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Abstract. Kuhn-Tucker approach and its dual have been proposed to the demand system estimation when there are non-negativity bindings. However, empirical researchers have been struggling two decades in applying this method into practice due to: (1) the difficulty in derivation of a coherent econometric model, and (2) the cumbersome evaluation of high order probability integrals needed in parameter estimation. In this paper, we avoid the above two issues by using the Amemiya-Tobin demand system approach and the simulation procedure to evaluate the probability integrals. An AIDS model is estimated and the elasticities are obtained that are impossible to achieve when using Kuhn-Tucker approach. The model is applied to an analysis of Canadian household food demand.

Key words: Amemiya-Tobin, censored demand system, AIDS model, probability simulation, Canadian household food demand

JEL classification: C34, D12,

Estimation of a Censored AIDS Model: A Simulated Amemiya-Tobin Approach

1. Introduction

Even though it has been two decades since Wales and Woodland (1983) first introduced the Kuhn-Tucker approach to estimate micro-level (censored) demand systems, empirical researchers today are still struggling in applying this method into practice. The Kuhn-Tucker approach derives demand (share) equations from maximizing an explicitly specified random utility function after incorporating non-negativity and budget constraints. For some systems, the direct utility functions are not easy to specify. The dual approach suggested later by Lee and Pitt (1986) avoids the specification of direct utility function by deriving the demand (share) equations using Roy's Identity from the indirect utility function. However, for some systems (e.g., the widely used AIDS model), an estimable empirical format accounting for non-negativity is still impossible to obtain either from direct or indirect utility functions.

In addition to the specification problem, the Kuhn-Tucker approach and its duality generally entail incoherency problem in model estimation. Incoherency implies that the sum of the demand regimes' probabilities is not equal to one. For example, in Lee and Pitt indirect utility approach, demand regimes are determined in the way that consumers compare virtual (reservation) prices to actual market prices in making purchase decisions. This regime-switching rule cannot mutually exclude regimes from each other without restricting a priori the parameters. As found by van Soest, Kapteyn, and Kooreman

(1993), van Soest and Kooreman (1990), and Ransom (1987), an incoherent demand system will lead to inconsistent results when using the maximum likelihood estimator.

Another issue relative to the estimation of a censored demand system when using the maximum likelihood method is the need of evaluating multivariate probabilities, which occurs with truncated correlated multivariate error structure. Those probabilities are given in the form of high order integrals. Conventional numerical procedures usually used in evaluating the multivariate probability integrals such as Gauss quadrature are extremely time consuming and inaccurate. Furthermore, the adding-up condition among demand (share) equations imposed via the budget constraint raises additional computational burden due to the singularity of the error structure.

However, along with the Kuhn-Tucker approach, Wales and Woodland (1983) also proposed a different procedure to estimate the censored demand systems using Amemiya and Tobin's model. Yet not much attention has been paid to this approach relative to the Kuhn-Tucker approach. In contrast to the above random utility approach, under the Amemiya-Tobin's approach, the demand (share) equations are derived from a non-stochastic utility function and the derived expenditures (shares) differ from observed values due to errors of maximization by the consumer, errors of measurement of the observed shares, and other random disturbances which influence the consumer's decisions (Wales and Woodland, 1983). To account for these differences, error terms are added to the deterministic shares. Given the assumed normality of equation error terms, observed expenditures (shares) are thus normally distributed about the deterministic expenditures (shares). Non-negativity constraints are incorporated via the truncation of the above equation error terms similar to the censored multivariate Tobit model proposed by Amemiya(1974).

In comparison to the random utility approaches: the Kuhn-Tucker and its duality, Amemiya-Tobin's method is easy to specify, and incoherency no longer exists. However, the evaluation of high order probability integrals and the adding-up issue still persist under this approach.

In this paper we extend and transform Amemiya-Tobin's approach proposed by Wales and Woodland (1983) to an estimable set-up using an AIDS model specification. The AIDS specification used here incorporates both non-negativity and budget constraints. Even though the estimation requires the use of simulated maximum likelihood techniques when the number of commodities analyzed is large, we need simulate orthogonal (rectangle) multivariate probabilities only, which are easy to obtain and the accuracy and speed are relatively high.

In the next section, we develop an estimable censored AIDS model imposing adding-up and other theoretical constraints. In this section, we build up the likelihood function based on Wales and Woodland (1983) and transform it so that it can be easily simulated. This is followed by a section on model prediction and elasticity evaluation. We then briefly present an empirical application using a Canadian household meat demand structure.

2. Censored AIDS Model

Following Deaton and Muellbauer (1980), and Heien and Wessells (1990), we assume the consumer's utility function can be represented by a PIGLOG class from which the

AIDS demand system is derived (Pollack and Wales, 1992). The following system of M+1 latent share (W^*) equations can be expressed as:

$$W^* = U + \varepsilon \, , \tag{1}$$

where $U = A + \gamma \ln P + \eta \ln Y$, $A = \alpha + \beta X$, $Y = \frac{y^*}{P^*}$, P is a [M + 1] column vector of commodity prices, X is a [L x 1] vector of demographic characteristics, y^* is a [(M + 1) x 1] vector of total expenditures, ε is a [(M + 1) x 1] vector of equation error terms, and P^* is a translog price index defined by:

$$\ln P^* = \alpha_0 + \alpha' \ln P + \frac{1}{2} (\ln P)' \gamma \, (\ln P) \, . \tag{2}$$

The equation parameters are: α [(M+1) x 1], β [(M + 1) x L], γ [(M+1) x (M+1)], η [(M+1) x 1], and α_0, a scalar parameter.

Theoretical constraints such as homogeneity and symmetry can be imposed on (1). Notice however there are no non-negativity constraints imposed on these latent shares. There is nothing in the formulation to ensure that the elements of W^* lie between 0 and 1.

Given the budget constraint, we know the latent shares must sum to one, and therefore, the joint density function of ε is singular. Consequently one of the [M + 1] latent share equations must be dropped during estimation. Dropping the last equation from the estimation, we assume the first M share equations' error terms, ε in (1), are distributed multivariate normal with a joint probability density function (*PDF*). That is, ε ~N(0, Σ), where Σ is an [M x M] error variance-covariance matrix.

The mapping of the vector of latent, W^*, to observed shares, W, must take into account that the elements of W: (i) lie between 0 and 1, and (ii) sum to unity. Following Wales and Woodland (1983), the following mapping rule imposes these characteristics:

$$W_i = \begin{cases} W_i^* / \sum_{j \in S} W_j^*, & \text{if} \quad W_i^* > 0, \\ 0, & \text{if} \quad W_i^* \leq 0, \end{cases} \tag{3}$$

where S is a set of all positive shares' subscripts. As pointed out by Wales and Woodland, though there may be ways other than (3) in mapping W^* to W, the one we have chosen is both simple and has the property that the resulting density function is independent of whatever set of the W^*'s is used in its derivation.

Assuming that at least one commodity is purchased, we can partition observed purchase patterns into three general purchase regimes: (i) at least one commodity is purchased, but the total number of purchased commodities is less then M, (ii) M commodities are purchased, and (iii) all M +1 commodities are purchased. For each of these regimes we can develop regime-specific likelihood functions that can be used to obtain demand system parameter estimates. Since a particular household is associated with only one purchase regime, the regime that encompasses the particular purchase pattern of a household determines the contribution this household makes to the overall sample likelihood function value.

Derivation of Regime i Likelihood Function: Some Commodities Not Purchased

To facilitate the presentation, for households where K commodities are purchased and $M > K \geq 1$, we can rearrange the ordering of the M commodities so that the first K are purchased. Accordingly, Σ, the error term covariance matrix can be partitioned as:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{10} \\ \Sigma_{10}^{'} & \Sigma_{00} \end{bmatrix}, \tag{4}$$

where Σ_{11} is a K x K submatrix associated with the purchased commodities, Σ_{00} is a (M-K) x (M-K) submatrix associated with the non-purchased commodities, and Σ_{10} is a (M-K) x K submatrix of covariance across purchase and non-purchase commodities. With this rearrangement, the likelihood of a household being in a purchase regime where the first K commodities are positive and zero for the remaining can be represented via the following (Wales and Woodland, 1983):[1]

$$\begin{aligned} & L(W_1, W_2, \cdots, W_k > 0; W_{k+1} = W_{k+2} \cdots = W_M = 0) \\ & = \int_{W_1}^{+\infty} \int_{1-\frac{W_1^*}{W_1}}^{0} \int_{1-\frac{W_1^*}{W_1}-W_{k+1}^*}^{0} \cdots \int_{1-\frac{W_1^*}{W_1}-W_{k+1}^*-\cdots-W_M^*}^{0} \phi(W_1^*, W_2^*, \cdots, W_M^*; U, \Sigma) dW_M^* \cdots dW_{k+1}^* dW_1^* \cdot \end{aligned} \tag{5}$$

The integral in (5) is [M–K+1] fold, which is the number of non-purchased commodities. In order to evaluate the multivariate integrals, as we will discuss below, we transform equation (5) as follows by reducing the dimension of $\phi\,(\cdot)$ from M to [M–K+1]:

$$\begin{aligned} & L(W_1, W_2, \cdots, W_k > 0; W_{k+1} = W_{k+2} \cdots = W_M = 0) \\ & = B \cdot \int_{W_1}^{+\infty} \int_{1-\frac{W_1^*}{W_1}}^{0} \int_{1-\frac{W_1^*}{W_1}-W_{k+1}^*}^{0} \cdots \int_{1-\frac{W_1^*}{W_1}-W_{k+1}^*-\cdots-W_M^*}^{0} \phi(W_1^*, W_{k+1}^*, \cdots, W_M^*; U^*, \Omega_{11}) dW_M^* \cdots dW_{k+1}^* dW_1^* \end{aligned} \tag{6}$$

where $U^* = \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} = \Omega_{11}\Omega_{10}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}$, an [($M$-$K$+1) x 1] vector, and

$$B = (2\pi)^{\frac{1-k}{2}} \cdot |\Sigma|^{-\frac{1}{2}} \cdot |\Omega_{11}|^{\frac{1}{2}} \cdot e^{-\frac{1}{2}\left\{ \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix}' \Omega_{00}^{-1} \begin{pmatrix} U_1 \\ U_{k+1} \\ \vdots \\ U_M \end{pmatrix} - \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix}' \Omega_{11}^{-1} \begin{pmatrix} U_1^* \\ U_{k+1}^* \\ \vdots \\ U_M^* \end{pmatrix} \right\}}.$$

[1] Notice that the sum of the K observed (positive) shares is one.

The above Ω_{ij}'s are [(*M*-*K*+1) x (*M*-*K*+1)] matrixes, and defined as:

$$\Omega_{11} = \begin{bmatrix} I'\sigma_{11}I & I'\sigma_{10} \\ \sigma_{10}'I & \sigma_{00} \end{bmatrix},\ \Omega_{00} = \begin{bmatrix} J'\sigma_{11}J & J'\sigma_{10} \\ \sigma_{10}'J & \sigma_{00} \end{bmatrix}, \text{ and } \Omega_{10} = \begin{bmatrix} I'\sigma_{11}J & I'\sigma_{10} \\ \sigma_{10}'J & \sigma_{00} \end{bmatrix},$$

where *I* is a [*K* x 1] vector of ones, and J is a [*K* x 1] vector with the elements:

$(1, \frac{U_2}{(\frac{W_2}{W_1})U_1}, \frac{U_3}{(\frac{W_3}{W_1})U_1}, \frac{U_4}{(\frac{W_4}{W_1})U_1}, \cdots, \frac{U_k}{(\frac{W_k}{W_1})U_1})'$. The σ_{ij}'s are defined via the following [M x M]

matrix: $\begin{bmatrix} \sigma_{11} & \sigma_{10} \\ \sigma_{10}' & \sigma_{00} \end{bmatrix} = \begin{bmatrix} A\Sigma_{11}^{-1}A' & A\Sigma_{10}^{-1} \\ \Sigma_{10}^{-1'}A' & \Sigma_{00}^{-1} \end{bmatrix}^{-1}$. The [*K* x *K*] matrix *A* is a diagonal matrix

with elements: $(1, \frac{W_2}{W_1}, \frac{W_3}{W_1}, \cdots, \frac{W_k}{W_1})$. Finally, the Σ_{ij}^{-1} matrices are obtained from the full

error variance matrix, Σ, in (4).

From the results shown in Tallis (1965), the likelihood function represented by (6) can be further transformed to:

$$L(W_1, W_2, \cdots, W_k > 0; W_{k+1} = W_{k+2} \cdots = W_{M+1} = 0) = B \cdot \Phi_{M-k+1}(b; R_C)\,, \tag{7}$$

where $\Phi_{M-k+1}(b; R_C)$ is a [*M*-*K*+1] dimensional multivariate standard normal cdf with correlation coefficient matrix as R_C and evaluated at vector *b*. Vector *b* is [(*M*-*K*+1) x 1] and can be shown to be equal $E \cdot G$, where *E* is a [*M*-*K*+1] diagonal matrix with diagonal elements equal to $(\,(C_1RC_1')^{-1/2}, (C_{k+1}RC_{k+1}')^{-1/2}, \cdots, (C_MRC_M')^{-1/2}\,)$; where

$C = \begin{pmatrix} C_1 \\ C_{k+1} \\ \vdots \\ C_M \end{pmatrix} = H \cdot D^{\frac{1}{2}},\ H = \begin{bmatrix} \frac{1}{W_1} & 1 & 1 & \cdots & 1 \\ 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{bmatrix}$, a [*M*-*K*+1] square matrix, *R* is the correlation coefficient matrix derived from $\Omega_{11,}$ and D the diagonal elements of Ω_{11}.

Term $G = \begin{pmatrix} 1 - H_1 U_1^* \\ -U_{k+1}^* \\ \vdots \\ -U_M^* \end{pmatrix}$, where H_1 is the first row of matrix H. The new correlation coefficient matrix (R_C) is given as $R_C = ECRC'E'$ (Tallis, 1965).

Equation (7) represents a rectangular standard multivariate normal probability, which can be conveniently evaluated using standard simulation procedures. The smooth recursive conditioning simulator (GHK) suggested by Geweke(1991), Hajivasiliou and McFadden(1990), and Keane(1994) is adopted for this analysis to simulate this multivariate normal probability.

Derivation of the Regime ii Likelihood Function: Only One Commodity Not Purchased

Regime *ii* is characterized by the number of commodities actually purchased, K, equaling M. This implies that (6) can be restated as:

$$L(W_1, W_2, \cdots, W_M > 0; W_{M+1} = 0) = B \cdot \int_{W_1}^{+\infty} \phi(W_1^*; U^*, \Omega_{11}) dW_1^* , \qquad (8)$$

where $U^* = U_1^* = \Omega_{11}\Omega_{10}^{-1}U_1$, and $\Omega_{11} = I'\sigma_{11}I$, $\Omega_{00} = J'\sigma_{11}J$, $\Omega_{10} = I'\sigma_{11}J$, are all scalars now with $\sigma_{11} = (A\Sigma^{-1}A')^{-1}$, A is an M x M diagonal matrix with diagonal elements: $(1, \frac{W_2}{W_1}, \frac{W_3}{W_1}, \cdots, \frac{W_M}{W_1})$, and I a [M x 1] vector of ones,

$$J = \left(1, \frac{U_2}{(\frac{W_2}{W_1})U_1}, \frac{U_3}{(\frac{W_3}{W_1})U_1}, \frac{U_4}{(\frac{W_4}{W_1})U_1}, \cdots, \frac{U_M}{(\frac{W_M}{W_1})U_1}\right)', \text{ and}$$

$$B = (2\pi)^{\frac{1-k}{2}} \cdot |\Sigma|^{-\frac{1}{2}} \cdot |\Omega_{11}|^{\frac{1}{2}} \cdot e^{-\frac{1}{2}\{U_1' \Omega_{00}^{-1} U_1 - U_1^{*'} \Omega_{11}^{-1} U_1^{*}\}} .$$

Thus, under purchase regime *ii*, the likelihood function requires the integration of a univariate *PDF*.

Derivation of the Regime iii Likelihood Function: All Commodities Purchased

For households where all commodities are purchased ($K = M+1$), the likelihood function of this regime is just the [M x 1] multivariate *PDF* of error term, ε, which is defined in (1) and distributed as $MN(0,\Sigma)$, where Σ is given by (4). That is:

$$L(W_1, W_2, \cdots, W_{M+1} > 0) = \phi(\varepsilon) \tag{9}$$

Consistent and efficient estimates of parameters can be obtained by maximizing the sum of log likelihood function over all households, which fall into one of the three demand regimes, i.e., equations (7), (8), and (9).

3. Evaluation of Predicted Shares and Demand Elasticities

Expected values of observed expenditure shares can be obtained from our censored demand system by summing the products of each regimes' probability and expected conditional share values over all possible regimes. Let R_k represent the k^{th} demand regime that is characterized as:

$$R_k = (W_1 = W_2 = \cdots = W_k = 0; W_{k+1} > 0, \cdots, W_{M+1} > 0).$$ [2]

[2] This is the regime of the first *k* *W*'s are zeros and the rest are positive. Given *k* zero *W*'s, other possible regime can be transformed to this pattern by rearranging the ordering of the *W*'s so that the first *k* are zeros

The expected value of the j^{th} observed share is:

$$E(W_j) = \sum_{k=1}^{M+1} \alpha_{R_k} E(W_j \mid R_k), \tag{10}$$

where α_{R_k} is the probability of regime R_k occurring, and

$$\alpha_{R_k} = prob(R_k) = prob(W_1 = W_2 = \cdots = W_k = 0; W_{k+1} > 0, \cdots, W_{M+1} > 0) \tag{11}$$

$$= \int_{-\infty}^{-U_1} d\varepsilon_1 \int_{-\infty}^{-U_2} d\varepsilon_2 \cdots \int_{-\infty}^{-U_k} d\varepsilon_k \int_{-U_{k+1}}^{\sum_{i=k+2}^{M+1} U_i - \sum_{i=2}^{k} \varepsilon_i} d\varepsilon_{k+1} \cdots \int_{-U_{M-1}}^{\sum_{i=M}^{M+1} U_i - \sum_{i=2}^{M-2} \varepsilon_i} d\varepsilon_{M-1} \int_{-U_M}^{U_{M+1} - \sum_{i=2}^{M-1} \varepsilon_i} \phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M) d\varepsilon_M,$$

where $\phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M)$ is the multivariate normal pdf with mean vector of zeros and variance-covariance matrix Σ. The expected share value conditional on purchase regime R_k can be represented as:

$$E(W_j \mid R_k) = \begin{cases} \dfrac{E(W_j^* \mid R_k)}{\sum_{i=k+1}^{M+1} E(W_i^* \mid R_k)}, & if \;\; j > k, \\ \\ 0, & if \;\; j \le k; \end{cases} \tag{12}$$

with $E(W_j^* \mid R_k) = U_j + E(\varepsilon_j \mid R_k) = U_j + \dfrac{\alpha_{R_k}^{\varepsilon_j}}{\alpha_{R_k}}$, where,

$$\alpha_{R_k}^{\varepsilon_j} = \int_{-\infty}^{-U_1} d\varepsilon_1 \int_{-\infty}^{-U_2} d\varepsilon_2 \cdots \int_{-\infty}^{-U_k} d\varepsilon_k \int_{-U_{k+1}}^{\sum_{i=k+2}^{M+1} U_i - \sum_{i=2}^{k} \varepsilon_i} d\varepsilon_{k+1} \cdots \int_{-U_{M-1}}^{\sum_{i=M}^{M+1} U_i - \sum_{i=2}^{M-2} \varepsilon_i} d\varepsilon_{M-1} \int_{-U_M}^{U_{M+1} - \sum_{i=2}^{M-1} \varepsilon_i} \varepsilon_j \phi(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_M) d\varepsilon_M \tag{13}$$

From (10) the impact of changes in prices, demographics or expenditures on food demand can be obtained but one needs to evaluate M-dimension integrals represented by (11) and (13). Given that there are 2^{M+1}-1 purchase regimes, one needs to evaluate (11) and (13) 2^M times. Phaneuf, Kling, and Herriges (2000) in an analysis of recreation demand, develop a simulation procedure to evaluate expressions similar to (10)-(13). We modify their procedure to our application. This procedure is designed below.

Assume we have R replicates of the $[M+1]$ error term vector, ε in (1). The r^{th} simulated latent share, $(W^*)_r$, evaluated at sample means of our exogenous variables (indicated by a bar over a variable) is

$$(W^*)_r = \alpha + \gamma \ln \overline{P} + \beta \ln \frac{\overline{Y}}{\overline{\overline{P^*}}} + \varepsilon_r \tag{14}$$

where ε_r is the r^{th} replicate of ε. The r^{th} replicate of the i^{th} observed share then is

$$(W_i)_r = \begin{cases} (W_i^*)_r / \sum_{j \in S} (W_j^*)_r, & \text{if} \quad (W_i^*)_r > 0, \\ 0, & \text{if} \quad (W_i^*)_r \leq 0, \end{cases} \tag{15}$$

where the subscript i of W represents the i^{th} element in the vector of W. The expected observed share vector for R replicates is then calculated as simple average of these simulated values:

$$E(W) = \frac{1}{R} \sum_{r=1}^{R} (W)_r \ . \tag{16}$$

Suppose we have a small change in price j, ΔP_j, the elasticity vector with respect to this price change is:

$$\eta_j = -\delta_j + \frac{\Delta E(W)}{\Delta P_j} \cdot \frac{P + \Delta P_j / 2}{E(W) + \Delta E(W)/2}, \tag{17}$$

where δ_j is a vector of 0's with the j^{th} element 1, and $\Delta E(W)$ is the change of the simulated $E(W)$ given the change of price, ΔP_j.

4. An Empirical Illustration: Canadian Household Food Demand Systems

In this empirical application, we investigate a food demand system for Canadian household using the above-developed approach. Canada represents a significant export market for raw and processed U.S. food products, and is the U.S.'s second largest trading partner after the European Union. The study of the effects on Canadian household food purchase patterns will provide valuable information for food marketing managers crafting export policies.

Data

The data is obtained from the nationwide 1996 *Canadian Family Food Expenditure Survey*. This survey contains two-week diaries of food expenditures and quantity purchased, where each expenditure item is coded according to a four-digit food code. In addition to purchase information, other data included in the survey are household member age distribution, pre-tax household income, male and female head country of birth, residential province, degree of urbanization and month during which the survey was undertaken. There are near 10,000 households in the base data set. For this analysis we use a random sample of 2,905 households to reduce the estimation time.

Table 1 provides the definition of exogenous variables used in our econometric model along with sample means and standard deviations. Among these demographic variables, there are household size, spending rate of food away from home, race, household composition, season, and region. The average size of the Canadian household is slightly less than 3, and they spend 22% of their food expenditures away from home. We categorize Canadian households as four ethnic groups: (1) Canada, USA, north and west Europe; (2) Asia; (3) south and east Europe; and (4) Others. In model estimation, we use group (1) as the base. Three dummy variables are defined to indicate the household types: single, single parent, and married couple without kids. There are four dummies for seasons and seven for regions. The region Atlantic includes: Newfoundland, Prince Edward Island, Nova Scotia, and New Brunswick. Due to singularity, we drop the fourth Quarter and Ontario from the estimation variables and treat them as base.

Table 2 presents purchase frequencies, means, and standard deviations of expenditures on six meat (including fish) commodities consumed by Canadian households. It appears that “other meat” is the most frequently purchased commodity. However, according to the expenditures, Canadian households spent, on average, the most on pork.

Prices are not observed directly. They are derived and aggregated from the quantities and expenditures over all the products in the corresponding composite commodity category. For those non-purchase households, the missing prices are replaced with the average price from the purchase households that reside in the same particular geographic location, i.e., the seven provincial areas. It appears that fish is the most expensive commodity, while beef is the second, and ground beef is the cheapest meat to Canadian households.

Estimated Coefficients

The AIDS model defined in equation (1) is estimated using the GAUSS software system and BHHH optimal algorithm (Berndt, Hall, Hall and Hausman; 1974). The price index parameter α_0 in (2) is normalized to 0 due to the identification problem with parameter α. As we mentioned above, the GHK simulation procedure is adopted to simulate the high order probability integrals. The number of simulation replicates is set to 200. Table 3 shows the maximum likelihood parameter estimates for the demographic, expenditure and price related coefficients. The equation omitted during estimation is the one corresponding to "other meat".[3] The associated parameters for this omitted equation are retrieved from the AIDS adding-up, symmetry, and homogeneity constraints.

Of the demographic related parameters estimated, we found total expenditure has significant effects on all the 6 commodities. We also found the effects of household size are statistically significant on poultry, beef, ground beef, and other meat. However, most other demographic variables were found statistically insignificant at the level of 0.05. For example, there is no evidence of significant differences in purchase patterns found over seasons for all the meat commodities except beef.

In addition to the expenditure and demographic related parameters, Table 3 also shows the estimated own and cross-price coefficients. All the own-price coefficients were found to be statistically different from zero at the 0.05 level of significance. Of the 14 cross-price coefficients estimated, 4 were found statistically significant at the 0.05 level.

Estimated Elasticities

The estimated parameters themselves are of little interest. From these parameters however we estimate uncompensated, unconditional own and cross-price elasticities by the simulation procedures outlined via equations (14)-(17). The resulting elasticity estimates are shown in Table 4. As expected, all own-price elasticities were found to be negative with a range of -1.1491 for other meat products to –1.6082 for beef.

[3] We compared the results by dropping different commodity and confirmed that these results asymptotically converge to the same.

Comparing these results with those from the aggregate time series model, it appears that the own price elasticity from household level model are much higher than those from aggregate time series model. For example, Chalfant, Gray, and White (1991) found the own price elasticities for meats using Canadian time series data ranging from –0.554 for fish to –0.955 for beef. The high numbers may come from the quality effects mingled with the price effects when using household level data (Cox and Wohlgenant; 1999). Capps and Havlicek (1984) found the similar high results of own price elasticities ranging from –1.2810 for poultry to –2.0264 for seafood in a study of US household's meat demand.

Table 6 shows estimated demographic elasticities for the continuous demographic variables used in our analysis along with the percentage point change in shares due to a discrete change in the set of dichotomous demographic characteristics.[4] The sign of elasticities was found consistent with that of the associated coefficients estimated.

5. Conclusions and Future Research

In this paper, we developed an estimable household's level AIDS model using an adapted Amemiya-Tobin approach to account for the censoring of commodity purchases. The model is estimated using simulated maximum likelihood techniques. The use of this technique has enabled us to evaluate a large censored demand system, which would have been impractical under traditional maximum likelihood techniques.

This research represents a first attempt at estimating a disaggregated food demand system and evaluating its elasticities. At least three extensions can be undertaken that

[4] Except for the exogenous variable of concern, all exogenous variables are set at their mean values.

could improve the quality of this research. First, a methodological improvement to the current specification would be the endogenization of product quality similar to the procedures outlined in the single equation approaches of Dong, Shonkwiler and Capps(1998). That is, in spite of the estimation of a disaggregated demand system, there continues to be a range of product quality within each commodity group where this product quality is an endogenous variable that is part of the household's purchase experience. Second, in this paper we only evaluated the uncompensated price elasticities. However, the compensated elasticities could be also obtained from Slutsky equations using a similar simulation procedure.

Finally, the above analysis has quantified the unconditional impacts of the change in prices and household demographic characteristics on commodity demand. However, we may be interested to know how the regime dependent (conditional) demand levels and the probability of purchase given a particular regime are impacted by these changes. This will enable us to quantify both the intensive and extensive consumer response on a given commodity in the system to changes in these variables.

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Table 1. Descriptive Statistics of Exogenous Variables Used in Econometric Models

Variable	Description	Mean	Standard Deviation
HHSIZE	Number of household members (#)	2.71	1.34
HOMERAT	Rate of FAFH to total food expenditures	0.22	0.20
ASIA	Household of Asian origin (0/1)	0.04	0.19
SEURP	Household of South and East Eroupe origin (0/1)	0.03	0.17
OTHCNT	Household of other origin (0/1)	0.03	0.17
SINGLE	Single household (0/1)	0.10	0.30
SINGLPAR	Single parent household (0/1)	0.07	0.25
MARRNOKD	Married couple without children (0/1)	0.19	0.39
QRT_1	First quarter of the year (0/1)	0.24	0.43
QRT_2	Second quarter of the year (0/1)	0.25	0.43
QRT_3	Third quarter of the year (0/1)	0.26	0.44
ATLANTIC	Atlantic area (0/1)	0.22	0.42
QUEBEC	Quebec (0/1)	0.15	0.36
MANITOB	Manitoba (0/1)	0.06	0.24
SASK	Saskatchewan (0/1)	0.08	0.28
ALB	Alberta (0/1)	0.09	0.28
BC	British Columbia (0/1)	0.12	0.33

Table 2. Descriptive Statistics of Purchase Variables Used in Econometric Models

Commodity	Mean Share	Purchase Frequency	Price (kg/$)		Expenditure ($)	
			Mean	St.deviation	Mean	St.deviation
Poultry	0.18	57 %	5.94	2.73	8.70	13.94
Fish	0.12	49 %	9.73	3.85	5.43	15.31
Beef	0.15	48 %	8.21	2.48	8.42	20.25
Ground Beef	0.10	50 %	5.22	1.46	4.36	6.80
Pork	0.21	70 %	7.60	2.81	8.84	11.29
Other Meat	0.24	77 %	7.19	3.22	8.79	11.50

Table 3. Censored Demand System Parameter Estimates (t-ratios are in parenthesis)

	Poultry	**Fish**	**Beef**	**Ground Beef**	**Pork**	**Other Meat**
Int	0.0509 (1.399)	**0.2235** **(6.423)**	0.0611 (1.687)	-0.0405 (-1.439)	**0.2931** **(8.269)**	**0.4118** **(10.99)**
			Demographic Characteristics			
Log(Hhsize)	**-0.0418** **(-1.976)**	-0.0237 (-1.175)	**-0.0875** **(-3.994)**	**0.0540** **(3.257)**	0.0143 (0.702)	**0.0847** **(3.828)**
HOMERAT	**0.1046** **(2.594)**	-0.0266 (-0.668)	0.0072 (0.184)	-0.0577 (-1.838)	-0.0245 (-0.616)	-0.0029 (-0.069)
ASIA	0.0714 (1.424)	**0.2179** **(5.013)**	-0.0257 (-0.488)	**-0.1194** **(-2.809)**	-0.0398 (-0.715)	-0.1045 (-1.686)
SEURP	0.0161 (0.328)	0.0145 (0.247)	-0.0521 (-1.004)	-0.0719 (-1.268)	0.0035 (0.072)	0.0899 (1.510)
OTHCNT	**0.1321** **(2.372)**	0.0645 (1.079)	0.0653 (1.259)	0.0418 (1.242)	**-0.1762** **(-2.853)**	-0.1276 (-1.945)
SINGLE	-0.0377 (-1.150)	-0.0450 (-1.427)	-0.0103 (-0.324)	0.0400 (1.438)	-0.0241 (-0.742)	**0.0771** **(2.390)**
SINGLPAR	-0.0040 (-0.114)	-0.0546 (-1.600)	-0.0277 (-0.748)	**0.1330** **(6.002)**	-0.0384 (-1.095)	-0.0082 (-0.214)
MARRNOKD	0.0080 (0.349)	0.0070 (0.301)	-0.0207 (-0.934)	0.0166 (0.914)	0.0208 (0.880)	-0.0317 (-1.158)
QRT_1	-0.0265 (-1.058)	-0.0008 (-0.033)	**0.0736** **(3.035)**	-0.0017 (-0.095)	-0.0435 (-1.770)	-0.0010 (-0.040)
QRT_2	-0.0367 (-1.502)	0.0215 (0.910)	**0.0604** **(2.514)**	-0.0010 (-0.055)	-0.0296 (-1.195)	-0.0146 (-0.535)
QRT_3	-0.0133 (-0.559)	-0.0441 (-1.812)	**0.0500** **(2.062)**	0.0009 (0.049)	0.0160 (0.673)	-0.0095 (-0.357)
ATLANTIC	-0.0347 (-1.335)	**0.0846** **(3.398)**	0.0072 (0.286)	0.0014 (0.074)	0.0168 (0.641)	-0.0754 (-2.574)
QUEBEC	-0.0503 (-1.703)	-0.0212 (-0.720)	**0.0897** **(3.341)**	0.0359 (1.682)	0.0026 (0.081)	-0.0566 (-1.663)
MANITOB	0.0103 (0.278)	-0.0715 (-1.778)	0.0286 (0.775)	-0.0380 (-1.304)	0.0283 (0.754)	0.0423 (1.063)
SASK	-0.0343 (-1.054)	-0.0172 (-0.498)	-0.0494 (-1.446)	-0.0188 (-0.754)	0.0544 (1.779)	0.0654 (1.923)
ALB	-0.0567 (-1.726)	-0.0297 (-0.854)	**0.0859** **(2.969)**	-0.0091 (-0.374)	**0.0681** **(2.114)**	-0.0585 (-1.634)
BC	-0.0323 (-1.0821)	**0.0594** **(2.141)**	-0.0217 (-0.706)	-0.0424 (-1.865)	0.0412 (1.401)	-0.0043 (-0.138)
			Total Expenditures			
	0.0903 **(7.065)**	**-0.0596** **(-6.366)**	**0.0775** **(6.629)**	**0.0226** **(2.378)**	**-0.0292** **(-2.916)**	**-0.1016** **(-9.745)**
			Price			
Poultry	**-0.1129** **(-5.936)**					
Fish	-0.0027 (-0.158)	**-0.1445** **(-7.392)**				
Beef	**0.0444** **(2.529)**	0.0321 (1.649)	**-0.2268** **(-10.30)**			
Ground Beef	0.0172 (1.166)	0.0121 (0.708)	**0.0407** **(2.189)**	**-0.0886** **(-4.751)**		
Pork	0.0352 (1.900)	**0.0675** **(3.328)**	0.0366 (1.769)	0.0222 (1.303)	**-0.1605** **(-6.307)**	
Other Meat	0.0186 (0.985)	0.0356 (1.875)	**0.0730** **(3.767)**	-0.0036 (-0.033)	-0.0009 (-0.051)	**-0.1226** **(-5.214)**

Table 5. Expenditure and Marshallian (Uncompensated) price Elasticities

	Poultry	**Fish**	**Beef**	**Ground Beef**	**Pork**	**Other Meat**
Poultry	**-1.2811**	-0.0366	0.0717	0.0169	0.0488	-0.0102
Fish	0.0260	**-1.3672**	0.1141	0.0473	0.2222	0.1260
Beef	**0.0799**	0.0490	**-1.6082**	0.0841	0.0625	0.1401
Ground Beef	0.0515	0.0282	**0.1383**	**-1.3226**	0.0674	-0.0338
Pork	0.1009	**0.1642**	0.0932	0.0580	**-1.3792**	0.0281
Other Meat	0.0485	0.0510	**0.1314**	0.0055	0.0662	**-1.1491**
Total Expend	**1.1904**	**0.8316**	**1.1926**	**1.0710**	**0.9348**	**0.8466**

Note: Numbers in bold indicate the associated coefficients are statistically significant at the level of 0.01.

Table 6: Elasticity and Unconditional Predicted Share Impacts of Changes in Demographic Characteristics

Exogenous Variable	Poultry	Fish	Beef	Ground Beef	Pork	Other Meat
	Elasticities					
HHSIZE	**-0.0878**	-0.0576	**-0.2232**	**0.1957**	0.0281	**0.1261**
HOMERAT	**0.0505**	-0.0164	0.0037	-0.0463	-0.0149	-0.0004
	Change in Shares From Discrete Change in Dichotomous Exogenous Variable					
ASIA	2.61	**8.24**	-1.15	**-4.05**	-1.88	-3.78
SEURP	0.68	0.51	-1.96	-2.48	-0.16	3.41
OTHCNT	**5.42**	2.01	2.46	1.36	**-7.21**	-4.04
SINGLE	-1.44	-1.42	-0.41	1.54	-1.27	**3.00**
SINGLPAR	-0.29	-1.84	-1.17	**5.41**	-1.77	-0.34
MARRNOKD	0.26	0.19	-0.82	0.61	1.03	-1.28
QRT_1	-1.03	-0.01	**2.95**	-0.07	-1.96	0.13
QRT_2	-1.44	0.75	**2.42**	-0.03	-1.28	-0.41
QRT_3	-0.60	-1.50	**1.92**	0.00	0.71	-0.53
ATLANTIC	-1.45	**2.92**	0.28	0.04	1.00	-2.80
QUEBEC	-2.10	-0.78	**3.57**	1.31	0.22	-2.23
MANITOB	0.34	-2.33	1.03	-1.41	1.07	1.29
SASK	-1.38	-0.57	-1.89	-0.68	2.35	2.18
ALB	-2.42	-1.12	**3.35**	-0.42	**3.24**	-2.63
BC	-1.32	**2.06**	-0.83	-1.52	1.92	-0.32

Note: Numbers in bold indicate the associated coefficients are statistically significant at the level of 0.01.

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