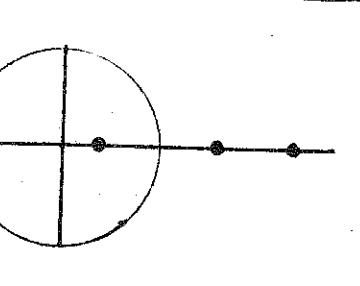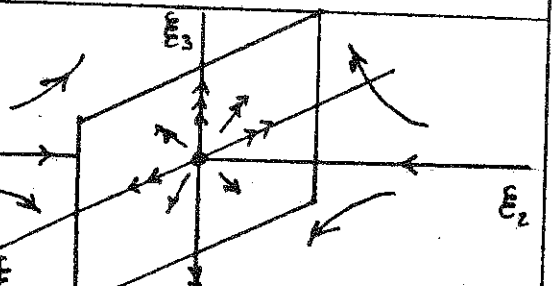
In this form we may define pristine, open acess, and managed systems. By a pristine system we mean a system with no commercial harvest, that is, $E_t = 0$ for the interval under consideration. In this case it is assumed that $H_1(X_{1,t}, 0) = H_2(X_{2,t}, 0) = 0$, and (5) reverts to a two-equation dynamical system.

By open access we mean a system where the level of effort is determined by a large number of competitive, unregulated fishermen. It is assumed that effort will expand as long as net revenue (fishery rent) is positive. In open access $\pi(\cdot)$ will represent net revenue and $\eta > 0$ will determine the responsiveness of effort in t+1 to net revenues in period t.

In the managed system, the first two equations, describing the bio-technical aspects of the system, are assumed the _same_ as under

TABLE 1

CHARACTERISTIC EQUATION: $\lambda^2 + a\lambda^2 + b\lambda + c = 0$

Case 1 $\lambda_1 \quad \lambda_2 \quad \lambda_3 \; 1$ Gen. SINK Point	$\varsigma \, \|\lambda\|=1$	
Case 2 $1 \quad \lambda_1 \quad \lambda_2 \quad \lambda_3$ Gen. Unstable Node	$\varsigma \, \|\lambda\|=1$	
Case 3 $\lambda_3 \quad \lambda_2 \quad 1 \quad \lambda_1$ Gen. Saddle Point	$\varsigma \, \|\lambda\|=1$	
Case 4 : $\|\lambda_1\|, \|\lambda_2\| < \lambda_3 < 1$ Gen. Spiral SINK	$\varsigma \, \|\lambda\|=1$	
Case 5. $\|\lambda_1\|, \|\lambda_2\| < 1 < \lambda_3$ Gen. of Spiral SADDLE POINT	$\varsigma \, \|\lambda\|=1$	
Case 6 $\lambda_3 < \|\lambda_1\|, \|\lambda_2\| < 1$ Gen. SPIRAL SINK		
Case 7 $\lambda_2 \; 1 \quad \lambda_1 \quad \lambda_3$ Gen. Saddle Point		

open access. The third, behavioral equation will change in that $\pi(\cdot)$ will now reflect marginal market value, marginal harvest cost, and user cost. The management authority will allow effort to increase as long as marginal market value net of harvest and user costs is positive. The adjustment parameter (η), while still positive, will probably differ from that which results from the actions of competitive fishermen, and the management authority may wish to regard η as a choice variable. The magnitude of η will influence the stability of open access and management equilibria.

When considering the harvest of two competing species, one might logically ask the following questions: (1) What stationary points exist for the pristine, open access, and managed systems; (2) what is the local stability of such points; and (3) if reachable, what is the best way for the system to move from a pristine or open access equilibrium to a bio-economic optimum? These questions are addressed in the next section where two Gaussian competitors are harvested according to two simple transcendental production functions.

III. A Numerical Example

The discrete-time analogue of the Gaussian system of interspecific competition takes the form

$$X_{1,t+1} = X_{1,t} + r_1 X_{1,t}(1 - X_{1,t}/K_1) - \alpha_1 X_{1,t} X_{2,t} \quad ,$$

$$X_{2,t+1} = X_{2,t} + r_2 X_{2,t}(1 - X_{2,t}/K_1) - \alpha_2 X_{1,t} X_{2,t} \quad ,$$

(6)

where r_1 and r_2 are referred to as intrinsic growth rates, K_1 and K_2 are the environmental carrying capacities if no competitor were present, and α_1 and α_2 are coefficients of competition or interference. All parameters are assumed positive.

Clark (1976a, pp. 205-207) reviews the type of stationary points which can result in the continuous time (differential equation) system. If $r_1/\alpha_1 > K_2$ and $r_2/\alpha_2 > K_1$, species will coexist at a stable node where

$$X_1^P = r_2(\alpha_1 K_2 - r_1)K_1/(\alpha_1\alpha_2 K_1 K_2 - r_1 r_2)$$

$$X_2^P = r_1(\alpha_2 K_1 - r_2)K_2/(\alpha_1\alpha_2 K_1 K_2 - r_1 r_2)$$

(7)

If $r_1/\alpha_1 < K_2$ and $r_2/_2 < K_1$, the pristine equilibrium defined by (7) will be a saddle point, and there will exist two stable nodes $(K_1, 0)$ and $(0, K_2)$ where one or the other species may be driven to extinction (competitive exclusion) depending on the initial population levels.

In the discrete-time analogue employed here, we will assume that the underlying pristine system exhibits stable, competitive coexistence. In addition, it is assumed that $0 < r_i < 2$, i=1,2, thus avoiding the problem of excessive overshooting which can arise in difference equation models of this sort.

Effort will produce a yield of species one and two according to

$$Y_{1,t} = X_{1,t}(1 - e^{-\beta_1 E_t}) \quad ,$$

$$Y_{2,t} = X_{2,t}(1 - e^{-\beta_2 E_t}) \quad ,$$

(8)

where $0 < \beta_i < 1$ are production parameters indicating the relative efficiency of fishing effort in harvesting each species. Thus, with $X_{1,t}$ and $X_{2,t}$ both positive, a positive level of effort will yield something of both species. In the open access and managed systems, yields of species one and two are deducted from the right-hand-side (RHS) of their respective difference equations to determine stock in t+1.

Net revenue is determined according to

$$\pi_t = p_1 X_{1,t}(1 - e^{-\beta_1 E_t}) + p_2 X_{2,t}(1 - e^{-\beta_2 E_t}) - cE_t \quad , \qquad (9)$$

where p_1 and p_2 are per unit prices for species one and two, and c is the per unit cost of effort. Under open access, effort will increase if net revenue is positive, decrease if net revenue is negative, and remain unchanged if net revenue is zero, according to the last equation in (5). After a bit of algebra, it can be shown that the open access equilibrium with both species present is defined by

$$X_1^0 = \frac{r_2(\alpha_1 K_2 - r_1)K_1 - \alpha_1 K_1 K_2(1 - e^{-\beta_2 E^0}) + r_2 K_1(1 - e^{-\beta_1 E^0})}{(\alpha_1 \alpha_2 K_1 K_2 - r_1 r_2)} \quad ,$$

$$X_2^0 = \frac{r_1(\alpha_2 K_1 - r_2)K_2 - \alpha_2 K_1 K_2(1 - e^{-\beta_1 E^0}) + r_1 K_2(1 - e^{-\beta_2 E^0})}{(\alpha_1 \alpha_2 K_1 K_2 - r_1 r_2)} \quad ,$$

$$E^0 = \frac{p_1 X_1^0(1 - e^{-\beta_1 E^0}) + p_2 X_2^0(1 - e^{-\beta_2 E^0})}{c} \qquad (10)$$

Given the parameters r_1, r_2, K_1, K_2, α_1, α_2, β_1, β_2, p_1, p_2, and c, we can numerically solve for the open access equilibrium by selecting an initial value for E, solving the first two equations in (10) for X_1^0 and X_2^0, substituting the resulting stocks and initial effort into (9), determining the sign of π_t and adjusting E accordingly. The stability of the open access equilibrium can be determined by calculating the Eigen values of (4) evaluated at $(X_1^0, X_2^0; E^0)$. The adjustment parameter η will influence the stability of the open access equilibrium, but in a complex way. Computer analysis to date indicates that η must be very small; that is, effort must adjust slowly when moving from an initial effort level less than the equilibrium level E^0.

Under management, it is assumed that a sole owner or regulatory agency can control effort and would seek to

$$\underset{\{E_t\}}{\text{maximize}} \ \sum_{t=0}^{\infty} \rho^t \{ p_1 X_{1,t}(1 - e^{-\beta_1 E_t}) + p_2 X_{2,t}(1 - e^{-\beta_2 E_t}) - cE_t \} \ ,$$

subject to

(11)

$$X_{1,t+1} = X_{1,t} + r_1 X_{1,t}(1 - X_{1,t}/K_1) - \alpha_1 X_{1,t} X_{2,t} - X_{1,t}(1 - e^{-\beta_1 E_t}) \ ,$$

$$X_{2,t+1} = X_{2,t} + r_2 X_{2,t}(1 - X_{2,t}/K_2) - \alpha_2 X_{1,t} X_{2,t} - X_{2,t}(1 - e^{-\beta_2 E_t}) \ ,$$

where $\rho = 1/(1 + \delta)$ is a discount factor and δ is the discount rate. In addition to the difference equations describing stock dynamics, the first order conditions for an interior maximum require

$$(p_1 - \rho\lambda_{1,t+1})X_{1,t}\beta_1 e^{-\beta_1 E_t} + (p_2 - \rho\lambda_{2,t+1})X_{2,t}\beta_2 e^{-\beta_2 E_t} = c \ , \quad (12)$$

$$\lambda_{1,t} = p_1(1 - e^{-\beta_1 E_t}) + \rho\lambda_{1,t+1}(r_1 - 2r_1 X_{1,t}/K_1 - \alpha_1 X_{2,t} + e^{-\beta_1 E_t})$$
$$- \rho\lambda_{2,t+1}\alpha_2 X_{2,t} \ , \quad (13)$$

$$\lambda_{2,t} = p_2(1 - e^{-\beta_2 E_t}) + \rho\lambda_{2,t+1}(r_2 - 2r_2 X_{2,t}/K_2 - \alpha_2 X_{1,t} + e^{-\beta_2 E_t})$$
$$- \rho\lambda_{1,t+1}\alpha_1 X_{1,t} \quad (14)$$

where $\lambda_{i,t}$ is the multiplier or shadow price associated with an additional unit of the ith species (in the water) in period t. At a stationary state with positive stocks and effort, the bioeconomic optimum is defined by

$$X_1 = \frac{r_2(\alpha_1 K_2 - r_1)K_1 - \alpha_1 K_1 K_2(1 - e^{-\beta_2 E}) + r_2 K_1(1 - e^{-\beta_1 E})}{(\alpha_1\alpha_2 K_1 K_2 - r_1 r_2)} \ ,$$

$$X_2 = \frac{r_1(\alpha_2 K_1 - r_2)K_2 - \alpha_2 K_1 K_2(1 - e^{-\beta_1 E}) + r_1 K_2(1 - e^{-\beta_2 E})}{(\alpha_1 \alpha_2 K_1 K_2 - r_1 r_2)} \tag{16}$$

$$\lambda_1 = \frac{p_1(1-e^{-\beta_1 E})[1-\rho(r_2-2r_2 X_2/K_2-\alpha_2 X_1+e^{-\beta_2 E})]-\rho\alpha_2 X_2 p_2(1-e^{-\beta_2 E})}{[1-\rho(r_1-2r_1 X_1/K_1-\alpha_1 X_2+e^{-\beta_1 E})][1-\rho(r_2-2r_2 X_2/K_2-\alpha_2 X_1+e^{-\beta_2 E})]-\rho^2\alpha_1\alpha_2 X_1 X_2} \tag{17}$$

$$\lambda_2 = \frac{p_2(1-e^{-\beta_2 E})[1-\rho(r_1-2r_1 X_1/K_1-\alpha_1 X_2+e^{-\beta_1 E})]-\rho\alpha_1 X_1 p_1(1-e^{-\beta_1 E})}{[1-\rho(r_1-2r_1 X_1/K_1-\alpha_1 X_2+e^{-\beta_1 E})][1-\rho(r_2-2r_2 X_2/K_2-\alpha_2 X_1+e^{-\beta_2 E})]-\rho^2\alpha_1\alpha_2 X_1 X_2} \tag{18}$$

$$(p_1 - \rho\lambda_1)X_1\beta_1 e^{-\beta_1 E} + (p_2 - \rho\lambda_2)X_2\beta_2 e^{-\beta_2 E} - c = 0 \quad . \tag{19}$$

Equations (15) and (16) defining the optimal stock levels are the same as the first two equations in (10) defining the open access equilibrium. This is to be expected because the bio-technical relationship between stocks and effort is not affected by the property rights status of the multispecies fishery. What differs is the criterion for setting the level of fishing effort. The sole owner or manager takes into account user costs represented by $\rho\lambda_1$ and $\rho\lambda_2$ in equation (19). Given the previous parameters and a rate of discount δ, the sole owner or management agency equipped with a micro computer could: (a) select an initial effort level E, (b) solve equations (15) and (16) for X_1 and X_2, (c) substitute E and the resultant X_1 and X_2 into (17) and (18) to obtain λ_1 and λ_2, (d) substitute E, X_1, X_2, λ_1, and λ_2 into the LHS of (19) to determine its sign, and (e) adjust effort according to the rule:

$$\text{If the sign of the LHS of (19) is } \begin{cases} \text{positive, } \Delta E > 0 \\ \text{zero, } \Delta E = 0 \\ \text{negative, } \Delta E < 0 \quad . \end{cases} \tag{20}$$

Thus, the LHS of (19) plays the role of $\pi(\cdot)$ in the last equation in (5). The local stability of the bioeconomic optimum could be determined by calculating the Eigen values of (4) evaluated at (X_1^*, X_2^*; E*). The presumed stability of the pristine system, coupled with the strict concavity of the production functions, imply that the solution to (15) - (19) will be a strict global maximum.

To illustrate the above methods of analysis, consider the numerical example where

$$r_1 = 1.300 \qquad K_1 = 220 \qquad \alpha_1 = 0.002 \qquad \beta_1 = 0.016$$

$$r_2 = 1.500 \qquad K_2 = 180 \qquad \alpha_2 = 0.001 \qquad \beta_2 = 0.014 \qquad (21)$$

$$p_1 = 1,300 \qquad p_2 = 1,200 \qquad c = 3,000 \qquad \delta = 0.100$$

The pristine, open access, and bioeconomic equilibria and the algorithm used to derive them are given in Table 2. At the pristine equilibrium, interspecific competition reduces species' stocks below their respective K-values such that $X_1^p \simeq 165.811$ and $X_2^p \simeq 160.103$. The open access equilibrium is associated with an effort of $E^0 \simeq 40.900$ and stocks $X_1^0 \simeq 99.553$ and $X_2^0 \simeq 115.740$. Net revenue is very sensitive to changes in effort, and to drive net revenues closer to zero would involve variations in E on the order of 10^{-6}. The interactive search, done via the solution algorithm, was terminated at N = 0.004785. The characteristic roots of the open access equilibrium were 0.7758, and $0.2164 \pm 0.8967i$ corresponding to case 4 in Table 1.

Under management a bioeconomic optimum was obtained at $E^* \simeq 18.657$, $X_1^* = 130.019$, and $X_2^* = 136.813$. Effort was more than halved in comparison to open access. The LHS of equation (19) was designated M (see statement 180 in the solution algorithm). It too was sensitive to slight variations

TABLE 2

THE PRISTINE, OPEN ACCESS, AND BIOECONOMIC EQUILIBRIA AND

THE ASSOCIATED SOLUTION ALGORITHM

PRISTINE	OPEN ACCESS	BIOECONOMIC
E= 0	E= 40.90048	E= 18.65725
X1= 165.8114176	X1= 99.55336273	X1= 130.0187718
X2= 160.1026299	X2= 115.7399795	X2= 136.8128342
Y1= 0	Y1= 47.81051941	Y1= 33.55545121
Y2= 0	Y2= 50.45647463	Y2= 31.44975125
N= 0	N= 0.004785	N= 25390.03808
M= 3138.601668	M=-1835.345717	M= 0.00353169

SOLUTION ALGORITHM USED TO CALCULATE

THE PRISTINE, OPEN ACCESS, AND BIOECONOMIC EQUILIBRIA

```
10:DATA 1.3,1.5,2
   20,180,0.002,0
   .001,0.016,0.0
   14
20:READ R1,R2,K1,
   K2,A1,A2,B1,B2
30:DATA 1300,1200
   ,3000,0.1
40:READ P1,P2,C,D
50:INPUT "E=";E
60:R=1/(1+D)
70:Z0=A1*A2*K1*K2
   -R1*R2
80:Z1=1-EXP (-B1*
   E)
90:Z2=1-EXP (-B2*
   E)
100:X1=(R2*(A1*K2-
    R1)*K1-A1*K1*K
    2*Z2+R2*K1*Z1)
    /Z0
110:X2=(R1*(A2*K1-
    R2)*K2-A2*K1*K
    2*Z1+R1*K2*Z2)
    /Z0
120:Z3=1-R*(R1-2*R
    1*X1/K1-A1*X2+
    EXP (-B1*E))
130:Z4=1-R*(R2-2*R
    2*X2/K2-A2*X1+
    EXP (-B2*E))
140:Z5=(R^2)*A1*A2
    *X1*X2
150:L1=(P1*Z1*Z4-R
    *A2*X2*P2*Z2)/
    (Z3*Z4-Z5)
160:L2=(P2*Z2*Z3-R
    *A1*X1*P1*Z1)/
    (Z3*Z4-Z5)
170:N=P1*X1*Z1+P2*
    X2*Z2-C*E
180:M=(P1-R*L1)*X1
    *B1*EXP (-B1*E
    )+(P2-R*L2)*X2
    *B2*EXP (-B2*E
    )-C
190:Y1=X1*Z1
200:Y2=X2*Z2
210:LPRINT "E=";E
220:LPRINT "X1=";X
230:LPRINT "X2=";X
    2
240:LPRINT "Y1=";Y
    1
250:LPRINT "Y2=";Y
    2
260:LPRINT "N=";N
270:LPRINT "M=";M
280:INPUT "CHANGE?
     YES=1,NO=0.";
    W
290:IF W=1GOTO 50
300:LPRINT "BYE-BY
    E"
310:END
```

in E, and the interactive convergence process was terminated at M = 0.00353169. For $\eta = 0.001$, characteristic roots at the bioeconomic optimum were 0.9208, 0.3364, and -0.0934 corresponding to case 1 in Table 1.

Table 3 contains a phase-plane diagram showing the movement from the pristine equilibrium to the open access equilibrium when $E_0 = 1$ and $\eta = 0.001$. After an initial period of irregular movement, the point $(X_{1,t}, X_{2,t})$ followed a convergent spiral. After 101 periods, effort had moved to $E_{101} \simeq 40.729$, and the stocks of species one and two had been reduced to 95.727 and 111.191 respectively. For $\eta \geqslant 0.022$, however, the system failed to converge to the open access equilibrium. Thus, it would appear that effort must adjust "slowly" if it is not to destabilize the system. This result was also noted by May et al. (1979) in their analysis of multispecies systems in the Southern Ocean.

The optimal approach from either the pristine or open access equilibrium to the bioeconomic optimum will be asymptotic. The authors were unable to derive closed form solutions for the infinite-horizon problem. Alternatively, if one knew the values for effort and stocks at the bioeconomic optimum, one could specify a finite horizon problem with the optimal stocks as terminal values. The optimal approach might be determined for numerical examples using the discrete-time maximum principle or dynamic programming. While the most rapid (or minimum time) approach paths will not be optimal, Clark (1976b) has found instances in problems with delayed recruitment where such paths were only slightly suboptimal.
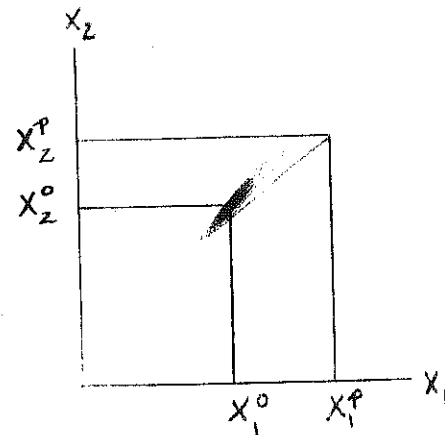
TABLE 3

PHASE-PLANE ALGORITHM AND DIAGRAM OF MOVEMENT FROM

THE PRISTINE EQUILIBRIUM TO THE OPEN ACCESS EQUILIBRIUM

```
10:DATA 1.3,1.5,2       140:N(T)=P1*X1(T)*
   20,180,0.002,0          (1-EXP (-B1*E(
   .001,0.016,0.0          T)))+P2*X2(T)*
   14                      (1-EXP (-B2*E(
20:READ R1,R2,K1,         T)))-C*E(T)
   K2,A1,A2,B1,B2     150:E(T+1)=E(T)+V*
30:DATA 1300,1200          N(T)
   ,3000,0.001        160:NEXT T
40:READ P1,P2,C,V      170:GRAPH
50:DIM X1(101),X2      180:COLOR 0
   (101),N(101),E      190:LINE (0,0)-(0,
   (101)                  220)
60:INPUT "E(0)=";     200:LINE (0,0)-(22
   E(0)                   0,0)
70:INPUT "X1(0)="     210:COLOR 1
   ;X1(0)             220:LINE (X1(0),0)
80:INPUT "X2(0)="        -(X1(0),X2(0))
   ;X2(0)             230:LINE (0,X2(0))
90:INPUT "X10=";X        -(X1(0),X2(0))
   10                 240:COLOR 2
100:INPUT "X20=";X     250:LINE (X10,0)-(
   20                     X10,X20)
110:FOR T=0TO 100     260:LINE (0,X20)-(
120:X1(T+1)=X1(T)*        X10,X20)
   (R1*(1-X1(T)/K     270:COLOR 3
   1)-A1*X2(T)+       280:FOR T=0TO 100
   EXP (-B1*E(T))     290:LINE (X1(T),X2
   )                     (T))-(X1(T+1),
130:X2(T+1)=X2(T)*        X2(T+1))
   (R2*(1-X2(T)/K     300:NEXT T
   2)-A2*X1(T)+       310:COLOR 0
   EXP (-B2*E(T))     320:END
   )
```


E(0)
                                    1

X1(0)
              165.8114176

X2(0)
              160.1026299

X10
              99.5336273

X20
              115.7399795

E(101)
              40.72896348

X1(101)
              95.72697675

X2(101)
              111.1909488

IV.   Conclusions and Policy Implications

The joint harvest of two or more interacting species is a problem of both theoretical and practical interest. While the problem is complex, and this paper has by no means provided complete analysis, it would appear that the problem may be more tractable than considered initially. We have attempted to cast the problem in a general way amenable to fixed-point and stability analysis. A particular specification employing a Gaussian model of interspecific competition and transcendental joint production was evaluated by analytical and numerical techniques. Two simple algorithms were developed. The first allowed for the interactive determination of the pristine, open access, and bioeconomic equilibria. The second plotted the movement in the X_1-X_2 phase space of open access dynamics from an initial state, such as the pristine equilibrium. The optimal approach to the bioeconomic optimum from either the pristine or open access equilibria will be asymptotic, and a closed form solution for the infinite-horizon problem could not be determined. While not optimal, minimal-time (or most-rapid) approach paths provide a tractable approach strategy that may be near optimal. Alternatively, the bioeconomic optimum might be calculated using the algorithm in Table 2, and X_1^*, X_2^* might be specified as terminal conditions for a finite horizon problem.

Within the basic (single-species) bioeconomic model, economists have recommended landings taxes, transferrable quotas, or both as management policies which would guide the system toward, and maintain it at, the bioeconomic optimum. When jointly harvesting two competing species, catch related management policies run into the difficulty that the tax

or aggregate quota for one species may not be consistent with the tax

or quota on the competing species. While effort is notoriously difficult

to measure and control, it would seem the more logical management instru-

ment when species are jointly harvested. Direct regulation of effort or

a tax on effort (thereby increasing the per unit cost of effort to $c + \tau$)

would in theory permit a management agency to achieve the optimal level

of effort implied by equation (19). In designing a dynamic tax policy,

an equilibrium tax τ^* might be calculated which would choke-off effort

until stocks reach X_1^*, X_2^*, or a dynamic tax policy might be determined

for a finite-horizon problem employing X_1^* and X_2^* as terminal conditions.

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