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OPTIMAL FUNDING STRATEGIES FOR FINANCIAL COOPERATIVES[1/]

by

Loren Tauer and Michael Boehlje[2/]

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Department of Agricultural Economics
Cornell University Agricultural Experiment Station
New York State College of Agriculture and Life Sciences
A Statutory College of the State University
Cornell University, Ithaca, New York, 14853

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2/ Assistant Professor, Department of Agricultural Economics, Cornell University and Professor, Department of Economics, Iowa State University.

OPTIMAL FUNDING STRATEGIES FOR FINANCIAL COOPERATIVES

The cooperative Farm Credit System has been established to accomplish the objective of improving the income and well-being of U.S. farmers and ranchers by furnishing sound, adequate and constructive credit. As a member of the System, a district Federal Intermediate Credit Bank (FICB) serves as the intermediary between national sources of money and local Production Credit Associations (PCAs) and other qualified financial institutions within that district. The FICB obtains most of its funds by routinely issuing bonds and notes on the national money market.

The FICBs through local PCAs have become significant suppliers of nonreal estate credit to farmers, increasing their market share from 16 percent of total nonreal estate farm debt in 1968 to 24 percent in 1978. The volume during the same period increased from $3.5 billion to $13.5 billion. This increase in volume has made cost control crucial to the System. An increase in interest costs of ten basis points (one tenth of a percentage point) on $13 billion amounts to $13 million additional costs a year.

The large volume and increased cost have been accompanied by fluctuations in interest rates and funding needs. During 1978, the interest rate on FICB consolidated nine-month bonds, a primary source of funds, flutuated between 7.15 and 10.00 percent. During the same year the total consolidated bonds outstanding for the Omaha FICB, for example, ranged from $1,193 million to $1,299 million, a difference of $106 million. Interest on bonds typically accounts for 88-90 percent of the interest rate charged to the farmer-borrower, and because of the cooperative nature of the Farm Credit Banks and their use of variable rate interest programs increases or reductions in rates will quickly be reflected in their costs of borrowing. Because of the fluctuation in interest rates and funding needs, and the large volume of debt outstanding, a procedure that can determine optimal funding strategies in an uncertain environment and reduce the average cost of funds would be a useful management aid.

Debt Management Decisions

To provide adequate credit at a reasonable cost, a FICB must make two primary debt management decisions. They are:

1. The amount of debt which should be issued at a specific point in time to meet the anticipated needs of the PCAs and to refinance maturing debt instruments before another opportunity to issue debt arises.
2. The term structure of the debt issued.

The decision to participate in any given debt issue will be influenced by past debt issuance and the possible participation in future debt issues.

The first decision is difficult because of the uncertainty in the needs of the PCAs in future periods between debt issuances. Issuing debt in any amount below evolving actual needs of the PCAs would require short-term borrowing, normally at a cost above System-wide debt cost. In contrast, debt in an amount greater than actual needs requires excess funds to be invested, normally at a rate below the cost of the funds.

The second decision is difficult because of the uncertainty in future interest rates. The debt term structure selected not only depends upon the present known yield curve, but also yield curves that may develop in the future which affect future financing and refinancing decisions.

The objective of this study is to develop a liability management model that can aid in the debt management decisions of a FICB. The analysis will be structured to determine the optimal borrowing activities that would minimize the expected cost of credit at various levels of cost variance. More specifically, given the expected value of cost and variance-covariance of cost of various debt instruments that can be issued, and the stochastic demand for funds by the PCAs in the future, the optimal (in terms of minimum cost at various levels of cost-risk) maturity distribution and time issuance of debt instruments will be determined for a multiperiod planning horizon.

A number of studies have analyzed the debt selection activities and policies of the Farm Credit System. Hollenhorst analyzed the Federal Land Banks' debt management policies for the period 1947 to 1961. Brake, Boger, and Swortzel and Jensen have completed studies that project the funding needs of the various banks of the System. Bildersee, Percival, Morris, and Smith evaluated the financing needs of the Farm Credit System in a Wharton School of Finance study in 1973. They concluded that the lowest cost debt structure changes frequently and argued for flexibility in the timing and placement of debt issues. The model developed in this study can aid in making optimal timing and placement decisions.

The Conceptual Framework

Expected Cost-Variance of Cost Model

The mathematical form of the expected cost - variance of cost liability model (EC-VC) is identical to the expected return - variance of return asset selection model (E-V) except that now the objective is to minimize the variance of cost subject to an expected level of debt cost. Rather than having a given level of funds to invest, it is now necessary to generate a specified level of funds to meet loan demands. Mathematically, the model can be stated as:

$$\text{Minimize} \qquad Z = X'QX \qquad (1)$$

$$\text{subject to} \qquad AX \leq B \qquad (2)$$

$$CX \leq k \qquad (3)$$

$$X \geq 0 \qquad (4)$$

where

$C_{(1,m)}$ = the expected discounted cost vector for the planning horizon,

$Q_{(m,m)}$ = the discounted variance-covariance matrix of C,

$A_{(n,m)}$ = the technical matrix,

$B_{(n,1)}$ = the funding requirements and debt policy constraints,

k = the cost constant which is varied parametrically,

$X_{(m,1)}$ = the debt activity levels found by solution after each change in k.

Since the procedure is now applied to the liability rather than the asset side of the balance sheet, at any level of expected cost one unique liability structure is determined that minimizes the variance of cost. As expected cost is reduced by moving to another liability structure, the minimum variance at that cost increases (Figure 1).

The right-hand side (the B vector) of the EC-VC model contains the deterministic estimates of the financing requirements of the FICB. But since the financing needs are stochastic, it is necessary to convert these stochastic variables into deterministic values. One possibility is to use the expected values of these needs as the values for the right-hand side. However, in some instances it may be optimal to plan for bond debt outstanding to be an amount greater or less than the expected debt needs. This would depend upon the cost of short-term debt and short-term investment return.

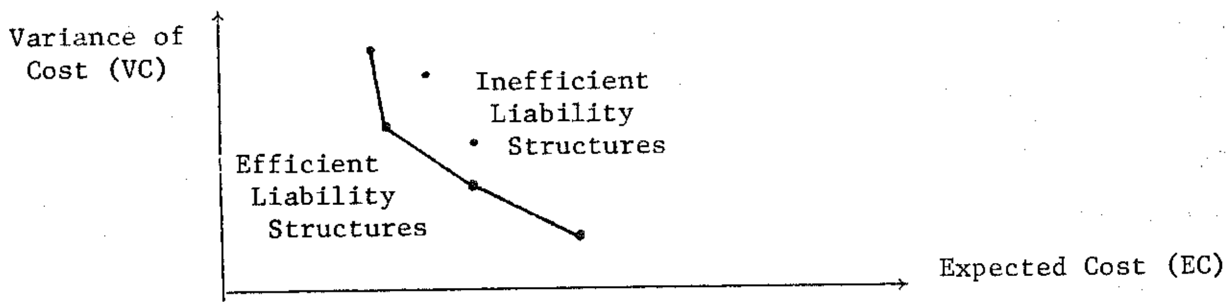

Figure 1. Expected cost and variance of cost (EC-VC) frontier.

Inventory Model

With the use of an inventory model it is possible to estimate the optimal bond purchases for a time period given an estimate of probable demand, cost of bonds, cost of inventory (funds) deficits, and return from excess inventory balances. After the optimal bond quantities are determined for each time period, these values can be inserted as the right-hand side of the mean-variance model and the optimal term structure of bond debt can be determined.

Debt demand for each period is defined as the amount of debt funds necessary to service the loans outstanding for that period. This definition of demand involves a stock rather than a flow concept. Debt outstanding during any period not only depends upon new loans granted during the period, but also loans made in previous periods which have a maturity of more than one period. Since demand is defined as a stock, the vast majority of demand occurs instantaneously at the start of the period as outstanding loans are carried into the new period. The model assumes for simplicity that all of the stochastic debt demand for each period occurs immediately after a bond is issued at the beginning of the period. Thus, after demand occurs there will be either an excess or a shortage of funds for the remainder of the period. Excess funds are invested in short-term investments; deficits are covered by short-term borrowing. The objective is to minimize the expected cost of funds for the period; the control variable is the quantity of bonds to be outstanding for the period. More explicitly, we want to minimize:

$$E\{c(y)\} = c \cdot y + p \int_{y}^{\infty} (v-y)\ f(v)\ dv - h \int_{-\infty}^{y} (y-v)\ f(v)\ dv \qquad (5)$$

where:

v = amount of debt demanded for a given time period,

$f(v)$ = probability density function for the possible values of v,

c = bond cost,

p = short-term debt cost,

h = short-term excess funds return,

y = amount of bonds outstanding.

The first derivative of equation (5) with respect to the control variable y set equal to zero is:

$$\frac{dE\{c(y)\}}{dy} = c - p \int_{y}^{\infty} f(v)\ dv - h \int_{-\infty}^{y} f(v)\ dv = 0. \qquad (6)$$

By definition,

$$\int_{y}^{\infty} f(v)\ dv = 1 - \int_{-\infty}^{y} f(v)\ dv.$$

Inserting this identity into equation (6) and solving for the minimum cost yields:

$$\int_{-\infty}^{y^*} f(v)\ dv = \frac{p - c}{p - h}\ . \qquad (7)$$

If f(v) is estimated, then y* can be determined as the optimal quantity of bonds outstanding for the period. But y* is only defined if $0 \leq \frac{p - c}{p - h} \leq 1$. This can occur only under either of two conditions:

(a) $p \geq c > h$,

(b) $p \leq c < h$.

For a minimum cost, the second derivative of equation (6) valued at y* must be greater than zero, or

$$\frac{d^2E\{c(y)\}}{dy^2} = (p - h)\ f(y^*) > 0. \qquad (8)$$

Since $f(y^*) > 0$, for the second order condition of (8) to be fulfilled, $p > h$, so condition (a) above must hold and $p \geq c > h$.

The expected inventory (funds) shortage for a given period will be

$$E(s) = \int_{y^*}^{\infty} (v-y^*)\ f(v)\ dv. \qquad (9)$$

This shortage can be multiplied by p for the expected funds shortage cost. The expected cost of not financing all debt by the lower bond cost is $(p - c)\ E(s)$.

Similarly, the expected excess funds for a given period will be

$$E(x) = \int_{-\infty}^{y*} (y* - v)\ f(v)\ dv. \tag{10}$$

This excess can be multiplied by h for the expected return on excess funds. The expected cost of over-financing with bonds will be (c - h) E(x).

The FICB is assumed to meet all of the financing needs of the PCAs so that none of the debt financing activity levels will affect the probability distribution of debt demand in any successive periods. Thus, there will be no correlation between y_i^*'s.

The Empirical Model

Planning Horizon

The planning horizon of the model is three years. Three years enables analysis of the impact of sequential funding with discount notes, six-month and nine-month bonds. The sequential impact of the longer-term bonds with various terms-to-maturity (two-year to twelve-year bonds have been used) would have required a substantially longer planning horizon.

The model is multiperiod; the first 18 periods are monthly periods, the last six periods are quarterly periods. Monthly periods were selected since the six-month and nine-month bonds are issued at the beginning of each month. The last half of the planning horizon was separated into quarters to reduce the number of activities in the model and still provide adequate detail. Transition to quarters required aggregating the monthly funding activities into quarters during the last half of the three year planning horizon.

The model terminates at the end of the three year horizon. Termination activities (such as complete liquidation by repurchasing all debt) were not included. Ending the model abruptly after three years can create myopic solutions. But as a planning aid, the purpose of the model is to derive optimal funding activities for the immediate future. Funding activities in future periods are only important and included because of their impact on immediate funding decisions (Boussard).

Activities

Eighteen nine-month bond activities were defined for the 18 monthly periods, and 6 nine-month bond activities were defined for the last six quarters. Six-month bond activities were defined in a similar manner. Long-term bond activities were defined as bonds issued on a quarterly basis at the beginning of the month for the first 18 months and then the beginning of each quarter for the last half of the planning horizon. Since the long-term bonds that are issued normally have a term-to-maturity beyond three years, they will provide funds for the duration of the planning horizon regardless of when they are issued. Discount notes can be issued by the FICB almost daily in maturities of 5 to 270 days, but were defined in the model as notes issued at the beginning of each monthly or quarterly period with a maturity of one month. The unit of size of all funding activities is one thousand dollars, since all the securities are issued in denominations of at least that minimum size.

Constraints

The model contains 24 rows which incorporate the funding needs (loan demand) of the FICB. The first 18 rows correspond to the first 18 monthly periods; the last six rows correspond to the six quarterly periods of the last half of the planning horizon. Transfer rows and columns were used to bring into solution the debt structure outstanding at the beginning of the planning horizon, since initial outstanding debt obligations will provide for some of the funding needs of the bank.

Additional constraints were placed on selected activities in some of the solutions to ascertain the effects of various debt management policies. One policy restriction is that no more than 10 percent of debt outstanding can be acquired by a single bond issue. Also, no more than 10 percent of the debt can be held as discount notes. A third restriction is that at least 30 percent of debt must be held in term bonds.

Coefficient Estimation Procedures

The model was applied to the debt selection process faced by the Omaha FICB. For this application it was assumed that the debt cost probability density function is multivariate normal. Thus, estimation of expected values of debt costs and the variance-covariance of these costs will completely define the probability density function. Although it is not necessary, since any functional form can be used, the probability density function for future FICB debt needs was also assumed to be normally distributed.

The variance-covariance matrix: Monthly observations of secondary market yields on all federal government agency securities from the period 1965 to 1977 were used to derive the variance-covariance matrix.[1/] Secondary market yields were used rather than initial placement interest rates because initial rates were not available for all currently used securities over

[1/] The security yield data were obtained from the Fiscal Agency of the Farm Credit System.

a sufficient observation period. The variance-covariance matrix was calculated as deviations of actual values from expected values. The deviations were squared and divided by the number of observations. To obtain expected values for the various interest rate variables, the expectations theory of the term structure of interest rates was used. The expectations theory states that the current long-term spot rate is the geometric mean of the current short-term spot rate and future short-term rates that are expected to occur during the duration of the long-term security (Sharpe, p. 226). It is assumed that the variance-covariance matrix is the same regardless of the expected values of bond and note costs.

Expected costs: Two projections of interest rates for the January 1979 to December 1981 time span were used.[2/] Both forecasts were obtained from the same national econometric model. A most probable interest rate forecast calls for interest rates to decrease during 1979, increase during 1980, and then fall again during 1981. A recession forecast simulated interest rates to increase the first two quarters of 1979, to fall drastically during the third and fourth quarters of 1979 as the recession develops and then decrease moderately during 1980 and 1981. To derive expected debt costs the cost of debt issuance in basis points was added to the interest rates. These adjusted rates were multiplied by $1000, and then by the term-to-maturity of the debt activity, or the time left until the end of the planning horizon if that time was less than the term of the debt activity. Since

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[2/] The projections were provided by the FICB of Omaha.

the model is multiperiod the expected cost and variance-covariance coefficients were discounted to the present.[3/]

Debt requirements: It was necessary to estimate probability density functions of FICB debt for each month for 18 months into the future, and then for each quarter for an additional six quarters into the future. To formulate normal probability density function estimates, the two parameters of the distribution, the mean and the standard deviation, were obtained by a linear regression of FICB debt on selected regressors as noted below. The forecasted values from the estimated regression equations were used as the means for the future periods. The variances of the error of forecast were used as the measure of variances for the distributions.

Two separate linear regressions were estimated to obtain two different forecasts of FICB debt. The first equation was estimated by a time series regression of FICB debt; the second equation was estimated by a regression of FICB debt upon PCA loans outstanding. Each equation generated a slightly different type of projection and allowed testing the sensitivity of the model to various debt projections. The time series equation provided a projection that increased every month, but with the greatest increase occurring the first quarter of each year as farmers prepared for the crop

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[3/] Since the expectations theory was used to compute the variance-covariance measures, the time preference for money, expected inflation, and the expected component of the default risk premium would be inherent in the forward interest rates. Deviations (variance-covariance) from the forward rates would include unexpected debt cost due to inflation and the unexpected variation of the default risk premium. The risk that money flows (costs) will not materialize because of changes in loan demand is incorporated in the inventory component of the model procedures. Therefore, the only elements of the discount rate that are not already incorporated in the model are the pure time preference of money and expected inflation. The interest rate that should most closely approximate the pure time preference of money and expected inflation would be short-term U.S. Treasury Bills. In this study projected three-month bills were used. A separate bill rate projection was used for each projection of expected interest rates.

season. The second equation projects debt to generally increase over the three year horizon with larger increases occurring during the first quarter and decreases occurring during the fourth quarter of each year. This decrease occurs as farmers sell part of their crops to reduce their debt at the end of the crop season.

The probability density functions of FICB debt along with the average cost of bond debt, short-term debt cost, and excess funds return were used to derive the optimal level of bond debt for each period with application of the inventory model. With this model the optimal FICB bond debt for each period was determined exogenously from the quadratic program and the values were inserted into the program. The FICB was restricted to a limit of planned excess or deficit funds of $25 million from the expected value for each period. The $25 million is the line of credit that the FICB has established with commercial banks. Although there is no formal limitation to the investment portfolio of the bank, the same $25 million restriction was applied to excess funds.

The computer program used to derive solution values for the model was the RAND QPF4. This program solves the problem of minimizing a quadratic function subject to linear constraints, and has a parametric procedure for the linear portion of the objective function. The program utilizes the Wolfe solution algorithm (Wolfe).

Empirical Results

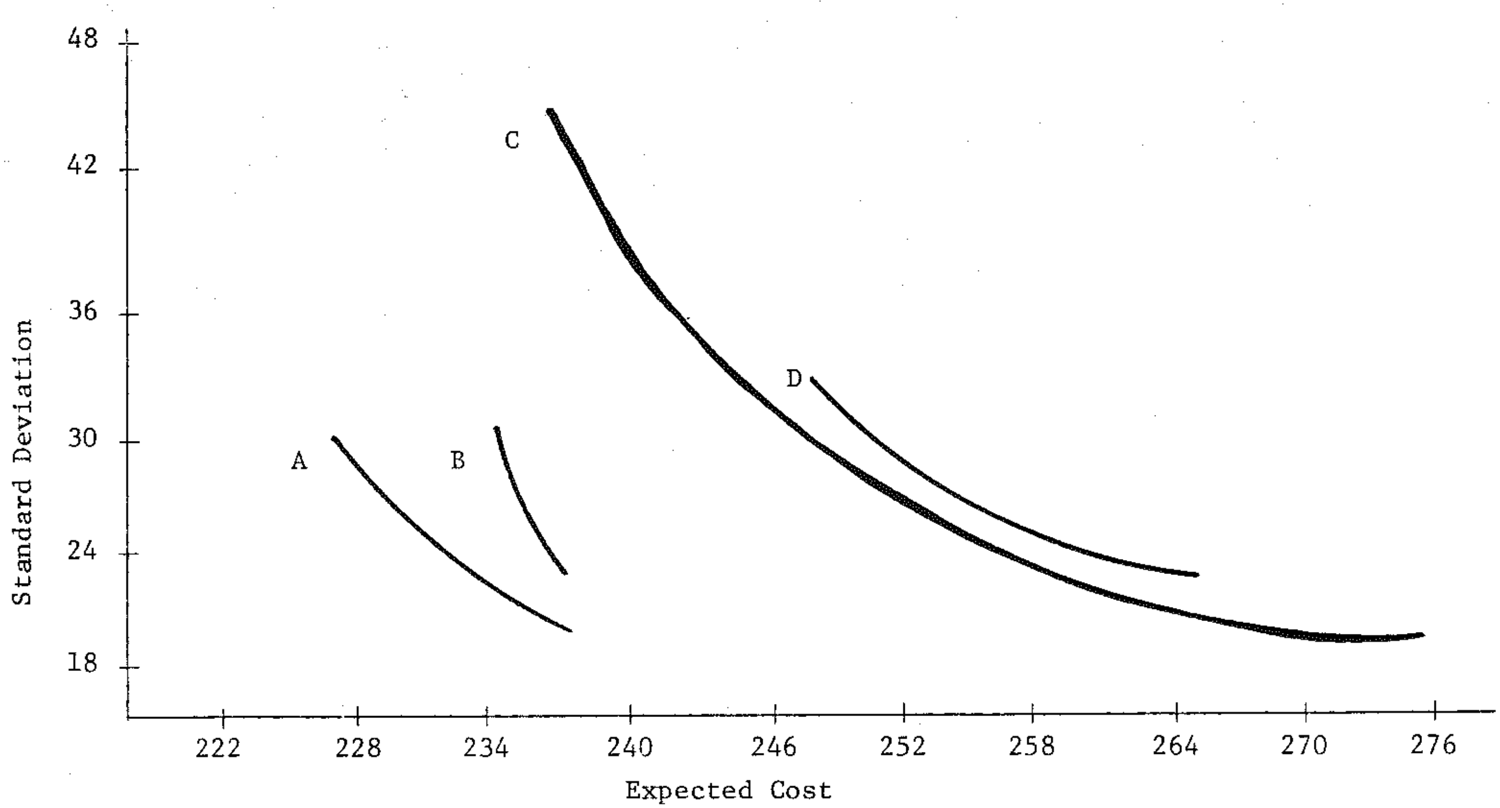
The empirical model was applied to the future three-year period of 1979-1981. A number of applications were performed with different projections of expected debt cost and optimal debt requirements.[4/] In most instances the applications were first made without debt policy constraints, and then debt policy constraints were imposed.

Most Probable Expected Debt Cost

The results generated by using the most probable forecast of interest rates to derive expected debt costs, and debt forecasts from PCA loan projections to obtain optimal debt requirements will be discussed in detail. Additional applications assuming alternative interest rate and debt need projections will then be briefly reviewed.

No debt policy constraints: The model excluding the debt policy constraints generated 25 individual portfolios on the frontier ranging from a low expected discounted cost of $226.602 million (high discounted standard deviation of $28.195 million) to a high expected discounted cost of $237.035 million (low discounted standard deviation of $18.805 million). The efficiency frontier is plotted in Figure 2. This frontier illustrates the tradeoff between expected cost and standard deviation--as a movement up the frontier to a lower expected cost portfolio occurs, a higher level of standard deviation of cost must be assumed.

[4/] To verify the realism of the model it was applied to the two historical periods of 1975 to 1977 and 1976 to 1978. Because actual historical debt amount and costs were used in these tests, the portfolios generated correspond closely to the actual debt issued by the bank. Some portfolios on the efficiency frontiers had lower expected costs than the cost actually incurred by the bank. One major difference between the portfolios generated and the actual debt issuance is that the bank typically participated in the term and nine-month bond when they were both offered. In the portfolios generated by the model typically only one of the two bond types was selected even though both were available. Hence, it would appear that the model does not diversify debt as well as the bank actually did. However, diversification can be accomplished over time with a similar or different bond type as well as by the use of different bond types at a point in time. Since interest rates are more variable over time than between securities at a point in time, it would be natural for the diversification of debt to occur over time more than by maturity at issuance.



A = Most probable expected debt cost, no debt policy constraints
B = Most probable expected debt cost, with debt policy constraints
C = Recession forecast of expected debt cost, no debt policy constraints
D = Recession forecast of expected debt cost, with debt policy constraints

Figure 2. Expected cost and standard deviation efficiency frontiers (in millions of dollars).

The portfolios with the highest expected cost, an intermediate expected cost and the lowest expected cost are shown in Table 1. The highest cost portfolio (lowest standard deviation) entails extensive use of the term bonds and moderate use of discount notes the first year. For the months in the first year that term bonds are not available, discount notes are used to provide funds until another term bond can be used. The second year of this portfolio involves the use of some nine-month bonds, as well as term bonds and discount notes. The term bonds and some of the nine-month bonds are carried into the last quarter of the second year, but no new debt is issued that quarter. In fact, during the fourth quarter of the second year discount notes that were issued the third quarter, and some nine-month bonds issued nine months previously will mature and not be refinanced. This occurs because the optimal debt needs of the bank decrease from the third to the fourth quarter. During the third year of the planning horizon only discount notes are used. The dominance of discount notes during the last year, and especially in the last two quarters of the planning horizon, is evidenced here and in portfolios presented later. This phenomenon may be myopic; however, tests indicate that the myopic terminal year condition does not appear to be transmitted into the first and second years.

In the intermediate cost and standard deviation portfolio as in the highest cost (lowest standard deviation) portfolio, only term bonds and discount notes are used the first year; in fact, there are no differences in the debt portfolios for the first six months of the first year between these two portfolios. However, in July of the first year, discount notes are substituted for term bonds in the intermediate cost portfolio. Then

Table 1. Debt issuance for the planning horizon: Most probable expected debt cost and debt requirements from PCA loan projections, no debt policy constraints (in millions of dollars).

	Lowest Standard Deviation Portfolio				Intermediate Standard Deviation Portfolio				Highest Standard Deviation Portfolio			
	Nine-month bonds	Term bonds	Six-month bonds	One-month Discount notes	Nine-month bonds	Term bonds	Six-month bonds	One-month Discount notes	Nine-month bonds	Term bonds	Six-month bonds	One-month Discount notes
1979												
January	0.0	126.073	0.0	0.0	0.0	126.073	0.0	0.0	0.0	0.0	0.0	126.073
February	0.0		0.0	105.853	0.0		0.0	105.853	0.0		0.0	231.926
March	0.0		0.0	218.863	0.0		0.0	218.863	0.0		0.0	344.936
April	0.0	305.441	0.0	0.0	0.0	305.441	0.0	0.0	0.0	0.0	0.0	431.514
May	0.0		0.0	111.807	0.0		0.0	111.807	0.0		0.0	543.321
June	0.0		0.0	228.154	0.0		0.0	228.154	0.0		0.0	659.668
July	0.0	358.906	0.0	0.0	0.0	36.620	0.0	322.285	0.0	0.0	0.0	790.420
August	0.0		0.0	99.587	0.0		0.0	421.872	0.0		0.0	890.007
September	0.0		0.0	189.675	0.0		0.0	511.960	0.0		0.0	980.095
October	0.0	169.762	0.0	0.0	0.0	492.047	0.0	0.0	0.0	960.182	0.0	0.0
November	0.0		0.0	21.119	0.0		0.0	21.119	0.0		0.0	21.119
December	0.0		0.0	42.238	0.0		0.0	42.238	0.0		0.0	42.238
1980												
January	0.0	107.472	0.0	0.0	0.0	107.472	0.0	0.0	0.0	107.472	0.0	0.0
February	38.163		0.0	0.0	38.163		0.0	28.033	38.163		0.0	0.0
March	38.164		0.0	0.0	10.131		0.0	0.0	0.0		0.0	38.164
April	12.366	25.797	0.0	0.0	0.0	66.196	0.0	0.0	0.0	72.949	3.378	0.0
May	8.513		2.499	0.0	0.0		0.0	11.012	0.0		0.0	11.012
June	0.0		0.0	11.012	0.0		0.0	22.024	0.0		0.0	22.024
Third Quarter	0.0	0.0	0.0	144.161	0.0	0.0	0.0	177.201	0.0	0.0	0.0	177.201
Fourth Quarter	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1981												
First Quarter	0.0	0.0	0.0	433.578	6.198	0.0	100.752	0.0	100.197	0.0	0.0	0.0
Second Quarter	0.0	0.0	0.0	756.909	0.0	0.0	0.0	314.818	0.0	0.0	0.0	314.818
Third Quarter	0.0	0.0	0.0	945.049	0.0	0.0	0.0	805.245	0.0	0.0	0.0	502.958
Fourth Quarter	0.0	0.0	0.0	520.291	0.0	0.0	0.0	399.082	0.0	0.0	0.0	378.820
	Expected Discounted Cost = 237.062				Expected Discounted Cost = 234.684				Expected Discounted Cost = 226.602			
	Standard Deviation = 18.803				Standard Deviation = 19.924				Standard Deviation = 28.195			

in August a greater use of discount notes occurs. The discount notes issued in September mature at the end of the month and a large volume of term bonds is used to refinance the debt in October. Although various factors influence the choice of activities, it appears that since interest rates are projected to reach their lowest level during October of 1979, discount notes are issued the months immediately prior to October so that a large amount of debt can be refinanced with the term bond at the lowest interest rate of the planning horizon. This phenomenon did not occur in the earlier low variance portfolio since the discount notes have high variances. During the second year fewer nine-month bonds and more discount notes and term bonds are used than in the lowest variance portfolio. For the first quarter of the third year, nine-month bonds and six-month bonds replace the discount notes of the low variance portfolio. This occurs because projected interest rates rise slightly during the early part of 1981, and the six-month and nine-month issues lock in a low debt cost before rates begin to rise.

For the lowest cost portfolio of the fontier, activities are selected on the basis of their expected cost without regard to variance. Therefore this portfolio is the same that would result from linear programming. For the first year only discount notes are used during the first nine months, and then in October $960.182 million of term bonds are used to refinance all new debt accumulated since the beginning of the planning horizon. Discount notes are again used during November and December until another term bond can be issued in January of the second year. In February of the second year a nine-month bond is issued before interest rates begin to increase. As new debt needs increase early in the second year, a term bond and a small volume of discount notes are first used, and then discount notes are issued during the third quarter since debt needs fall at the end of the year and the discount notes will mature at that time. For the third year discount notes are again used extensively, but now a nine-month bond rather than the six-month bond is used the first quarter since it has a lower expected cost.

A summary of activities that are used the first year for the 25 individual portfolios on the efficiency frontier is shown in Table 2. The table indicates the monthly average percentage of bond and note debt acquired the first year using the four types of debt securities. Every fifth portfolio is summarized with the portfolios listed in order of descending expected costs and ascending standard deviation. A movement from higher to lower expected cost portfolios results in a shift from term bonds to discount notes augmented with six-month bonds, and then to discount notes exclusively. Nine-month bonds are never included in the portfolios during the first year on this efficiency frontier.

If the model is used on an operational basis, a bank would be especially interested in the activities for the first period because a decision to participate in the debt issues of that period would be eminent. Many of the portfolios on the efficiency frontier have the same first period debt activities (Table 3), and the range of expected cost and standard deviation is quite large before there is a change in the first period's activities. For example, expected cost varies from the highest cost of \$237.062 million to \$231.446 million before a change occurs in the activities for the first period; this change is from \$125.073 million in term bonds to \$96.665 million in term bonds and \$29.408 million

Table 2. Average monthly new debt outstanding for the first year: Most probable expected debt cost and debt requirements from PCA loan projections, no debt policy constraints.

Portfolio number	Nine-month bonds	Term bonds	Six-month bonds	Discount notes	Expected cost[a/]	Standard deviation[a/]
			Percent of Yearly Total			
1	0	87.2	0	12.8	237.062	18.803
5	0	86.7	.5	12.8	236.972	18.816
10	0	77.8	0	22.2	235.185	19.598
15	0	73.6	0	26.4	234.358	20.161
20	0	43.6	25.3	31.1	228.733	25.735
25	0	36.3	0	63.7	226.602	28.195

[a/] In millions of dollars.

Table 3. Activities into solution the first period: Most probable expected cost and debt requirements from PCA loan projections, no debt policy constraints (in millions of dollars).

Portfolio numbers	Range in expected cost	Range in standard deviation	Nine-month bonds	Term bonds	Six-month bonds	Discount notes
1 to 18	237.062 to 231.446	18.803 to 22.904	0	126.073	0	0
19	230.781	23.597	0	96.665	29.408	0
20 to 24	228.733 to 226.800	25.735 to 27.910	0	0	126.073	0
25	226.602	28.195	0	0	0	126.073

in six-month bonds. The next change at an expected cost of $228.733 million is to $126.073 million in six-month bonds. The final change at the lowest expected cost solution is to $126.073 million in discount notes.

With debt policy constraints: The model incorporating the debt policy constraints generated 116 individual portfolios on the efficiency frontier. The portfolios ranged from a low expected discounted cost of $234.310 million (high standard deviation of $28.578 million) to a high expected discounted cost of $237.184 million (low standard deviation of $22.021 million). This efficiency frontier is also plotted in Figure 2. As illustrated, the addition of debt policy constraints shifts the efficiency frontier to the right; at any level of standard deviation, the portfolio with policy constraints has a higher expected cost than the portfolio without policy constraints. At the low standard deviation of $22.021 million, the increase in expected cost is approximately $5 million. At the high standard deviation of $28.578 million, the increase in expected cost is approximately $7.7 million. With the projections of expected costs and debt requirements used in this model, it appears that the addition of policy constraints imposes a greater penalty cost at the higher standard deviation levels.

One purpose of debt policy constraints is to reduce the volatility of the cost of debt. However, application of the policy constraints does not necessarily accomplish this objective. The highest standard deviation for the portfolios generated with the constraints is $28.578 million, which is one percent greater than the highest standard deviation ($28.195 million) without the debt policy constraints. Unfortunately, in addition to failing to limit potential high cost volatility, the constraints raise the minimum standard deviation attainable from $18.803 million without the debt policy constraints to $22.021 million with the constraints, a 17 percent increase.

The highest expected cost, an intermediate expected cost, and the lowest expected cost portfolios on the efficiency frontier generated with constraints are shown in Table 4. The low standard deviation portfolio with the constraints, like the analogous portfolio without the constraints, includes a large amount of term bonds the first year of the planning horizon. However, the value is now reduced because no more than 10 percent of the total debt can be held in any specific bond issue. As a substitute for the term bonds the next lowest variance bonds are selected, which in this case are the nine-month bonds. Some discount notes are also included in the portfolio, mostly entering the month before a term bond is issued. The debt acquired by the discount notes is again refinanced with term bonds when possible. In the second year of the constrained low standard deviation portfolio, more debt issuance activities are included than in the analogous nonconstrained portfolio. This occurs because the nine-month bonds issued during the first year must be refinanced; when these bonds mature, they are refinanced with additional nine-month bonds and discount notes. For the last year of the portfolio, since discount notes are constrained, some six-month bonds and nine-month bonds are included.

A move from the high expected cost to the intermediate expected cost portfolio results in a shift from nine-month bonds to six-month

Table 4. Debt issuance for the planning horizon: Most probable expected debt cost and debt requirements from PCA loan projections, debt policy constraints (in millions of dollars).

	Lowest Standard Deviation Portfolio				Intermediate Standard Deviation Portfolio				Highest Standard Deviation Portfolio			
	Nine month bonds	Term bonds	Six-month bonds	One-month Discount notes	Nine month bonds	Term bonds	Six-month bonds	One-month Discount notes	Nine-month bonds	Term bonds	Six-month bonds	One-month Discount notes
1979												
January	0.0	126.073	0.0	0.0	0.0	93.479	32.594	0.0	0.0	93.479	0.0	32.594
February	105.853		0.0	0.0	0.0		88.789	17.064	0.0		19.772	118.675
March	51.109		0.0	61.901	0.0		0.0	130.074	0.0		101.611	130.074
April	0.0	130.615	17.992	0.0	0.0	130.615	86.094	0.0	0.0	5.353	130.615	80.741
May	111.750		0.0	0.0	0.0		93.393	18.357	0.0		59.803	132.689
June	110.402		0.0	5.945	0.0		0.0	134.704	0.0		114.331	134.704
July	0.0	136.697	0.0	0.0	136.697	136.697	0.0	24.656	0.0	20.275	108.485	136.697
August	58.010		0.0	39.768	0.0		77.749	133.475	0.0		120.773	133.475
September	0.0		0.0	131.664	7.243		86.463	131.664	63.653		131.664	131.664
October	0.0	129.673	0.0	0.0	68.172	129.673	0.0	0.0	0.0	129.673	0.0	112.693
November	126.972		0.0	0.0	40.943		0.0	73.569	61.830		0.0	131.785
December	1.453		0.0	70.775	0.0		0.0	94.688	133.338		0.0	133.897
1980												
January	0.0	136.009	0.0	0.0	0.0	136.009	0.0	23.913	73.263	136.009	0.0	98.344
February	139.825		0.0	10.088	139.825		0.0	0.0	139.825		0.0	117.455
March	49.359		0.0	109.295	122.413		0.0	2.214	143.642		0.0	143.642
April	0.0	147.458	0.0	0.0	29.616	147.458	0.0	0.0	34.347	147.458	0.0	0.0
May	50.325		1.122	17.576	11.012		0.0	0.0	11.012		0.0	0.0
June	0.0		0.0	28.588	0.0		0.0	18.255	74.665		0.0	0.0
Third Quarter	0.0	150.761	0.0	0.0	0.0	150.761	0.0	0.0	0.0	0.0	122.714	0.0
Fourth Quarter	0.0	65.832	0.0	0.0	59.798	0.0	0.0	0.0	235.983	0.0	0.0	0.0
1981												
First Quarter	53.931	0.0	0.0	440.979	252.640	0.0	0.0	0.0	419.129	0.0	0.0	0.0
Second Quarter	0.0	0.0	200.047	202.919	0.0	0.0	0.0	322.827	0.0	0.0	0.0	472.158
Third Quarter	0.0	0.0	0.0	502.070	0.0	99.764	0.0	502.070	158.365	167.336	0.0	502.070
Fourth Quarter	0.0	0.0	94.162	448.798	0.0	0.0	92.824	448.798	109.715	149.598	0.0	448.798
	Expected Discounted Cost = 237.184 Standard Deviation = 22.021				Expected Discounted Cost = 235.590 Standard Deviation = 23.755				Expected Discounted Cost = 234.310 Standard Deviation = 28.578			

bonds and discount notes for the first year. Except for the first month, the amount of term bonds to be issued the first year in this portfolio is not altered from the high cost (low standard deviation) portfolio. In the second year a move to the intermediate from the high expected cost portfolio entails very little change in term bond usage but includes more nine-month bonds, fewer discount notes and no six-month bonds. The debt structure of the third year is also restructured with the inclusion of nine-month and term bonds because more maturing debt is refinanced and the use of the discount notes is constrained.

The first year of the low expected cost portfolio on the frontier differs substantially from the nonconstrained low expected cost portfolio. Now not only are discount notes included in the first year, but because of the 10 percent constraint limiting the use of any specific issue, six-month bonds are included as well since they are the next lowest cost debt activity. Term bonds are also included in the portfolio during the first year because of the requirement to maintain 30 percent of the debt in term issues. Unlike the nonconstrained portfolio where $960.182 million of term bonds were issued in October because of that issue's low cost, now only $129.673 million can be issued because of the 10 percent constraint. Because of the limits on the issuance of term bonds, beginning with November of the first year and into the second year nine-month bonds are issued, subject to the 10 percent constraint. Interest rates are projected to increase and low debt costs are being locked in with the long-term securities--term and nine-month bonds. During the third year low cost notes, term bonds and nine-month bonds are used subject to the 10 percent limit constraint.

Similar to the first period's activities for the unconstrained model, a movement to lower costs entails a shift from term bonds to six-month bonds to discount notes. Now however, \$93.479 million of term bonds is always issued the first period to comply with the 30 percent minimum debt to be held in term bonds. Although there are 116 different portfolios for the entire planning horizon, there are a significantly reduced number of first period options, in this case, five.

Recession Forecast of Debt Costs

Additional analyses were completed using the recession forecast of debt cost. The expected cost for the efficiency frontier generated ranged from \$237 to \$275 million for the three-year period, and the standard deviation ranged from \$45 to \$18 million. This frontier lies to the right of the frontier generated with the most probable forecast of interest rates. At low variance (high cost) levels on the efficiency frontier, the portfolios contain primarily term bonds and discount notes the first year. At the higher variance (lower cost) levels, nine-month and six-month bonds are used the first few months of the first year to lock in a low cost as interest rates increase during the first half of the first year. As interest rates fall during the second and third years, shorter-term securities are used extensively; nine-month and six-month bonds at the lower variance levels and discount notes at the higher variance levels.

Comparison of the Efficiency Frontiers

As noted, changes in the coefficients of the model and the addition of debt policy constraints shifts the efficiency frontier. The addition of debt policy constraints shifts the frontier to the right such that at any level of standard deviation, a greater expected cost is incurred. This is reflected by a shift from frontier A to frontier B (Figure 2), the frontiers obtained using the most probable debt forecast without and with the debt policy constraints respectively, and from frontier C to D, the frontiers obtained using the recession debt forecast without and with the debt policy constraints respectively.

The shift from frontier A to frontier B is not a parallel shift--the increase in expected cost is greater at high standard deviation levels. This occurs because the term bond in October of the first year is used at a volume as large as $960.182 million in the nonconstrained frontier, but is restricted to a maximum of $129.673 million in the constrained frontier. Thus, the increase in expected cost is greater at the higher standard deviation levels when the use of that bond is extensive because of its low expected cost. The shift from frontier C to D is more nearly parallel because no prevalent bond is used on the nonconstrained frontier.

The shift from C to D also entails a reduction in the highest cost risk exposure (standard deviation) of the Bank. This did not occur with the shift from A to B. The reduction occurs because a large volume of discount notes, which have a high standard deviation, were used in the low cost portfolio on frontier C but were limited in use on frontier D.

The shift from A to C, which results from a change in expected debt cost coefficients and debt requirements, is much more drastic than the shift due to the debt constraints. This implies that the debt policy constraints have a relatively small impact on the portfolios on the efficiency frontiers compared to the effects of a change in the expected debt cost coefficients and debt needs.

Implications

In general, with all applications, a movement along the efficient frontier from low expected cost and high cost variance portfolios to higher expected cost but lower cost variance portfolios entails a shift from one-month discount notes to six-month bonds to nine-month bonds to term bonds. A projected increase in expected interest rates over the planning horizon will cause longer-term bonds to be used to lock in a low debt cost. A projected decrease in expected interest rates will cause shorter-term bonds and notes to be used to take advantage of the decrease. However, the specific maturities used depend upon the duration of the movement and variance level on the efficiency frontier. The long-term activities used at lower variances will be term bonds; long-term activites at higher variances will be nine-month bonds. Short-term activities used at lower variances will be nine-month and six-month bonds; short-term activites at higher variances will be discount notes. The fluctuation of debt needs over the planning horizon requires the use of some short-term securities which mature when debt needs decrease at all variance levels. A steady growth in debt needs permits the use of all terms-to-maturity, the selection of which depends upon the expected cost and variance of the portfolio.

As expected, the addition of policy constraints shifted the efficiency frontier to the right--expected debt cost was higher at each level of variance. With the most probable forecast of debt cost, expected cost was $5 to $8 million higher on the frontier with policy constraints; with the recession forecast, expected cost was $2 to $3 million higher. The impact of policy limits was greater for the most probable forecast of debt cost because the debt policy constraints limited the extensive use of a low cost term bond and there was no other long-term bond to serve as a replacement. With the recession forecast the use of the discount notes was constrained, but they were replaced at a small penalty cost with six-month and nine-month bonds. The policy constraints limited the high levels of variance (cost risk) that the bank could be exposed to with the recession forecast of interest rates by limiting the use of discount notes. Unfortunately, they also truncated the lower section of the efficiency frontier and eliminated alternative low variance solutions as well. The results also indicate that changes in expected costs and debt needs result in dramatic shifts in the efficiency frontier.

It is impossible to compare the solution results for 1979-1981 to past financing activities of the bank because of the transition to System-wide securities which were included in the model but were not completely available to the bank before 1979. However, with the coefficient values used in the model, it would appear that more extensive usage of both long-term bonds and discount notes should occur. The nine-month and six-month bonds are very similar as to expected costs, variance-covariance, and duration. It therefore appears that they are good substitutes for each other, depending upon funding needs of the bank and relative costs.

The model appears to derive realistic optimal solutions. Whether these solutions are the best solutions obtainable depends upon the accuracy of the forecasted coefficients used in the model. The historical tests generated some solutions that had lower expected costs than what the bank actually experienced over the test periods. However, those solutions were obtained using the actual debt costs that occurred over the period. Since the model is an optimization model which selects from an enormous but finite set of feasible solutions, a task which would not be possible without a decision model, it would seem plausible that it could be useful as a day-to-day management aid in analyzing potential liability structures.

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