

A USER'S GUIDE FOR TWO PROGRAMS TO ESTIMATE
AND ANALYZE THE MULTINOMIAL LOGIT
ALLOCATION MODEL

by

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I. Introduction

This guide describes two programs for estimating and analyzing multinomial logit allocation models. The first program (MLML) performs the maximum likelihood estimation of the model and produces estimates of the parameters, their asymptotic standard errors and t-values, and the determinant and information limit to their variance-covariance matrix. The second program (MLAM) performs sensitivity analyses on an already-estimated multinomial logit model. It produces tables and graphs of the changing allocations and elasticities of the model over user-specified ranges of the explanatory variables. Both programs assume that a model is defined by equations:¹

$$\pi_i = \frac{e^{f_i(X)}}{\sum_j e^{f_j(X)}} \quad i = 1, \dots, N$$

where π_i is the probability of an allocation to the i'th group,

$$f_i(X) = b_{i0}X_0 + b_{i1}X_1 + \dots + b_{iK}X_K,$$

and $X_k = g_k(y)$, $k = 0, \dots, K$, a specific transformation of the set of explanatory variables y (e.g. $X_k = \ln(y)$).

In large samples, the estimating equations can be approximated by

$$(1) \quad \ln(w_i/w_c) = B_{i0}X_0 + B_{i1}X_1 + \dots + B_{iK}X_K \quad i \neq c$$

or

$$(2) \quad \ln(w_c/w_i) = -B_{i0}X_0 - B_{i1}X_1 - \dots - B_{iK}X_K \quad i \neq c$$

¹It is assumed throughout that the user is already familiar with the multinomial logit model. If not, see Tyrrell and Mount, "An Application of the Multinomial Logit Model to the Allocation of Budgets in the U.S. Consumer Expenditure Survey for 1972", Cornell Agricultural Economics Staff Paper No. 78-20.

where $B_{ik} = b_{ik} - b_{ck}$ and w_i is the proportion actually allocated to group i . The difference between (1) and (2) is whether a common denominator or a common numerator is used to define the $N-1$ equations. For the maximum likelihood estimator the default normalization is that the N 'th share is a common denominator.

II. A Maximum Likelihood Computer Program for Estimating the Multinomial Logit Allocation Model (MLML)

The multinomial logit model can be applied to many different allocation problems. The correct maximum likelihood estimation procedure for the parameters depends on the assumed relationship between observations. The assumption here is that all observations are weighted equally. For a discussion of the alternatives see Chapter III of "An Application of the Multinomial Logit Model to Predicting the Patterns of Food and Other Household Expenditures in the Northeastern United States", T. J. Tyrrell, unpublished Ph.D. thesis, Cornell University, 1979.

1. The Algorithm

The program described here searches for the maximum of the likelihood of the multinomial logit model. The log likelihood function is given by:

$$(3) \ln(L) = \text{constant} + \sum_{t=1}^T \left[\sum_{i=1}^N w_{it} \left\{ f_i(X_t) - \ln \left(\sum_{j=1}^N e^{f_j(X_t)} \right) \right\} \right]$$

where T is the number of observations.

First order conditions are given by:

$$(4) \frac{\partial \ln(L)}{\partial B_{ik}} = \sum_{t=1}^T X_{ikt} [w_{it} - \pi_{it}]$$

$i = 1, \dots, N$ and $k = 1, \dots, K$

$$\text{where } \pi_{it} = \frac{e^{f_i(X_t)}}{\sum_{j=1}^N e^{f_j(X_t)}}$$

Second order conditions are given by:

$$(5) \frac{\partial^2 \ln(L)}{\partial B_{ik} \partial B_{ik'}} = \sum_{t=1}^T X_{ikt} X_{ik't} (\pi_{it}^{-1} - \pi_{it})$$

where both parameters (B_{ik} and $B_{ik'}$) appear in the same equation, and

$$(6) \quad \frac{\partial^2 \ln(L)}{\partial B_{ik} \partial B_{jk'}} = \sum_{t=1}^T X_{ikt} X_{jk't} \pi_{it} \pi_{jt}$$

where each parameter comes from a different equation ($i \neq j$).

The search employs both first and second order derivatives in an extended Newton-Raphson procedure. If the matrix of second order partials is singular because the same variable appears in each equation, the parameter of the last (N^{th}) equation is constrained to zero. Therefore, the maximum number of parameters to be estimated is no greater than $R \times (N-1)$.

Coefficients can be initialized by the user and the default values are that the constant term of each equation is initialized at the logarithm of the average budget share and other coefficients are initialized at zero. At each iteration the vector of coefficients is adjusted by the vector of changes given by:

$$(7) \quad \Delta B = - \left[\left\{ \frac{\partial^2 \ln(L)}{\partial B_{ik} \partial B_{jk'}} \right\} \right]^{-1} \left[\left\{ \frac{\partial \ln(L)}{\partial B_{ik}} \right\} \right]$$

where braces $\{\}$ indicate a typical element of the matrices and all derivatives are evaluated at the vector of coefficients of the previous iteration.

The non-constant portion of the log likelihood function is computed at each iteration. As long as this computed value is increasing iterations continue until the absolute relative change in the log-likelihood is less than the tolerance level set by the user. If the computed value decreases at any iteration the vector of changes is multiplied by $(1/3)^Y$ where $Y = 1, \dots, 10$, or until the likelihood increases.

Final output consists of a vector of estimated coefficients, first and second order derivatives of the likelihood function with respect to each, asymptotic standard errors and t-statistics.

2. Program Input

The user is expected to run the Fortran source deck of MLML (at least the first time) to set the maximum dimensions of the arrays in the program. This is done to economize on space since the program holds all observations in main storage rather than on temporary disk storage like most maximum likelihood estimation programs. As a result, the program is quite efficient for small and medium sized problems, but may exceed storage capacity of some machines for large problems.

The model dimensions are specified on four cards in the main program (each begins in column 7):

```
REAL*8 B(NU) , W(N,NØBS) , X(NA,NØBS) , XO(NB), EF(N)
```

```
REAL*8 G(NU) , GG(NU,NU) , WW(N) , XX(NA) , BR(NR) , BO(NU)
```

```
INTEGER IU(NA,N)
```

```
NUMAX = NU
```

where the user's problem determines the arguments:

NU is the number of unrestricted parameters - those to be estimated.
If two or more coefficients will be constrained to the same value,
count the parameter only once.

N is the number of allocation groups.

NØBS is the number of observations.

NB is the number of data items in the input data list for each observation,
including allocations and explanatory variables (this is used for
reading primary input).

NA is the number of explanatory variables in the model after transformations. If a transformed explanatory variable is found in more than
one estimating equation, count it only once.

NR is the number of restricted parameters - those set to specified levels.

The dimensions set by these cards must equal or exceed the values read on the first input card (see below). The user must also provide a fortran subroutine (TRAN) to transform the input data list (stored in NB locations of XO for each observation) into N allocations W (which sum to unity) and NA explanatory variables stored in X.

In addition to the data there are five types of control cards for the program. These are described on the following pages. Table 1 shows how to set up the card deck.

TABLE 1 Input for MLML^a

Hasp Job Card

```
/*LIMITS_REGION=115K,CLASS=A
//_EXEC_FTG1CLG,PARM.FORT='DECK'
//FORT,SYSPUNCH_DD_SYSOUT=(F,MLML)
```

} first run only

Fortran Deck of MLML

```
//_EXEC_FTG1LG
//LKED.SYSIN_DD_*
/*I_MLML
/*
//GØ.SYSIN_DD_*
```

} subsequent runs

- | | |
|--------------------------------------|---------------------|
| 1. Size and Definitions Card | (10I5,D10.0) |
| 2. Parameter Restrictions Cards | (40I2) (N of these) |
| 3. Restricted Parameter Values Cards | (8F10.0) (optional) |
| 4. Initial Parameter Estimates Cards | (8F10.0) (optional) |
| 5. Data Format | (20A4) |

Data Cards

/*

//

^aAlphabetic Ø's are slashed; zeros (0) are not.

Blanks are denoted by _.

Right-adjust everything.

The JCL is specific for the Cornell University computer system.

Description of Input Cards 1. - 5.

1. Size and Definitions Card (Format = (10I5,D10.0))

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-5	NB	number of data items ²
6-10	NA	number of X's after transformations ²
11-15	NØBS	number of observations ²
16-20	N	number of groups ²
21-25	NU	number of unrestricted parameters ²
26-30	NR	number of restricted parameters ²
31-35	INIT	indicator for reading initial values of unrestricted parameters. Zero or blank means initialize parameters at zero. Nonzero means read NU initial values on Card 4.
36-40	ITMAX	maximum number of iterations to be performed (ten has been more than enough for problems tested.)
41-45	NC	the number of the constant term in the variable list after transformations.
46-50	NP	the number of observations to be printed after transformations.
51-60	TOL	tolerance criterion for absolute relative change in the likelihood function (1.0D-05 has worked well, right adjust).

²The values given on input card 1 may describe problems less than or equal to the size of the dimensions of the main program.

2. Parameter Restrictions Card

(40I2)

There are N of these cards, one for each $f_i(X)$ of the model. Together they contain all positive integers from 1 to NU (corresponding to the unrestricted parameters) and all negative integers from -1 to -NR (corresponding to the restricted parameters.) The NA columns (of width 2) correspond to the set of transformed explanatory variables. The location of an integer on a card links the parameter to an explanatory variable for an equation. A parameter may appear in more than one equation (an integer may appear on more than one card) in order to impose constraints across equations. All variables needn't be used in every equation (some of the NA columns may be blank) in order to constrain some parameters to zero. Negative integers fix a parameter at its initial value given on card 3.

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-2	IU	parameter which is the coefficient of X(1) in this equation
3-4		parameter which is the coefficient of X(2) in this equation
...		
2NA-1 - 2NA		parameter which is the coefficient of X(NA) in this equation

3. Restricted Parameter Values Card (optional)

(8F10.0)

This card is read when NR on Card 1 is not zero. These restrictions may be used to identify parameters of the model (in the econometric sense).

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-10	BR(1)	value for first restricted parameter
11-20	BR(2)	value for second restricted parameter
...		
?	BR(NR)	value for NR th restricted parameter

4. Initial Parameter Estimates Card (optional)

(8F10.0)

If INIT on Input Card 1 is not zero then initial parameter estimates will be read (eight-to-a-card) corresponding to the NU parameters. Since the model is identified only up to differences between two parameters, when a variable is present in all equations these initial values will be converted to differences from the value of a corresponding parameter in the last equation of the model.

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-10	BO(1)	initial value for first parameter
11-20	BO(2)	initial value for second parameter
...		
?	BO(NU)	initial value for NU th parameter

5. Data Format

(20A4)

This card describes the input of all NB values for input on each observation. It is a Fortran-type format beginning with a left parenthesis and ending with a right parenthesis.

3. An Example Run of MLML

To illustrate the use of the programs, the input for and output from MLML are presented in Tables 2 and 3 for a household budget allocation model. To summarize: total expenditures for 456 households were allocated into six expenditure groups: Food at home, Food away from home, Clothing, Transportation, Housing and Other Goods and Services. Forty five parameters were estimated defined by five (N-1) functions with nine parameters each as described by:

$$\begin{aligned}
 (8) \quad f_{it} = & B_{i0} + B_{i1} \ln(M_t) + B_{i2} \ln(S_t) \\
 & + \sum_{r=1,2,4} B_{i2r} \sum_{s=1}^{S_m} L_r(a_s) + \sum_{r=1,2,4} B_{i2(r+4)} \sum_{s=1}^{S_f} L_r(a_s)
 \end{aligned}$$

where M_t is total expenditure, S_t is household size and $L_r(a_s)$ are Lagrangian interpolation variables^t (see Tyrrell, 1979, op. cit., Chapter IV.)

Input is shown in Table 2. Output is shown in Table 3 and consists of:

1. Verification of the Size and Definitions Card.
2. The first 20 observations on allocated shares and the average shares over all observations.
3. The corresponding first 20 observations on the explanatory variables and the average values over all observations.
4. A verification of the parameters to be estimated, the initial value of the likelihood function, and the initial parameter estimates.
5. At each iteration: the value of the non-constant portion of the likelihood function for the estimates from the previous iteration and the absolute relative change in this value (TOL) between iterations.
6. The parameters, efficient scores (first order derivatives), and negative of the estimated information limit to the variance-covariance matrix (second order derivatives) for the last iteration as well as a summary of the parameters and asymptotic t-statistics.

This particular run from the object deck of the MLML program used a region of 114 K, 39 CPU seconds and 9 I/O seconds on Cornell's IBM 360.

TABLE 2 Example Input for MLML

```

___14___9___456___6___45___0___0___10___1___20___0.1D-05
_1_2_3_4_5_6_7_8_9
101112131415161718
192021222324252627
282930313233343536
373839404142434445
-----
(9D15.6)

```

TABLE 3 Example Output From MLML

SIZE AND DEFINITIONS CARD
14 9 456 6 45 0 0 10 1 20 0.1000-05

ALLOCATED SHARES

0.25135400+00	0.46163600-02	0.57435100+00	0.20993300-01	0.0	0.14647900+00
0.18024700+00	0.47672400-01	0.14281100+00	0.24229700-01	0.12397000+00	0.41106900+00
0.67294200-03	0.16615800-01	0.30739300+00	0.13007000+00	0.56567700-01	0.48868100+00
0.30389600+00	0.0	0.14831100+00	0.38740100-01	0.14192400+00	0.36712800+00
0.68772400-01	0.34461800-01	0.29292500+00	0.48633400-01	0.12182500+00	0.43338300+00
0.14267300+00	0.0	0.67355700-01	0.10017000+00	0.54579600+00	0.14400500+00
0.45279900+00	0.0	0.60944000-01	0.10990100-01	0.0	0.47526700+00
0.42048000+00	0.0	0.10201400+00	0.82690000-02	0.77818200-01	0.39141900+00
0.11484100-02	0.10617000-01	0.40371100+00	0.41676100-01	0.29823900+00	0.24460800+00
0.14625900+00	0.27719500+00	0.11262800+00	0.17565900-01	0.15148200+00	0.29465000+00
0.21645900+00	0.15236400+00	0.12233000+00	0.92148900-01	0.12416200+00	0.29253700+00
0.27433100+00	0.19657000-01	0.22865800+00	0.32136400-01	0.0	0.44521800+00
0.0	0.91464100-01	0.26796700+00	0.49859100-01	0.0	0.50709000+00
0.23356500+00	0.37470300-02	0.15924900+00	0.34626800-01	0.11782300+00	0.45098900+00
0.36536000+00	0.13407700-01	0.42617400-01	0.35207200-01	0.17070900+00	0.37269900+00
0.23709100+00	0.23660500-01	0.58178800-01	0.19130300+00	0.12754000+00	0.36222800+00
0.12337200+00	0.47494600-01	0.30094200+00	0.56587100-01	0.15218200+00	0.31942200+00
0.20311600+00	0.11001500+00	0.13082000+00	0.57086400-01	0.88610600-01	0.41035200+00
0.15861900+00	0.30816900-01	0.22519000+00	0.10598600+00	0.11046100+00	0.37092600+00
0.17037000+00	0.01377700-01	0.11851500+00	0.12119600+00	0.26222500+00	0.26629100+00

AVERAGE VALUES

0.17897890+00	0.47947520-01	0.19597250+00	0.79449930-01	0.15958000+00	0.35807110+00
---------------	---------------	---------------	---------------	---------------	---------------

TABLE 3 (continued)

EXPLANATORY VARIABLES AFTER TRANSFORMATIONS

0.10000000+01	0.81083250+01	0.0	0.0	0.0	0.0	0.14758800-01	0.95978600+00	0.63737500+00
0.10000000+01	0.91767930+01	0.16094400+01	0.40225900+00	0.59831400+01	0.10699800+01	0.45682900-02	0.63362900+00	0.34462200+01
0.10000000+01	0.92903520+01	0.69314700+00	0.13916200-01	0.12830100+01	0.16629800+00	0.62416800-02	0.78714400+00	0.50792500+01
0.10000000+01	0.86370720+01	0.16094400+01	0.26971600-01	0.10365000+01	0.37370900+00	0.17928100-01	0.39028400+00	0.31914400+00
0.10000000+01	0.89721210+01	0.10986100+01	0.62416800-02	0.78714400+00	0.50792500-01	0.10521400+00	0.31768000+01	0.23587700+00
0.10000000+01	0.89721210+01	0.10986100+01	0.62416800-02	0.78714400+00	0.50792500-01	0.20343600-01	0.17286500+01	0.37361500+00
0.10000000+01	0.75935910+01	0.0	0.0	0.0	0.0	0.15794100-01	0.10460400+01	0.59525500+00
0.10000000+01	0.80256500+01	0.69314700+00	0.16941800-01	0.11497400+01	0.53919800+00	0.18307000-01	0.13390500+01	0.40044300+00
0.10000000+01	0.93327880+01	0.69314700+00	0.54076900-02	0.71345200+00	0.42383800-01	0.37311600-02	0.54726400+00	0.27071500+01
0.10000000+01	0.73626320+01	0.0	0.15455940-01	0.10180500+01	0.60929100+00	0.0	0.0	0.0
0.10000000+01	0.86053180+01	0.0	0.18288900-01	0.13622000+01	0.37311700+00	0.0	0.0	0.0
0.10000000+01	0.81488270+01	0.69314700+00	0.17354700-01	0.11962000+01	0.51124900+00	0.16407100-01	0.10946000+01	0.56720600+00
0.10000000+01	0.72039930+01	0.0	0.14983300-01	0.13255300+01	0.19043600+00	0.0	0.0	0.0
0.10000000+01	0.83771630+01	0.69314700+00	0.16684400-01	0.11531100+01	0.53319800+00	0.17911900-01	0.12583500+01	0.46947500+00
0.10000000+01	0.83372770+01	0.10986100+01	0.22648700-01	0.18867400+01	0.61799900+00	0.18277100-01	0.13254200+01	0.41417500+00
0.10000000+01	0.87275560+01	0.10986100+01	0.37311600-02	0.54726400+00	0.27071500-01	0.14697400+00	0.37422600+01	0.24882500+00
0.10000000+01	0.88552810+01	0.10986100+01	0.37311600-02	0.54726400+00	0.27071500-01	0.33580900+00	0.40029700+01	0.30145800+00
0.10000000+01	0.88070110+01	0.16094400+01	0.62416800-02	0.78714400+00	0.50792500-01	0.55008100+00	0.11722300+02	0.68987100+00
0.10000000+01	0.89221090+01	0.16094400+01	0.40523700-01	0.20637700+01	0.19689400+00	0.36047900+00	0.64177700+01	0.38262300+00
0.10000000+01	0.95060950+01	0.17917600+01	0.25606500-01	0.28828900+01	0.21683700+00	0.13997400-01	0.10413800+01	0.18959400+00

AVERAGE VALUES

0.10000000+01 0.90178290+01 0.95667440+00 0.36898590+01 0.12056230+00 0.26899460+00 0.46408910-01 0.66533340-02 0.28892430+00

POSITIVE INDEXES FOR PARAMETERS TO BE ESTIMATED

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
0	0	0	0	0	0	0	0	0

INITIAL LIKELIHOOD = -0.10000000+09

INITIAL PARAMETER ESTIMATES

1 -0.17204870+01

2	0.0
3	0.0
4	0.0
5	0.0
6	0.0
7	0.0
8	0.0
9	0.0
10	-0.30376480+01
11	0.0
12	0.0
13	0.0
14	0.0

TABLE 3 (continued)

15	0.0
16	0.0
17	0.0
18	0.0
19	-0.1629781D+01
20	0.0
21	0.0
22	0.0
23	0.0
24	0.0
25	0.0
26	0.0
27	0.0
28	-0.2532628D+01
29	0.0
30	0.0
31	0.0
32	0.0
33	0.0
34	0.0
35	0.0
36	0.0
37	-0.1835210D+01
38	0.0
39	0.0
40	0.0
41	0.0
42	0.0
43	0.0
44	0.0
45	0.0

ITERATION NUMBER 1	
VALUE OF NON-CONSTANT PORTION OF THE LIKELIHOOD FUNCTION = -0.8094136D+03,	TOLERANCE = 0.9999919D+00, FAC = 0.1000000D+01

ITERATION NUMBER 2	
VALUE OF NON-CONSTANT PORTION OF THE LIKELIHOOD FUNCTION = -0.7387817D+03,	TOLERANCE = 0.9726303D-01, FAC = 0.1000000D+01

ITERATION NUMBER 3	
VALUE OF NON-CONSTANT PORTION OF THE LIKELIHOOD FUNCTION = -0.7350993D+03,	TOLERANCE = 0.4984360D-02, FAC = 0.1000000D+01

ITERATION NUMBER 4	
VALUE OF NON-CONSTANT PORTION OF THE LIKELIHOOD FUNCTION = -0.7350872D+03,	TOLERANCE = 0.1651992D-04, FAC = 0.1000000D+01

ITERATION NUMBER 5	
VALUE OF NON-CONSTANT PORTION OF THE LIKELIHOOD FUNCTION = -0.7350872D+03,	TOLERANCE = 0.7622452D-09, FAC = 0.1000000D+01

TABLE 3 (continued)

PARAMETERS		EFF. SCORE	
- ESTIMATED INFORMATION LIMIT TO THE VARIANCE-COVARIANCE MATRIX (DET = -0.1746303D+34)			
-0.3099000+01-0.1840145D-06			
-0.6106476D+00-0.1341131D-05			
-0.5151680D+00-0.1756931D-06			
-0.1122807D+01-0.7961805D-08			
0.3769041D-01-0.4516300D-07			
-0.4290171D-01-0.3979202D-07			
-0.7440023D+00-0.8593756D-08			
-0.2766158D-02-0.3159325D-07			
0.3747709D+00-0.3765508D-07			
-0.4250442D+01 0.4684548D-06			
0.3341426D+00 0.4366702D-05			
-0.3053233D+00 0.6402437D-06			
-0.4337716D+00 0.2222825D-07			
0.1341752D-01 0.2810616D-06			
-0.1989130D+00 0.1179673D-06			
0.4988190D+00 0.3034732D-07			
-0.6320917D-02 0.6001638D-07			
-0.1467553D+01 0.1654347D-06			
0.7803233D+00 0.2739207D-06			
-0.9853670D-01 0.2429142D-05			
-0.2974633D+00 0.2026070D-06			
0.1918969D+00 0.9753651D-08			
0.7656942D-01 0.1672786D-07			
- ESTIMATED INFORMATION LIMIT TO THE VARIANCE-COVARIANCE MATRIX (DET = -0.1746303D+34)			
-0.5851474D+01			
-0.8662380D+00-0.8265864D-01			
-0.4551138D+00 0.6297911D-01-0.1373864D+00			
-0.4131814D+00 0.6339718D-01-0.1012898D+00-0.9972096D+01			
0.271724D-01-0.5763900D-02 0.1188309D-01-0.2263104D+00-0.9887652D-02			
0.2282450D+00-0.1476430D-01 0.9124444D-01 0.1199581D+01-0.2458786D-01-0.7769346D+00			
0.9178583D-01-0.2928662D-02 0.2104759D-01 0.5106782D+00-0.1311472D-02-0.2170401D+00-0.4812607D-01			
0.1992813D-01-0.3143757D-02 0.1044922D-01 0.1540047D-01-0.5390039D-03-0.1499934D-01 0.14266809D+00			
-0.9350186D-02			
-0.2327698D+00-0.1135950D-01 0.6279024D-02-0.2793999D+00-0.8488609D-02 0.1538084D+00 0.5913759D+06			
-0.1647555D-01-0.6416794D+00			
-0.2053917D+01 0.2363301D+00-0.1403808D+00-0.6830844D-01 0.1324073D-01 0.4349219D-01 0.16666934D-01			
0.6982305D-02 0.1195778D+00-0.1786337D+02			
0.2371820D+00-0.2793728D-01 0.1912759D-01 0.1256926D-01-0.1799378D-02-0.3059647D-02 0.3535522D-03			
-0.1056080D-02-0.8151776D-02 0.2043329D+01-0.2381214D+00			
-0.1422528D+00 0.1920449D-01-0.4335970D-01-0.2279817D-01 0.3843819D-02 0.2852410D-01 0.7039621D-02			
0.3456573D-02 0.7533473D-02-0.1167132D+01 0.1538321D+00-0.3418873D+00			
-0.7313307D-01 0.1315375D-01-0.2319950D-01-0.2933441D+01 0.7018287D-01 0.3583842D+00 0.2526293D+00			
0.9128346D-03-0.1081760D+00-0.4156725D+00 0.8797524D-01-0.1798209D+00-0.2429798D+02			
0.1349327D-01-0.1826296D-02 0.3845580D-02 0.7029738D-01-0.3482385D-02-0.7593151D-02-0.1949147D-02			
-0.8360787D-04-0.2913649D-02 0.1075381D+00-0.1484269D-01 0.5392345D-01 0.5738131D+00-0.2929578D-01			
0.4218908D-01-0.2598343D-02 0.2408030D-01 0.3572668D+00-0.7548412D-02-0.2619926D+00-0.8250387D-01			
-0.4761396D-02 0.6226444D-01 0.4116073D+00-0.2783637D-01 0.2141257D+00 0.3028435D+01-0.5637032D-01			
-0.2281689D+01			
0.1983347D-01 0.7573047D-04 0.7070537D-02 0.2501247D+00-0.1860536D-02-0.8306076D-01-0.1457337D+01			
0.4394250D-01 0.1894456D+00 0.2036624D+00-0.4296055D-02 0.4388929D-01 0.1897005D+01-0.5472689D-02			
-0.7242350D+00-0.1176851D+02			
0.6956076D-02-0.1049320D-02 0.3459375D-02 0.1034348D-02-0.9062830D-04-0.4790652D-02 0.4418229D-01			
-0.3116305D-02-0.5524126D-02 0.6494255D-01-0.9460091D-02 0.2832437D-01 0.1919391D-01-0.1325806D-02			
-0.4009417D-01 0.3540477D+00-0.2727507D-01			
0.9783825D-01-0.6778187D-02 0.6334200D-02-0.1072072D+00-0.2909611D-02 0.6376284D-01 0.1908569D+00			
-0.5481696D-02-0.2218414D+00 0.7949860D+00-0.5970485D-01 0.1286390D+00-0.1035229D+01-0.2935202D-01			
0.6316305D+00 0.1728867D+01-0.4726707D-01-0.2248848D+01			
-0.2176532D+01 0.2504972D+00-0.213735D+01 0.2468174D+00-0.1503520D+00-0.6969456D-01 0.1421314D-01			
0.6995769D-02 0.1046372D+00-0.213735D+01 0.2468174D+00-0.1503520D+00-0.6969456D-01 0.1421314D-01			
0.3707396D-01 0.1769393D-01 0.7357425D-02 0.1027616D+00-0.1672994D-02-0.5370786D-02-0.2123866D-03			
-0.2413195D+00-0.2950430D-01 0.1971047D-01 0.7357425D-02 0.1027616D+00-0.1672994D-02-0.5370786D-02-0.2123866D-03			
-0.1048182D-02-0.7143394D-02 0.2461924D+00-0.2916912D-01 0.2048349D-01 0.1297710D-01-0.1927845D-02			
-0.1916016D-02 0.3000317D-03-0.1120764D-02-0.6717377D-02 0.6650282D+00-0.7871700D-01			
-0.14921044D+00 0.2018891D-01-0.4487245D-01-0.2114309D-01 0.3789462D-02 0.2966406D-01 0.7295900D-02			
0.3559203D-02 0.6110867D-02-0.1474539D+00 0.2015635D-01-0.4562828D-01-0.2489737D-01 0.4019707D-01			
0.2877462D-01 0.6756056D-02 0.3631517D-02 0.5497412D-02-0.4183359D+00 0.5592764D-01-0.1249929D+00			
-0.5224081D-01 0.1071290D-01-0.2252891D-01-0.2996461D+01 0.7254352D-01 0.3694819D+00 0.2642886D+00			
0.6492866D-03-0.1263150D+00-0.6600647D-01 0.1288966D-01-0.2522543D-01-0.2966785D+01 0.7117398D-01			
0.3643465D+00 0.2581121D+00 0.1133693D-02-0.1136569D+00-0.2252416D+00 0.3902174D-01-0.7757405D-01			
-0.7352361D+01			
0.1297043D-01 0.1765477D-02 0.3865091D-02 0.7207286D-01-0.3600879D-02-0.7817104D-02-0.2192662D-02			
-0.6915225D-04-0.2960378D-02 0.1377441D-01-0.1871454D-02 0.4003505D-02 0.7113055D-01-0.3542076D-02			
-0.7591123D-02-0.1940721D-02-0.5058269D-04-0.3091091D-02 0.4098260D-01-0.5439399D-02 0.11266838D-03			
0.1800646D+00-0.9014917D-02			

-0.71113240+00 0.69765030-07
-0.46880580-01 0.11012150-07
0.24854160-01-0.15629610-07
0.17137490-01 0.73312050-07
-0.18940810+01-0.50653560-07
0.74909130-01-0.46213390-06
0.82762970-01-0.63646860-07
-0.19259240+00-0.20070920-08
0.76833400-02-0.32308430-07
-0.42719950+00-0.11617220-07
0.18880120+00-0.32270400-08
-0.10893740-01-0.29501800-08
0.38333570-01-0.20811230-02 0.28559190-01 0.36601550+00-0.77550310-02-0.27081330+00-0.86442670-01
-0.48864930-02 0.65966270-01 0.42713330-01-0.27403950-02 0.29373240-01 0.36565580+00-0.75828620-02
-0.27042460+00-0.84910490-01-0.50231850-02 0.67931430-01 0.12728720+00-0.98561430-02 0.96345590-01
0.92790300+00-0.20334490-01-0.71786540+00
-0.19164460-01 0.17644150-03 0.66083130-02 0.27094310+00-0.72789100-02-0.87064580-01-0.14843720+01
0.44839150-01 0.19613460+00 0.17404180-01 0.22133530-03 0.72417970-02 0.25941390+00-0.20648380-02
-0.83170900-01-0.14768660+01 0.44525340-01 0.19555340+00 0.43339030+01 0.57152430-03 0.23245120-01
0.64219880+00-0.67541660-02-0.21066380+00-0.38706740+01
0.72992370-02-0.10990470-02 0.35641020-02 0.17018610-03-0.58193320-04-0.49397520-02 0.44584940-01
-0.31700730-02-0.54250120-02 0.72249350-02-0.11078700-02 0.36364290-02 0.10664350-02-0.84866600-04
-0.49860730-02 0.44708990-01-0.31707640-02-0.52012390-02 0.19611400-01 0.30269500-02 0.10419910-01
0.34246370-02-0.33852330-03-0.13362720-01 0.12102840+00-0.87474810-02
0.10365260+00-0.71555580-02 0.66877130-02-0.11615350+00-0.28920330-02 0.68140570-01 0.19713030+00
-0.54713300-02-0.23301360+00 0.10745430+00-0.70370120-02 0.47491660-02-0.11311320+00-0.28839910-02
0.63620940-01 0.19499040+00-0.51962280-02-0.23901530+00 0.31614580+00-0.22136250-01 0.15000490-01
-0.27611350+00-0.66929060-02 0.13017520+00 0.49764800+00-0.14234700-01-0.60644730+00
-0.21133530+01 0.29382440+00-0.21651730+01 0.25004210+00-0.14836540+00-0.66823660-01 0.13730850-01
0.74132360-02 0.10695510+00-0.21651730+01 0.71324250-02 0.28908450-01-0.21319000-01 0.24620300+00-0.14760760+00
0.41072830-01 0.21898600-01 0.71324250-02 0.28908450-01-0.21319000-01 0.24620300+00-0.14760760+00
-0.65709260-01 0.13369240-01 0.42935570-01 0.18096000-01 0.72286500-02 0.10526950+00-0.11635390+02
0.24405620+00-0.28015630-01 0.20060370-01 0.12453320-01-0.18195350-02-0.33019750-02 0.41191160-03
-0.11146820-02-0.75539220-02 0.25004500+00-0.29569020-01 0.20235450-01 0.12631350-01-0.18646550-02
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-0.15768710+00
-0.14626160+00 0.19865810-01-0.44788750-01-0.23077860-01 0.39103810-02 0.29323460-01 0.70916210-02
0.35609660-02 0.68440070-02-0.14943060+00 0.23828290-01-0.45510460-01-0.24475260-01 0.39850340-02
0.28409850-01 0.71359370-02 0.36277830-02 0.46613500-02-0.14784120+00 0.20150140-01-0.45180910-01
-0.24673680-01 0.39615660-02 0.29094460-01 0.70726830-02 0.36037310-02 0.51125220-02-0.76935820+00
0.10496600+00-0.23354820+00
-0.66693570-01 0.12452820-01 0.23363630-01-0.29615110+01 0.71107520-01 0.36316410+00 0.25744210+00
0.88250800-03-0.11034300+00-0.66395590-01 0.12650620-01-0.24775980-01-0.29763290-01 0.71808260-01
0.36781010+00 0.26419970+00 0.10120290-02-0.11658080+00-0.66998040-01 0.12635530-01-0.24245970-01
0.29690520+01 0.71217790-01 0.36508120+00 0.26061900+00 0.96906600-03-0.11332600+00-0.22611430+00
0.68621980-01-0.14740430+00-0.15016850+02
0.13490670-01-0.18316650-02 0.39168790-02 0.71358880-01-0.35415600-02-0.76891100-02-0.19885080-02
-0.84770610-04-0.29806380-02 0.11376130-01-0.18660630-02 0.39854200-02 0.71391820-01-0.35606680-02
-0.76158780-02-0.20188260-02-0.86601810-04-0.30996400-02 0.13675780-01-0.18585800-02 0.39464830-02
0.71275750-01-0.35448760-02-0.76231370-02-0.20385900-02-0.83726850-04-0.29223570-02 0.71968470-01
-0.98093160-02 0.21223740-01 0.35214950+00-0.17718160-01
0.41821560-01-0.25412350-02 0.28663200-01 0.36111200+00 0.76141150-02-0.26716060+00-0.84278700-01
-0.48796390-02 0.64561440-01 0.43168750-01-0.27252690-02 0.29443260-01 0.36781460+00-0.76475110-02
-0.27319060+00-0.86787780-01-0.50409530-02 0.68307260-01 0.40455800-01-0.24486370-02 0.29174310-01
0.36564870+00-0.76886690-02-0.26384930+00-0.84569490-01-0.49809830-02 0.65325660-01 0.19489530+00
-0.11314900-01 0.15611700+00 0.19322720+01-0.35175100-01-0.15332020+01
0.19476080-01 0.12012940-03 0.69701490-02 0.25476470+00-0.19173920-02-0.84429360-01-0.14732070+01
0.44479790-01 0.19321840+00 0.21034310-01-0.12493080-03 0.70520690-02 0.26403960+00-0.20214900-02
-0.86580400-01-0.14904330+01 0.44486460+00 0.19890230+00 0.19057640-01 0.12033780-03 0.69787600-02
0.25374120+00-0.19931620-02-0.85176930-01-0.14791310+01 0.44686550-01 0.19558600+00 0.10194430+00
0.22609890-03 0.29482130-01 0.12354350+01-0.53062970-02-0.45676130+00-0.72750020+01
-0.71692370-02-0.10851190-02 0.35553100-02 0.93516240-03-0.67433220-04-0.49294870-02 0.44553840-01
-0.31541500-02-0.44524400-02 0.72334240-02-0.11002560-02 0.36383560-02 0.95179440-03-0.86260290-04
0.10433340-02-0.87611570-04-0.49594610-02 0.44603000-01-0.31683430-02-0.52468280-02 0.56603740-01
-0.56566110-02 0.18911030-01 0.12774450-01-0.68108300-03-0.28382250-01 0.21809520+00-0.16201650-01

TABLE 3 (continued)

-0.70265000+00-0.19151380-07	0.10105960+00-0.69202050-02	0.65444580+00-0.10998180+00-0.29733510-02	0.65622370-01	0.19357510+00
	0.54337260-02-0.22880690+00	0.10104970+00-0.63577250-02	0.51681980-02-0.11529910+00-0.30489310-02	0.67248380-01
	0.19867750+00-0.52472680-02	0.24341430+00-0.10226060+00-0.67006080-02	0.57072300-02	0.11309860+00-0.30511400-02
	0.668034610-01	0.19623280+00-0.53046170-02-0.23771250+00	0.57896150+00	-0.40136760-01
	0.33683720-01	0.63615730+00-0.18244670-01	0.41753850+00	0.10489310+01-0.24247790-01
	0.13998050+01	0.24049340+00-0.14614940+00-0.82368160-01	0.14314740-01	0.46761410-01
	0.18725340-01	0.20849520+01	0.10857040+00-0.21832000+01	0.25216890+00-0.14907650+00-0.62456010-01
	0.13430320-01	0.71640950-02	0.21064330-02	0.21151230+01
	0.24405400+00-0.16619060+00	0.19793120-01	0.72164330-02	0.97666800-01
	0.69470420-01	0.13792070-01	0.42493080-01	0.19200900-01
	0.24884580+00-0.14835560+00-0.66567810-01	0.13627500-01	0.42830640-01	0.28050910-01
	0.71927210-02	0.10166560+00-0.65152570+01	0.14149900-01	0.19249410-02-0.51734470-02
	0.11730640-03	0.24077490+00-0.28427480-01	0.19866190-01	0.12433000-01
	0.18275270-02	-0.10423540-02-0.78283720-02	0.25230740+00-0.29832890-01	0.20347950-01
	0.12143000-01	-0.25383850-02-0.12529320-03	0.11089210-02-0.58103090-02	0.24439560+00-0.28881980-01
	0.19943410-01	0.12963290-01-0.18649240-02	0.28268530-02	0.55156500-04-0.10853460-02-0.69457440-02
	0.24887830+00	-0.29421390-01	0.20222570-01	0.12594650-01-0.18508330-02-0.26765880-02
	0.10070660-02	-0.64819290-02	0.79381250+00-0.93148170-01	0.24334770-01
	0.39557570-02	-0.14473000+00	0.19664820-01-0.44467650-01	0.28942160-01
	0.71287100-02	0.35231790-02	0.71781480-02-0.15057590+00	0.24570140-01
	0.45781430-01	0.29244430-01	0.71426100-02	0.36698850-02
	0.40971700-02	0.40971700-02	0.14720700+00	0.20075220-01
	0.45010120-01	-0.24522340-01	0.39307830-02	0.29118530-01
	0.71352730-02	0.39714690-02	0.29149300-01	0.47984450-02-0.14809530+00
	0.36195800-02	0.20194640-01-0.45346960-01	0.24852130-01	0.13671740+00
	0.52066050-02-0.44957360+00	0.60965530-01	0.13671740+00	0.68677710-01
	0.35302050+00	0.25923330+00	0.73426110-01	0.13145220-01
	0.23456860-01-0.28738340+01	0.68677710-01	0.12664370-01	0.24878400-01
	0.29309080+01	0.71633490-01	0.74573350-03	0.10868640+00-0.66103860-01
	0.13025360-01-0.13025360-01	0.70372470-01	0.11890300+00	0.11946700+00
	0.66745800-01	-0.29613600-01	0.71113180-01	0.36599800+00
	0.71230660-01	0.12677980-02	0.12677980-02	0.11946700+00
	0.66745800-01	0.12626810-01	0.24556130-01	0.71230660-01
	0.84167460+01	-0.11512210+00-0.16244880+00	0.33552320-01	0.65251270-01
	0.84167460+01	0.13826880-01-0.18698460-02	0.39131030-02	0.68744470-01
	0.34462260-02-0.71768470-01	0.34462260-02	0.10647870-02	0.39861300-02
	0.39861300-02	-0.86284000-04-0.29719290-02	0.13704610-01	0.18647870-02
	0.18647870-02	-0.76221330-02-0.20728010-02	0.82922770-04-0.30061070-02	0.13859590-01
	0.18800150-02	0.39449060-02	0.76221330-02	0.29205490-02
	0.13642900-01	0.70742720-01	0.35163630-02	0.75319980-02
	0.91864800-02	0.19166480-02	0.19166480-02	0.20301010-02
	0.65264210-04	-0.18541350-02	0.39616280-02	0.71383730-01
	0.55514990-02	0.76231140-02	0.20301010-02	0.11416780-01
	0.11416780-01	-0.30124800-02	0.40161940-01	0.55803730-02
	0.12320590-01	0.20705300+00-0.11416780-01	0.35215580+00	0.73525920-02
	0.28415680+88-8	0.83429550-01	0.42237080-01	0.26613830-02
	0.28415680+88-8	0.48340340-02	0.63688930-01	0.42685240-01
	0.28415680+88-8	-0.27647520+00-0.87837370-01	0.51192490-02	0.29645520-01
	0.57138480+00-0.76776360-02	0.36460410+00-0.76378340-02	0.26957750+00-0.83894830-01	0.50006230-02
	0.49812040-02	-0.25423820-02	0.29020650-01	0.36754340+00-0.27207060+00-0.86332700-01
	0.49812040-02	0.66965880-01	0.14599500+00-0.89751250-02	0.82059020-01
	0.20933860-01	0.20216150-01	0.21992430-06	0.70631940-02
	0.23061300-02	0.85316150-01	0.26857140+00-0.23061300-02	0.85316150-01
	0.14637260+01	0.44115810-01	0.19187790+00	0.21745130-01
	0.20075540-02	0.20075540-02	0.20075540-02	0.20075540-02
	0.20075540-02	0.18995730-01	0.18995730-01	0.15266270-03
	0.69166280-02	0.87893800-01	0.14895460+01	0.44688460-01
	0.19943410-01	0.87893800-01	0.14895460+01	0.44688460-01
	0.44688460-01	0.26087420+00-0.19877200-02	0.85396950-01	0.44547910-01
	0.19487730+00	0.20775410-01	0.70443500-02	0.26437780+00-0.20520400-02
	0.14836810+01	0.84949250-01	0.19724930+00	0.10613930+00-0.49359790-02
	0.19529190-01	0.26510510+00-0.77176250-02	0.31881020+00	0.478499040+01
	0.478499040+01	-0.69997040-02	0.10630760-02	0.35179760-02
	0.74468120-03-0.86261990-04-0.48422050-02	0.48422050-02	0.48422050-02	0.48422050-02
	0.48422050-02	-0.51340900-02-0.54799830-02	0.72661960-02-0.11377200-02	0.36707750-02
	0.10359150-02	0.10359150-02	0.36707750-02	0.36707750-02
	0.09610650-04	-0.31212030-02	0.46494010-01	0.32012930-02
	0.51426380-02	0.51426380-02	0.71188800-02	0.10894050-02
	0.35794210-02	0.10353270-02	0.87683510-04	0.49615080-02
	0.49615080-02	0.49615080-02	0.49615080-02	0.31922990-02
	0.71751490-02	-0.10978160-02	0.36166510-02	0.98192330-03
	0.87841040-04	0.87841040-04	0.50047150-02	0.44630950-01
	0.31943300-02	0.32513620-02	0.21516210-01	0.32857780-02
	0.10955070-01	0.27948510-02	0.14053060-03	0.13793060-01
	0.13793060-01			

TABLE 3 (continued)

-0.1047459D+01-0.1228181D-06
 0.1370355D+00-0.9878522D-02
 0.1003927D+00-0.6925554D-02
 -0.3430112D-02-0.2259047D+00
 0.7009400D-01 0.1992100D+00-0.5154110D-02
 -0.1132597D+00-0.3049004D-02
 -0.6369358D-02 0.5251617D-02
 -0.2410745D+00 0.3018302D+00
 0.6665220D+00-0.1623559D-01
 0.9842418D-01-0.5995988D-02
 0.4810771D-02-0.1168798D+00
 0.1018999D+00-0.6680200D-02
 0.1953733D+00-0.5218580D-02
 0.6739721D-01 0.1147578D+00
 0.3069184D-02 0.6779704D-01
 0.3201049D-01-0.4110071D+00
 0.1123550D-01 0.2723792D+00
 0.2973665D-02 0.6460909D-01
 0.1922372D+00
 0.5478091D-02
 0.1003753D+00
 0.5265801D-02
 0.2723792D+00

TOLERANCE OF 0.7622452D-09 MEETS CRITERION OF 0.1000000D-05

***** PROGRAM ENDED AFTER ITERATION NO. 5. MAXIMUM ALLOWED = 10 *****

PARAMETERS	STANDARD ERRORS	T-VALUES
1 0.4309900D+01	0.2410982D+01	0.1781700D+01
2 -0.6106476D+00	0.2875042D+00	0.2123961D+01
3 0.5151680D+00	0.3706567D+00	0.1349881D+01
4 -0.1122807D+01	0.3157863D+01	0.355591D+00
5 0.376901D-01	0.9943667D-01	0.3790393D+00
6 -0.4290171D-01	0.8814389D+00	0.4467236D-01
7 -0.7840023D+00	0.2193765D+01	0.3391440D+00
8 -0.2766158D-02	0.9469636D-01	0.2660665D-01
9 0.3747790D+00	0.8010490D+00	0.4678502D+00
10 -0.4250482D+01	0.4226508D+01	0.1005663D+01
11 0.3341426D+00	0.4879768D+00	0.6847510D+00
12 -0.3053233D+00	0.5847115D-00	0.5221779D+00
13 -0.4337716D+00	0.4429299D+01	0.8799865D-01
14 0.1341752D-01	0.1711601D+00	0.7839160D-01
15 -0.1989130D+00	0.1510526D+01	0.1316846D+00
16 0.4988190D+00	0.3430527D+01	0.1454060D+00
17 -0.6328917D-02	0.1651517D+00	0.3832185D-01
18 -0.1467553D+01	0.1499616D+01	0.9786192D+00
19 0.7803233D+00	0.2395861D+01	0.3256963D+00
20 -0.9853630D-01	0.2805655D+00	0.3512975D+00
21 -0.2974633D+00	0.3535534D+00	0.8413771D+00
22 0.1918969D+00	0.2711524D+01	0.7077086D+01
23 0.7656942D-01	0.9494692D+01	0.8064445D+00
24 -0.7111354D+00	0.8472694D+00	0.8393226D+00
25 -0.4688059D-01	0.1467403D+01	0.2382866D-01
26 0.2485416D-01	0.9352797D-01	0.2657404D+00
27 0.1713749D-01	0.7787475D+00	0.2200649D-01
28 -0.1894081D+01	0.3413999D+01	0.5547984D+00
29 0.7490913D-01	0.3970989D+00	0.1886412D+00
30 0.8276297D-01	0.4832682D+00	0.1712568D+00
31 -0.1925924D+00	0.5875158D+01	0.4969923D-01
32 0.7683340D-02	0.1331096D+00	0.5772191D-01
33 -0.4271995D+00	0.1238225D+01	0.3450095D+00
34 0.1888012D+00	0.2697221D+01	0.6999841D-01
35 -0.1089574D-01	0.1272857D+00	0.8560066D-01
36 -0.7026500D+00	0.1183132D+01	0.5938897D+00
37 -0.1964636D+01	0.2629687D+01	0.7470988D+00
38 0.1653724D+00	0.3852016D+00	0.5418460D+00
39 0.6450794D-01	0.3697532D+00	0.1744622D+00
40 0.8416883D+00	0.2901163D+01	0.2901210D+00
41 -0.6863219D-01	0.1068494D+00	0.6423267D+00
42 -0.2996562D+00	0.9511129D+00	0.3150589D+00
43 -0.6552369D-01	0.2187462D+01	0.2995421D-01
44 -0.3458014D-02	0.9939076D-01	0.3479211D-01
45 -0.1047459D+01	0.9302779D+00	0.1125963D+01

cont'd.

III. A Computer Algorithm for Computing Elasticities and Performing Sensitivity Analysis on the Multinomial Logit Allocation Model (MLAM)

1. The Algorithm

The program described here calculates the elasticities and allocations from estimated coefficients of the multinomial logit model at user-specified levels of the explanatory variables. The user provides estimates of the coefficients and indicates the normalization which was used in their estimation.

For example, suppose that the model was normalized by constraining all parameters of the N^{th} equation to zero. (This is equivalent to using the N^{th} category as the denominating allocation when a least squares approximation is used.) The ratio of each of the other $N-1$ allocations to the N^{th} allocation is predicted from the input coefficients (B_{ik}) and specified levels of the explanatory variables (X_k):

$$(9) \quad \ln(\widehat{w_i/w_N}) = \hat{B}_{i1}X_1 + \hat{B}_{i2}X_2 + \dots + \hat{B}_{iK}X_K \quad i = 1, \dots, N-1$$

The N^{th} allocation is predicted by:

$$(10) \quad \hat{w_N} = 1 / (1 + \sum_{j=1}^{N-1} e^{\ln(\widehat{w_j/w_N})})$$

and the other $N-1$ allocations are predicted by:

$$(11) \quad \hat{w_j} = \hat{w_N} e^{\ln(\widehat{w_j/w_N})} \quad j = 1, \dots, N-1$$

For the budget allocation application, price (P_N), expenditure (M) and other (P_j, Z , etc.) elasticities for the N^{th} good are predicted by:

$$(12) \quad \hat{\eta}_{Ny} = -y \sum_{j=1}^{N-1} \hat{w_j} \frac{\partial \hat{f}_j}{\partial y} + \begin{cases} 1 & \text{if } y = M, \\ -1 & \text{if } y = P_N, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

and for other $N-1$ goods by:

$$(13) \quad \hat{\eta}_{jy} = \hat{\eta}_{Ny} + y \frac{\partial \hat{f}_j}{\partial y} \quad j = 1, \dots, N-1$$

$$\text{where } \frac{\partial \hat{f}_j}{\partial y} = \frac{\partial (\hat{B}_{j1}X_1 + \hat{B}_{j2}X_2 + \dots + \hat{B}_{jK}X_K)}{\partial y}$$

and $y = M, P_1, \dots, P_N$ or Z .

The calculation of the elasticities requires the derivatives of the function f_i with respect to each of the explanatory variables. To accomplish this, the derivatives of a number of commonly-used transformations are built into the program. Thus the calculations are currently limited to functions f_i which are linear in the variables themselves, their logarithms, their inverses and a cubic Lagrangian interpolation polynomial. Additional transformations may be added only by editing the Fortran source program.

The output of the program consists of a verification of the input coefficients of the model, and tables and graphs of the calculated elasticities and budget shares.

2. Program Input

The input for the program MLAM consists of 9 types of control cards in addition to standard job control language. The program currently allows for up to 20 allocation categories, 20 explanatory variables and 10 terms in each equation. The control cards are described on the next few pages; Table 4 shows how to set up the card deck when the source deck of MLAM has been previously stored in the user's library.

TABLE 4 Input for MLAM^a

Hasp Job Card

/*LIMITS_REGION=60K,CLASS=A

//_EXEC_FTG1CLG

/*I_MLAM

//GØ.SYSIN_DD_*

1. Size and Definitions Card (3I4)

2. Group Labels Card (20A4)

3. Average Group Allocations Card (10F8.0) (1 or 2 of these)

4. Variable Labels Card (20A4)

5. Average Variable Values Card (10F8.0) (1 or 2 of these)

6. Types of Variables Card (20I4)

7. Equation Definitions Card (2I4) } N-1 pairs of these

8. Coefficients Card (8(F6.0,2I2)) }

9. Sensitivity Analysis Card (2I2,2F8.0,20I2,2F10.0) (Optional number of these).

/*

//

^a Alphabetic Ø's are slashed, zeros (0) are not.
Blanks are denoted by_.

Right-adjust everything except labels of cards 2 and 4.

The JCL is specific for the Cornell University Computer System.

Description of Input Cards 1. - 9.

1. Size and Definitions Card (Format = 3I4)

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-4	N-1	number of estimated equations
5-8	ND	number of different variables (before transformation) ≤ 20
9-12	c	number of the common group, positive if a common denominator (equation (1)), negative of a common numerator (equation (2)). c = N for estimates from MLML.

Group numbers correspond to the labels of the next card.

2. Group Labels Card (20A4)

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-4	ALAB	four character label for group number 1
5-8	BLAB	four character label for group number 2
...		
77-80	TLAB	four character label for group number 20

The first letters of these labels are used as the symbol for the group in the plots of shares and elasticities, so at least make the first letters different.

3. Average Group Allocations Card (10F8.0)

<u>Card</u>	<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1	1-8	w(1)	approximate value of share allocated to group 1.
1	9-16	w(2)	approximate value of share allocated to group 2.
...			
2	73-80	w(20)	approximate value of share allocated to group 20.

The second card is only needed if there are more than ten groups. These values should add to unity. At present these values are not used in the calculations so they may be rough estimates.

4. Variable Labels Card

(20A4)

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-4	ANAM	name of the 1st untransformed variable
...		
77-80	TNAM	name of the 20 th untransformed variable

5. Average Variable Values Card

(10F8.0)

These are average or typical values of the variables at which elasticities and allocations will be computed (maximum 20). For sensitivity analysis, these define the initial fixed levels of variables not under analysis - they may be changed later. These are values of the untransformed variables.

<u>Card</u>	<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1	1-8	VALA	typical value of 1st variable
1	9-16	VALB	typical value of 2nd variable
...			
2	73-80	VALT	typical value of 20 th variable

6. Types of Variables Card

(20I4)

If the dependent variable (allocation) is composed of variables which are also included on the right-hand side (e.g. $w_i = P_i Q_i / M$, or $w_i = R_i / R$ where $f(X) = f(P, M, \text{ or } R)$) the corresponding elasticities and budget shares will be affected. These special cases can be handled by using this card. The 20 columns (I4) correspond to the NA variables. If the j^{th} explanatory variable is also a multiplier in the numerator of the i^{th} dependent share $w(i)$ (e.g. $X(j) = P_i$ and $w_i = P_i Q_i / M$) then put an "1" in the j^{th} column of this card. If the j^{th} explanatory variable is a multiplier in the denominator of every dependent allocation, then a "-1" should appear in the m^{th} column (e.g. $X(j) = M$ and $w_i = P_i Q_i / M$).

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-4	IVCA	"1", "-1" or blank indicating type of first variable,
5-8	IVCB	"1", "-1" or blank indicating type of second variable,
...		
77-80	IVCT	"1", "-1" or blank indicating type of 20 th variable, where i is the number of the share where this variable is in the numerator.

7. Equation Definition Card

(2I4)

There are N-1 pairs of cards 7. and 8. This card describes the equation whose coefficients are given on the next card.

Col.	Internal Label	Description
1-4	i	number of uncommon share in the estimating equation ($i \neq c$)
5-8	K(i)	number of terms in the estimating equation (including the constant), ≤ 10 .

8. Coefficients Card

(8(F6.0,2I2))

This card always follows a card of type 7. For each term in the i^{th} equation it gives an estimated linear coefficient, a transformation code and the original variable code number. There will be two of these cards for one card no. 7 when $K(i) > 8$.

Col.	Internal Label	Description
1-6	COEF1	coefficient of the first term in this equation
7-8	FN	transformation code (see below)
9-10	IV	variable number (according to card 4.)
11-16	COEF2	coefficient of the second term
17-18	FN	transformation code
19-20	IV	variable number
...	...	...

Transformation codes available at the present time are:

Code	Transformation	Description
1	$X = 1.0$	Constant
2	$X = y$	linear
3	$X = \ln(y)$	logarithmic
4	$X = 1/y$	inverse
5-8	$L_1(y) - L_4(y)$	logarithmic interpolation variables for a cubic function with reference points at 0.0, 2.7568, 3.0796 and 4.6225

9. Sensitivity Analysis Card

(2I2,2F8.0,20I2,2F10.0)

The above cards completely specify the model and one level of each variable for calculations of a typical or average level for allocations and elasticities. This card permits changes in the fixed levels of the variables if ID = 0, or directs output of the tables and graphs of 50 allocations and elasticities over a user-specified range of the abscissa of one variable (From X1 to X2) if ID = 1, 2 or 3. For most visually appealing graphs design the range X2-X1 to be equally divisible by 49. (e.g. 200. to 10000., rather than 0. to 10000.) The PA-PT permits the user to choose the order for cumulative share plotting. Default for plotting (all blanks) is the order implied by card 2. E1 and E2 define the range of the ordinate for plotting elasticities.

<u>Col.</u>	<u>Internal Label</u>	<u>Description</u>
1-2	ID	identification code (see below)
3-4	IV	variable code number (see card 4.)
5-12	X1	minimum of X(IV) or new fixed value
13-20	X2	maximum of X(IV)
21-22	PA	no. of share to be plotted first
23-24	PB	no. of share to be plotted second
...	...	...
59-60	PT	no. of share (if 20) to be plotted last
61-70	E1	minimum of range for plotting elasticities
71-80	E2	maximum of range for plotting elasticities

Identification codes

<u>ID</u>	<u>Definition</u>
0	set fixed value of X(IV) to X1 and print all own and cross elasticities at the new set of fixed values.
1	compute and print table of 50 allocations and elasticities varying X(IV) from X1 to X2.
2	same as 1 but graph <u>instead</u> of table output.
3	same as 1 <u>plus</u> graph of results.

3. An Example Run of MLAM

To illustrate the use of this program, the complete output from MLAM is presented in Table 6 for the analysis of the model estimated using MLML in Part II.

Six categories of expenditures were defined by equation (8) with three explanatory variables: total expenditures, household size and age. The single age variable limits this analysis to households of same-aged individuals, but permits easy calculation of household age elasticities. Constants were generated by transformation number 1, the total expenditures variable was transformed into its logarithm by transformation number 3, and the age variable was transformed into two pairs of three Lagrangian interpolation variables for males and females.

In addition to the automatic output of predicted budget shares and elasticities for average values, a sensitivity analysis was requested on total expenditures (INCM) for the range of values from \$500 to \$25000.

Input is shown in Table 5. The output presented as Table 6 consists of:

1. A verification of the input share categories, average values and codes; the input variables and codes; and a listing of currently available transformations and their codes.
2. A verification of the complete model with coefficients, transformations and variables.
3. Predicted allocation shares and elasticities with respect to each variable for each category evaluated at the initial values of the variables.
4. Tables of all allocated shares and elasticities with respect to INCM over the range from \$500 to \$25000.
5. A plot of cumulative shares over the range \$500 to \$25000 where incremental shares are designated by the first letter of their label.
6. A plot of INCM elasticities for the range from \$500 to \$25000 with the range of the ordinate specified as .52 to 1.5.

The GO step for this particular run used a region of 58K for 1.5 CPU seconds and 2.5 I/O seconds on Cornell's IBM 360.

TABLE 5 Sample Input for MLAM

___5___3___6

FØØDAWAYHØUSCLTHTRNSØTHER

___0.1790___0.0479___0.1960___0.0794___0.1596___0.3381

INCMSIZEPAGEFAGE

___10000___4___45.

___1___9

_4.310_1_0_0.334_3_1-0.305_3_2-0.434_5_3_.0134_6_3-0.711_8_3-.0469_5_3_.0249_6_3

_1.468_8_3

___3___9

_0.780_1_0-.0985_3_1-0.297_3_2_0.192_5_3_.0766_6_3-0.711_8_3-0.469_5_3_.0249_6_3

_.0171_8_3.

___4___9

-1.894_1_0_.0749_3_1_.0828_3_2-0.193_5_3_.0077_6_3-0.427_8_3_0.189_5_3-.0109_6_3

-0.703_8_3

___5___9

-1.964_1_0_0.165_3_1_.0645_3_2_0.841_5_3-0.686_6_3-0.300_8_3-0.655_5_3-.0035_6_3

-1.047_8_3

_3_1___500.0_25000.0_____0.52_____1.50

TABLE 6 Output from MLAM

MULTINOMIAL LOGIT ALLOCATION MODEL (MLAM)

PROGRAM TO COMPUTE, PRINT AND PLOT ELASTICITIES AND ALLOC. SHARES

TIM TYRRELL, DEPT. OF AG. ECON., CORNELL U., 5/22/78

COMPONENTS OF THIS MLAM

INPUT SHARES				INPUT VARIABLES		AVAILABLE FUNCTIONS	
CODE	LABEL	MEAN	CODE	LABEL	FIXED VALUE	CODE	FUNCTION
1	FOOD	0.179	1	INCM	0.10000E 05	1	CONSTANT
2	AWAY	0.048	2	SIZE	0.20000E 01	2	*LN()
3	HOUS	0.196	3	MAGE	0.45000E 02	3	*LN()
4	CLTH	0.079	4		0.0	4	*LN()
5	TRNS	0.160	5		0.0	5	*LN()
6	OTHER	0.338	6		0.0	6	*LN()
7		0.0	7		0.0	7	*LN()
8		0.0	8		0.0	8	*LN()

MODEL			
(1)	LN(W /W)	= 4.3100	-0.6110*LN(INCM) 0.5150*LN(SIZE) -1.1230*LO(MAGE) 0.0377*LO(L1(MAGE)
	LN(W /W)	= -4.2500	-0.0429*LO(L3(MAGE) -0.7440*LO(MAGE) -0.0028*LO(L1(MAGE) 0.3750*LO(L3(MAGE)
(2)	LN(W /W)	= -4.2500	0.3340*LN(INCM) -0.3050*LN(SIZE) -0.4340*LO(MAGE) 0.0134*LO(L1(MAGE)
	LN(W /W)	= 0.7800	-0.1990*LO(L3(MAGE) 0.4990*LO(MAGE) -0.0063*LO(L1(MAGE) -1.4680*LO(L3(MAGE)
(3)	LN(W /W)	= -1.8940	-0.0985*LN(INCM) -0.2970*LN(SIZE) 0.1920*LO(MAGE) 0.0766*LO(L1(MAGE)
	LN(W /W)	= -0.7110*LO(L3(MAGE)	-0.0469*LO(MAGE) 0.0249*LO(L1(MAGE) 0.0171*LO(L3(MAGE)
(4)	LN(W /W)	= -1.8940	0.0749*LN(INCM) 0.0828*LN(SIZE) -0.1930*LO(MAGE) 0.0077*LO(L1(MAGE)
	LN(W /W)	= -0.4270*LO(L3(MAGE)	0.1890*LO(MAGE) -0.0109*LO(L1(MAGE) -0.7030*LO(L3(MAGE)
(5)	LN(W /W)	= -1.9640	0.1650*LN(INCM) 0.0645*LN(SIZE) 0.8410*LO(MAGE) -0.0686*LO(L1(MAGE)
	LN(W /W)	= -0.3000*LO(L3(MAGE)	-0.0655*LO(MAGE) -0.0035*LO(L1(MAGE) -1.0470*LO(L3(MAGE)

ALLOC. SHARES AND ELASTICITIES FOR FIXED VALUES

ALLOC. SHARES					
ELASTICITY WRT					
AT					
INCM	AWAY	HOUS	CLTH	TRNS	OTHER
10000.000	1.3769	0.9444	1.1178	1.2079	1.0429
2.000	-0.3206	-0.3126	0.0672	0.0489	-0.0156
45.000	-0.0581	-0.0116	-0.0302	-0.0401	0.0274

TABLE 6 (continued)

INCH	FOOD			AWAY			HOUS			CLTH			TRNS		
	SHARE	ELAST	ELAST	SHARE	ELAST	ELAST	SHARE	ELAST	ELAST	SHARE	ELAST	ELAST	SHARE	ELAST	ELAST
500.00	0.50988	0.6963	0.6258	0.01304	1.6413	1.5155	0.17092	1.1383	1.2088	0.05146	0.4146	0.822	0.7064	0.9564	0.721
1000.00	0.34370	0.5854	0.5579	0.02481	1.5304	1.17930	0.18369	1.0979	1.1383	0.05234	0.4273	0.317	0.1163	0.9564	0.4018
1500.00	0.30745	0.5374	0.5174	0.03213	1.4824	1.18617	0.18758	1.0499	1.0704	0.06331	0.5394	0.224	0.1133	0.9564	0.3339
2000.00	0.25196	0.5282	0.5082	0.03760	1.4532	1.18877	0.18888	1.0307	1.0338	0.06700	0.5700	0.207	0.1020	0.9564	0.2847
2500.00	0.21809	0.4979	0.4798	0.04203	1.4229	1.18881	0.18881	1.0094	1.0094	0.07266	0.5756	0.183	0.0963	0.9564	0.2655
3000.00	0.19519	0.4726	0.4525	0.04577	1.4111	1.18861	0.18861	0.9851	0.9851	0.07820	0.5820	0.165	0.0914	0.9564	0.2442
3500.00	0.17730	0.4605	0.4405	0.04745	1.4055	1.18794	0.18794	0.9780	0.9780	0.08021	0.5851	0.142	0.0859	0.9564	0.2253
4000.00	0.16315	0.4553	0.4353	0.04952	1.3956	1.18708	0.18708	0.9631	0.9631	0.08197	0.5871	0.125	0.0800	0.9564	0.2086
4500.00	0.15161	0.4462	0.4262	0.05196	1.3833	1.18612	0.18612	0.9548	0.9548	0.08238	0.5883	0.108	0.0744	0.9564	0.1941
5000.00	0.14198	0.4386	0.4186	0.05457	1.3801	1.18511	0.18511	0.9444	0.9444	0.08338	0.5893	0.093	0.0688	0.9564	0.1819
5500.00	0.13378	0.4319	0.4119	0.05809	1.3769	1.18460	0.18460	0.9386	0.9386	0.08439	0.5903	0.080	0.0633	0.9564	0.1719
6000.00	0.12666	0.4261	0.4061	0.06112	1.3718	1.18358	0.18358	0.9334	0.9334	0.08538	0.5913	0.068	0.0578	0.9564	0.1629
6500.00	0.12047	0.4209	0.4009	0.06402	1.3653	1.18256	0.18256	0.9287	0.9287	0.08636	0.5923	0.057	0.0523	0.9564	0.1549
7000.00	0.11501	0.4162	0.3962	0.06639	1.3592	1.18154	0.18154	0.9246	0.9246	0.08733	0.5933	0.047	0.0468	0.9564	0.1479
7500.00	0.11017	0.4120	0.3920	0.06840	1.3535	1.18051	0.18051	0.9208	0.9208	0.08830	0.5943	0.038	0.0413	0.9564	0.1419
8000.00	0.10570	0.4083	0.3883	0.07029	1.3482	1.17948	0.17948	0.9174	0.9174	0.08926	0.5953	0.030	0.0358	0.9564	0.1369
8500.00	0.10176	0.4049	0.3849	0.07169	1.3432	1.17845	0.17845	0.9142	0.9142	0.09021	0.5963	0.023	0.0313	0.9564	0.1319
9000.00	0.09846	0.4027	0.3827	0.07306	1.3384	1.17742	0.17742	0.9112	0.9112	0.09116	0.5973	0.017	0.0268	0.9564	0.1269
9500.00	0.09488	0.4010	0.3810	0.07436	1.3339	1.17638	0.17638	0.9085	0.9085	0.09208	0.5983	0.012	0.0223	0.9564	0.1219
10000.00	0.09183	0.3966	0.3766	0.07554	1.3296	1.17535	0.17535	0.9057	0.9057	0.09298	0.5993	0.008	0.0178	0.9564	0.1169
10500.00	0.08897	0.3932	0.3732	0.07673	1.3256	1.17432	0.17432	0.9035	0.9035	0.09386	0.6003	0.005	0.0133	0.9564	0.1119
11000.00	0.08636	0.3887	0.3687	0.07784	1.3217	1.17328	0.17328	0.9012	0.9012	0.09473	0.6013	0.003	0.0088	0.9564	0.1069
11500.00	0.08393	0.3846	0.3646	0.07897	1.3180	1.17225	0.17225	0.8981	0.8981	0.09559	0.6023	0.001	0.0043	0.9564	0.1019
12000.00	0.08166	0.3806	0.3606	0.07951	1.3146	1.17122	0.17122	0.8952	0.8952	0.09644	0.6033	0.000	0.0000	0.9564	0.0969
12500.00	0.07951	0.3772	0.3572	0.08000	1.3114	1.17019	0.17019	0.8923	0.8923	0.09729	0.6043	0.000	0.0000	0.9564	0.0919
13000.00	0.07747	0.3743	0.3543	0.08044	1.3084	1.16916	0.16916	0.8894	0.8894	0.09814	0.6053	0.000	0.0000	0.9564	0.0869
13500.00	0.07554	0.3719	0.3519	0.08083	1.3056	1.16813	0.16813	0.8865	0.8865	0.09899	0.6063	0.000	0.0000	0.9564	0.0819
14000.00	0.07370	0.3698	0.3498	0.08117	1.3030	1.16710	0.16710	0.8836	0.8836	0.09984	0.6073	0.000	0.0000	0.9564	0.0769
14500.00	0.07195	0.3679	0.3479	0.08146	1.3006	1.16607	0.16607	0.8807	0.8807	0.10069	0.6083	0.000	0.0000	0.9564	0.0719
15000.00	0.07029	0.3661	0.3461	0.08170	1.2984	1.16504	0.16504	0.8778	0.8778	0.10154	0.6093	0.000	0.0000	0.9564	0.0669
15500.00	0.06870	0.3644	0.3444	0.08189	1.2963	1.16401	0.16401	0.8749	0.8749	0.10239	0.6103	0.000	0.0000	0.9564	0.0619
16000.00	0.06716	0.3628	0.3428	0.08204	1.2944	1.16298	0.16298	0.8720	0.8720	0.10324	0.6113	0.000	0.0000	0.9564	0.0569
16500.00	0.06567	0.3613	0.3413	0.08215	1.2926	1.16195	0.16195	0.8691	0.8691	0.10409	0.6123	0.000	0.0000	0.9564	0.0519
17000.00	0.06422	0.3599	0.3399	0.08222	1.2909	1.16092	0.16092	0.8662	0.8662	0.10494	0.6133	0.000	0.0000	0.9564	0.0469
17500.00	0.06281	0.3586	0.3386	0.08226	1.2894	1.16000	0.16000	0.8633	0.8633	0.10579	0.6143	0.000	0.0000	0.9564	0.0419
18000.00	0.06144	0.3574	0.3374	0.08227	1.2880	1.15907	0.15907	0.8604	0.8604	0.10664	0.6153	0.000	0.0000	0.9564	0.0369
18500.00	0.06011	0.3563	0.3363	0.08225	1.2867	1.15814	0.15814	0.8575	0.8575	0.10749	0.6163	0.000	0.0000	0.9564	0.0319
19000.00	0.05882	0.3553	0.3353	0.08220	1.2855	1.15721	0.15721	0.8546	0.8546	0.10834	0.6173	0.000	0.0000	0.9564	0.0269
19500.00	0.05757	0.3544	0.3344	0.08212	1.2844	1.15628	0.15628	0.8517	0.8517	0.10919	0.6183	0.000	0.0000	0.9564	0.0219
20000.00	0.05635	0.3536	0.3336	0.08201	1.2834	1.15535	0.15535	0.8488	0.8488	0.11004	0.6193	0.000	0.0000	0.9564	0.0169
20500.00	0.05516	0.3529	0.3329	0.08187	1.2825	1.15442	0.15442	0.8459	0.8459	0.11089	0.6203	0.000	0.0000	0.9564	0.0119
21000.00	0.05400	0.3522	0.3322	0.08170	1.2817	1.15350	0.15350	0.8430	0.8430	0.11174	0.6213	0.000	0.0000	0.9564	0.0069
21500.00	0.05287	0.3516	0.3316	0.08150	1.2810	1.15257	0.15257	0.8401	0.8401	0.11259	0.6223	0.000	0.0000	0.9564	0.0019
22000.00	0.05177	0.3510	0.3310	0.08127	1.2804	1.15164	0.15164	0.8372	0.8372	0.11344	0.6233	0.000	0.0000	0.9564	0.0000
22500.00	0.05070	0.3505	0.3305	0.08101	1.2800	1.15071	0.15071	0.8343	0.8343	0.11429	0.6243	0.000	0.0000	0.9564	0.0000
23000.00	0.04966	0.3500	0.3300	0.08072	1.2797	1.14978	0.14978	0.8314	0.8314	0.11514	0.6253	0.000	0.0000	0.9564	0.0000
23500.00	0.04864	0.3496	0.3296	0.08041	1.2795	1.14885	0.14885	0.8285	0.8285	0.11599	0.6263	0.000	0.0000	0.9564	0.0000
24000.00	0.04764	0.3492	0.3292	0.08008	1.2794	1.14792	0.14792	0.8256	0.8256	0.11684	0.6273	0.000	0.0000	0.9564	0.0000
24500.00	0.04666	0.3489	0.3289	0.07973	1.2794	1.14700	0.14700	0.8227	0.8227	0.11770	0.6283	0.000	0.0000	0.9564	0.0000
25000.00	0.04570	0.3486	0.3286	0.07937	1.2795	1.14607	0.14607	0.8198	0.8198	0.11855	0.6293	0.000	0.0000	0.9564	0.0000

Appendix A. The Fortran Listing of MLML

The program for the maximum likelihood estimator was written in Fortran, and was compiled by the G-level compiler in 2.5 CPU seconds with 2.7 I/O seconds in a region of 122 K on Cornell's IBM 360. The Fortran listing is given in Table 7.

TABLE 7 A Fortran Listing of the Maximum Likelihood Program

```

C
C ----- MAXIMUM LIKELIHOOD ESTIMATOR FOR THE MULTINOMIAL LOGIT MODEL
C              T.J. TYRRELL, MAY 16, 1978
C              DEPT. OF AGRICULTURAL ECONOMICS
C              CORNELL UNIVERSITY
C
C      REAL*8 L,F,SEP,EFF,TOL,TOLA,LQ,DET,Y1,FAC,DD
C      REAL*4 FMT(20)
C      INTEGER*4 T
C
C      THIS PROGRAM HOLDS ALL OBSERVATIONS IN MAIN; THEREFORE IT
C      IS DESIREABLE AND IN SOME CASES NECESSARY TO SET DIMENSIONS
C      AS CLOSE AS POSSIBLE TO THE SIZE OF THE PROBLEM. THE FOLLOWING
C      FOUR STATEMENTS SHOULD AGREE WITH THE FIRST CARD OF INPUT.
C      PROBLEMS MAY BE LESS THAN THE SIZE SPECIFIED IN THESE CARDS.
C
C      REAL*8 B(NU),W(NEQ,NOBS),X(NA,NOBS),X0(NB),EF(NEQ)
C      REAL*8 G(NU),GG(NU,NU),WW(NEQ),XX(NA),BR(NR),BO(NU)
C      INTEGER IU(NA,NEQ)
C      NUMAX=NU
C
C      REAL*8 B(45),W(6,456),X(9,456),X0(14),EF(6)
C      REAL*8 G(45),GG(45,45),WW(6),XX(9),BR(1),BO(45)
C      INTEGER IU(9,6)
C      NUMAX=45
C      READ(5,1000) NB,NA,NOBS,NEQ,NU,NR,IT,ITMAX,NC,NP,TOL
1000 FORMAT(10I5,D10.0)
C
C      WHERE      NB = THE NUMBER OF VARIABLES BEFORE TRANSFORMATIONS
C                NA = THE NUMBER OF VARIABLES AFTER TRANSFORMATIONS
C                NOBS = THE TOTAL NUMBER OF OBSERVATIONS
C                NEQ = THE NUMBER OF ALLOCATION ALTERNATIVES (EQUATIONS)
C                NU = NO. OF UNRESTRICTED (FREE) PARAMETERS (+S)
C                NR = NO. OF RESTRICTED PARAMETERS (-S)
C                IT = NON-ZERO TO READ INITIAL VALUES OF B(NU)
C                ITMAX = MAXIMUM NUMBER OF ITERATIONS TO BE PERFORMED
C                NC = THE NUMBER OF THE CONSTANT VARIABLE
C                NP = THE NUMBER OF OBSERVATIONS TO BE PRINTED
C                TOL = TOLERANCE CRITERION FOR (L-L0)/L
C
C      THE USER MUST ALSO SUPPLY A *TRAN* PROGRAM (SEE BELOW)
C
C      WRITE(6,1011)NB,NA,NOBS,NEQ,NU,NR,IT,ITMAX,NC,NP,IOL
1011 FORMAT('1  SIZE AND DEFINITIONS CARD: ',1X,10I5,D10.3,///)
C      IEND=1
C      DO 4 I=1,NA
C      4  XX(I)=0.0D0
C      L=-100000000.0D0

```


TABLE 7 (continued)

```

DO 5 N=1,NEQ
  WW(N)=0.000
  5 READ(5,1001)(IU(I,N),I=1,NA)
1001 FORMAT(40I2)
  IF(NR)99,17,10
  10 READ(5,1002)(BR(I),I=1,NR)
1002 FORMAT(8F10.0)
  17 IF(IT)9,18,9
  9 READ(5,1002)(B(I),I=1,NU)
  GO TO 20
  18 DO 19 I=1,NU
  19 B(I)=0.000
C
  20 READ(5,1003)FMT
1003 FORMAT(20A4)
  WRITE(6,1014)
1014 FORMAT('1 ALLOCATED SHARES',///)
  DO 30 T=1,NOBS
  READ(5,FMT,END=99)(X0(I),I=1,NB)
  CALL TRAN(NB,NA,NEQ,X0(1),W(1,T),X(1,T))
  DO 32 I=1,NA
  32 XX(I)=XX(I)+X(I,T)
  DO 35 N=1,NEQ
  35 WW(N)=WW(N)+W(N,T)
  IF(T.GT.NP)GO TO 30
  WRITE(6,1015)(W(N,T),N=1,NEQ)
1015 FORMAT(1X,9D14.7)
  30 CONTINUE
  T1=NOBS
  DO 31 N=1,NEQ
  31 WW(N)=WW(N)/T1
  DO 34 I=1,NA
  34 XX(I)=XX(I)/T1
  WRITE(6,1023)
1023 FORMAT('1 AVERAGE VALUES')
  WRITE(6,1018)(WW(N),N=1,NEQ)
1018 FORMAT('1X,9D14.7)
  WRITE(6,1017)
1017 FORMAT('1 EXPLANATORY VARIABLES AFTER TRANSFORMATIONS',///)
  DO 40 T=1,NP
  40 WRITE(6,1015)(X(I,T),I=1,NA)
  WRITE(6,1023)
  WRITE(6,1018)(XX(I),I=1,NA)
C
C   CONSTRAIN LAST PARAMETER TO 0.0 IF SAME TERM IS IN ALL EQUATIONS
C
  KK=0
  DO 14 I=1,NA
  NBS=0
  DO 12 N=1,NEQ
  IB=IU(I,N)
  22 IF(IB)12,12,11
  11 NBS=NBS+1
  IF(I.NE.NC)GO TO 12
  B(IB)=DLOG(WW(N))
  DO 16 J=1,NA
  IF(J.EQ.I)GO TO 16
  KB=IU(J,N)
  IF(KB)15,16,16
  15 B(IB)=B(IB)-BR(-KB)*XX(J)
  16 CONTINUE
  12 CONTINUE
  IF(NBS.NE.NEQ)GO TO 14
  DO 13 N=1,NEQ
  JB=IU(I,N)
  13 B(JB)=B(JB)-B(IB)
  NR=NR+1

```

TABLE 7 (continued)

```

IU(I,NEQ)=-NR
DO 21 N=1,NEQ
DO 21 J=1,NA
JB=IU(J,N)
IF(JB.GT.IB)IU(J,N)=JB-1
21 CONTINUE
WRITE(6,1022)I,NEQ
1022 FORMAT(/,' COEFFICIENT OF VAR, ',I2,' IN EQUATION NO, ',I2,
2 'HAS BEEN CONSTRAINED TO ZERO',/)
KK=KK+1
14 CONTINUE
WRITE(6,1016)
1016 FORMAT('1 POSITIVE INDEXES FOR PARAMETERS TO BE ESTIMATED',/)
NK=NU-KK
DO 25 N=1,NFQ
25 WRITE(6,1012)((IU(I,N),I=1,NA)
1012 FORMAT(1X,40I3)
WRITE(6,1010)L
1010 FORMAT(/,' INITIAL LIKELIHOOD = ',D14.7,/)
WRITE(6,1009)((I,B(I),I=1,NU)
1009 FORMAT(/,' INITIAL PARAMETER ESTIMATES',/,40(1X,I2,2X,D14.7,/)
IF(NR)99,49,48
48 WRITE(6,1024)
WRITE(6,1025)((I,BR(I),I=1,NR)
1025 FORMAT(/,40(1X,I2,2X,D14.7,/)
C
C COMMENCE ITERATIONS
C
49 IT=0
WRITE(6,1021)
1021 FORMAT('1')
50 IT=IT+1
LO=L
FAC=1.0D0
51 DO 60 I=1,NK
G(I)=0.0D0
DO 60 J=1,I
60 GG(I,J)=0.0D0
L=0.0D0
C
C CALCULATE FIRST AND SECOND ORDER DERIVATIVES OF LIKELIHOOD
C FUNCTION(GINK) AND GG(NK,NK)) AND COMPUTE VALUE OF NON-CONSTANT
C PORTION OF LIKELIHOOD FUNCTION FOR INITIAL OR PREVIOUS SET
C OF PARAMETER ESTIMATES.
C
DO 100 T=1,NOBS
SEF=0.0D0
DO 70 N=1,NEQ
F=0.0D0
DO 68 I=1,NA
IB=IU(I,N)
IF(IB)66,68,65
66 F=F+BR(-IB)*X(I,T)
GO TO 68
65 F=F+B(IB)*X(I,T)
68 CONTINUE
L=L+F*W(N,T)
IF(F.LT.-150.0D0)GO TO 69
IF(F.GT.150.0D0)F=150.0D0
EF(N)=DEXP(F)
GO TO 70
69 EF(N)=0.0D0
70 SEF=SEF+EF(N)
L=L-DLOG(SEF)
72 DO 80 N=1,NEQ
IF(EF(N),LE,0.0D0)GO TO 73

```

TABLE 7 (continued)

```

DD=DLOG(EF(N))-DLOG(SEF)
IF(DD.LT.-150.000)EF(N)=0.000
EF(N)=EF(N)/SEF
73 DO 80 I=1,NA
  IB=IU(I,N)
  IF(IB)80,80,71
71 G(IB)=G(IB)+(W(N,T)-EF(N))*X(I,T)
  DO 79 N1=1,NEQ
  DO 79 J=1,NA
  JB=IU(J,N1)
  IF(JB)79,79,75
75 IF(IB-JB)79,76,76
76 EFF=EF(N)*EF(N1)
  IF(N-N1)79,77,78
77 EFF=EFF-EF(N)
78 GG(IB,JB)=GG(IB,JB)+X(I,T)*X(J,T)*EFF
79 CONTINUE
80 CONTINUE
100 CONTINUE
  IF(L-L0)95,101,101
95 DO 97 I=1,NK
97 B(I)=0.5*B(I) + 0.5*B0(I)
  FAC=FAC*0.500
  IF(FAC=0.0100)102,51,51
102 IEND=3
101 TOLA=DABS(1.000-L/L0)
  WRITE(6,1004)IT,L,TOLA,FAC
1004 FORMAT(/,7X,'ITERATION NUMBER',I4,/, 'VALUE OF NON-CONSTANT',
2 ' PORTION OF THE LIKELIHOOD FUNCTION =' ,D15.7, ' TOLERANCE =' ,
3 ' D15.7, ' FAC =' ,D15.7,/)
  DO 108 I=1,NK
108 B0(I)=B(I)
  IF(IT-1)99,112,109
109 IF(TOL.GE.TOLA)IEND=2
  IF(IT.GE.ITMAX)IEND=3
C
C      FISHER SCORING (NEWTON-RAPHSON) CALCULATION FOR CHANGE (D) TO PARAM (B)
C
112 DO 120 I=2,NK
  I1=I-1
  DO 120 J=1,I1
120 GG(J,I)=GG(I,J)
  CALL INVERT(GG,DET,NK,NUMAX)
  IF(DET.EQ.0.000)GO TO 199
  DO 125 I=1,NK
  D=0.000
  DO 124 J=1,NK
124 D=D+GG(I,J)*G(J)
125 B(I)=B0(I)-D
  IF(IEND-1)99,50,126
126 WRITE(6,1005)DET
1005 FORMAT(
2 /, ' PARAMETERS      FFF SCORE',10X, ' - ESTIMATED INFORMATION',
3 ' LIMIT TO THE VARIANCE-COVARIANCE MATRIX (DET = ',D14.7,')' )
  DO 115 I=1,NK
115 WRITE(6,1006)B(I),G(I),(GG(I,J),J=1,I)
1006 FORMAT(1X,2D14.7,4X,7D14.7,5(/,32X,7D14.7 ))
  GO TO(50,130,140),IEND
130 WRITE(6,1007)TOLA,TOL
1007 FORMAT(/,8X,'TOLERANCE OF ',D15.7, ' MEETS CRITERION OF ',D15.7)
140 WRITE(6,1008)IT,ITMAX
1008 FORMAT(/,/, '***** PROGRAM ENDED AFTER ITERATION NO.,',I4,
2 ' , MAXIMUM ALLOWED =',I4, ' *****')
  WRITE(6,1019)
1019 FORMAT('1',/, ' I PARAMETERS STANDARD ERRORS T-VALUES')
  DO 150 I=1,NK
  G(I)=DSQRT(-GG(I,I))
  GG(I,1)=B(I)/G(I)

```

TABLE 7 (continued)

```

150 WRITE(6,1020)I,B(I),G(I),G6(I,1)
1020 FORMAT(1X,I2,2X,3D14,7)
      IF(NR,99,99,155)
155 WRITE(6,1024)
1024 FORMAT(/, ' (RESTRICTED PARAMETERS)',/)
      DO 160 I=1,NR
160 WRITE(6,1020)I,BR(I)
      99 CONTINUE
      STOP
199 WRITE(6,1026)
1026 FORMAT('1',/, ' I PARAMETERS EFF. SCORE',/)
      DO 165 I=1,NK
165 WRITE(6,1020)I,B(I),G(I)
      GO TO 155
      END

```

```

      SUBROUTINE INVERT(A,DET,N,NMAX)
C*****
C      INVERTS A (NXN) MATRIX (A) INTO (A) AND RETURNS THE DETERMINANT
C      (DET) OF (A) . NMAX IS THE DIMENSION OF (A) IN THE CALLING PROGRAM
C*****
      IMPLICIT REAL*8(A-H,O-Z)
      LOGICAL COL
      DIMENSION A(NMAX,NMAX),NROW(100),COL(100)
      DET=1.0D0
      DO 5 I=1,N
      NROW(I)=I
      5 COL(I)=.FALSE.
      DO 40 I=1,N
      PIVOT=0.0D0
      DO 15 J=1,N
      IF(COL(J))GO TO 15
      DO 10 K=1,N
      IF(COL(K).OR.(DABS(A(J,K)).LT.DABS(PIVOT)))GO TO 10
      PIVOT=A(J,K)
      JROW=J
      KCOL=K
10 CONTINUE
15 CONTINUE
      DET=DET*PIVOT
      IF(DET.EQ.0.0D0)GO TO 61
      IF(JROW.EQ.KCOL)GO TO 25
      DET=-DET
      NEMP=NROW(KCOL)
      NROW(KCOL)=NROW(JROW)
      NROW(JROW)=NEMP
      DO 20 K=1,N
      TEMP=A(JROW,K)
      A(JROW,K)=A(KCOL,K)
20 A(KCOL,K)=TEMP
25 COL(KCOL)=.TRUE.
      A(KCOL,KCOL)=1.0D0
      DO 30 K=1,N
30 A(KCOL,K)=A(KCOL,K)/PIVOT
      DO 40 J=1,N
      IF(J.EQ.KCOL)GO TO 40
      TEMP=A(J,KCOL)
      A(J,KCOL)=0.0D0
      DO 35 K=1,N
35 A(J,K)=A(J,K)-A(KCOL,K)*TEMP
40 CONTINUE
      DO 60 J=1,N
      IF(NROW(J).EQ.J)GO TO 60
      DO 45 K=J,N

```

TABLE 7 (continued)

```

      I=K
      IF(NROW(K).EQ.J)GO TO 50
45  CONTINUE
50  DO 55 K=1,N
      TEMP=A(K,J)
      A(K,J)=A(K,I)
55  A(K,I)=TEMP
      NROW(I)=NROW(J)
60  CONTINUE
      RETURN
61  WRITE(6,1001)JROW,KCOL
1001 FORMAT(//,' DETERMINANT = 0.0, PROGRAM STOPS, PARTIALLY INVERTED',
2    ' INFORMATION MATRIX FOLLOWS, (JROW,KCOL)= ',2I3,/)
      DO 63 K=1,N
63  WRITE(6,1002)(A(J,K),J=1,N)
1002 FORMAT(1X,9D14.7)
      RETURN
      END

```

```

SUBROUTINE TRAN(NB,NA,NEQ,X0,w,X)
REAL*8 X0,w,X,M,T
DIMENSION X0(NB),W(NEQ),X(NA)

```

```

C      INPUT DATA IN X0 NEEDS TO BE TRANSFORMED INTO SHARES (W) AND VAR. X
C      WHERE W(1) + W(2) + ... + W(NEQ) SHOULD EQUAL 1, AND
C      THE X'S ARE THE SPECIFICALLY TRANSFORMED X0'S FOR F(X)'S
C
      X(1)=1.000
      X(2)=DLOG(X0(1))
      X(3)=X0(2)
      DO 10 I=1,6
      X(I+3)=X0(I+2)
10  W(I)=X0(I+8)
      RETURN
      END

```

Appendix B. A Fortran Listing of MLAM

The program for the analysis of the multinomial logit was written in Fortran with a Main routine, a Block Data segment, four subroutines (RD, WRT, COMP, and PLT) and a function (F). The Fortran listing is given in Table 8.

TABLE 8 A Fortran Listing of MLAM

```
C
      MLAM   T.J.TYRRELL.    5/22/78

COMMON /NAMES/NAMBUD,NAMVAR,NAMFN
COMMON /VALUES/VALBUD,VALVAR,COEF
COMMON /TYPES/IFN,NFN,IVC,NO
COMMON /SIZES/NOBUD,NOVAR,NOFN,NOEQ
COMMON /RESULT/WI(20,50),EI(20,50)
COMMON /FORCST/X1,X2,XD,Y1,Y2,YD,NREPS,IV,NP(20)
REAL*4 VALBUD(20),VALVAR(20),COEF(20,10)
INTEGER*4 NAMBUD(20),NAMVAR(20),NAMFN(2,20)
INTEGER*2 IFN(20,20),NFN(20,2),IVC(20)
        IFN(20EQ,10*(FNCODE+VRCODE)), NFN(20EQ,BUDCODE + NOFUNCTIONS)
CALL RO(1)
CALL WRT(1)
5 CALL COMP(O)
CALL WRT(3)
10 READ(5,1000,END=99)ID,IV,X1,X2,NP,Y1,Y2
1000 FORMAT(I2,2FB,O,2O I2,2FI O.O)
        ID = MEANS
            0 SET VALVAR(IV) = X1, PRINT FIXED VALUE
              BUDGETS AND ELASTICITIES WRT ALL VARIABLE
                1 50 REPS, PRINT BUDGETS AND ELASTICITIES
                  2 50 REPS, PLOT BUDGETS AND ELASTICITIES
                    3 50 REPS, PRINT AND PLOT BUDGETS AND ELASTICITIES
NP GIVES ORDER OF SHARES FOR CUMULATIVE PLOTS, IF ZEROS ORDER IS OF BUD
IF(ID.NE.O)GO TO 15
VALVAR(IV)=X1
GO TO 5
15 NREPS=50
CALL COMP(1)
IF(ID.EQ.2)GO TO 20
CALL WRT(4)
IF(ID.EQ.1)GO TO 10
20 CALL PLT(1)
CALL PLT(2)
GO TO 10
99 CONTINUE
STOP
END


BLOCK DATA A
COMMON /NAMES/NAMBUD,NAMVAR,NAMFN
COMMON /VALUES/VALBUD,VALVAR,COEF
COMMON /TYPES/IFN,NFN,IVC,NO
COMMON /SIZES/NOBUD,NOVAR,NOFN,NOEQ
COMMON /RESULT/WI(20,50),EI(20,50)
COMMON /FORCST/X1,X2,XD,Y1,Y2,YD,NREPS,IV,NP(20)
INTEGER*2 IFN(20,20)/400*0/,NFN(20,2)/40*0/,IVC(20)/20*0/
REAL VALBUD(20)/20*0.0/,VALVAR(20)/20*0.0/,COEF(20,10)/200*0.0/
INTEGER NAMFN(2,20)/CONS,TANT/,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*,*LN(*),*,*,*
**((1/.)) ,,,LO('') ,,,L1(''),,,L2('')
A **L3('') ,,24*%/,
3 NOFN/B/,
4 NAMBUD(20)/20*%, %/,
5 NAMVAR(20)/20*%, %/
END
```

TABLE 8 (continued)

```

SUBROUTINE RD(ND)
COMMON /NAMES/NAMBUD,NAMVAR,NAMFN
COMMON /VALUES/VALBUD,VALVAR,COEF
COMMON /TYPES/IFN,NFN,IVC,NO
COMMON /SIZES/NOBUD,NOVAR,NOFN,NOEQ
COMMON /FORCST/X1,X2,XD,Y1,Y2,YD,NREPS,IV,NP(20)
REAL*4 VALBUD(20),VALVAR(20),COEF(20,10)
INTEGER*4 NAMBUD(20),NAMVAR(20),NAMFN(2,20)
INTEGER*2 IFN(20,20),NFN(20,2),IVC(20)
READ(5,1001)NOEQ,NOVAR,ND
1001 FORMAT(3I4)
NOBUD=NOEQ+1
READ(5,1002)(NAMBUD(N),N=1,NOBUD)
1002 FORMAT(20A4)
READ(5,1003)(VALBUD(N),N=1,NOBUD)
1003 FORMAT(10F8.0)
READ(5,1002)(NAMVAR(I),I=1,NOVAR)
READ(5,1003)(VALVAR(I),I=1,NOVAR)
READ(5,1005)(IVC(I),I=1,NOVAR)
1005 FORMAT(20I4)
DO 10 N=1,NOEQ
READ(5,1001) NFN(N,1),NFN(N,2)
NF=NFN(N,2)
READ(5,1004)(COEF(N,J),IFN(N,2*J-1),IFN(N,2*J) ,J=1,NF)
1004 FORMAT(8(F6.0,2I2))
10 CONTINUE
RETURN
END

```

```

SUBROUTINE WRT(ND)
C 8 1/2 * 11 PAGE WITH MARGINS: //,52(8X,60A1,/)
COMMON /NAMES/NAMBUD,NAMVAR,NAMFN
COMMON /VALUES/VALBUD,VALVAR,COEF
COMMON /TYPES/IFN,NFN,IVC,NO
COMMON /SIZES/NOBUD,NOVAR,NOFN,NOEQ
COMMON /RESULT/W1(20,50),E1(20,50)
COMMON /FORCST/X1,X2,XD,Y1,Y2,YD,NREPS,IV,NP(20)
REAL*4 VALBUD(20),VALVAR(20),COEF(20,10)
INTEGER*4 NAMBUD(20),NAMVAR(20),NAMFN(2,20),BL
INTEGER*4 A(3,10)
INTEGER*2 IFN(20,20),NFN(20,2),IVC(20)
DATA BL/0/
MAXNO=MAX0(NOBUD,NOVAR,NOFN)
GO TO (10,20,30,40),NO
10 WRITE(6,2001)
2001 FORMAT('1',//,14X,'MULTINOMIAL LOGIT ALLOCATION',
2 'MODEL (MLAM)',//,7X,'PROGRAM TO COMPUTE, PRINT AND PLOT ',
3 'ELASTICITIES AND ALLOC. SHARES',//,12X,'TIM TYRRELL, DEPT. ',
4 'OF AG. ECON., CORNELL U., 5/22/78')
WRITE(6,2002)
2002 FORMAT('1',//,26X,'COMPONENTS OF THIS MLAM',//,14X,'INPUT',
2 19X,'INPUT',15X,'AVAILABLE',//,13X,'SHARES',16X,'VARIABLES',
3 13X,'FUNCTIONS',//,8X,'CODE LABEL MEAN CODE LABEL ',
4 'FIXED VALUE CODE FUNCTION',//)
WRITE(6,2003) (K,NAMBUD(K),VALBUD(K),K,NAMVAR(K),VALVAR(K),K,
2 NAMFN(1,K),NAMFN(2,K) ,K=1,MAXNO)

```

TABLE 8 (continued)

```

2003 FORMAT(8X, I3, 3X, A4, F7.3, 4X, I3, 3X, A4, E14.5, 3X, I3, 2X, 2A4)
20 CONTINUE
WRITE(6, 2010)
2010 FORMAT(///, 35X, 'MODEL', //)
DO 25 N=1, NOEQ
NF=NFN(N, 2)
NF1=NF-5
NB=NFN(N, 1)
DO 23 J=1, NF
J1=IFN(N, 2+J-1)
IF(J1.EQ.1) J1=NOFN+1
J2=IFN(N, 2+J)
A(1, J)=NAMFN(1, J1)
A(3, J)=NAMFN(2, J1)
23 A(2, J)=NAMVAR(J2)
IF(NF1.GT.0) NF=5
2011 FORMAT(/, 8X, '(1, I2, ' 1', 5X, 'LN(W /W ) =', 5(F8.4, 2A4, A2))
IF(ND) 21, 99, 22
21 NN=-NO
WRITE(6, 2012) NAMBUD(NN), NAMBUD(NB)

GO TO 24
22 WRITE(6, 2012) NAMBUD(NB), NAMBUD(ND)
24 CONTINUE
2012 FORMAT(22X, A4, 1X, A4)
IF(NF1.LE.0) GO TO 7
WRITE(6, 2013) (COEF(N, J+5), A(1, J+5), A(2, J+5), A(3, J+5), J=1, NF1)
2013 FORMAT(36X, 5(F10.4, 3A4))
C CONVERTS ALL PROBLEMS TO COMMON DENOMINATOR TYPE
7 IF(ND) 8, 99, 25
8 DO 9 J=1, NF
9 COEF(N, J)=-COEF(N, J)
25 CONTINUE
ND=IABS(ND)
RETURN
30 NN=NOBUD
IF(NOBUD.GT.10) NN=10
32 WRITE(6, 2005)
WRITE(6, 2006) (NAMBUD(N), N=1, NN)
WRITE(6, 2007) (W1(N, 1), N=1, NN)
WRITE(6, 2008) (BL, N=1, NN)
DO 34 I=1, NOVAR
34 WRITE(6, 2009) NAMVAR(I), VALVAR(I), (E1(N, I), N=1, NN)
NN=NOBUD-10
IF(NN) 36, 36, 32
36 RETURN
2005 FORMAT('1', //, //, //, //, //, //, //, 24X, 'ALLOC. SHARES AND ELASTICITIES FOR ',
2 'FIXED VALUES')
2006 FORMAT(///, 10X, ' ALLOCATION GROUPS: ', 10(6X, A4))
2007 FORMAT(/, 13X, 'ALLOC. SHARES: ', 10(F10.5))
2008 FORMAT(/, ' ELASTICITY WRT', 9X, 'AT ', 10(A4, '-----'))
2009 FORMAT(/, 13X, A4, 3X, F10.3, 10(F10.4))
RETURN
40 CONTINUE
NB1=NOBUD-5
NB2=NOBUD
IF(NB1.GT.0) NB2=5
WRITE(6, 2014) NAMVAR(IV), X1, X2, (NAMBUD(N), N=1, NB2)

```


TABLE 8 (continued)

```

2014 FORMAT('1', ///, 20X, 'ALLOC. SHARES AND ELASTICITIES WRT ',
2. A4, ' FROM ', F8.2, ' TO ', F8.2, ///, 30X, 5(A4, 16X))
WRITE(6, 2016) NAMVAR(IV), (BL, N=1, NR2)
2016 FORMAT(1/, 11X, A4, 5X, 5(1X, A4, ' SHARE', 5X, 'ELAST', 1), /)
X=X1-XD
DO 45 K=1, NREPS
X=X+XD
45 WRITE(6, 2015) X, (W1(N, K), E1(N, K), N=1, NR2)
2015 FORMAT(5X, F10.2, 5X, 5(F10.5, F10.4))
IF(NB1.LE.0) RETURN
WRITE(6, 2014) NAMVAR(IV), X1, X2, (NAMBUD(N+5), N=1, NB1)

WRITE(6, 2016) NAMVAR(IV), (BL, N=1, NB1)
X=X1-XD
DO 50 K=1, NREPS
X=X+XD
50 WRITE(6, 2015) X, (W1(N+5, K), E1(N+5, K), N=1, NB1)
RETURN
99 STOP 1
END

```

```

SUBROUTINE COMPTND)
COMMON /VALUES/ VALBUD, VALVAR, COEF
COMMON /TYPES/ IFN, NFN, IVC, ND
COMMON /SIZES/ NOBUD, NOVAR, NOFN, NOEQ
COMMON /RESULT/ W1(20, 50), E1(20, 50)
COMMON /FORCST/ X1, X2, XD, Y1, Y2, YD, NREPS, IV, NP(20)
REAL*4 VALBUD(20), VALVAR(20), COEF(20, 10)
INTEGER*4 NAMBUD(20), NAMVAR(20), NAMFN(2, 20)
INTEGER*2 IFN(20, 20), NFN(20, 2), IVC(20)
DIMENSION W(20), E(20), C(20), D(20)
C NO=0 : ALL VAR, ONE REP NO=1 : VAR(IV) , NRPS
IF(NOI) 3, 2, 3
2 NREPS=1
ASSIGN 20 TO M1
ASSIGN 40 TO M2
IV=1
GO TO 5
3 ASSIGN 12 TO M1
ASSIGN 290 TO M2
C CALCULATIONS UNCHANGED WITH VALUE OF VAR(IV)
5 CONTINUE
DO 10 N=1, NOEQ
C(N)=0.0
D(N)=0.0
NF=NFN(N, 2)
DO 10 J=1, NF
NV=IFN(N, 2*J)
IF(NV.EQ.IV) GO TO 10
B=COEF(N, J)
J1=IFN(N, 2*J-1)
IF(NV.EQ.0) GO TO 9
X=VALVAR(NV)
9 C(N)=C(N)+F(B, J1, X, 1)

```

TABLE 8 (continued)

```

10 CONTINUE
11 GO TO M1,(12,20)
12 XD=(X2-X1)/(NREPS-1)
   X=X1-XD
C  CALCULATIONS CHANGE WITH VAR(IV)
20 EO=0.0
   IV1=IVC(IV)
   IF(IV1.EQ.-1)EO=1.0
   DO 30 K=1,NREPS
     X=X+XD
     IF(ND.EQ.0)X=VALVAR(IV)
C  DERIVATIVES EVALUATED AT X
   DO 25 N=1,NOEQ
     NB=NFN(N,1)
     W(NB)=C(N)

     E(NB)=D(N)
22 NF=NFN(N,2)
   DO 25 J=1,NF
     NV=IFN(N,2*J)
     IF(NV.NE.IV)GO TO 25
     B=COEF(N,J)
     J1=IFN(N,2*J-1)
     W(NB)=W(NB)+F(B,J1,X,1)
     E(NB)=E(NB)+F(B,J1,X,2)
C  ALLOCATIONS AND ELASTICITY CALCULATIONS
25 CONTINUE
26 W(ND)=1.0
   E(ND)=EO
   DO 27 N=1,NOEQ
     NB=NFN(N,1)
     W(NB)=EXP(W(NB))
27 W(ND)=W(ND)+W(NB)
   W(ND)=1.0/W(ND)
   DO 28 N=1,NOEQ
     NB=NFN(N,1)
     W(NB)=W(ND)*W(NB)
28 E(ND)=E(ND)-E(NB)*W(NB)
   DO 29 N=1,NOEQ
     NB=NFN(N,1)
29 E(NB)=E(ND)+E(NB)
   IF(IV1.GT.0)E(IV1)=E(IV1)-1.0
   GO TO M2,(290,40)
290 DO 291 N=1,NOBUD
     W1(N,K)=W(N)
291 E1(N,K)=E(N)
30 CONTINUE
   RETURN
40 DO 45 N=1,NOBUD
   E1(N,IV)=E(N)
45 W1(N,IV)=W(N)
   IV=IV+1
   IF(IV.LE.NOVAR)GO TO 5
   RETURN
END

```

TABLE 8 (continued)

```

FUNCTION F(B,J1,X0,ID)
X=X0
G=0.0
5 GO TO(10,20),ID
F=0.0
RETURN
10 GO TO(11,12,13,14,15,16,17,18),J1
F=0.0
RETURN
11 F=B
RETURN
12 F=B*X
RETURN
13 F=B*ALOG(X)
RETURN
14 F=B/X
RETURN
15 X1=ALOG(X+1.75)
F=-B*(X1-2.7568)*(X1-3.0796)*(X1-4.6225)/39.2452
GO TO 40
16 X1=ALOG(X+1.75)
F=B*X1*(X1-3.0796)*(X1-4.6225)/1.6601
GO TO 40
17 X1=ALOG(X+1.75)
F=-B*X1*(X1-2.7568)*(X1-4.6225)/1.5337
GO TO 40
18 X1=ALOG(X+1.75)
F=B*X1*(X1-2.7568)*(X1-3.0796)/13.3062
GO TO 40
C
20 DERIVATIVES OF ABOVE FUNCTIONS * X
GO TO (21,12,11,22,23,24,25,26),J1
21 F=0.0
RETURN
22 F=-B/X
RETURN
23 X1=ALOG(X+1.75)
F=(3.*X1*X1-20.9178*X1+35.4685)/(39.2452*X1)*X1*B
GO TO 40
24 X1=ALOG(X+1.75)
F=(3.*X1*X1-15.4042*X1+14.2354)/(1.6601*X1)*X1*B
GO TO 40
25 X1=ALOG(X+1.75)
F=-(3.*X1*X1-14.7586*X1+12.7433)/(1.5337*X1)*X1*B
GO TO 40
26 X1=ALOG(X+1.75)
F=(3.*X1*X1-11.6728*X1+8.4898)/(13.3062*X1)*X1*B
GO TO 50
40 G=F

F=0.0
X=X-25.
GO TO 5
50 F=F+G
RETURN
END

```

TABLE 8 (continued)

```

SUBROUTINE PLY(NO)
COMMON /NAMES/NAMBUD,NAMVAR,NAMFN
COMMON /SIZES/NJBUD,NOVAR,NOFN,NOEQ
COMMON /RESULT/W1(20,50),E1(20,50)
COMMON /FORCST/X1,X2,XD,Y1,Y2,YD,NREPS,IV,NP(20)
INTEGER*4 NAMBUD(20),NAMVAR(20),NAMFN(2,20)
DIMENSION XX(10)
INTEGER*2 IX(50,50),MB(2,20),BL,ST
EQUIVALENCE (MB,NAMBUD)
DATA BL/' ','/ST/'**'/
C      NO = 1 : PLOT BUDGETS,      NO = 2 : PLOT ELASTICITIES
      GO TO(3,2),NO
2  ASSIGN 132 TO M1
   IF(Y2-Y1)22,21,22
21 Y0=-2.4
   YN=2.5
   GO TO 5
22 Y0=Y1
   YN=Y2
   GO TO 5
3  ASSIGN 131 TO M1
   Y0=0.02
   YN=1.00
5  DO 10 I=1,50
   DO 10 J=1,50
10  IX(I,J)=BL
   IF(NO.EQ.1.OR.Y2.NE.Y1)GO TO 12
   DO 11 I=1,50
11  IX(I,26)=ST
12  XD=(X2-X1)/49.0
   YD=(YN-Y0)/49.0
   DO 14 I=1,50
   Y=Y0
   DO 14 N=1,NJBUD
   GO TO M1,(131,132)
131 N1=NP(N)
   IF(N1.EQ.0)N1=N
   Y=Y+W1(N1,I)
   GO TO 133
132 Y=E1(N,I)
   N1=N
133 J=51.0-(Y-Y0)/YD
   IF(J.LT.1)J=1
   IF(J.GT.50)J=50
14  IX(I,J)=MB(I,N1)
   WRITE(6,3001)
3001 FORMAT('1',//)
   WRITE(6,3004)
   YD5=5.0*YD
   Y=YN+YD5
   DO 50 J1=1,50,5
   Y=Y-YD5
   WRITE(6,3002)Y,(IX(I,J1),I=1,50)
3002 FORMAT(10X,F10.4,4X,'+',50(1X,A1),'+')
   DO 50 J2=1,4
50  WRITE(6,3003)(IX(I,J1+J2),I=1,50)
3003 FORMAT(24X,'+',50(1X,A1),'+')
   WRITE(6,3004)
3004 FORMAT(24X,'-',10('-----+',),'-')
   XD5=XD*5.0
   XX(1)=X1+4.0*XD
   DO 55 I=2,10
55  XX(I)=XX(I-1)*XD5
   WRITE(6,3005)(XX(K),K=1,10)
3005 FORMAT(/,27X,10F10.2)
   RETURN
   END

```