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DECOMPOSING THE INDUCED INCOME CHANGES IN INPUT-OUTPUT MODELS

by

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Decomposing the Induced Income Changes in Input-Output Models

Input-output (I-O) models continue to be used by many applied economists to estimate impacts on output, employment and income in an economy stemming from economic development, public investment or other change in public policy (e.g. Bills and Barr; Schreiner, Muncrief and Davis; McKusic). Other applications have incorporated the basic I-O structure into an optimizing framework or as part of a dynamic interindustry model (e.g. Penn, McCarl, Brink and Irwin; Tung, MacMillan and Framingham).

One topic frequently discussed in empirical work or in the classroom is the reduction in the secondary economic impacts due to "leakage" from an open economy. For regional models, much of the discussion focuses on "leakage" due to imports, but in general, these economic impacts are affected by any portion of a spending stream that goes to any exogenous final payments sector rather than being respent locally. Despite its importance, much of the discussion has focused on the qualitative differences in the impacts resulting from "leakage" or attempts to estimate its magnitude for a particular empirical application (e.g. Little and Doeksen). One exception is in Mustafa where a general expression for leakage is defined as "the direct plus indirect import requirement for different sectors" (p. 681).

The purpose of this note is to develop analytically the relationship between two income multipliers in an economy: the first assuming that household final demand is exogenous (type I) and the second assuming that the economy is "closed" with respect to the household sector (type II). Numerous empirical studies, as well as the derivations by Sandoval and

Bradley and Gander, have demonstrated that for a given I-O system, type II multipliers for all sectors are a constant proportion different from the type I multipliers in the corresponding sectors. What is not clear is how this difference is related to the marginal propensity to consume locally and to leakages to imports or other final payments sectors. This latter relationship should be helpful to researchers in understanding how sensitive empirical results are to particular I-O coefficients. It is also a convenient and rigorous method by which instructors can help students understand the subtle interactions among production, consumption and leakages in an I-O model.

The note begins with a characterization of the I-O model and definitions of appropriate terms. A second section outlines briefly Bradley and Gander's derivation of the relationship between the two multipliers, while the third section reinterprets it in terms of local propensities to consume and leakages to final payments. The final section contains a numerical illustration.

The I-O Model

One can begin to define the I-O model from the transactions in Table 1. The first n sectors are the usual production sectors for the I-O model; sector n+1 is the household sector, while D is exogenous final demand and Q is the payment to primary factors other than labor and imports.

Assuming that each production sector has fixed coefficient technology and is subject to constant returns to scale, one can define B (the matrix of direct input coefficients), w (the vector of direct payments to labor and k (the vector of direct household consumption coefficients) by:

Table 1: Interindustry Transactions Table

	1	2	. . .	n	n+1	D	Total
1	Y_{11}	Y_{12}	. . .	Y_{1n}	$Y_{1,n+1}$	D_1	Y_1
2	Y_{21}	Y_{22}	. . .	Y_{2n}	$Y_{2,n+1}$	D_2	Y_2
⋮	⋮	⋮		⋮	⋮	⋮	⋮
n	Y_{n1}	Y_{n2}	. . .	Y_{nn}	$Y_{n,n+1}$	D_n	Y_n
n+1	$Y_{n+1,1}$	$Y_{n+1,2}$	. . .	$Y_{n+1,n}$	$Y_{n+1,n+1}$	D_{n+1}	Y_{n+1}
Q	Q_1	Q_2	. . .	Q_n	Q_{n+1}		Q
Total	Y_1	Y_2	. . .	Y_n	Y_{n+1}	D	

Note: The elements of the table represent the dollar transactions in an economy from the sector in the row to the sector in the column. Initially, there are n producing sectors, sector n+1 is household consumption (column) and payments to households (row). Other final demand has been aggregated into D and other final exogenous payments is given in Q. Later in the paper Q is disaggregated into its components such as value added other than labor and imports into the region. If one is willing to assume that each production sector has fixed input coefficients and is subject to constant returns to scale, the economy can be described analytically by (for households exogenous)

$$Y - BY = F$$

where $Y' = (Y_1, \ldots, Y_n)$; $B = Y_{ij}/Y_j$; and

$$F' = (Y_{1,n+1} + D_1, \ldots, Y_{n,n+1} + D_n).$$

That is, output by sector less intermediate inputs is equal to final demand. Alternatively for a given vector F, one can solve for the required output by:

$$Y = [I-B]^{-1} F.$$

See Yan, Chenery and Clark or Miernyk for details.

$$(1) \quad B = \begin{bmatrix} Y_{11}/Y_1 & \cdots & Y_{1n}/Y_n \\ \vdots & & \vdots \\ Y_{n1}/Y_1 & \cdots & Y_{nn}/Y_n \end{bmatrix} = b_{ij} \; ;$$

$$(2) \quad w = \begin{bmatrix} \frac{Y_{n+1,1}}{Y_1}, & \frac{Y_{n+1,2}}{Y_2}, \ldots, \frac{Y_{n+1,n}}{Y_n} \end{bmatrix} = (w_1, w_2, \ldots, w_n); \text{ and}$$

$$(3) \quad k = \frac{1}{Y_{n+1}} \begin{bmatrix} Y_{1,n+1} \\ \vdots \\ Y_{n,n+1} \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix}$$

Assuming that household final demand is exogenous, the vector of type I income multipliers is given by:

$$(4) \quad M^I = w(I-B)^{-1} \, w^*,$$

where

$$w^* = \begin{bmatrix} 1/w_1 & & 0 \\ & \ddots & \\ 0 & & 1/w_n \end{bmatrix}$$

When households are endogenous, the matrix of direct coefficients is

$$(5) \quad \overline{B} = \begin{bmatrix} B & k \\ w & k_{n+1} \end{bmatrix}$$

and

$$(6) \quad (I-\overline{B}) = \begin{bmatrix} I-B & -k \\ -w & G \end{bmatrix} = H;$$

where these are partitioned matrices of dimension (n+1 x n+1) and

$G = (1-k_{n+1})$ and $k_{n+1} = Y_{n+1,n+1}/Y_{n+1}$.

If the elements of H^{-1} are h_{ij}, the vector of type II income multipliers is

$$(7)\quad M^{II} = [h_{n+1,1},\ldots,h_{n+1,n}]\ [w^{*}].$$

Proportionality of the Multipliers

Bradley and Gander have established that

$$(8)\quad \frac{M_j^{II}}{M_j^{I}} = \theta.$$

Because the denominators of (4) and (7) are the same, this is equivalent to proving that

$$(9)\quad h_{n+1,j} \Big/ \sum_{i=1}^{n} w_i e_{ij} = \theta \qquad \text{for } (j=1,\ldots,n);$$

where e_{ij} is the i,jth element of $(I-B)^{-1}$.

The denominator of (9) is the direct plus indirect income per dollar of final demand in sector j but the numerator of (9) also includes the income induced by additional consumption per dollar of final demand in sector j. Thus, one would also expect that $\theta \geq 1$.

Letting the partitioned inverse be

$$(10)\quad H^{-1} = \begin{bmatrix} P & Q \\ R & S \end{bmatrix}$$

one knows that

$$(11)\quad H\,H^{-1} = \begin{bmatrix} (I-B) & -k \\ -w & G \end{bmatrix} \begin{bmatrix} P & Q \\ R & S \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

From the partitioned products

(12) $(I-B)P - kR = I$;

(13) $(I-B)Q - kS = 0$;

(14) $-wP + GR = 0$; and

(15) $-wQ + GS = I$.

From (14),

(16) $GR = wP$.

From (12), one can solve for P and substitute into (16) to obtain

(17) $P = (I-B)^{-1} [I+kR]$

(18) $GR = w(I-B)^{-1}[I+kR] = w(I-B)^{-1} + w(I-B)^{-1}kR$.

Rearranging

(19) $GR - w(I-B)^{-1}kR = w(I-B)^{-1}$; or

(20) $[G - w(I-B)^{-1}k]R = w(I-B)^{-1}$.

From (4), (7) and (11), one knows that R contains the numerators of the type II multipliers and $w(I-B)^{-1}$ are the numerators of the type I income multipliers. Recognizing that

(21) $G = (1-k_{n+1})$,

one can write

(22) $(G - w(I-B)^{-1}k) = [1 - (k_{n+1}+w(I-B)^{-1}k)]$.

Substituting (22) into (20) and rearranging, one has

(23) $R = [\theta]w(I-B)^{-1}$

where $\theta = \left[\frac{1}{1-\lambda}\right]$ and $\lambda = k_{n+1}+w(I-B)^{-1}k$ Q.E.D.

<u>An Interpretation of θ and λ</u>

Although equation (23) establishes the fact that type II multipliers are a constant multiple of the type I multiplier, as currently written, it is not possible to establish that $\theta \geq 1$, nor is the relationship between the magnitude of exogenous final payments other than labor and θ entirely obvious.

From Table 1, if households consume some fraction of local production and if some local labor is employed by every sector, then

$$(24)\quad 0 < w_i < 1;\ 0 \leq b_{ij} < 1;\quad 0 \leq k_j < 1;\ \sum_{i=1}^{n} b_{ij} < 1;\ \sum_{j=1}^{n} k_j < 1.$$

With this information, one can establish $0 \leq \lambda \leq 1$ (and therefore $\theta \geq 1$) by redefining the direct income (payments to labor) coefficients in (2) in an alternative, but equivalent, fashion:

$$(25)\quad w_i = \frac{Y_i - Q_i - 1'\tilde{y}_i}{Y_i} = 1 - q_i - 1'\tilde{b}_i;\ \text{or}$$

$$(26)\quad w = 1' - \tilde{q}' - 1'B = 1'\ [I-B] - \tilde{q};$$

where $\tilde{y}_i$ is ith column of the transactions matrix; $1'$ is $[1, 1, 1, \ldots, 1]$; $\tilde{b}_i$ is ith column of B; $q_i = Q_i/Y_i$; and $\tilde{q}' = [q_1, q_2, \ldots, q_n]$.

Substituting (26) into the expression for λ:

$$(27)\ k_{n+1} + w(I-B)^{-1}k = k_{n+1} + \{1'(I-B) - \tilde{q}'\}(I-B)^{-1}k = k_{n+1} + 1'k - \tilde{q}'(I-B)^{-1}k.$$

From equation (24) one knows that[1]

$$(28)\ w > 0;\quad (I-B)^{-1} > 0;\quad k > 0;\ \tilde{q}' \geq 0;\ k_{n+1} \geq 0;$$

and the left-hand side of (27) is non-negative. However, the right-hand side of (27) is the difference between two non-negative numbers. The first term, $k_{n+1} + 1'k$, is equal to the marginal propensity to consume (MPC) locally and logically can be assumed to be within the closed interval between zero and unity. Because the left-hand side of (27) is non-negative, $k_{n+1} + 1'k \geq \tilde{q}'(I-B)^{-1}k$ and

$$(29)\quad 0 \leq \lambda \leq \text{MPC} \leq 1.$$

This analysis proves that under all conditions normally found in empirical I-O models, λ is between zero and unity and θ is greater than unity. One limiting, but not realistic, case is where the propensity to consume locally is zero (i.e., $k_{n+1} = 0$ and $k = 0$). Thus, $\lambda = 0$ and $\theta = 1$; there is no induced income effect when the model is closed with respect to households and the two income multipliers are equal.

The second limiting case is more important to understanding how "leakages" from the region and to final payments other than households affect the induced income impact. When exogenous final payments to sectors other than labor are zero, $\tilde{q}' = 0$ and $\lambda = MPC$. This implies that the induced income effect is at its maximum. There is no leakage out of the economy and the difference between the two multipliers is 1/(1-MPC), a value equivalent to the simple Keynesian consumption multiplier for a closed economy.

The intermediate situation is the most relevant for empirical analysis. Here, $\tilde{q}' \neq 0$ and $k \neq 0$ and λ falls below MPC and θ is less than the Keynesian multiplier. This "leakage" can be examined in greater detail. Because the difference between the type I and type II multipliers is constant, the absolute size of the induced income effect is a function of θ, as well as the magnitude of the type I multipliers themselves (M^I). That is, for a given value of θ, the induced income effect varies directly with M^I (e.g., induced income = $M^{II} - M^I = \theta M^I - M^I = (\theta-1)M^I$). One can also determine quite easily the change in θ as λ changes in response to increases or decreases in the leakage to primary factors $L = \tilde{q}'(I-B)^{-1}k$:

$$(30) \quad \frac{\partial\theta}{\partial\lambda} \quad \frac{\partial\lambda}{\partial L} = \frac{\partial(1-\lambda)^{-1}}{\partial\lambda} (-1) = -(1-\lambda)^{-2} \ .$$

Thus, as L increases, the difference between the type I and the type II multipliers falls and the rate at which it falls varies directly with λ.

More importantly, one must also understand the individual componets of L and how each of them affects the size of the leakage, λ, and θ. To do this, it is useful to recall the individual components of $\tilde{q}$. As defined in Table 1, q is the vector of direct payments to all exogenous final payments sectors other than labor. It could include value added other than labor, indirect business taxes and imports of labor or intermediate inputs from outside the region. In examining these final payments, it makes sense to distinguish three separate categories a) direct payments to imported labor per dollar of sales by sector $i = \alpha_i$; b) direct value of intermediate inputs imported per dollar of sales by sector $i = \gamma_i$; c) direct value of all other final payments per dollar of output in sector $i = v_i = q_i - \alpha_i - \gamma_i$. The reason for making this distinction has to do with the way in which changes in these items affect the size of L.

Because matrix operations are distributive by addition, one can segment L into three components.

(31) $L = \alpha'(I-B)^{-1}k + \gamma'(I-B)^{-1}k + v'(I-B)^{-1}k$

where

$$\alpha' = (\alpha_1,\ldots,\alpha_n);\ \gamma' = (\gamma_1,\ldots,\gamma_n);\ v' = (v_1,\ldots,v_n).$$

Although each of these components has the same algebraic form, changes in each of the direct payments vectors are likely to affect L differently. As an example, as long as the technology in an economy remains unchanged, the vector v remains constant. However, the elements of the vectors α and γ are affected by changes in the economy other than technology and in the region's dependence on imports. The simplest change in these vectors is where imports into the economy include some payments to labor living outside the region. If for some reason, these payments were shifted to the regional workforce, elements of α_i would fall (e.g. from $\alpha_i^o > \alpha_i^-$ for at least one i), but there would be an increase in the corresponding elements of w. Because w appears in the numerators and the denominators of both multipliers, it is difficult to determine the effects on their absolute sizes, but from equation (31), one knows that:

(32) $L^- < L^o; \lambda^- > \lambda^o; \theta^- > \theta^o; \alpha^{-'}(I-B)^{-1}k < \alpha^{o'}(I-B)^{-1}k;$

The leakage in the economy is reduced and the relative difference between the two multipliers increases.

Perhaps the most interesting analysis concerns the distribution of intermediate input purchases. As the proportion of intermediate inputs falls, <u>ceteris paribus</u> elements of γ would fall (e.g. assume from $\gamma_i^o > \gamma_i^+$ for at least one i). These inputs, if now purchased locally, are distributed among some or all elements of the corresponding column of B. The new Leontief matrix is

(33) $[I - (A+B)]^{-1}$

where the elements of A, $0 \leq a_{ij} \leq 1$ and $\sum_{i=1}^{n} a_{ij} = \gamma_j^o - \gamma_j^+$. By construction, $0 \leq a_{ij} + b_{ij} < 1$ and $\sum_{i=1}^{n} (a_{ij} + b_{ij}) < 1$. One, but perhaps not the most obvious, conclusion of this change is obtained by examining equations (33) and (23). From equation (33) and the logic in footnote 1, the individual elements of the new Leontief matrix $[I-(A+B)]^{-1}$ are at least as large as in the original inverse $[I-B]^{-1}$ and some are larger. Substituting the new inverse into the expression for λ in equation (23) one knows that λ increases and θ increases. Because λ increases and the propensity to consume locally does not change, one can also conclude that the leakage (as measured by equation (31)) must decline. In summary, one knows that

(34) $L^+ < L^o;\ \lambda^+ > \lambda^o;\ \theta^+ > \theta^o;$

and because α' and v' do not change

(35) $\alpha'[I-(A+B)]^{-1}k > \alpha'(I-B)^{-1}k;$

(36) $v'[I-(A+B)]^{-1}k > v'(I-B)^{-1}k;$

(37) $\gamma^{+'}[I-(A+B)]^{-1}k < \gamma^{o'}(I-B)^{-1}k.$

The relative increase in the elements depends on how these new purchases are distributed locally and the net contribution of the three terms to the leakage is an empirical question. All that one knows for sure is that the contribution of payments to imported labor and other value added to the remaining leakage increases. The absolute, as well as the relative, contribution of imported intermediate inputs to the leakage falls.

Empirical Example

The most effective way to illustrate these ideas is through a small empirical example. The transactions table is given in Table 2. The direct requirements data (matrix B and the vectors α, γ, and v) are in Table 3, as are the two relevant Leontief inverses.[2] From the data in these tables, one can apply equations (4) and (7) to obtain the type I and type II multipliers for the four production sectors:

Sector	Type I	Type II
1	1.249851	1.688456
2	1.342961	1.814242
3	1.248147	1.686155
4	1.194655	1.613890

Table 2. Transactions Table for Empirical Example

	Output / Inputs	Agriculture	Manufacturing	Transportation	Services	Households	Exports	Government	Total Sales
Processing Sectors	Agriculture	34	290	0	0	7	137	1	469
	Manufacturing	25	1,134	5	201	607	12,303	37	14,312
	Transportation	6	304	54	105	22	111	8	610
	Services	48	962	71	877	2,558	1,539	842	6,897
Final Payments	Households w	208	3,242	252	2,697	869	1,203	244	8,715
	Imports labor	40	700	40	300	0			
	Imports other	37	5,012	43	1,048	2,539			
	Other Value Added v	71	2,668	145	1,669	2,113			
	Total Purchases	469	14,312	610	6,897	8,715			

Table 3. Direct Requirements Data for I-O Example

$$B = \begin{bmatrix} 0.072495 & 0.020263 & 0.0 & 0.0 \\ 0.053305 & 0.079234 & 0.008197 & 0.029143 \\ 0.012793 & 0.021241 & 0.088525 & 0.015224 \\ 0.102345 & 0.067216 & 0.116393 & 0.127157 \end{bmatrix}; \quad \begin{bmatrix} w \\ \alpha \\ \gamma \\ v \end{bmatrix} = \begin{bmatrix} 0.443497 & 0.226523 & 0.413115 & 0.391040 \\ 0.085288 & 0.048910 & 0.065574 & 0.043497 \\ 0.078891 & 0.350196 & 0.070492 & 0.151950 \\ 0.151386 & 0.186417 & 0.237705 & 0.241989 \end{bmatrix}$$

$$(I-B)^{-1} = \begin{bmatrix} 1.079623 & 0.023824 & 0.000317 & 0.000801 \\ 0.066920 & 1.090536 & 0.014489 & 0.036664 \\ 0.018955 & 0.027258 & 1.099934 & 0.020095 \\ 0.134273 & 0.090409 & 0.147829 & 1.151278 \end{bmatrix}; \quad (I-\overline{B})^{-1} = \begin{bmatrix} 1.081691 & 0.024960 & 0.002241 & 0.002544 & 0.003732 \\ 0.131924 & 1.126211 & 0.074957 & 0.091448 & 0.117271 \\ 0.026884 & 0.031610 & 1.107309 & 0.026777 & 0.014304 \\ 0.392391 & 0.232069 & 0.387937 & 1.368815 & 0.465661 \\ 0.748825 & 0.410968 & 0.696575 & 0.631095 & 1.350926 \end{bmatrix}$$

Note: The sectors are in the same order as in Table 3. See equations (1), (2), (5), (6) and (31) for definitions of variables. The matrix inversions and other calculations were performed by a program documented by Lassiter and Boisvert.

It is easy to verify that each type II multiplier is a constant proportion higher than the type I multipliers; and $\theta = 1.350926$. This means that

$$\lambda = \frac{\theta - 1}{\theta} = 0.259767,$$

which is considerably less than MPC = 0.466208. Consequently, θ is less than 1.874839, the value of θ if the economy were closed and λ were equal to MPC. If one argues that this is the maximum value of θ, then the actual value of θ is only 72.1% of the maximum. The difference is due to leakage and the magnitude of the "leakage coefficient" is L = 0.206442. According to equation (31) this leakage can be decomposed into three components, one due to labor imports, one due to intermediate factor imports and one due to other value added. These three components are 0.020170; 0.083741 and 0.102531, respectively. Thus, in this example, 49.7% of the difference between the maximum and actual θ is due to other value added; 9.8% is due to imports of labor; and 40.6% is due to imports of intermediate inputs.

Summary

The purpose of this short note is to establish formally the relationship between type I and type II income multipliers in an input-output system and relate the relationship directly to the marginal propensity to consume locally and leakages from the regional economy. It is shown that the maximum difference between the two multipliers is equal to 1/(1-MPC), the simple macro multiplier. This maximum "induced" income effect is attained under conditions where all spending streams are respent locally and there is no leakage from the economy to final payments, other than local labor nor to regional imports of intermediate goods.

As leakage occurs, the "induced" income effect falls and the difference between the type I and type II multiplier is less than 1/(1-MPC). However, as the empirical example illustrated, it is quite easy to determine the relative importance of each type of leakage in reducing the size of the induced income effect. Other comparative static analysis is also possible to determine the change in this induced income effect as the magnitude of the leakages change.

It is anticipated that these results will be useful in the classroom to explain the interactions in an input-output system as well as in research where it is necessary to understand how different types of leakage affect the induced income effect. This is particularly true where the cost improving the quality of data on leakages from the region must be balanced against its impact on the empirical results.

Footnotes

[1]This stems from the fact that if $0 \leq b_{ij} < 1$ and $\sum_{i=1}^{n} b_{ij} < 1$, then as a positive integer, $m \to \infty$,

$$B^m \to 0 \text{ and } I + B + B^2 + B^3 + \ldots + B^m \to [I-B]^{-1} \geq 0$$

(see Waugh for details). This means that a sufficient condition for $[I-B]^{-1}$ to exist is that the economy be open. In the case where households are exogenous, payments to labor in each sector must be positive. In the case where the economy is closed with respect to households, payments to other final payments sectors such as other value added or imports must be positive. Neither of these assumptions is unrealistic in empirical applications.

[2]The data are reported to six decimals so that others can verify the calculations.

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