Abstract

This paper demonstrates how linear programming can be used to derive optimal values of the parameters in formulas allocating state aid to public schools in New York. Although disparities in expenditures and tax rates among districts are not easily reduced by formula modifications, the model identifies tradeoffs between distributions of tax burdens and educational resources.

A PROGRAMMING APPROACH TO FINANCING PUBLIC EDUCATION

by

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Given current efforts to shift financing of public services to the states and localities, primary and secondary education and other activities traditionally financed locally face increased competition for financial resources. For education, the situation is exacerbated in New York and other states where challenges to the existing financing arrangements remain unresolved. The reality of providing additional resources to poorer districts combined with political pressure not to reduce expenditures for richer districts, places a premium on budget allocation efficiency by government officials. There is no evidence to suggest that formulas for state aid to education are optimal with respect to reasonable objectives such as providing the necessary aid at minimum state cost, given expenditure levels and tax constraints, or maximizing the minimum total expenditures by any school district.

This paper develops a linear programming (LP) model to evaluate strategies for modifying public school financing arrangements in light of tax rate limits and expenditures that may be consistent with court decisions mandating equal educational opportunity. The model is applied in New York where a successful court challenge to the school financing system for not providing equal opportunity was still under appeal at the time the study was initiated.[1] Bruno used a similar programming approach

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[1]On June 23, 1982, the New York State Court of Appeals overturned lower court decisions in Levittown v. Nyquist which challenged the constitutionality of the state's current method of financing public education (New York Times). However, it is unlikely that the debate over equal educational opportunity in New York will subside. Efforts to modify the ways in which schools are financed will continue as they have for years.

for funding the California Junior College System, but the analysis was limited to a simple foundation-type program and allowed for few changes in the program's parameters. The model designed in the present study accommodates a two-tier system of state aid ratios and foundation levels. With only minor modifications, the model can be used to examine the implications of changing from the current two-tier aid formula to a district power equalization formula.

The LP Model of New York's School Financing System

The model of New York's school financing system deals only with operating expenditures. They account for over 70% of all spending and are financed at the district level through local property tax and state aid revenues (Colburn). As of 1978-79, state aid is provided through a two-tier aid formula, based on both spending levels and property wealth per aidable pupil unit (The University of the State of New York, 1978-79). Second tier aid is available to districts spending at a relatively high level, but with little wealth per aidable pupil unit. Districts (i=1...m) eligible to receive both tiers of aid can be represented by:

$$(1)\quad \frac{AID_{it}}{TAPU_{it}} = C_1\left(1 - \frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{AFVRTAPU_{t-2}}\,X_1\right) + C_2\left(1 - \frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{AFVRTAPU_{t-2}}\,X_2\right);$$

where $AID_{it}/TAPU_{it}$ is district i's operating aid per total aidable pupil unit (TAPU) in year t; C_1 and C_2 are first and second tier expenditure ceilings per TAPU; X_1 and X_2 are portions of C_1 and C_2 raised by local district of average wealth; $AFVRTAPU_{t-2}$ is the state average full value of property per RTAPU, lagged two years; and $FV_{i(t-2)}/RTAPU_{i(t-2)}$ is the ith district's property wealth per resident TAPU.[2] For districts of average wealth and spending at least $C_1 + C_2$ per TAPU, equation (1) suggests that the state aid per TAPU is equal to $(1-X_1)C_1 + (1-X_2)C_2$. As wealth increases, the state's share declines. For programming purposes, it is useful to rewrite (1) as:

$$(2)\quad \frac{AID_{it}}{TAPU_{it}} + C_1X_1\frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{AFVRTAPU_{t-2}} + C_2X_2\frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{AFVRTAPU_{t-2}} = C_1+C_2.$$

Other districts receive only tier 1 aid either because their operating expenses per TAPU are less than C_1 or their wealth per RTAPU is too high. Following the convention above, state aid for these districts $(i=m+1,...,n)$ can be written as:

$$(3)\quad AID_{it}/TAPU_{it} + C_1\left(\frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{AFVRTAPU_{t-2}}X_1\right) = C_1 .$$

The third class of districts is the richest in terms of property wealth per TAPU and receives a flat grant per TAPU. For flat grant districts $(i=n+1,...,r)$ state aid is given by a constant (FG):[3]

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[2]See the University of the State of New York, 1978-79 for procedures used in defining TAPU, the number of pupil units in a district on which state aid is based and on how RTAPU is calculated for measuring a district wealth.

[3]The analysis in this paper is conducted under the assumption that this simple two-tier aid formula is applied without modification across all school districts, while in reality provisions such as the 1980-81 flat grant taper formula, the save-harmless provisions, low income and high tax rate aid are also important to many districts (The State University of New York, 1979-80). These other provisions are ignored so that the implications for the basic aid structure spending alternatives designed to equalize educational opportunity can be isolated. By conducting the initial analysis in this way, one can understand better the potential magnitude of the adjustments required by the state and local districts in response to attempts at "equalizing" educational opportunity. Work is currently underway to incorporate these other provisions into a more comprehensive programming model.

(4) $AID_{it}/TAPU_{it} = FG$.

Given this existing wealth measure, the aid formula is characterized by three types of policy variables: the wealth levels by which school districts are grouped, the expenditure ceilings, C's, and the aid ratios, X's. In 1980-81 (t=0), the first expenditure ceiling, C_1, was set at \$1,600 per TAPU (The State University of New York, 1979-80). The second tier ceiling (C_2) was \$100 and the flat grant (FG) provision was set at \$360 per TAPU. In continuing to use the full value of property wealth as a measure of wealth, the statewide average ($AFVRTAPU_{t-2}$ = \$69,472) was used in the denominator of the aid formula. School districts with full value of property per RTAPU less than \$86,840 received tier 1 and tier 2 aid. Districts with wealth between \$86,840 and \$105,000 per RTAPU were eligible for only tier 1 aid.[4] Flat grant districts were those with full value of property per RTAPU in excess of \$105,000. State shares of expenditure ceiling levels for districts of average wealth remained at 49% for tier 1 and 20% for tier 2 aid (e.g. $X_1 = 0.51$ and $X_2 = 0.8$).

For programming purposes, a number of other constraints for each district is needed. To analyze equity issues from both student and taxpayer perspectives, one can specify minimum (MNE_{it}) and maximum (MXE_{it}) expenditure levels (E_{it}) per TAPU by district:

(5) $MNE_{it} \leq E_{it} \leq MXE_{it}$ (i=1...r);

and maximum (MXR_{it}) and minimum (MNR_{it}) tax rates (R_{it}):

(6) $MNR_{it} \leq R_{it} \leq MXR_{it}$ (i=1...r).

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[4]Because the flat grant taper is ignored and because these limits are calculated from data on unpublished state aid worksheets from the Department of Education, these limits differ slightly from the ones published in the State University of New York's 1980-81 supplement.

Total expenditures per TAPU (E_{it}) must equal the sum of local expenditures (L_{it}), plus state aid:

(7) $AID_{it}/TAPU_{it} + L_{it} = E_{it}$; and

local expenditures equal tax effort applied to full value of property:

(8) $-L_{it} + R_{it}\ (FV_{i(t-2)}/RTAPU_{i(t-2)}) = 0.$ [5]

Total state aid (TS_t) and total local expenditures (TL_t) are:

(9) $\sum_{i=1}^{r} AID_{it}/TAPU_{it}\ (TAPU_{it}) - TS_t = 0$; and

(10) $\sum_{i=1}^{r} L_{it}(TAPU_{it}) - TL_t = 0.$

Within this programming context, a number of alternative objective functions could be explored. To illustrate the usefulness of the model, minimizing total aid (TS_t) subject to specified maximum tax efforts and minimum expediture levels allows one to examine some important tradeoffs among tax effort, state costs, and expenditure levels. In all specifications, district aid levels and local tax rates are decision variables. In others, the levels of X_1 and X_2, are decision variables as well.[6]

Allowing X_1 and X_2 to vary introduces potential problems into the model because as X_2 increases, tier 2 aid can become negative and imply

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[5]For purposes of state aid ratios, it was appropriate to use the full value of property in year t-2 but because of lack of more recent data, these same tax rolls were also used to establish tax rates in the model.

[6]Because equations (2) and (3) are multiplicative functions of the C's and the X's, optimal levels of both policy variables cannot be obtained simultaneously using linear programming. In contrast to the experimentation above, one could also fix the X's and solve for optimal levels of the C's.

that districts receiving tier 1 and tier 2 aid can receive less aid than relatively richer districts presently receiving only tier 1 aid. This inconsistency is ruled out by placing an upper bound on X_2.[7]

If no upper bound is placed on X_2, District Power Equalization (DPE), as implemented in Wisconsin in 1973 (Johnson and Collins), can be examined for districts receiving both tiers of aid. Under such a scheme, the state effectively guarantees a fiscal capacity for each local district equal to a specified level of property valuation per pupil. Once the desired level of spending is determined, the district must impose the tax rate needed to raise this revenue if the guaranteed value of property per pupil were actually available. This tax rate is applied to the actual valuation of property in the district and the state pays the difference. In terms of the model, equation (2) would become:

$$(11)\quad \frac{AID_{it}}{TAPU_{it}} + C_1 \frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{GV_1} + C_2 \frac{FV_{i(t-2)}/RTAPU_{i(t-2)}}{GV_2} = C_1+C_2$$

where $GV_1 = AFVRTAPU_{t-2}/X_1$ and $GV_2 = AFVRTAPU_{t-2}/X_2$ are two guaranteed valuation levels set by the state. The importance of this flexibility is that negative tier 2 aid is equivalent to a transfer of funds from rich to poor districts. To study this alternative for New York would require respecifying the model to accommodate 2-tier aid for all districts and is beyond the scope of this paper.

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[7] The upper bound in this model is calculated by setting the tier 2 aid ratio equal to zero for the richest district receiving tier 2 aid, and solving for X_2. Using the aid ratio for the richest group receiving tier 2 aid, $X_2 = 0.87$.

Empirical Application

To illustrate the utility of the model, the state aid formula described above is examined first in conjunction with projected school expenditures for 1980-81. Projected base-run expenditures were obtained by inflating the 1979-80 approved operating expenses (AOE) by the consumer price index. Several adjustments to equalize expenditures and/or account for cost differentials among school districts or level up expenditures to some minimum level are also examined to understand the financial implications of providing equal or improved educational opportunity as defined narrowly by spending levels. (See Colburn for a discussion of the problems in defining educational opportunity.) The results from seven programming solutions are summarized in table 1 and reported in detail by Colburn.[8]

For the base run, total school operating expenditures would be approximately $7.7 billion. Expenditures per TAPU would be increased to almost $2,500. The range across the 79 school district aggregates used for programming purposes is from $1,600 to $4,036 per TAPU and state aid accounts for 35% of total expenditures. Average local property tax rates are nearly $22 per $1,000 of full value and range from just over $8 to just over $39. Because the aid ceilings (C_1 and C_2) and the aid ratios (X_1 and X_2) are fixed, the programming model effectively solves only for

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[8]New York's more than 700 school districts are aggregated for programming purposes into 79 groups, based on similarities in size and wealth. New York and the five largest upstate cities are treated separately (Colburn). Data to estimate the model's parameters were from unpublished state aid worksheets from the New York State Department of Education. The cost adjustments were made using the cost indices developed by Wendling which reflect the differential costs of hiring teachers and non-classroom professionals after controlling for personal and other characteristics.

Table 1. Alternative Assumptions in the Linear Programming Model Solutions of New York State

Number	Assumptions: (a) Expenditure/TAPU (b) Tiers 1 and 2 aid ratio (c) Tax rate ceiling	Solution: Total (millions)	Expenditures: Per TAPU: Average	Expenditures: Per TAPU: Range[a]	State Share: Per TAPU	State Share: % Total	Local Share: Per TAPU	Local Share: % Total	Tax Rates Per $1,000 Full Value: Average	Tax Rates Per $1,000 Full Value: Range[a]
1	(1979-80 AOE)(1.1125) $X_1 = 0.51$, $X_2 = 0.80$ $45/$1,000 FV	$7,692	$2,492	$1,600-$4,036	$ 883	35.4	$1,609	64.6	$21.96	$ 8.4-$39.4
2	$2,336[b] $X_1 = 0.51$, $X_2 = 0.80$ $45/$1,000 FV	7,209 (-6.3)[c]	2,336 (-6.3)	2,336- 2,336	883 (0)	37.8	1,453 (-9.7)	62.2	19.84 (-9.7)	7.4- 36.7
3	(1979-80 AOE)(1.1125)(CERI)[d] $X_1 = 0.51$, $X_2 = 0.80$ $45/$1,000 FV	7,766 (1.0)	2,516 (1.0)	1,406- 4,271	883 (0)	35.1	1,633 (1.0)	64.9	22.29 (1.5)	3.4- 42.9
4	Leveled-up to 65th percentile[e] $X_1 = 0.51$, $X_2 = 0.80$ $45/$1,000 FV	8,209 (6.7)	2,660 (6.7)	2,546- 4,036	883 (0)	33.2	1,777 (10.4)	66.8	24.26 (10.5)	11.9- 44.5
5	(1979-80 AOE)(1.1125) $X_1 = 0.37$, $X_2 = 0.0$ $35/$1,000 FV	7,692 (0)	2,492 (0)	1,600- 4,036	1,096 (24.1)	44.0	1,396 (-13.2)	56.0	19.06 (-13.2)	6.4- 35.0
6	$2,336 $X_1 = 0.49$, $X_2 = 0.0$ $35/$1,000 FV	7,209 (-6.3)	2,336 (-6.3)	2,336- 2,336	955 (8.2)	40.9	1,381 (-14.2)	59.1	18.85 (-14.2)	7.4- 35.0
7	(1979-80 AOE)(1.1125)(CERI) $X_1 = 0.22$, $X_2 = 0.0$ $35/$1,000 FV	7,766 (1.0)	2,516 (1.0)	1,406- 4,271	1,279 (44.8)	50.8	1,237 (-23.1)	49.2	16.89 (-23.1)	0.0- 35.0

Note: In all programming runs, combined state aid (TS_t) to all districts is being minimized, subject to constraints 2-10. See Colburn for details.

[a]Range across the 79 district aggregates.

[b]Simple average of district expenditures so it is less than the $2,492 in the base case because that is effectively an average weighted by TAPU.

[c]Numbers in parentheses indicate percent changes from the base run.

[d]CERI are cost of educational resource indices (Wendling).

[e]The leveled-up expenditure level was $2,546 per TAPU.

tax rates and expenditure levels. Thus, in comparing the base solution with solutions 2 and 3, local tax rates are all that can adjust to the new equalized or cost-adjusted expenditure levels. To guarantee feasible solutions, in all three cases, tax rate ceilings were set at $45 per $1,000 of full value. This limit was never reached but in solution 3 at least one district aggregate would have required a tax rate of nearly $43 per $1,000 of full value to finance the difference between state aid and projected spending levels.

Because solution 3 reflects adjustments for inflation and cost differences, the fact that total expenditures rise only slightly suggests that the additional expenditures in high cost districts relative to the state norm more than offset expenditure decreases justified in low-cost areas. The implications of these new spending levels are particularly significant for some district groups. Were the state to attempt to minimize further "real" disparities in educational opportunity by encouraging districts to adjust any anticipated expenditure increases by this cost of services index, the 56 districts with the lowest wealth (taken as a group) would have to reduce expenditures by an amount nearly equal to the general rate of inflation. (See Colburn for details). Other groups of districts in upstate New York would be unable to raise expenditures sufficiently to compensate entirely for general inflationary trends. While placing a burden on the budgets of local school districts, such cost adjustments would mean significant property tax reductions relative to the base case. These reductions are reflected by the fact that the low end of the range in tax rates falls to less than $4 per $1,000 of full value. In New York City and other large cities around the state, the situation would be reversed. Cost adjustments, implying expenditures in excess of those needed to keep up with the general rate of inflation, would be required.

Although the idea of equalizing expenditure levels at some state average, either with or without a cost adjustment, is appealing, from a political perspective any reduction in school expenditures is difficult to implement. These adjustments might ultimately be mandated by the courts, but an alternative might be to define equal educational opportunity in terms of some minimum level of expenditures. Such a strategy was recommended by the Fleischmann Commission in the early 1970's in which operating expenditures would be leveled up to the 65th percentile of districts when ranked from low to high in terms of spending per pupil. In the situation where 1979-80 AOE are adjusted for general inflation, this 65th percentile would be at $2,546 per TAPU. Under this scheme, all districts would spend at least this amount, but those spending more could continue to do so.

As reflected by solution 4 (table 1) such a scheme would have a significant impact on school spending statewide. Total operating expenses would rise to $8.2 billion, a 19% increase over and above the 1979-80 levels, or 13% above the existing expenditure levels adjusted for inflation. With no change in the state aid formula, the state share of school expenditures would fall from 35% in the base case to 33% and average tax rates would increase to $24.26 per $1,000 full value. Most of this increase in tax rates occurs in poorer districts previously spending at levels well below $2,546 per TAPU.

Although the spending scenarios in solutions 2-4 may be realistic attempts to equalize education opportunity, it is unrealistic to assume that they would be financed without an updating of the aid formula. By letting either the C's or the X's be decision variables, the model can determine the appropriate modifications in the aid formula to minimize state aid subject to lower ceilings on local tax rates.

By imposing tax rate ceilings of \$35 per \$1,000 on school districts and solving for optimal levels of X_1 and X_2, state aid increased under the first three expenditure scenarios (table 1). As indicated by solution 5, optimal levels of X_1 and X_2 under the base expenditure levels fall to 0.37 and 0.0, respectively, indicating that the state pays 64% of the tier 1 ceiling level and the entire tier 2 ceiling level of \$100 to the district of average wealth. The state's share of the total expenditures rises from 35% in the base case to 44%. Similar patterns are noticed if fixed X_1 and X_2 is compared with optimal levels of X_1 and X_2 for equalized expenditure level assumptions (solution 2 vs. solution 6). Optimal levels of X_1 and X_2 fall to 0.49 and 0.0, respectively. The state pays 51% of the tier 1 ceiling to the district of average wealth while all districts eligible for tier 2 aid receive a "flat grant" of \$100 per TAPU, the tier 2 ceiling level. The share of total expenditures, \$7,209 million, paid by the state is increased from 38% under the current aid formula to 41%. When expenditures are adjusted for differentials in costs of providing services (solution 7), X_1 falls to 0.22, X_2 remains at 0.0 and the average state share increases to 51%. Property taxes, relative to the base case, fall by 23%. Tax rate reductions are more modest for solutions 5 and 6.

Conclusion

This paper demonstrates how combined systems of state and local school finance can be studied within a linear programming framework. For

New York State, the parameters of an existing state aid formula can be modified to minimize state aid required to meet new equalized, cost-adjusted or leveled-up expenditure levels. As one varies ceilings on local property tax rates, the optimal values of the parameters of the state aid system change dramatically, as does the state's relative share of total expenditures. Although significant disparities in expenditures and tax rates among districts are not easily reduced under the modified aid formulas examined, the model helps identify the tradeoffs between state and local taxpayer equity and the resources provided students in the state's 700+ school districts. From the magnitude of the changes involved, adjustments based on an optimizing strategy may be more effective than those based on ad hoc analysis.

In this paper, optimal levels of only the aid distribution parameters were obtained. It is clear that a simple change in the parameters of the aid formula cannot resolve the current problems. In a more extensive analysis, one would also want to: vary the aid ceilings systematically and place constraints on minimum levels of local tax effort to see how a combined change in both sets of parameters affects the distribution of costs between the state and local taxpayers; examine the implications of district power equalization in the state; and perhaps, investigate the implications of utilizing personal income as a substitute for or a supplement to property values as a measure of wealth (Education Unit). The flexibility of the programming structure described above, would accommodate this extended analysis within a consistent frame of reference.

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