AN APPLICATION OF THE MULTINOMIAL LOGIT MODEL

TO THE ALLOCATION OF BUDGETS IN THE U.S.

CONSUMER EXPENDITURE SURVEY FOR 1972

by

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I. Introduction

Empirical studies of consumer demand have traditionally taken one of two approaches: the theoretical or the practical.[1] The theoretical approach can be characterized by the specification of a utility function and the estimation of the demand equations which satisfy the first order conditions for maximizing utility subject to a budget constraint. The practical approach has been to directly select flexible forms of demand equations using goodness-of-fit criteria. In spite of the tremendous body of literature that exists on these subjects, there is still a surprisingly large difference between the characteristics of the models typifying the two approaches. It is the purpose of this paper to describe a model, based on a multinomial logit specification, that brings the two approaches closer together, and provides a reasonable compromise between flexibility of form and consistency with the constraints implied by theory.

The problems associated with both the theoretical and practical approaches to estimating demand relationships can conveniently be illustrated in terms of modelling Engel curves. If it is assumed for this example that income is the only variable determining demand, then the estimation problem is to predict the quantity of each commodity demanded in terms of the level of income. The emphasis of the practical approach is generally on getting a good fit for each relationship over the observed range of incomes. As Prais and Houthakker show, some commodities require distinctly different forms than other commodities. Many commodities, such as food, exhibit decreasing income elasticities

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[1]This dichotomy has been discussed generally by Brown and Deaton (1972), Phlips (1974, Ch. 4) and Barten (1977) and specifically with respect to food demand studies, by Hassan and Johnson (1977), Prato (1977) and Tomek (1977).

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as income rises, and the semi-log form, in which this elasticity is inversely proportional to income, performs well. In contrast, with the theoretical approach the requirement that predicted expenditures on all commodities sum to total expenditures (additivity) is an important constraint that, by common sense as well as by theory, should hold for any level of income. With most practical approaches, this constraint is generally valid at only one or possibly more levels of income, assuming that nonlinear functions are used for some commodities. To be fair to the practical approach, such analyses are often limited to consideration of a single commodity, and in these circumstances, the fit of the models may be a sensible basis for selection. Even in this restricted situation, however, there are cases when fit proves to be misleading.[1]

When constraints implied by theory, such as additivity, are imposed on an estimated system of demand equations, the resulting system generally has a very restrictive form. For example, the Linear Expenditure System (Stone, 1954), which is still widely used, implies that the Engel curves are linear so that each additional dollar of income above a certain minimum is allocated in exactly the same way among different commodities. The characteristics implied by a semi-log function do not hold with this model. Even the Rotterdam model (Theil, 1975), which is not initially limited to a specific utility function, is generally estimated in a particular form which restricts all income elasticities to one. Once again, the established characteristics of the demand for food, for example, are not reflected by the model.

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[1]An example of this is provided by contrasting the use of extrapolation and an "economic" model to forecast the demand for electricity (Chapman, et al., 1972).

It is important to realize that in spite of the obvious limitations of the theoretical approach, constraints such as additivity have direct bearing on practical considerations. In particular, assessing the effects of changing the distribution of income on the composition of demand through modifying income tax schedules, for example, is difficult with models that violate additivity. In this situation, it is desirable to have a model that makes "sensible" predictions regardless of the specified level of income. If, on the other hand, the purpose of the analysis is to estimate a single set of representative elasticities for a given sample, then it is not nearly so critical that the model performs well at the extremes of income observed in the sample. Since the aggregation of micro demand relationships for a given distribution of income is an important objective for our analysis of the demand for food, additivity and a reasonably flexible form are both desirable characteristics of the model. In a recent survey of the food demand literature, Tomek (1977) has discussed the merits of a flexible functional form versus those of a form which satisfies the constraints of demand theory. In summarizing the compromise taken by empirical analysts, Tomek describes three major concerns about the selection of functional forms:

> (a) the logic of the relationship; e.g., the possibility of an initial income below which the commodity is not purchased, a declining mpc as income grows, and a possible satiety level in consumption; (b) the validity and usefulness of the equation over the plausible range of expenditures; i.e., predictions and elasticities should be reasonable at the end-points of the data; and (c) simplicity and convenience of estimation. (p. 10)

The multinomial logit model appears to offer an attractive compromise among these conflicting objectives.

Before describing the model itself, it is useful to illustrate its difference from other models. This can be done most easily by describing its relationship to the simple exponential function. Consider the expenditure function given by

$$(1) \qquad P_i Q_i = e^{f_i + \varepsilon_i}$$

where

$P_i Q_i$ is expenditures on good i,

f_i is an unspecified function of income, prices and other variables and

ε_i is a random error.

This function has the useful properties that predicted expenditures will always be positive and that a wide range of flexibility is provided through the specification of f_i . It can be made linear by the usual transformation:

$$(2) \qquad \ln(P_i Q_i) = f_i + \varepsilon_i .$$

The adding-up condition (budget constraint) is given by

$$(3) \qquad \sum_{j=1}^{N} P_j Q_j = M .$$

Predictions made from (1), i.e. $e^{\hat{f}_i}$, do not satisfy (3) since

$$(4) \qquad \sum_{j=1}^{N} e^{\hat{f}_j} \neq \sum_{j=1}^{N} e^{f_j + \epsilon_j} = \sum_{j=1}^{N} P_j Q_j = M$$

The error is "expected" to be zero but is not always identical to zero so that the sum of predicted expenditures may not equal the total budget.

By constraining the exponential expenditure function (1) to the budget constraint (3) we can derive the allocation model:

$$(5) \qquad w_i = \frac{P_i Q_i}{M} = \frac{e^{f_i + \epsilon_i}}{\sum e^{f_j + \epsilon_j}} \qquad\qquad i = 1, \ldots, N$$

which satisfies the adding up property since

$$(6) \qquad \sum_{j=1}^{N} w_j = \frac{\sum e^{f_j + \epsilon_j}}{\sum e^{f_j + \epsilon_j}} = 1 .$$

The remainder of this paper describes a model with the form of equation (5). Section II describes its properties; III describes its estimation and IV illustrates its use in a preliminary analysis of U.S. household expenditures for 1972. The advantages of this model for budget allocations are that it still has the flexibility of the exponential function, predicted expenditures add to income, and it can be transformed into a linear model of the form:

$$(7) \qquad \ln(w_i / w_j) = f_i - f_j + \epsilon_i - \epsilon_j \qquad\qquad i \neq j$$

This model has been suggested elsewhere in the literature. When f_i is assumed to be linear in its parameters and ϵ_i is distributed normally with mean zero (5) is equivalent to the multinomial logit model suggested by Theil (1969). McFadden (1974) has derived the multinomial logit as a model of qualitative choice based on a set of behavioral axioms for the consumer. Apart from these references, the authors are unaware of the development of the multinomial logit as a budget allocation model or its use in related empirical analysis of household expenditures.

In summary, a multinomial logit budget allocation model seems to offer much to the empirical analyst in search of a compromise between theoretical and practical considerations. The form of the model is such that budget shares will always lie between 0 and 1, and add to unity. Thus pre-dictions will always be of reasonable magnitudes for any level of income. Also, the model is quite flexible in characterizing responses to the explana-tory variables. This allows, for example, Engel functions for different goods to have distinctly different forms. Finally, the model can be transformed into a linear regression framework. Unlike other flexible forms, there is no need for additional simplifications or the use of nonlinear estimators.

II.   The Multinomial Logit Budget Allocation Model (MLBAM)

As a model to explain the allocation of income, the f_i of the multinomial logit in (5) are assumed to be functions of prices, income, household character-istics and other factors which affect allocation decisions. In addition, total expenditures and income are assumed identical or, at least, the former is a predetermined portion of the latter.

For the general case where a household allocates its expenditures to

N different goods or groups of goods the MLBAM is given by

$$(8) \qquad w_i = \frac{e^{f_i(M,\vec{P},\vec{Z})}}{\sum\limits_{j=1}^{N} e^{f_j(M,\vec{P},\vec{Z})}} \qquad\qquad i = 1, \ldots, N$$

where

w_i is the budget share allocated to the i'th good or group of goods,

M is total expenditures on all goods,

$\vec{P}$ is a vector of prices $(P_1, \ldots, P_N)$, and

$\vec{Z}$ is a vector of household characteristics (e.g. measures of family composition) and other factors.

Since the form of f_i is linear in unknown parameters for the multinomial logit model, it can be written as

$$(9) \qquad f_i(M,\vec{P},\vec{Z}) = \beta_{i1} X_1 + \beta_{i2} X_2 + \ldots + \beta_{iR} X_R$$

where X_r, $r = 1, \ldots, R$ are specified transformations of the explanatory variables, i.e. $(X_r = g_r(M,\vec{P},\vec{Z}))$. The choice of these provides the flexibility of the MLBAM.[1]

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[1] It is not necessary to have identical explanatory variables or to have the same number of variables in each f_i. This has been done here to simplify the notation.

Although predicted budget shares can be used directly as weights for prices in cost of living indices, they can also be transformed into quantities demanded at specified levels of M, $\vec{P}$ and $\vec{Z}$. Omitting the arguments of f_i for convenience, the demand for good i can be expressed by

$$(10) \qquad Q_i = \frac{M}{P_i} \frac{e^{f_i}}{\Sigma e^{f_j}} = \frac{e^{f_i + \ln(M) - \ln(P_i)}}{\Sigma e^{f_j}} \quad .$$

Consequently, the income (actually total expenditure) elasticity is given by

$$(11) \qquad E_{im} = M(\frac{\partial f_i}{\partial M} - \Sigma_j w_j \frac{\partial f_j}{\partial M}) + 1 \quad ,$$

the own-price elasticity is given by

$$(12) \qquad E_{ii} = P_i(\frac{\partial f_i}{\partial P_i} - \Sigma_j w_j \frac{\partial f_j}{\partial P_i}) - 1$$

and the cross-price elasticities by

$$(13) \qquad E_{ik} = P_k(\frac{\partial f_i}{\partial P_k} - \Sigma_j w_j \frac{\partial f_j}{\partial P_k}), \qquad\qquad k = 1, \ldots, N, \text{ and } k \neq i \ .$$

There is a definite intuitive appeal about the form of the elasticities. For income, the difference between $M \frac{\partial f_i}{\partial M}$ and the weighted sum of all such terms $(\Sigma_j w_j M \frac{\partial f_j}{\partial M})$ is equal to the amount by which the income elasticity differs from unity. The difference between $P_i \frac{\partial f_i}{\partial P_i}$ and the sum of all such terms is

exactly the difference between the own-price elasticity and -1 . For all other prices and other factors (Z) the difference equals the deviation of the elasticity from zero.

Demand theory implies four major constraints on price and income elasticities: Engel aggregation, Cournot aggregation, homogeneity and Slutsky symmetry. Of these, the first two are automatically satisfied by the MLBAM; the others may be imposed on the MLBAM by linear restrictions on the estimating equations.

Engel aggregation states that the weighted sum of the income elasticities of all goods (the weights being their budget shares) should equal unity. Cournot aggregation states that the weighted sum of cross-price elasticities of each other good to the price of good i should equal the negative of the budget share for the i'th good. These conditions are satisfied because they represent the N+1 derivatives of the budget constraint with respect to each price and income, and the MLBAM always satisfies the budget constraint.

Homogeneity of demand implies that for proportional changes in all prices and income, quantities demanded and budget shares do not change. The MLBAM may be constrained to satisfy homogeneity by deflating prices and income. This restriction could be accomplished by dividing all prices and income by one price or by income, or by deflating all prices and income by an all-item price index such as the Consumer Price Index. However, demand theory implies a pair of "correct" price indices for deflating income and prices which use budget shares and income slopes, respectively, as weights. These deflators permit the estimation of specific substitution effects for the typical consumer as described in Tyrrell (1978).

The fourth condition, Slutsky symmetry, implies that after being compensated for changes in purchasing power, the cross-price elasticity of good i to price j should equal the cross-price elasticity of good j to price i. The $(N^2 + N)/2$ such restrictions can be imposed on the MLBAM for the typical consumer by first imposing homogeneity as recommended above and then imposing symmetry on the cross-price parameters under the normalization that the weighted sum of parameters of price terms equals zero.

III. Estimation of the MLBAM

One of the major advantages of the MLBAM, mentioned earlier, is that it can be estimated by linear regression techniques. There is, however, a basic problem of the indeterminacy in the parameters. This can be shown by adding δX to each of the N functions, f_i, in (7), where δ is a constant and X is any variable

$$(14) \qquad \frac{e^{f_i + \epsilon_i + \delta X}}{\sum_j e^{f_j + \epsilon_j + \delta X}} = \frac{e^{f_i + \epsilon_i}}{\sum_j e^{f_j + \epsilon_j}} = w_i$$

Thus the N parameters for a variable that occurs in all N functions are identifiable only if a constraint is imposed, such as that they sum to zero or that one of the parameters is zero. This is similar to the problem of indeterminacy that occurs between a complete set of dummy variables and an intercept in a standard linear regression model. Nevertheless, predictions of the budget shares, w_i, are not affected by the choice of constraint.

By forming the ratio of one budget share to each of the N-1 other budget shares, the denominators of the MLBAM model cancel.[1] By taking the logarithm of each ratio, an equation that is linear in differences between parameters of the untransformed model is derived:

$$(15) \qquad \ln(w_1/w_j) = (\beta_{11} - \beta_{j1}) X_1 + \dots + (\beta_{1R} - \beta_{jR}) X_R + \varepsilon_1 - \varepsilon_j$$

$$j = 2, 3, \dots, N$$

where the notation is defined under (13).

This model could be estimated one equation at a time by OLS. However, since the error terms of the N-1 equations are generally correlated, this suggests that Zellner's (1962) GLS technique for the efficient estimation of "Seemingly Unrelated Regressions" is appropriate. It should be noted that if exactly the same set of regressors occurs in each of the N-1 equations, then the GLS and OLS procedures give identical estimates of the coefficients. In general, the regressors will differ among equations because of differences in the forms of the demand equations.

A related complication arises because the estimates of the parameters of the N-1 equations of the transformed model are not invariant with respect to the choice of a common numerator budget share when an estimated variance-covariance matrix of residuals is used. A similar problem exists when

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[1] Under the strict interpretations of the multinomial logit model these ratios are only approximations to the ratios of the probabilities of allocating dollars to different expenditure groups.

estimating a system of demand equations, and one equation must be dropped

to produce a nonsingular matrix (Barten, 1969). For that case, it has

been shown that an iterative version of Zellner's technique will converge

on a single set of estimates. For estimation of the MLBAM, the same tech-

nique is also expected to produce a unique set of estimates regardless

of the choice of numerator budget share. This point still needs to be in-

vestigated.

An additional problem arises in the estimation of the MLBAM if the

expenditure on a good for any sample observation is zero, since this implies

taking the logarithm of a zero-valued budget share. Several solutions have

been suggested. The first, and most frequently employed, is to add a con-

stant to all budget shares (Tyrrell, 1978, p. 79). A second possible solution,

which is an extension of the first, is to reestimate the model using different

magnitudes for the constant, and to extrapolate from the different estimates

of each parameter to the level where the constant is zero (Hu and Tseng, 1969).

A third is to use the maximum likelihood estimator of the MLBAM based on the

strict interpretation of the multinomial logit specification as has been

done by Tyrrell (1978, p. 76).

Budget shares and elasticities implied by an estimated model are derived

for specified levels of the explanatory variables using the corresponding

predicted values of the logarithms of the N-1 expenditure ratios, $\widehat{\ln(w_1/w_j)}$

$j = 2, 3, \ldots, N$. The budget share which serves as the common numerator

(w_1 in our example) is computed by

$$(16) \qquad \hat{w}_1 = 1/(1 + \sum_{j=2}^{N} e^{-\widehat{\ln(w_1/w_j)}}) \ .$$

The other N-1 budget shares are then computed by

$$(17) \qquad \hat{w}_j = \hat{w}_1 \, e^{-\widehat{\ln(w_1/w_j)}} \qquad\qquad j = 2, \ldots, N \; .$$

The price, income and other demand elasticities are also predicted at specified levels of the explanatory variables. The elasticity for good 1 with respect to y is computed by

$$(18) \qquad \hat{E}_{1y} = y \left[\sum_{j=2}^{N} \hat{w}_j \sum_r \left[\frac{\partial \widehat{\ln(w_1/w_j)}}{\partial X_r} \; \frac{\partial X_r}{\partial y} \right] \right] + \begin{cases} 1 & \text{if } y = M \\ -1 & \text{if } y = P_1 \text{ and} \\ 0 & \text{otherwise} \end{cases}$$

where

$X_r = g(M, \vec{P}, \vec{Z})$, a specified transformation of the explanatory

variables, and

y is an element of $M, \vec{P}$ or $\vec{Z}$.

elasticities for other goods are then computed by

$$(19) \qquad \hat{E}_{jy} = \hat{E}_{1y} - y \sum_r \frac{\partial \widehat{\ln(w_1/w_j)}}{\partial X_r} \; \frac{\partial X_r}{\partial y} \; .$$

It should be noted that the form of these expressions implies that the elasticities are not affected by the choice of normalization rule used to obtain the estimated coefficients.

IV. An Analysis of U.S. BLS Consumer Expenditures Survey Data

An analysis of household expenditures in the U.S. for 1972-73 using the MLBAM is currently in progress.[1] To illustrate the model we have performed a preliminary analysis of the allocation of expenditures to food consumed at home and away from home as part of a complete system of budget expenditures. From Engel's law, the proportion of total expenditures allocated to all food decreases as income increases. A concomitant change is the shift from food consumed at home to food consumed away from home. Results for a subset of the data show that the MLBAM has captured both phenomena.

The sample data used in the preliminary analysis is comprised of 516 consumer units interviewed during 1972 and residing in central cities of SMSA's of at least one million people in the Northeast. Of these, 21 were eliminated because they were not in existence as consumer units for an entire year.

In an analysis of household data, it is necessary to account for a wide variety of factors other than income and prices which influence consumer expenditures. Since this was a preliminary analysis the emphasis was primarily on technique. Therefore, the more difficult problems relating to food stamps and household composition have been treated in a somewhat ad hoc fashion: First, 55 food stamp recipients were eliminated from the data set. Second, 118 single-person consumer units (mostly retired) were eliminated. Third, the income measure was converted into a per capita value, and five units for which this exceeded $10,000 were eliminated.

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[1] The survey data used in this study was collected during 1972-73 by the U.S. Bureau of the Census under contract to the Bureau of Labor Statistics. There are 9869 observations for 1972 and 10,106 for 1973.

Six groups of expenditures were selected for the analysis:
Food consumed in the home (F), food consumed away from home (A), housing
(H), clothing (C), transportation (T), and other expenditures (O). The
function f_i chosen to reflect the influence of income (the only explanatory
variable)[1] on these budget shares was

$$f_i = \beta_1 + \beta_2 \ln\left(\frac{M}{FS}\right) + \beta_3\left(\frac{FS}{M}\right)$$

where

ln(M/FS)   is the natural logarithm of total expenditures divided
           by family size and

FS/M   is the inverse of per capita expenditure, included for
           added flexibility.

A nonlinear least squares procedure which converges to the results
of an iterative Zellner procedure was used to estimate the model.[2]
Since it didn't accept zero observations for budget shares, an additional
64 observations were eliminated reducing the total number of useful obser-
vations to 253.

Food consumed in the home has been used as the common numerator thus

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[1] Data for prices were not available. Their influence was assumed to
be captured by the constant term.

[2] The computer program was written by G.T. Gruvaeus and K.G. Jöreskog
at the Educational Testing Service, Princeton, New Jersey and adapted for
the "Seemingly Unrelated Regressions" problem by W.G. Greene of the Dept.
of Economics, Cornell University. The algorithm was based on the Davidson-
Fletcher-Powell procedure (Fletcher and Powell, 1963).

the estimating equations are:

$$\ln(w_F/w_j) = \beta_{Fj1} + \beta_{Fj2} \ln(\frac{M}{FS}) + \beta_{Fj3}(\frac{FS}{M}) + \epsilon_{Fj}$$

$$j = A, H, C, T \text{ and } 0 .$$

Results of the estimation are given in Table 1. The constant terms
and the coefficients of the logarithm of per capita expenditures are all
relatively precise: the ratio of the coefficients to their estimated
asymptotic standard errors is very close to 2.0 in each case. The coef-
ficients of the inverse term, however, are uniformly close to zero with
large standard errors. This seems to imply that this term, added for extra
flexibility, was not needed for this particular grouping of expenditures.
This apparent lack of precision may also be due to the high correlation
(-.95) between the two income terms.

Income elasticities and budget shares are given in Table 2 for house-
holds with per capita total annual expenditures from one to ten thousand
dollars. In addition, the values of the elasticities and budget shares
for each type of expenditure are plotted against the level of expenditures
in Figure 1 and 2.

All elasticities are shown to decline as one moves to higher levels of
income. Households with per capita expenditures of $1000 allocate the lar-
gest portion of their budget (about 1/3) to food consumed at home. At
higher levels of income this share drops rapidly along with the elasticity.
Food consumed away from home receives a much smaller share of income at

Table 1. Preliminary Estimates of the MLBAM for Central Cities of SMSAs in the Northeast with Greater than One Million People in 1972

$$\ln(W_F/W_A) = \begin{matrix} -1.108 \ \ln(M/FS) \\ (.570)^* \end{matrix} + \begin{matrix} 0.008 \ FS/M \\ (1302.8) \end{matrix} + \begin{matrix} 10.107 \\ (5.026) \end{matrix}$$

$$\ln(W_F/W_H) = \begin{matrix} -0.899 \ \ln(M/FS) \\ (.452) \end{matrix} + \begin{matrix} 0.029 \ FS/M \\ (1033.7) \end{matrix} + \begin{matrix} 7.274 \\ (3.988) \end{matrix}$$

$$\ln(W_F/W_C) = \begin{matrix} -0.784 \ \ln(M/FS) \\ (.391) \end{matrix} + \begin{matrix} 0.018 \ FS/M \\ (893.2) \end{matrix} + \begin{matrix} 6.853 \\ (3.445) \end{matrix}$$

$$\ln(W_F/W_T) = \begin{matrix} -0.929 \ \ln(M/FS) \\ (.460) \end{matrix} + \begin{matrix} 0.019 \ FS/M \\ (1051.2) \end{matrix} + \begin{matrix} 7.758 \\ (4.054) \end{matrix}$$

$$\ln(W_F/W_O) = \begin{matrix} -0.887 \ \ln(M/FS) \\ (.349) \end{matrix} + \begin{matrix} 0.036 \ FS/M \\ (797.6) \end{matrix} + \begin{matrix} 6.462 \\ (3.077) \end{matrix}$$

The estimated variance-covariance matrix of equations:

$$\hat{\Omega} = \begin{bmatrix} 1.917 & & & & \\ .703 & 1.208 & & & \\ .759 & .621 & .899 & & \\ .633 & .395 & .635 & 1.240 & \\ .669 & .555 & .581 & .477 & .719 \end{bmatrix}$$
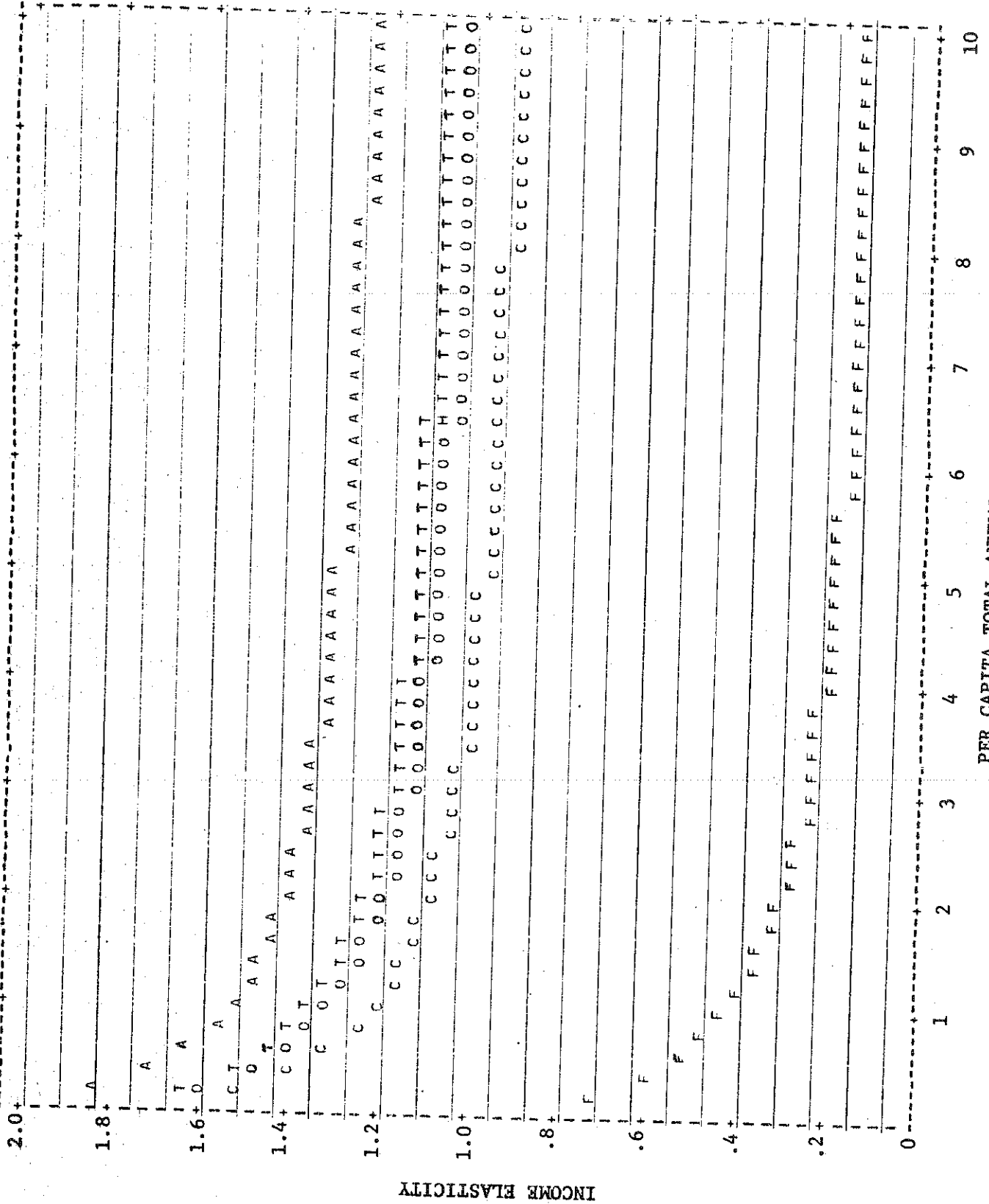
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*Approximate asymptotic standard errors in parentheses.

Table 2. Predicted Demand Elasticities and Budget Shares by Level of
Per Capita Household Expenditure

	Per Capita Expenditures (Thousands of Dollars)									
Expenditure Item	1	2	3	4	5	6	7	8	9	10
Food at Home	.44	.32	.27	.23	.21	.20	.18	.17	.17	.16
	(38)*	(24)	(19)	(15)	(13)	(11)	(10)	(9)	(8)	(7)
Food Away	1.55	1.43	1.38	1.34	1.32	1.30	1.29	1.28	1.27	1.27
	(3)	(5)	(5)	(6)	(6)	(7)	(7)	(7)	(8)	(8)
Housing	1.34	1.22	1.17	1.13	1.11	1.10	1.08	1.07	1.07	1.06
	(13)	(16)	(17)	(18)	(18)	(19)	(19)	(19)	(19)	(19)
Clothing	1.23	1.11	1.05	1.01	1.00	.98	.97	.96	.95	.94
	(9)	(10)	(10)	(11)	(11)	(11)	(10)	(10)	(10)	(10)
Transpor-tation	1.37	1.25	1.20	1.16	1.14	1.13	1.11	1.10	1.10	1.09
	(10)	(12)	(13)	(14)	(15)	(15)	(15)	(15)	(16)	(16)
Other	1.33	1.21	1.16	1.12	1.10	1.08	1.07	1.06	1.05	1.05
	(27)	(33)	(35)	(36)	(37)	(38)	(38)	(39)	(39)	(39)

*Percent of total expenditures in parentheses.

FIGURE 1. INCOME (TOTAL EXPENDITURE) ELASTICITIES

FIGURE 2. CUMULATIVE BUDGET SHARES

PERCENT OF TOTAL ALLOCATIONS
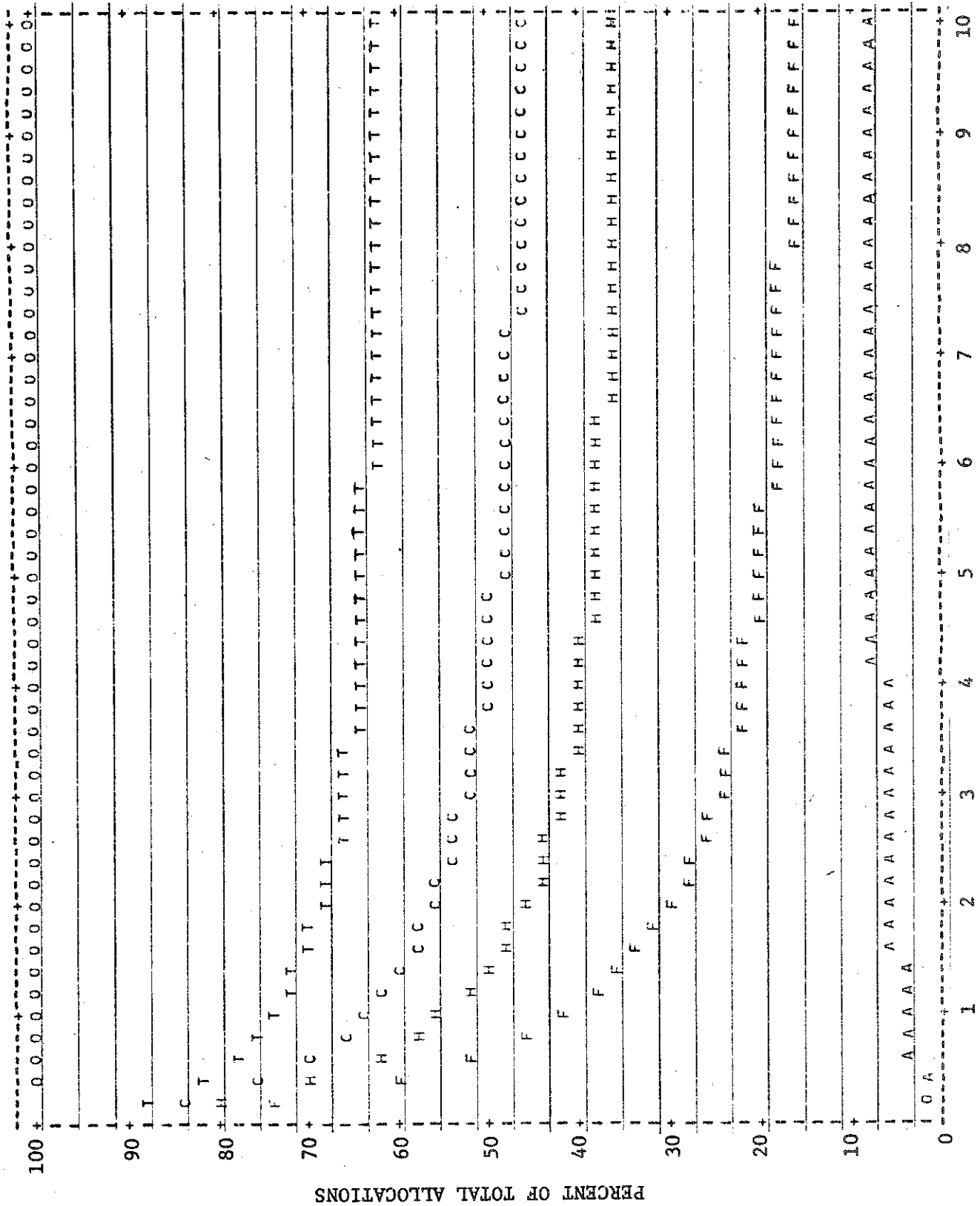
PER CAPITA TOTAL ANNUAL EXPENDITURES
(Thousands of Dollars)

Table 3. A Comparison of Income Elasticity Estimates

Expenditure category	Hassan, Johnson and Green (1977)*	Weiserbs (Phlips (1974))*	Lluch and Williams (1975)*	Tyrrell and Mount (1978)
Food	.64	.53	.41	At Home: .44 → .16 Away : 1.55 → 1.27
Housing	.46	.24	1.13	1.34 → 1.06
Clothing	1.13	1.04	.90	1.23 → .94
Transpor- tation	1.04	.80	–	1.37 → 1.09

*Taken from Hassan, Johnson and Green (November 1977, Table 9, p. 23-4)

low income levels but eventually overtakes expendiutres on food at home. As a luxury item, the income elasticity of food away from home is always greater than 1. In fact, it has the highest income elasticity of all the six categories of expenditures over all levels of income, while food consumed at home has the lowest.

Housing, transportation and other expenditures also appear as luxuries over the entire range of income levels, while clothing changes from luxury to necessity at about $5000.

Table 3 compares the estimated income elasticities with those of other studies of U.S. expenditure behavior which also employ the complete system of demand equations approach. Each of these studies used the Linear Expenditure System and annual national expenditure data.

Our food elasticity is a bit lower than the rest, but the others are based on aggregate food expenditures. Thus, our estimate for food consumed away from home seems to balance the discrepancy. Elasticities of clothing and transportation are in general agreement with the others and that of housing agrees with the estimate by Lluch and Williams. In general, all the estimates are in good accord with the other studies of U.S. expenditure patterns.

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