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Agricultural Risk Modeling

Using Mathematical Programming

Richard N. Boisvert
and
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AGRICULTURAL RISK MODELING USING MATHEMATICAL PROGRAMMING

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and

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FOREWORD

Like others in the Cooperative Series of the Southern Regional Association of State Agricultural Experiment Stations, this bulletin is in effect a separate publication issued by each of the cooperating stations. As such, it may be mailed under the frank and indicia of each of the stations listed below. Since the bulletin is identical for all stations, requests for copies originating within the cooperating states should be made to the appropriate Agricultural Experiment Station director. Requests from other states should be sent to the director of the New York Agricultural Experiment Station, Cornell University, Ithaca, NY 14853.

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ABSTRACT

Since the advent of linear programming, a body of literature has been developed focusing on techniques to incorporate uncertainty in the model parameters into the programming formulations. Many of the important applications in the early years were related to agricultural decision problems involving risk (Freund, 1956; Heady and Candler, 1958; Stovall, 1966; and Tintner, 1955). Since these early efforts, numerous studies have been completed, yielding a rich literature on risk in prices, production, costs, resource usage and resource availability. The purpose of this bulletin is to provide a survey of the literature on the variety of modeling techniques, theory and applications in a risky environment. It was written to be useful to researchers using mathematical programming methods in the study of risk, as well as in graduate courses in mathematical programming or risk analysis.

The bulletin is organized into four sections. The first briefly characterizes the risky nature of agricultural decisions, while the second reviews theoretical foundations for risk analysis. This second section is rather long and is included for completeness. It will be of most interest to students or researchers unfamiliar with the foundations underlying expected utility theory. Those familiar with the theory can move rapidly to the third section which provides some discussion of when risk and uncertainty should be incorporated explicitly into programming analyses and how it should be accomplished. Then, a number of programming techniques are discussed, as is their consistency with theoretical risk decision criteria. Emphasis is placed on models in which the objective function coefficients are not known with certainty, but considerable attention is also given to models in which the right-hand side values or the technical coefficients or some combination of all three types of parameters are uncertain. The fourth section contains a brief set of concluding comments.

The manuscript also contains two appendices. The first appendix should be most useful to students. It illustrates most of the programming models discussed in the text, using small empirical examples. In the models reflecting risk in the objective function, a portfolio problem, with only one financial constraint, is used. This model helps isolate the effect of the risk decision criteria on the optimal solution. More complicated examples are needed (and used) to illustrate the other risk models.

The second appendix provides a bibliography of recent applications of risk programming models in agricultural economics. Other less recent articles which, in our judgement, have made important contributions to the field have also been included, as have some other reviews of the literature. In this appendix we have made no attempt to provide a complete annotation for the papers listed. We have, however, placed them in a number of categories, depending on the type of risk being analyzed (e.g. whether the uncertainty is in the objective function, the technical coefficients, the right-hand side or some combination of the three). The citations are listed by technique, year and author, with the earlier works appearing first. The particular subject matter area or subarea of the application is listed as well.

INTRODUCTION

Since the advent of linear programming, a body of literature has been developed focusing on techniques to incorporate uncertainty in the model parameters into the programming formulations. Many of the important applications in the early years were related to agricultural decision problems involving risk (Freund, 1956; Heady and Candler, 1958; Stovall, 1966; and Tintner, 1955). Since these early efforts, numerous studies have been completed, yielding a rich literature on risk in prices, production, costs, resource usage and resource availability. The purpose of this bulletin is to provide a survey of the literature on the variety of modeling techniques, theory and applications in a risky environment.

The bulletin is organized into four sections. The first briefly characterizes the risky nature of agricultural decisions, while the second deals with theoretical foundations for risk analysis. The third section outlines a number of programming techniques and discusses their consistency with theoretical risk decision criteria.[1] The fourth presents a brief set of concluding comments.

THE RISKY ENVIRONMENT AND THE ROLE OF PROGRAMMING MODELS

Agricultural production occurs in a risky environment. The biological nature of crop and livestock production, interacting with variable weather and environmental conditions, and changing demand, as well as unpredictable government policies, affects agricultural prices and can lead to wide year-to-year and seasonal swings in agricultural incomes and the well being of farm decision makers. The severity of these "risks" varies from farming situation to situation, as do decision makers' responses. Unless these "risk" responses are adequately reflected in planning models, the results generated in empirical analysis may bear little resemblance to actual decisions and may be of little use either in direct decision making or in policy analysis.

The typical representation of a farm decision process as a linear programming (LP) model is:

$$\text{maximize} \sum_{j=1}^{n} c_j x_j$$

s.t.

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad (i=1...m)$$

$$x_j \geq 0 \quad (j=1...n)$$

where:

x_j is the j^{th} decision variable employed by the farmer;

---

[1]Two appendices are also included. The first contains numerical examples of many of the programming models described. The second contains a list of risk programming applications found in the agricultural economics literature, categorized by the type of risk and subject matter being examined.

c_j is the per unit profit contribution of x_j;
a_{ij} is the per unit use of the i^{th} resource by x_j; and
b_i is the fixed endowment of the i^{th} resource.

Ordinarily in LP models, the parameters c_j, a_{ij} and b_i are assumed to be known with certainty. In risk models, this assumption is relaxed and subsets of c_j's, a_{ij}'s and b_i's are treated probabilistically.[2] Under these conditions, the outcome from any choice of the decision variables depends on the values of the parameters actually realized and is itself a random variable. Thus, assuming the set of x_j's constitutes a farm plan, then the decision involves choosing the action x associated with the most desirable probability distribution of farm profits, net return or other appropriate measure of income or well-being.[3]

The times at which the various aspects of uncertainty are resolved are also important in the risky agricultural environment. This is perhaps illustrated best through a simple example. Consider corn production in the Midwest. At the time preceding planting, one has information about weather to date, futures prices, and most of the input costs, but one is uncertain about weather conditions from the post-planting period to the harvest season (including the planting season weather). After the planting season, one has gained additional information about planting season weather but remains uncertain about yields, harvest conditions, and prices. At the end of the summer, the forecasts of yields and prices become more accurate, but farmers still do not know them with certainty. Generally, as additional information becomes available over the growing season, the uncertainty surrounding the decision situation is gradually resolved. This suggests an adaptive process, farmers may alter or update their production and marketing plans as new information is received. For example, if the crop fails to produce an adequate stand after planting, it may be possible to reseed with the same or an alternative crop. Marketing plans can also be changed at harvest. Thus, in developing models which adequately represent the decision process in a risky agricultural environment, it is necessary to isolate the most important sources of uncertainty. It can also be important to account for the time at which information becomes available.

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[2]Throughout, the terms risk and uncertainty are used somewhat interchangeably both for convenience and variety of presentation. We make no attempt to distinguish between the two concepts on the basis of the degree of knowledge about the probability distributions (e.g. Knight, 1921), nor do we explicitly consider risk as that subset of uncertain events whose outcomes alter the decision maker's well-being (Robison and Barry, 1987). To conduct empirical risk analysis, one must have some estimates of characteristics of these distributions and the way in which this information is formulated, either subjectively or based on analysis of historical data. While this is an important issue, it is not the actual focus of the bulletin.

[3]In theoretical development as well as applications in risk models, the discussions have often referred to distribution of income, farm profits, net returns or gross margins. The appropriate measure of income depends on the application; these terms are also used somewhat interchangeably, sometimes for consistency with the original literature and other times merely for variety in presentation.

Given this characterization of risk, a framework for risk programming analysis contains a number of common elements (Boisvert, 1985). First, one must identify the set of alternative actions. Second, the set of resource constraints which may restrict these actions or decisions must be enumerated. Third, one must estimate distributions of possible parameter values (i.e. distribution of values for c_j, a_{ij}, and b_i), as well as describe both the time at which these various parameters become known and interrelationships among them. Fourth, one must identify the criteria by which decisions are made. Finally, this information must be combined to develop "optimal" or "best" decisions, recognizing that such decisions may not be "best" for any particular state of nature, but rather may be "robust" (e.g. exhibit good performance) across a wider range of uncertain parameters.

For many applications, this framework is implemented through mathematical programming models. In these models the x's describe the decision set, while the constraints delineate limits on resource supplies or other factors that restrict decision criteria. Criteria for choice of decision variables are reflected normally through the objective function. In many cases where there is incomplete knowledge of the decision makers attitudes toward risk or where it is important to generate results applicable to a diverse set of decision makers, one may solve the model a number of times to generate a set of possible plans for further analysis by the decision makers.

THEORETICAL FOUNDATIONS

Risk programming (RP) models are based on a number of different decision criteria. Some RP models are direct applications of expected utility theory and attempt to identify a single optimal decision given the utility function. Other models are consistent with expected utility maximization but only identify "efficient" portfolios of decision alternatives. Yet a third group is based on more *ad hoc* decision criteria.

Review of Expected Utility Theory

The principal theory of choice underlying risky decision making is expected utility theory which is based on the existence of an ordinal utility function by which alternatives can be ranked. The foundations of this theory, as developed by von Neumann and Morgenstern (1947), and somewhat more recently by Luce and Raiffa (1957), are found in a set of postulates or axioms, the most important of which include: ordering, transitivity, continuity and independence. The ordering axiom requires that for any two choices A_1 and A_2, the decision maker either prefers A_1 to A_2, A_2 to A_1 or is indifferent between them. Transitivity implies that if A_1 is preferred to A_2, and A_2 is preferred to A_3, then A_1 is preferred to A_3. Continuity implies that if A_1 is preferred to A_2 and A_2 to A_3, then there is a mixture of A_1 and A_3 that is preferred to A_2 and a mixture of A_1 and A_3 over which A_2 is preferred. The independence assumption requires that if A_1 is preferred to A_2 and A_3 is any other prospect, then the individual will prefer a mixture of A_1 and A_3 to the same mixture of A_2 and A_3.

Given the existence of an ordinal utility function, the expected utility maxim can be illustrated by a simple case situation. Suppose a decision maker is faced with the problem of choosing among alternative courses of action, the outcomes from which are determined by the state of an uncertain environment where:

A_j = the j^{th} act or alternative course of action;
s_i = the i^{th} possible risky outcome;
$p_i = P(s_i)$ = the probability that s_i occurs; and
y_{ij} = the outcome of A_j given that s_i occurs.

Then, for the utility function $U(y)$, we know:

a) if any risky action, A_1, is preferred to another, A_2, then $U(A_1) > U(A_2)$, and
b) $U(A_j) = E_i U(y_{ij}) = \sum_i p_i U(y_{ij})$.

Following expected utility theory, the optimal act, A_j^*, is the one which maximizes expected utility (Anderson, Dillon and Hardaker, 1977):

$$(1) \quad EU(A_j^*) = \max_j U(A_j) = \max_j [\sum_i p_i U(y_{ij})].$$

This theory, therefore, ranks alternatives according to the probability of states of nature occurring, and relative preferences regarding outcomes as represented in the utility function. The utility function $U(y)$ is assumed to be a single valued function of some measure of wealth, y. Several assumptions characterize $U(y)$.

First, it is assumed that the decision maker prefers more wealth to less; this implies a monotonically increasing utility function with marginal utility of wealth strictly positive, $U'(y) > 0$.[4] Second, it is generally assumed that the utility function exhibits decreasing marginal utility of wealth implying a concave function with $U''(y) < 0$; this is equivalent to assuming risk aversity.[5]

Because of the shape of the utility function, a risk averse individual prefers a sure amount to taking a risk, i.e., $U[E(\bar{y})] > E[U(\bar{y})]$. This is demonstrated, for a simple lottery, in Figure 1. Suppose an individual is given the choice of playing a lottery that pays y_1 units of y with probability p_1 and y_2 units of y with probability $p_2 = 1 - p_1$. The expected outcome is $E(y) = \sum_i p_i y_i$. As can be seen from Figure 1, when the utility function is concave, the expected utility of the lottery, $E[U(\bar{y})]$, is less than the utility of the expected outcome:

$$(2) \quad E[U(y)] = \sum_i p_i U(y_i) < U[E(\bar{y})] = U[\sum_i p_i y_i].$$

---

[4]Throughout, U' will denote the first derivative of a function, U" the second derivative, and so on.

[5]Risk neutrality and risk preference are represented by linear and convex utility functions, respectively. It is possible for an individual to be risk averse over some range of y, and risk preferring over another range of y (Friedman and Savage, 1948).

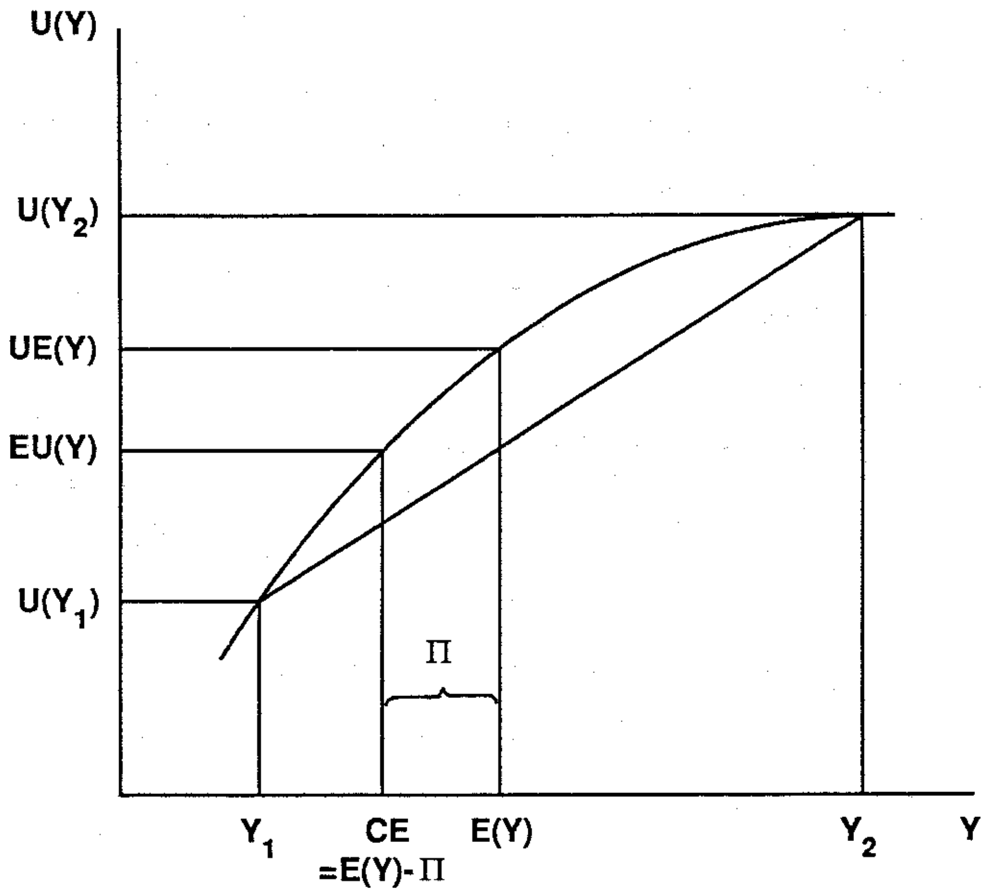

Figure 1. Illustration of the concepts of risk aversity, certainty equivalence, and the risk premium, Π

The certainty equivalent, CE, is the amount, in units of y, that will give the same utility as the lottery (i.e., $U(CE) = E[U(y)]$; it is the certain amount. Risk averse individuals are willing to pay an insurance premium to avoid the uncertainty involved in the lottery. Pratt's (1964) risk premium, $\pi(y)$, is the difference between the certainty equivalent and the expected outcome of the lottery such that:

(3) $U(CE) = U[E(y) - \pi(y)] = E[U(y)].$

If $U(y)$ is monotonically increasing ($U'(y) > 0$) then U^{-1} exists and

(4) $\pi(y) = E(y) - U^{-1}E[U(y)].$

The risk premium is the amount (of y) that will make an individual indifferent between receiving the certain amount, CE, and taking a gamble on the lottery. For individuals who are risk averse, the risk premium is positive, ($\pi(y) > 0$) (Anderson, Dillon and Hardaker, 1977; Cochran, Lodwick and Robison, 1982). Risk averse individuals prefer certain outcomes above the certainty equivalent to the lottery and prefer the lottery to any certain outcome below CE.

The single valued utility function $U(y)$ is not a unique representation of preferences; any positive monotonic transformation of a utility function leaves the ranking of certain outcomes unchanged.[6] However, expected utility rankings are invariant under any positive linear transformation of the form: $V(y) = a + bU(y)$, $b > 0$ (Henderson and Quandt, 1980). While the sign of the second derivative, $U''(y)$, provides an indication of an individual's attitudes toward risk, its magnitude is no indicator of the degree of risk aversity because $U''(y)$ is not invariant to such linear transformations. The degree of risk aversity is uniquely measured by the Arrow-Pratt absolute risk aversion function:

$$(5) \quad r_A(y) = - U''(y)/U'(y).$$

Values of $r_A(y)$ are local measures of the degree of concavity or convexity of a utility function and are unique measures of risk preference. The value of $r_A(y)$ is unchanged by any positive linear transformation of $U(y)$ as follows:

$$\text{if } V(y) = a + bU(y),\ b > 0,$$

$$V'(y) = bU'(y), \quad V''(y) = bU''(y), \text{ and}$$

$$r_A(y) = -V''(y)/V'(y) = -bU''(y)/bU'(y) = -U''(y)/U'(y).$$

Relative risk aversion is defined as:

$$(6) \quad r_R(y) = -yU''(y)/U'(y) = yr_A(y).$$

Arrow (1965) suggests that utility functions for risk averse individuals should display: a) decreasing absolute risk aversion (DARA), (i.e., the willingness to engage in small bets of fixed size increases as income rises) but b) increasing relative risk aversion, (i.e., as income and the size of the bet increase in the same proportion, the willingness to accept the bet falls). DARA requires the first derivative of the absolute risk aversion function to be negative:

$$(7) \quad r_A'(y) = \{U''(y)^2 - U'(y)U'''(y)\}/U'(y)^2 < 0.$$

Given the conditions for risk aversity: $U'(y) > 0$ and $U''(y) < 0$, this implies a further condition on the utility function; a positive third derivative is a necessary (but not sufficient) condition for DARA.[7]

Direct Application of Expected Utility

Although the expected utility maxim is based on a set of appealing axioms, its long-term acceptance as a theory of risky choice is based on considerations other than the fact it is consistent with the economists' concept of rationality. As early as 1948, Friedman and Savage, for example, demonstrated that an individual may have aversion to some risks and no aversion to others and still be behaving according to the expected utility

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[6]A positive monotonic transformation, $F(U)$, is defined such that $F(U_1) > F(U_0)$ whenever $U_1 > U_0$.

[7]A necessary and sufficient condition for DARA is $U'(y)U'''(y) > U''(y)^2$.

maxim. They were also able to reconcile gambling by a person who has a general predominance for risk aversion -- a condition which must hold if the utility function is to be bounded from above and below. The maxim, combined with the risk aversion hypothesis, also serves as a qualitative explanation of observed aversions toward risk such as the purchase of insurance or investment in a diversified portfolio (Tobin, 1958). Despite these attractive features of the expected utility maxim, its direct application in economic analysis using mathematical programming is limited by the need to know something about the decision maker's utility function.

In the direct application of the expected utility maxim, assumptions regarding the utility functions have been for the most part limited to those which would allow the decision problem to be formulated as a quadratic programming problem (e.g. maximizing a quadratic function subject to linear constraints). By assuming that utility can be written as:

(8) $U(y) = 1 - e^{-ay}$

where $a > 0$ is a measure of the decision maker's attitude toward risk, Freund (1956) demonstrated that if y is normally distributed with mean μ and variance σ^2 then maximizing expected utility is equivalent to maximizing:[8]

(9) $\mu - a/2\ (\sigma^2)$.

As an alternative, one can assume that a decision maker's utility function is quadratic in y:

(10) $U(y) = (1+b)y + by^2$,

where $b > -1$ and $(1+b) + 2by \geq 0$ so that $U'(y) \geq 0$. Under these conditions, Tobin (1958) showed that maximizing expected utility is equivalent to maximizing:

(11) $(1+b)\mu + b(\sigma^2 + \mu^2)$.

Farrar (1962) obtained a similar result by assuming that utility could be approximated adequately by a second-order Taylor series expansion.

When preferences are known and can be precisely formulated, the decision theoretic approach to maximizing expected utility gives a unique and complete ordering of actions, but in applied problems preferences are rarely known, are difficult to measure, and are unique to decision makers.[9] In many cases, however, individual decision makers' preferences may not be required. For instance, when dealing with policy questions, one is more interested in

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[8]This can be shown by completing the square on the exponent for the expression for E(U(y)) which produces a normal integral multiplied by $-\exp[a^2\sigma^2/2 - a\mu]$.

[9]Using methods originally put forward by von Neumann and Morgenstern (1947) efforts to measure risk preferences of farmers have been made by Officer and Halter, 1968; Halter and Dean, 1971; Lin *et al.* 1974; Dillon and Scandizzo, 1978; Binswanger, 1980; Halter and Mason, 1978 and Knowles, 1980.

specifying how a group of individuals with similar preferences might respond. Under these circumstances other ordering criteria can be specified. Such criteria, in the absence of complete information on preferences, provide a partial ordering of alternatives by identifying two subsets: those that are 'risk efficient', for which no clear preference can be determined without further information on preferences, and those that clearly would not be preferred by any individual in the group (Boisvert, 1985).

Risk Efficiency Analysis

Another approach to decision making under risk attempts to develop sets of efficient solutions. This approach, often called Risk Efficiency Analysis (REA), is based on the expected utility maximization framework but does not require full specification of the utility function. REA assumes all individuals' preferences can be represented by a utility function: groups of decision makers are then described in terms of properties of the utility function. An efficiency criterion is a decision rule that provides a partial ordering of choices for the decision makers whose preferences conform to a specified set of conditions placed on the utility function (King and Robison, 1981).

Generally, risk efficiency analysis involves imposing a set of conditions, or restrictions, on utility functions and/or the probability distributions of the choice set. Then for prospect A to be preferred to prospect B according to the risk efficiency condition, the expected utility of A must be greater than the expected utility of B, for every utility function satisfying the restrictions. Such REA criteria are sufficient conditions for expected utility maximization for that set of functions. The efficiency criterion is an optimal criterion if it is both a necessary and a sufficient condition for expected utility maximization. An optimal efficiency criterion minimizes the efficient set of choices by discarding those that are inefficient;[10] any further reductions in the efficient set require further restrictions on the admissible set of utility functions.

E-V Analysis

Perhaps the simplest and most widely used REA criterion includes the mean-variance (E-V) analysis. The E-V criterion is based on the proposition that, given any two distributions with equal means, a risk averter will prefer the distribution with the smallest variance. In effect, the E-V approach entails a trade-off between expected returns and risk, as measured by the variance (or the standard deviation) of returns. The E-V criterion can be stated as: if A and B are two uncertain actions, and $\mu_A \geq \mu_B$ while $\sigma^2_A \leq \sigma^2_B$, with at least one strict inequality, then A is preferred to B. By plotting each action in mean-variance space, the efficient set of actions can be identified as all those that maximize μ for a given σ^2, or minimize σ^2 for a given μ.

It is not surprising that under certain conditions, the E-V criterion is completely consistent with the expected utility maxim. It suggests that decisions can be ranked solely in terms of the first and second mo-

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[10]Prospects are inefficient in the sense that they would never be preferred by an expected utility maximizer in the group of decision makers defined by the restrictions on the utility function.

ments of the distribution (i.e., μ and σ^2); this is exactly the case in the objective functions in equations (9) and (11) above. By taking the total derivatives of these functions with respect to σ^2 and μ, it is evident that the iso-expected utility curves are positively sloped in μ,σ space. Thus, for a specific value of the risk aversion parameter, an optimal action can be identified from the E-V efficiency locus by finding the efficient point lying on the highest feasible iso-utility curve.[11]

In addition to its applications in agricultural economic problems, the E-V approach has been widely used in the financial literature as a means of choosing among portfolios of assets (Markowitz, 1952; 1959; Levy and Markowitz, 1979). Its development in a quadratic programming framework has allowed the incorporation of risk considerations into mathematical programming models. The conditions under which E-V is an acceptable REA criteria have been controversial. Proofs exist indicating it is exactly consistent with expected utility when a) returns are normal, and/or b) all the distributions differ only by location and scale (Meyer, 1987). In addition, Levy and Markowitz (1979) and Tsiang (1972,1974) argue that this criterion is acceptable where risks are small relative to total wealth.

Stochastic Dominance

Over time, a number of other risk efficiency criteria have appeared in the literature. Perhaps the most common is stochastic dominance analysis, which provide a means of selecting alternatives that are optimal, according to expected utility maximization, for a specified set of utility functions. Initially, two such criteria were developed (Quirk and Saposnik, 1962; Hadar and Russell, 1969, 1971; Hanoch and Levy, 1969). For first degree stochastic dominance (FSD), preferences are restricted to the set of utility functions, U_1, that are monotonically increasing: $U_1 = (u(y):u'(y) > 0)$; it follows that $-\infty \leq r(y) \leq \infty$ where r(y) is the absolute risk aversion function. The FSD ordering rule for two risky prospects F and G having cumulative frequency distributions (CDF) of F(y) and G(y), respectively, is: F dominates G by FSD if, and only if, $F(y) \leq G(y)$ for all y with a strict inequality for at least one value of y. Second degree stochastic dominance (SSD) assumes a further restriction, that of risk aversion. $U_2 = (u(y)\text{: } u'(y) > 0,\ u''(y) < 0)$ represents all risk averse individuals by restricting $0 < r(y) < \infty$. The ordering rule for SSD is: F dominates G by SSD if, and only if $F_2(y) \leq G_2(y)$ with a strict inequality for at least one value

of x, where $F_2(x) = \int_0^x F(t)dt$. Graphically, SSD is interpreted as F is

preferred to G, by risk averse decision makers if, and only if, the area under F(y) is less than that under G(y) for all y. When the CDF's cross,

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[11]These problems would actually have to be solved in two steps. The mean-variance efficiency locus (see the next section for details) would have to be generated first. Then, it would be necessary to find the point of tangency (for a given b) between the efficiency locus and equation (9).

the area between F(y) and G(y) when F(y) lies above G(y) must be less than the area between them when F(y) lies below G(y).[12]

Because stochastic dominance places few restrictions on the utility function and none on the probability distribution, it has some theoretical advantages over criteria such as E-V analysis. Unfortunately, stochastic dominance can not be applied directly in programming models. Two other criteria, however, the Mean-Gini and the Target MOTAD criteria, have been applied in programming applications and have been shown to be consistent with second degree stochastic dominance under certain conditions.

Mean-Gini Analysis

This approach to efficiency analysis, developed by Yitzhaki (1982), is based on mean income and Gini's mean absolute difference as a measure of income dispersion. The approach has the convenience of E-V analysis and can be applied using linear programming. It differs from E-V analysis in that risk is not equated with variance, and the decision rules are shown to be necessary conditions for SSD.

Gini's mean absolute difference is the expected value of the absolute differences between all pairs of values of a random variable with distribution F(y):

$$(12)\quad D = E[|y-x|] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |y-x|\,dF(y)\,dF(x).$$

This coefficient is dependent on the spread of the values among themselves and not on deviations from some constant value such as the mean.[13]

Yitzhaki (1982) proposes that a necessary condition for a distribution F_1 to dominate another, F_2, by FSD and SSD is:

$$(13)\quad \mu_1 \geq \mu_2 \text{ and } \mu_1 - \Gamma_1 \geq \mu_2 - \Gamma_2,$$

with at least one strict inequality, where Γ_i is defined as one half Gini's mean difference for the i^{th} distribution:

$$(14)\quad \Gamma_i = 1/2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |y-x|\,dF_i(y)\,dF_i(x),$$

which can be written as:

$$(15)\quad \Gamma_i = \int_{-\infty}^{\infty} F_i(x)[1-F_i(x)]\,dx.$$

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[12]Anderson, Dillon and Hardaker (1977) demonstrate how discrete distributions can be accommodated in applying stochastic dominance.

[13]As with stochastic dominance, this analysis could be accommodated for discrete probability functions as well (see Bailey and Boisvert, 1989).

$\Gamma = \Delta/2 = \mu G$, where G is Gini's coefficient of concentration. Proof of this proposition (termed M-G efficiency) is given in Yitzhaki (1982).

The construction of the M-G efficient set requires the calculation of means (μ) and Gini's mean differences (Γ). Further the M-G criterion is a necessary condition for second degree stochastic dominance (SSD). But, while all M-G efficient distributions are SSD efficient (by necessity), some SSD efficient distributions may be M-G inefficient. In other words the M-G criterion exhibits greater discriminatory power.

In contrast to E-V analysis, the M-G criterion allows prospects with a larger mean and variances to be preferred. This supports Hanoch and Levy's (1969) assertion that an increase in variability is not necessarily undesirable if it is accompanied by a shift to the right in the location of the distribution. The implications of this greater discriminatory power are not known. Since the M-G criterion reduces the SSD efficient set, some rejected choices may be preferred by some risk averse individuals. The implication is that the M-G criterion defines some subset of U_2.

Buccola and Subaei (1984) acknowledge that one shortcoming of the M-G approach is that one can not determine the absolute risk aversion interval being represented. Thus, the approach can not be used to derive efficient strategies for a precisely known class of utility functions or absolute risk aversion interval.

Buccola and Subaei (1984) and Bailey and Boisvert (1989), however, argue that the M-G criterion represents the preferences of relatively weakly risk averse decision makers. They showed the M-G efficient set is identical to the generalized SSD efficient set when $0 \le r_A \le 0.0015$. However, as r_A increased to 0.0045, they found the generalized SSD efficient sets increasingly diverge from the M-G efficient set.

Bey and Howe (1984) comparing M-G, E-V, mean-semivariance (ES), and stochastic dominance sets found the M-G efficient set to be the smallest. Namely, the average M-G efficient set consisted of only 19% of the average SSD efficient set with all SSD efficient members. There was a strong tendency for the M-G efficient set to contain mostly those portfolios with high returns and high variances. Bey and Howe conclude that the M-G criterion is potentially useful if the admissible set of decision makers could be more accurately defined.

Another feature of the M-G approach is that, if the return from a particular action is composed of returns from a number of different sources (or individual items in a portfolio), the effect of a particular prospect on the risk of a portfolio can be presented in a similar fashion to the E-V model.

Following Shalit and Yitzhaki (1984), Γ_F can be estimated as:

(16) $\Gamma_F = 2\text{cov}[y, F(y)]$.

Letting y_k be the return from prospect k in the portfolio, distributed F_k, then return from the entire portfolio is:

(17) $y = \sum_k s_i\, y_k$ for $\sum_k s_i = 1$,

then we obtain:

$$(18)\ \Gamma_F = 2\sum_k s_i \operatorname{cov}[y_k, F(y)];$$

that is, "the risk of the portfolio can be decomposed into the weighted sum of the covariance between variables $[y_k]$ and the cumulative distribution of the portfolio [F]" (Shalit and Yitzhaki, 1984, p. 1456). The variance of the portfolio can be decomposed by replacing F(y) in equation (18) by the returns (y). The difference in the two decompositions is that in equation (18) the portfolio is represented by the cumulative distribution of returns.

Target MOTAD

Another recent development in terms of REA decision rules is the one implied in the target MOTAD formulation by Tauer (1983). This is easily accommodated in a programming framework and is a two-attribute risk and return model. It is a member of the mean-target models by Fishburn (1977). In this model, μ is maximized subject to some level of risk where "... risk is measured as the expected sum of the negative deviations of the solution results from a target-return level, [T]" (Tauer, 1983, p. 607). An appropriate expected utility function based on expected returns and expected losses is given by $EU = a + b\mu + c \min(\mu - T, 0)$; for $b, c > 0$, the function is increasing and concave in μ (Tauer, 1983). The most interesting feature of this model is that target MOTAD solutions (except for those with equal means and deviations) are second-degree stochastic dominant (SSD). However, while most target MOTAD solutions are SSD, there is no guarantee that the target MOTAD model can be used to generate all SSD solutions.

Other Decision Criteria

The previous two sections have focused on the expected utility theory and decision criteria consistent with this theory. The discussion is not exhaustive; it focuses on those criteria which can be incorporated into risk programming models. There are, however, other criteria which have been incorporated into risk programming models. Some of these criteria reflect an attempt to overcome some of the objections to expected utility theory. These objections may be as much the result of the simplifying assumptions needed to apply expected utility theory as they are objections to the theory itself.[14]

Criticisms of the expected utility maxim have taken many forms. At one extreme, one can argue that utility functions involving only one attribute are a gross simplification of reality. Recent critics argue that

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[14]An example is the objection leveled at E-V analysis for treating positive as well as negative derivations about the mean as undesirable. However, this is not a limitation in the theory. To claim that E-V analysis is consistent with expected utility theory, one must assume utility is quadratic or returns are normally distributed. In the latter case, higher-order moments of the distribution vanish, but for more general classes of utility functions, the problem would be avoided because expected utility would involve higher-order moments.

while expected utility theory generalizes notions about economic behavior by relaxing the assumption of linearity in the payoffs, it retains the notion of linearity in probabilities, thus leading some to conclude that expected utility may be only a prescription of what is believed to be rational behavior rather than a description of observed behavior (Machina, 1987). Experimental investigations by psychologists have also uncovered instances where decision makers violate the postulates of expected utility theory (Weber, 1989). Modification or relaxation of the assumptions have led to a class of models (e.g. regret and disappointment theory) designed to describe observed behavior and accommodate psychological variables such as perceived riskiness and ambiguity. These models are not unlike the theories of "bounded rationality" of the behavioral theorists such as Cyert and March (1963), Machlup (1967) and Simon (1979). To date, little has been done to accommodate these theories into programming models and they are thus beyond the major focus of this report.

At a more practical level, alternative decision criteria have been suggested which, for computational reasons, are approximations to more theoretically acceptable alternatives. Due to recent advances in computing technology, these criteria are less important from a computational perspective. They are still widely used and their applicability is now judged best on their performance relative to other criteria. In addition, it is often argued that decision makers are concerned about income falling below some minimum level. Many of the models that embody this notion have been labeled "safety-first" but fall into a general class of mean-risk dominance models in which risk is measured by a probability weighted function of deviations below a specific target return (Fishburn, 1977).

Among the linear approximations, the most widely used criterion is MOTAD in which Hazell (1971) proposed that mean absolute deviations (MAD) be used as a measure of income variability in place of the sample variance. As is seen in subsequent sections of this report, this leads to a linear, rather than quadratic programming formulation. Even though MAD is a less efficient estimator of the population variance, the two risk formulations generate surprisingly similar results (Hazell and Norton, 1986). Thus, it appears that MAD does equally well or may even outperform sample variances (if incomes are skewed) in ranking alternatives (Thomson and Hazell, 1972).

Another decision rule closely related to E-V analysis is the expected gain-confidence limit (E,L) criterion proposed by Baumol in 1963. He argued that not all efficient plans on the E-V frontier are reasonable in that plans with lower μ or σ may not always be the most secure if one accounts for the probability of a large shortfall in income. Thus, decision makers are required to choose the alternative which maximizes μ for a given level of $L = \mu - \theta\sigma$, where $\theta > 0$ is a risk aversion parameter. In some cases where the probability distributions are known, θ may also reflect directly the probability of income falling below some particular level. In this sense, this (E-L) criterion is similar to the more general class of "safety-first" criteria and if one lets $\theta\sigma = \Gamma$, it is equivalent to the M-G criterion discussed above.

Safety-first rules focus attention on some critical (and generally arbitrary) 'disaster' level in the lower tails of the probability distributions, and in some sense minimize the probability of falling below this level. As such, any choice is critically dependent on the target level selected. Such approaches have been reviewed by Boisvert (1972) and Anderson (1979), among others.

One of the earliest safety-first principles was developed by Roy (1952) and involves minimizing the probability that the outcome of an action or prospect falls below some specified disaster level, y*, i.e., Min. $P[y < y^*]$ or, alternatively, Min. $F_i(y^*)$, where $F_i(y)$ is the CDF of the i^{th} prospect. It is difficult to incorporate this criterion into a mathematical programming model, but Hazell and Norton (1986) show that this safety-first alternative can be generated *ex post* from the E-V set if returns are normally distributed. Low (1974) proposed an alternative in which the decision is one which maximizes μ subject to income larger than some minimum level for all states of nature. While this is relatively easy to incorporate into a programming model, it could lead to an infeasible result if the target is set too high.

Katoaka (1963) is responsible for a criterion which suggests that an entrepreneur wishes to be assured of some non-negative income with some specified high probability $(1-\alpha)$. Each alternative can guarantee some income at this specified high probability level. Accordingly, he selects the portfolio which maximizes the income which can be assured $[(1-\alpha)100]$ percent of the time [i.e., max. R^* subject to $P(R \leq R^*) \leq \alpha$]. Appealing to Chebyshev's inequality, this criterion can be restated (for a given $\alpha = 1/k^2$) as max. $\mu - k\sigma$. The probability statement can be made more precise if one can assume normal returns, but in either case, the alternative that maximizes this function can be derived from the E-V efficient set.

A Summary

This section of the report has reviewed expected utility theory and outlined the important decision rules and risk efficiency criteria that are consistent with the theory and which can also be incorporated into mathematical programming models. Some additional decision rules, which have a slightly different or more pragmatic underpinning have also been discussed and compared briefly with those developed directly from expected utility. Again this was not an exhaustive list, but rather it focuses on those criteria that have been adapted to mathematical programming analysis. Perhaps the two most notable omissions are the game theory rules such as maximin and minimax (Hazell and Norton, 1986) and the focus loss model developed by Boussard and Petit (1967). These criteria have not been widely used and there are some major objections to their use. Nonetheless, the programming models consistent with these criteria are described briefly in the next section.

After reviewing these various decision rules and efficiency criteria, we examine the mathematical programming models which accommodate these various decision rules. To some extent, the model structures depend on the nature of the risk being examined.

TECHNIQUES FOR RISK PROGRAMMING

In most applications of risk programming techniques, the analyst chooses the key elements of risk to be studied and this in turn determines which parameters of the model (e.g. objective function coefficients, technical coefficients or right-hand sides) are to be considered uncertain. The next step is to develop probability distributions (or estimate moments of the distributions) for selected parameters and determine how these distributions as well as the behavioral response to risk can be adequately represented in the model. Although these distributions may be based on sample data or on subjective information, mathematical programming models usually treat these probability distributions as if they were known with certainty (i.e., as population distributions).

Most risk programming models focus on uncertain objective function parameters. These applications are often the easiest to formulate mathematically and to accommodate one or more of the decision criteria outlined above. Much of the discussion in this section will concentrate on this type of model. Other applications have dealt with risk as reflected by uncertain technical coefficients and right-hand side values separately, while others accommodate uncertainty in all three types of parameters. These applications are, however, more difficult both to formulate conceptually and to relate to well-known decision criteria. For these reasons, a logical place to begin is with a discussion of models in which the risk is reflected in the objective function.

Objective Function Risk

Several models have been proposed which deal with objective function coefficient uncertainty. Much of the initial work evolved around portfolio analysis where the major source of risk was in the variability of returns from individual stocks in a portfolio. In the agricultural literature, farm prices and yields have been major sources of risk that are manifest in the objective function as variability in gross margins for individual crop and livestock enterprises. This section reviews these various models and compares them, both analytically and empirically.

Mean-Variance (E-V) Programming

The general linear programming problem can be written as find $x_j \geq 0$ $(j=1,\ldots,n)$ which:

(19) $$\max. \sum_{j=1}^{n} c_j\, x_j = Z$$

s.t.

(20) $$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \quad (i=1,\ldots,m)$$

where the x_j's are decision variables and the c_j's are now uncertain parameters. If the c_j's are assumed to have means $\bar{c}_j$ and covariances σ_{ij} $(\sigma_{ii} = \sigma_i^2)$ then the mean and variance of the objective function (Z) are given by:

(21) $$\bar{Z} = \sum_{j=1}^{n} \bar{c}_j\, x_j; \text{ and}$$

(22) $$\sigma_Z^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}\, x_i x_j.$$

Using these relationships, the general formulation of the E-V problem due to Freund (1956) is:

$$(23) \quad \max. \; \bar{Z} - \phi\, \sigma_Z^2 = \sum_{j=1}^{n} \bar{c}_j\, x_j - \phi \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}\, x_i x_j$$

s.t.

$$(24) \quad \sum_{j=1}^{n} a_{ij}\, x_j \le b_i \quad (i=1,\ldots,m)$$

$$x_j \ge 0 \quad \text{all } j.$$

Here the objective function maximizes expected total profits less a "risk aversion coefficient" (ϕ) times the variance of total profits. In the original application of this model to a problem in farm planning, Freund (1956) assumed that gross margins on the farm activities were normally distributed. Thus, 2ϕ is equivalent to the risk aversion parameter in equation (8).

Markowitz (1952) presented a formulation of the E-V problem predating Freund, but this formulation minimized variance subject to a given level of expected income, λ. Algebraically, the model was:

$$(25) \quad \text{minimize} \sum_{j=1}^{n} \sigma_{ij}\, x_i x_j$$

$$(26) \quad \sum_{j=1}^{n} \bar{c}_j\, x_j \ge \lambda$$

$$(27) \quad \sum_{j=1}^{n} a_{ij}\, x_j \le b_i \; (i=1,\ldots,m)$$

$$x_j \ge 0.$$

The major difference between these two formulations is that for any single solution, one model requires the specifications of ϕ, while the other requires a specification of λ. In theory it is possible to estimate a decision maker's value for ϕ, which is largely a function of the decision maker's preference between income and risk, but in practice this can be quite difficult.[15] However, the value for λ to be adopted is a function of both

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[15] In the empirical literature, several strategies for estimating risk parameters have been used. First, one may subjectively elicit a risk aversion parameter (see Anderson, Dillon and Hardaker (1977) for details) or transform risk aversion coefficients from another study of decision makers thought to have similar risk preferences. Second, one may derive the efficiency frontier, have the decision maker select an acceptable point on the frontier, and use the implied risk aversion parameter in further analysis. Third, following Weins (1976) and assuming that the E-V rule was used by decision makers in generating their past choices, one can set the risk aversion coefficient equal to the difference between marginal revenue and marginal cost of resources which occurred in the past divided by the appropriate marginal variance. Fourth, one may estimate a risk aversion

the risk-income tradeoff and the values of all parameters in the model and these change from application to application as well as during a model based analysis (i.e., when changing parameters to test sensitivity). The major advantage of using Freund's formulation is that it determines directly the risk aversion parameter associated with points on the frontier.

As mentioned above, this E-V formulation leads to more diversified production plans or investment portfolios than would occur if expected income or revenue were maximized. This is illustrated in the empirical example in Appendix A, but other important characteristics of the optimal solutions may be examined through the Kuhn-Tucker conditions. Writing the problem first in matrix notation, the Lagrangian and the Kuhn-Tucker conditions are:

(28) $L^*(X,U) = \bar{C}X' - \phi XSX - U(AX-b)$

(29) $\partial L^*/\partial X = \bar{C} - 2\phi X'S - UA \leq 0$

(30) $\partial L^*/\partial X\ (X) = (\bar{C} - 2\phi X'S - UA)\ X = 0$

(31) $X \geq 0$

(32) $\partial L^*/\partial U = AX - b \leq 0$

(33) $(U)'\ \partial L^*/\partial U = U(AX - b) = 0$

(34) $U \geq 0$

where U is the vector of dual variables (Lagrangian multipliers) associated with the primal constraints $AX \leq b$, S is the variance-covariance matrix and $\bar{C}$ is the vector of expected returns.

A cursory examination of these conditions indicates two important things. First, the solution permits more variables to be non-zero than would a basic solution to constraints (32) alone; variables (X) may also be in the basis if equation (29) holds as an equality. Thus, a diversified solution involving more non-zero variables than the number of constraints may be achieved. Second, equation (29), which relates resource cost (UA) with marginal expected revenue ($\bar{C}$) also contains a marginal cost of bearing risk ($2\phi X'S$). Consequently, the optimal shadow prices are "risk-adjusted" as are the optimal decision variable values.

E-V Models and Other Decision Criteria

Once the E-V efficiency locus has been generated, one can use the information, particularly if returns are normally distributed, to apply decision criteria other than the expected utility maxim. As illustrated by Hazell and Norton (1986), these criteria are applied in an *ex post* fashion and rely on a one-to-one mapping from the E-V locus to the E-σ locus (where

parameter such that the difference between observed behavior and the model solution is minimized (Brink and McCarl (1978) or Hazell *et al.*, (1983)). Fifth, one can make probabilistic assumptions and derive one as in McCarl and Bessler (1989).

σ is standard deviation). For example, one can use this information to calculate $L = E - \phi\sigma$ and make application of Baumol's (1963) expected gain confidence limit criterion for ranking alternatives by finding the subset of the E-V efficient alternatives for which E is largest for a given L. This E,L efficient set, of course, depends on the value of ϕ. As another example, it may be reasonable to argue that a good risk decision strategy is to maximize the level below which income will fall α percent of the time. If incomes are normally distributed, there is a one-to-one correspondence between α and ϕ and for a given ϕ this decision criterion involves finding the alternative that maximizes L.

A Linear Approximation - MOTAD

Because quadratic programming (QP) problems historically were harder to solve than linear programs there was considerable effort in the past to develop linear programming approximations to the E-V model or alternative risk models that could be solved using LP procedures. These computational issues are now less important (McCarl and Onal, 1989). Several LP approximations have evolved (Hazell, 1971; Thomas *et al.* 1972; Chen and Baker, 1974; and others as reviewed in McCarl and Tice, 1982). Only Hazell's MOTAD is discussed here due to its extensive use and the apparent lack of adoption of the others.

The acronym MOTAD stands for Minimization of Total Absolute Deviations. In the MOTAD model, risk is measured by absolute deviations from mean returns rather than by the variance of total returns.[16] Thus, the original Hazell model depicts tradeoffs between expected profits and the absolute deviation of profits.

Since the absolute value operator is not linear in the x_j's, the model must be reformulated into an LP framework by recognizing that any number (A) can be written as the difference of two non-negative variables ($A = A^+ - A^-$). As long as we can be guaranteed that both these components can never appear in the basic solution, then $|A| = A^+ + A^-$. Hazell (1971) used this formulation in developing the MOTAD model. (The approach was suggested in Markowitz (1959, p. 187)). Formally, assuming that there are K states of nature, then the total absolute deviation of profits from the expected value under the k^{th} state of nature (D_k) is:

$$(35) \quad D_k = \left| \sum_{j=1}^{n} C_{kj} x_j - \sum_{j=1}^{n} \bar{C}_j x_j \right|$$

where C_{kj} is the per unit net return to x_j under the k^{th} state of nature and $\bar{C}_j$ the mean net return to x_j. The above equation gives the absolute value of the difference between income under the k^{th} state of nature

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[16]The MOTAD model has been rather widely used. Early uses were by Hazell, 1971; Hazell and Scandizzo, 1975; Hazell *et al.* 1983; Simmons and Pomareda, 1975; and Nieuwoudt, *et al.*, 1976. In the late 1970's the model saw much use. Articles from 1979 to mid-1980 in the *American Journal of Agricultural Economics* include Gebremskal and Shumway, 1979; Schurle and Erven, 1979; Pomareda and Samayoa, 1979; Mapp, *et al.*, 1979; Apland, McCarl and Miller, 1980; and Jabara and Thompson, 1980.

$(\sum_{j}^{n} C_{kj}\ x_j)$ and mean income $(\sum_{j}^{n} \bar{C}_j\ x_j)$.

Since both terms involve x_j and the sum is over the same index, this can be rewritten as:

$$(36)\quad D_k = \left| \sum_{j=1}^{n} (C_{kj} - \bar{C}_j)\ x_j \right|.$$

Total absolute deviation is the sum of the d_k over k. Now introducing variables to depict positive (d_k^+) and negative (d_k^-) deviations we obtain:

$$(37)\quad TAD = \sum_{k=1}^{K} D_k = \sum_{k=1}^{K} (d_k^+ + d_k^-),$$

where $d_k^+ + d_k^- = \left| \sum_{j=1}^{n} (C_{kj} - \bar{C}_j)x_j \right|$.

Substituting this expression for the variance in the objective function of the E-V model (equation (23)) we obtain the MOTAD model's objective function which maximizes expected net returns less some risk aversion coefficient (Ψ) times the TAD:

$$(38)\quad \text{maximize}\quad E - \Psi\ TAD = \sum_{j=1}^{n} C_j\ x_j - \Psi \sum_{k=1}^{K} \left| \sum_{j=1}^{n} (C_{kj} - \bar{C}_j)x_j \right|.$$

To convert this into an objective function that is linear in the decision variables, we can write:[17]

$$(39)\quad \text{maximize}\quad \sum_{j=1}^{n} \bar{C}_j\ x_j - \Psi \sum_{k=1}^{K} (d_k^+ + d_k^-)$$

$$(40)\quad \sum_{j=1}^{n} (C_{kj} - \bar{C}_j)x_j - d_k^+ + d_k^- = 0 \quad \text{for all } k$$

$$(41)\quad d_k^+\ ,\ d_k^- \geq 0; \quad x_j \geq 0.$$

The total MOTAD model then is:

$$(42)\quad \text{maximize}\quad \sum_{j=1}^{n} \bar{C}_j\ x_j - \Psi \sum_{k=1}^{K} (d_k^+ + d_k^-)$$

s.t.

$$(43)\quad \sum_{j=1}^{n} a_{ij}\ x_j \leq b_i \quad \text{for all } i$$

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[17]Because d_k^+ is the negative of d_k^-, at most only one will appear in the basic solution.

$$(44)\quad \sum_{j=1}^{n} e_{kj}\, x_j - d_k^{+} + d_k^{-} = 0 \qquad \text{for all } k$$

$$(45)\qquad x_j,\ d_k^{+},\ d_k^{-} \geq 0,$$

where e_{kj}'s are the deviations from the value expected for the j^{th} variables under the k^{th} observation ($e_{kj} = C_{kj} - \bar{C}_j$); d_k^{+} is the positive deviation of the k^{th} income occurrence from mean income and d_k^{-} is the associated negative deviation. Because the sum of positive deviations about the mean is always equal to negative deviations, this model is most often written just in terms of negative deviations from the mean:

$$(46)\quad \max. \sum_{j=1}^{n} \bar{C}_j\, x_j - 2\,\Psi \sum_{k=1}^{k} d_k^{-}$$

s.t.

$$(47)\qquad \sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \qquad \text{for all } i$$

$$(48)\qquad \sum_{j=1}^{n} e_{kj}\, x_j + d_k^{-} \geq 0 \qquad \text{for all } k$$

$$(49)\qquad x_j,\ d_k^{-} \geq 0.$$

Ignoring positive deviations in this case does not alter the solution.

In an attempt to make an analytical comparison between the measure of risk in this model and that in the E-V model, Hazell (1971) relied on Fisher's work showing that an estimate of the standard error of a normally distributed population can be formed from a sample of size n by multiplying Mean Absolute Deviation (MAD) by a constant:

$$(50)\qquad \sigma \approx \left[\frac{\Pi\, n}{2(n-1)}\right]^{0.5} \text{MAD}.$$

MAD is the total absolute deviation divided by n, which is twice the total negative deviation (TND) divided by n, i.e.,

$$(51)\quad \text{MAD} = \frac{\text{TAD}}{n} = \frac{2\ \text{TND}}{n}.$$

The total negative deviation is the sum of the negative deviations under each state of nature:

$$(52)\quad \text{TND} = \sum_{k=1}^{K} d_k^{-}.$$

Thus, the approximation of the standard error, assuming normality, can be written as follows:

(53) $$\sigma \simeq \left[\frac{\Pi\ n}{2(n-1)}\right]^{0.5} \text{MAD} = \left[\frac{\Pi\ n}{2(n-1)}\right]^{0.5} (2\sum d_k^-)/n = \left[\frac{\Pi\ n}{2(n-1)}\right]^{0.5} 2\text{TND}/n.$$

Conversely, the inverse formulation relates the total negative deviation to the standard error as follows:

(54) $$\text{TND} \simeq (2\Pi/(n(n-1))\ )^{-1/2}\ \sigma = \Delta\ \sigma.$$

This transformation is commonly used in MOTAD formulations, e.g., suppose we introduce an identity relating d_k^- and a new variable (σ) which is the approximate standard error of income. The problem is to find $x_j \geq 0$ that:

(55) $$\text{maximize} \sum_{j=1}^{n} \bar{C}_j\ x_j - \alpha\ \sigma$$

s.t.

(56) $$\sum_{j=1}^{n} a_{ij}\ x_j \leq b_i \quad \text{for all } i$$

(57) $$\sum_{j=1}^{n} e_{kj}\ x_j + d_k^- \geq 0 \quad \text{for all } k$$

(58) $$-\ \Delta\ \sigma + \sum_{k=1}^{K} d_k^- = 0,$$

where $\Delta = (2\Pi\ /\ (n\ (n-1)))^{-1/2}$ and $x_j \geq 0$ and $d_k^- \geq 0$.

(An application of the MOTAD formulation to the portfolio problem used to illustrate E-V analysis is also given in Appendix A.)

Comments on MOTAD

Because MOTAD is often thought of as a linear approximation to the E-V model, many of the comments regarding the strengths and limitations of E-V analysis are appropriate and are not repeated. Additional comments are also appropriate. First, a cursory examination of MOTAD might lead one to conclude that the model ignores covariance. However, it must be remembered that the deviations are totalled across all the activities, allowing negative deviations from the mean for one activity to cancel with the positive deviations for another. Thus, in minimizing total absolute deviations, the model has an incentive to "diversify" in much the same way as the E-V model which explicitly accounts for covariance. This similarity is seen more readily by realizing that the E-V model can be formulated in a fashion similar to MOTAD, only with the deviation variables d_k^- and d_k^+ from equation (39) being squared and divided by n.

Second, the equivalence of the formulations using total negative and total absolute deviations depends critically on the symmetry of the deviations. This symmetry occurs whenever the differences are taken from the mean. This, however, implies that the mean is the value expected in each observation. This may not always be the case and when the value expected is not the mean, moving averages or other expectation models should be used instead of the mean (see Brink and McCarl (1978) or Young (1980)). In such cases, the deviations are generally non-symmetric and consideration must be given to an appropriate measure of risk (for example, Brink and McCarl (1978) use the mean negative deviation formulation with a moving average expectation.)

A third set of comments on MOTAD (and E-V models) relates to the use of standard error as a measure of risk. When using this measure, the risk aversion parameters can be interpreted as the number of standard errors by which one wishes to discount income. This, and an assumption of normality, permits one to place confidence limits on income. For example, a risk aversion coefficient, α, equal to one means that level of income which occurs at one standard error below the mean is maximized. Assuming normality, this is a level of income that will occur 84% of the time. Thus, the use of the normality assumption and the standard error approximation allows a probabilistic interpretation of the risk aversion coefficient.

Fourth, to use this model one must have empirical values for the risk aversion parameter. The approaches discussed in the E-V section above are all applicable to its discovery. The most common approach with MOTAD models has been based on observed behavior. Assume the measure of risk is standard deviation, then the procedure has been to: a) take a vector of observed solution variables (i.e. acreages); b) parameterize the risk aversion parameter (ψ) in small steps (e.g., 0.25) from 0 to 2.5 at each point computing a measure of dispersion expressing the difference between the model solution and observed behavior; and c) select as the risk aversion parameter that which exhibits the smallest value for the measure of dispersion (for example see Hazell, *et al.*, 1983; Brink and McCarl, 1978; Simmons and Pomareda, 1975; or Nieuwoudt, *et al.*, 1976).

Fifth, the MOTAD model as presented above does not, to the authors' knowledge, have a direct relationship to a theoretical utility function. Some authors have discovered special cases under which there is a link (see the note by Johnson and Boehlje, 1981 and the subsequent exchange with Buccola, 1982). Viewed in terms of an approximation to the E-V model, Thomson and Hazell (1972) investigated the comparative efficiency of the two formulations and showed MOTAD to be relatively more efficient with small samples from non-normal distributions. Given the interpretation which can be placed on the risk aversion parameter outlined above, the model does not necessarily have to be viewed as an approximation. If it is not, one may find it easier to use with decision makers. Furthermore, with the advances in non-linear programming algorithms which have been realized with the release of codes such as MINOS (Murtaugh and Saunders, 1983), some would argue that the motivation for using MOTAD as an approximation is largely gone (McCarl and Onal, 1989). MOTAD models, however, are still being used frequently.

McCarl and Bessler (1989) derive a link between the MOTAD and the E-V risk aversion parameters, under the assumption that the link between mean absolute deviation and standard error holds. This may be developed as follows. Consider the models:

<u>E - Standard Deviation</u>
(as approximated by absolute deviations)

$$\text{Max} \quad CX - \alpha\,\sigma \quad \text{s.t.} \quad AX \le b, \quad X \ge 0$$

and

<u>E-V</u>

$$\text{Max} \quad CX - \phi\,\sigma^2 \quad \text{s.t.} \quad AX \le b, \quad X \ge 0.$$

The Kuhn-Tucker conditions with respect to X of these two models are:

<u>E - Standard Deviations</u>

$$C - \alpha \frac{\partial \sigma}{\partial X} - uA \le 0$$

$$\left(C - \alpha \frac{\partial \sigma}{\partial X} - uA\right) X = 0$$

$$X \ge 0$$

<u>E-V</u>

$$C - 2\phi\sigma \frac{\partial \sigma}{\partial X} - uA \le 0$$

$$\left(C - 2\phi\sigma \frac{\partial \sigma}{\partial X} - uA\right) X = 0$$

$$X \ge 0.$$

For these two solutions to be identical in terms of X and u, then

$$\alpha = 2\phi\sigma \text{ or } \phi = \frac{\alpha}{2\sigma}.$$

Thus, the risk aversion coefficient in the QP will equal the MOTAD standard error model risk aversion coefficient (α) divided by twice the estimated standard error. This explains why QP risk aversion coefficients are usually very small (i.e., a MOTAD risk aversion parameter range of 0-2 when the standard error of income is expected to be approximately $10,000 corresponds to a E-V range of 0 - 0.00002). Unfortunately, since ϕ is a function of σ, which is a function of X, this condition must hold *ex post* and can not be imposed *a priori*. However, one can develop an approximate *a priori* relationship between the risk aversion parameters given an estimate of the standard error.

The final comment on the MOTAD model relates to its sensitivity. Schurle and Erven (1979) show that the several plans with very different solutions can be feasible and close to the plans on the efficiency locus. Both results place doubt on strict adherence to the efficient frontier as a norm for decision making. (Actually the issue of near optimal solutions is much broader than just its role in risk models).

The Focus Loss Model

Boussard and Petit (1967) posed a different approach to handling uncertainty in the objective function. This approach, the focus loss model, assumes that decision makers are reluctant to accept levels of income below a minimum level M; i.e. a level at which "ruin" occurs.[18] The formulation can be described as follows:

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[18] Roy (1952) and Low (1974) present similar models based on "ruin" income levels.

Assume the average income level ($\sum_{j=1}^{n} \bar{C}_j x_j = \bar{C}X$) less the minimum income level (M) expenditures gives the maximum admissible loss ($L = \bar{C}X - M$). This loss may be spread across the risky activities. Assuming there are K activities over which to spread this loss and that each activity can have at most a 1/K share of the overall loss, then any activity is constrained to exhibit no more than $1/K^{th}$ the admissible (focus) loss. This is done by entering the constraints $F_j x_j \le L/K$, for all x_j that are risky; F_j is a measure of the risk level incurred when producing one unit of activity j. The programming problem becomes:

$$(59)\quad \text{Maximize} \sum_{j=1}^{n} \bar{C}_j x_j$$

s.t.

$$(60)\quad \sum_{j=1}^{n} a_{ij} x_j \le b_i \quad \text{for all } i$$

$$(61)\quad \sum_{j=1}^{n} \bar{C}_j x_j - L = M$$

$$(62)\quad F_j x_j - L/K \le 0 \quad \text{for all } j \in k$$

$$(63)\quad L;\ x_j \ge 0 \quad \text{for all } j,$$

where L is an endogenous variable giving the maximum admissible loss between expected return ($\bar{C}X$) and the level of ruin (M) and F_j, K and M are parameters.

This formulation requires specification of the new parameters F_j, K and M. Following Boussard and Petit: if the activity distributions are normal with zero covariance, the F_j can be written as $t\ \sigma_j$ where t is a value from the standardized normal or t distributions and σ_j is the standard deviation of the net return of the j^{th} activity. Under these conditions the probability level leading to the selection of t is the probability (one-tailed) that the loss will not be incurred (t=1 corresponds to an 84% chance under normality). Boussard and Petit also argue that K should be greater than or equal to the square root of the number of basic risky variables. The number of basic variables is never known *a priori*; Boussard and Petit (1967) and later Boussard (1971) suggest and provide justification of a value of K=3. M is not discussed here as its specification depends on the problem.

The focus loss model has not been extensively used in empirical research. There are several possible explanations. First, as Wicks (1978) suggests, MOTAD is easier to use. More importantly, the focus loss model ignores covariance. This may be a dominant concern in some empirical settings that lead to the choice of different techniques. As mentioned in Boussard and Petit (p. 873), however, when several activities are highly interrelated it may be beneficial to include these activities into a single focus loss constraint. This method introduces some consideration of covariance, but the fact remains that a universally accepted method for establishing the focus loss for a variable (F_j) has not been developed. In an empirical example, Wicks (1978) used several alternative methods based on criteria other than the probabilistic discounting discussed above. The alternatives were to set F_j to: a) the objective function value (C), b) one half the objective function value, and c) the variable cost component of the objective function value. Wicks provides no basis for choosing among the methods.

Target MOTAD

As suggested above, a more promising programming formulation combining the target income and MOTAD concepts is the so-called Target MOTAD model developed by Tauer (1983). The significance of this formulation is that the solutions to Target MOTAD are efficient according to second degree stochastic dominance. This has been shown for only one other risk formulation.

Target MOTAD is based on a target level of income and restrictions on the level of negative deviation from that target. Given a target level of T, the formulation becomes:

(64) Maximize $\sum_{j=1}^{n} \bar{C}_j x_j$

s.t.

(65) $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ for all i

(66) $\sum_{j=1}^{n} C_{kj} x_j + y_k \geq T$ for all k

(67) $\sum_{k=1}^{K} P_k y_k \leq \lambda$

(68) $x_j, y_k \geq 0$ for all j and k.

All definitions are as above except P_k is the probability of the k^{th} state of nature. T is the target income level (analogous to M in Focus Loss). The variable y_k is the negative deviation of income under the k^{th} state of nature below the target income; and λ is the maximum amount of the average income shortfall permitted. This model maximizes expected income subject to the normal resource constraints and two new constraints. Equation (66) gives the relationship between income under the k^{th} state of nature and a target income level. The variable y_k is non-zero if the k^{th} income state falls below T. The second additional constraint (67) requires the average shortfall to be no more than a parameter λ; thus, the target MOTAD model then has two parameters relating to risk (T and λ) which must be specified. These, in turn, are parameterized to yield different risk solutions. (An example is in Appendix A.)

Because target MOTAD is relatively new and has not been applied as widely as other risk programming models, it is too early to evaluate its long term contribution to the literature. Although, as stated above, target MOTAD is consistent with expected utility theory. It is true that target MOTAD requires specification of two parameters, T and λ. McCamley and Kliebenstein (1987) outline a strategy for generating all target MOTAD solutions, but it is still impossible to relate these solutions to more conventional measures of risk preferences. Despite the fact that target MOTAD solutions are SSD efficient, no attempt has been made to determine which of the solutions are consistent with which ranges of Arrow-Pratt measures of risk aversion. The only thing we know at this time is that target MOTAD and original MOTAD models can be related. If one solves for λ endogenously with a weighted objective function value and sets the target level to the endogenous level of mean income, this yields a model where λ equals total negative deviations. For these reasons, there are additional difficulties communicating the results of target MOTAD to decision makers in an attempt to identify the parameter values consistent with their own risk preferences.

The Mean-Gini Programming Problem

As with target MOTAD, the Mean-Gini programming model also generates only a subset of SSD portfolios and before either model can be given its rightful place in the risk literature, the relationship between these two models, to SSD alternatives and to other risk models must be understood. Previous work has suggested that the M-G efficient set corresponds to SSD alternatives preferred by those with only mild aversion to risk. This work, however, was not performed in a programming context. It remains to be seen if the same conclusions will hold in wider applications. Since M-G analysis has not been applied in a programming context to a significant empirical problem, our purpose here is to outline the procedures for constructing the model.

In a footnote to his 1982 paper, Yitzhaki first formulated a portfolio problem using the M-G decision criterion, although at that time he attempted no empirical application. More recently, Okunev and Dillon (1988) have independently developed and applied the model to a small example farm problem.

The construction of the M-G efficient set is facilitated by first finding the M-Γ efficient set by deleting from the M-Γ set all plans which have the same value for μ-Γ but a lower μ. This is demonstrated in Figure 2. The M-G efficient set consists of BC of the μ-Γ set ABC.[19]

To generate the M-Γ set within a programming context, we must recognize that Γ involves the sum of absolute values of period differences in returns (differences in returns between all pairs of periods). This sum can be minimized in a fashion similar to that used in the MOTAD model. To

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[19]As stated by Okunev and Dillon (1988), "[b]y virtue of the geometry implied by the 45°-degree line tangential to ABC at B, plans such as those represented by points D and E lying the same horizontal distance (D'D=E'E) from the 45°-degree line have the same μ-Γ value" (p. 10). However, E has a higher μ value and by equation (13) dominates D by M-G efficiency. The M-G efficient set is then BC. Point B is where μ-Γ is a maximum.

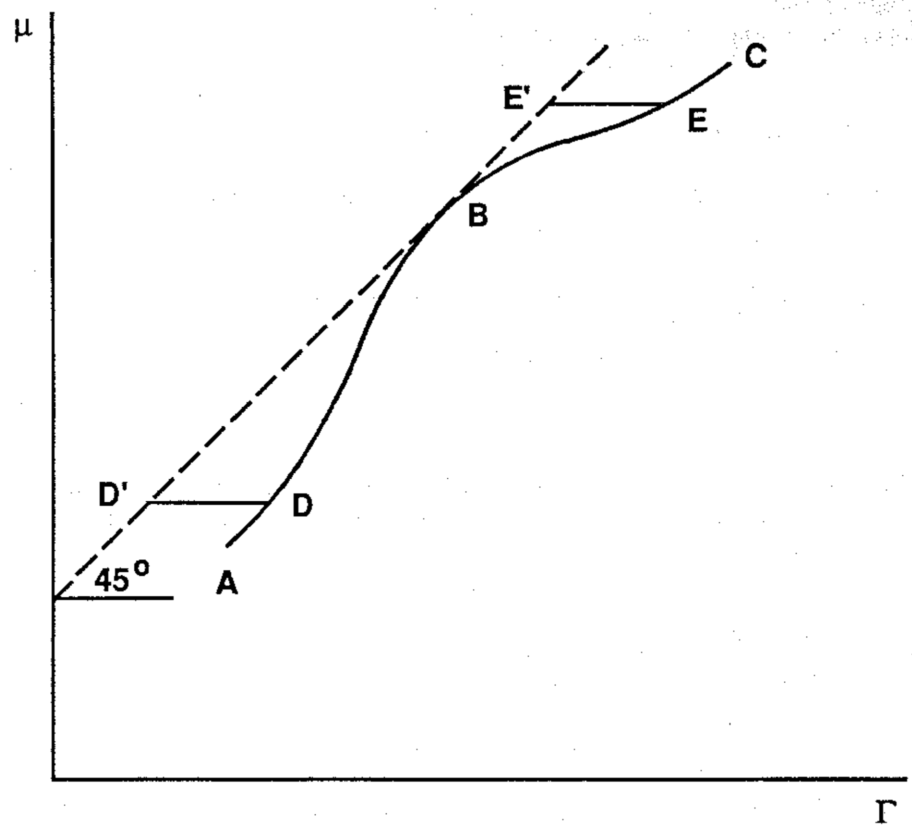

Figure 2. The mean-gini efficient frontier

get started, suppose a farmer has information on gross margins c_{kj} for a number of years $k = 1,\ldots,s$, for each activity $j = 1,\ldots,n$. Denoting the j^{th} activity as x_j then the total gross margin for any farm plan is (T_k).

$$(69)\quad T_k = \sum_{j=1}^{n} c_{kj}\, x_j .$$

Assuming that the returns in any year are equally likely, then:

$$(70)\quad \Gamma = \sum_{k=1}^{s} \sum_{r>k}^{s} |T_k - T_r|/s^2 .$$

Thus, the M-Γ can be found by minimizing equation (70) subject to:

$$(71)\quad \sum_{j=1}^{n} \bar{c}_j\, x_j = \mu \text{ (for all values of } \mu)$$

where $\bar{c}_j = \sum_{k=1}^{s} C_{kj}/s$; = expected returns for activity j, and a set of resource constraints:

(72) $\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad (i=1,\ldots,m),$

$x_j \ge 0$ all j.

To linearize equation (70), we must transform the problem by defining:

(73) $T_k - T_r = y_{kr}^+ - y_{kr}^-$; y_{kr}^+ and $y_{kr}^- \ge 0$,

for all k and r. Then we can write:

(74) $|T_k - T_r| = y_{kr}^+ + y_{kr}^-$,

provided that at most one of these variables will be in any basic feasible solution to the problem. The problem can now be written to minimize $s^2 \Gamma$ (since s^2 is a constant):

(75) minimize $\sum_{k=1}^{s} \sum_{r>k}^{s} (y_{kr}^+ + y_{kr}^-)$

s.t.

(76) $\sum_{j=1}^{n} (C_{kj} - C_{rj})x_j - y_{kr}^+ + y_{kr}^- = 0$ (for k=1,...,s; r>k)

(77) $\sum_{j=1}^{n} c_j x_j = \mu$ (all values of μ)

(78) $\sum_{j=1}^{n} a_{ij} x_y \le b_i \quad (i=1,\ldots,m)$

(79) $x_j \ge 0$; $y_{kr}^+ \ge 0$; $y_{kr}^- \ge 0$.

Once this problem is solved parametrically for all relevant values of μ, then the solutions can be ranked further by the M-G criterion given in equation (13). (Remember equation (75) minimizes $s^2\Gamma$, and equation (13) involves Γ.) In solving this programming model, it is clear that for large values of s, equation (76) generates a large number of programming constraints. Okunev and Dillon (1988) develop and simplify the dual to this problem for solution purposes. While this may lead to computational efficiencies, generating the data input for this model is a formidable task if it is to be solved by standard commercial codes unless a matrix generator is written to construct the model. Jefferson and Boisvert (1989) have demonstrated a simple way to construct the model within the "GAMS" programming language.

DEMP Model

Partly as a result of the increased availability of non-linear solvers such as MINOS, Lambert (1984) and Lambert and McCarl (1985) introduced the Direct Expected Maximizing Non-linear Programming (DEMP) formulation, which maximizes the expected utility of wealth. Their original application was to a problem in wheat sales. Kaylen, Preckel and Loehman (1987) employ a variation of DEMP where the probability distributions are of a known continuous form; numerical integration is used in the solution.

DEMP was designed as an alternative to E-V analysis, relaxing some of the restrictions regarding the underlying utility function. The basic DEMP formulation requires a utility of wealth function ($U(W)$) and a level of initial wealth (W_o) to which the income generated by the model is added. The basic formulation is:

$$(80) \quad \text{Maximize} \sum_{k=1}^{K} P_k \, U(W_k)$$

s.t.

$$(81) \quad \sum_{j=1}^{n} a_{ij} \, x_j \leq b_i \quad \text{for all } i$$

$$(82) \quad W_k - \sum_j C_{kj} \, x_j = W_o \text{ for all } k$$

$$(83) \quad W_k \geq 0; \quad x_j \geq 0 \quad \text{for all } k \text{ and } j,$$

where P_k is the probability of the k^{th} state of nature;
W_k is the wealth under the k^{th} state of nature; and
C_{kj} is the return to one unit of the j^{th} activity under the k^{th} state of nature.

While this model does allow one to relax some of the restrictive assumptions embodied in the utility functions underlying the E-V models, important data on the form of the utility and the risky parameters still are necessary. (An example application is given in Appendix A.)

EUMGF Model

Yassour, Zilberman and Rausser (1981) have also presented a direct expected utility maximizing model. It has been called the EUMGF model because it is based on the assumption of an exponential utility function and the maximization of expected utility takes the form of a moment generating function for a probability distribution. The model also requires that the probability distribution of outcomes be specified. Under these conditions, the expected utility function becomes:

$$(84) \quad EU = \int_{-\infty}^{\infty} -e^{-rw} \, f(w) \, dw,$$

where r is the risk aversion coefficient;
w is the level of wealth; and
$f(w)$ is the probability distribution of wealth.

These moment-generating functions implied by this model have been developed analytically for a number of specific distributions, including the Binomial, Chi Square, Gamma, Normal and Poisson distributions (Hogg and Craig, 1970). If, for example, one assumes that $f(w)$ is distributed Gamma:

(85) $f(w) = (\Gamma(\alpha)\ \beta^{\alpha})^{-1}\ \upsilon\chi^{\alpha-1}\ e^{-w/\beta}$,

then the moment-generating function for a given risk aversion coefficient r is:

(86) $(1-\beta r)^{-\alpha}$.

For the Gamma distribution, the mean equals $\alpha\beta$ and the variance $\alpha\beta^2$. Thus, $\beta = \sigma^2/u$ and the expression for the moment generating function becomes:

(87) $(1 - r\sigma^2/u)^{-\alpha}$.

Assuming that the decision model can be posed within a programming context with linear resource constraints, the model can be solved as a non-linear programming problem of the form:

(88) $\max\ (1 - r\sigma^2/u)^{-\alpha}$

(89) $u - \bar{c}X = 0$

(90) $\sigma^2 - (n-1)^{-1}\ X'\ \sum X = 0$

(91) $AX \leq b$

(92) $u,\ \sigma^2,\ X \geq 0$.

assuming wealth (u) is positive and where $\sum$ is the covariance matrix on returns to X. Collender and Zilberman (1985) apply the model to a problem of land allocation under stochastic yield. Moffit *et al.* (1984) apply this model to a problem in pest control on cotton, while Collender and Chalfant (1986) have proposed a version of the model no longer requiring that the form of the probability distribution be known.

Other Approaches to Objective Function Uncertainty

A number of other methods have appeared at various times in the past but none have been used extensively. In the 1960's and 1970's, a number of applications of game theory appeared in the literature (Dillon, 1963; Agrawal and Heady, 1968, 1972; Hazell, 1970; McInerney, 1967, 1969; and Maruyama, 1972). These were by and large linear programming models solving games against nature. The methods, however, have not been widely adopted.

Right-hand Side Uncertainty

Up to this point, attention has been focused on models that accommodate risk in the objective function coefficients. This emphasis is understandable given the historical importance both of yield and price risk in agriculture. This does not, however, mean that other sources of risk are unimportant in certain decision situations. By defining the programming model in particular ways, uncertainty in water supplies, field time or other important resources appear as right-hand parameters.

A number of approaches have been suggested for dealing with right-hand side (RHS) uncertainty but in several respects each is problematic. Since the uncertainty in the problem is not in the objective function, it is impossible to relate these risk decision models to the widely accepted expected utility or other risky decision criteria discussed above. A second difficulty is that solutions to these types of models fail to recommend how plans should be altered if resource supplies fall short of planned levels. This issue is discussed in greater detail below. However, these programming models have been used effectively in some decision situations. The following discussion is for completeness and to help the reader assess the applicability of these models.

Chance Constrained Programming

The most common approach used to deal with RHS uncertainty is the chance constrained programming formulation introduced by Charnes and Cooper in 1959. In this approach, it is assumed that the distribution of a RHS value (e.g. a b_i) is known and that the decision maker is willing to state a lower limit (α) on the probability (P) of a constraint being satisfied:

$$(93) \quad P\left(\sum_{j=1}^{n} a_{ij}\, x_j \le b_i\right) \ge \alpha.$$

If the average value of the RHS ($\bar{b}_i$) is subtracted from both sides of the inequality and in turn both sides are divided by the standard error of the RHS (σ_i) then (93) becomes:

$$(94) \quad P\left\{\left[\frac{\sum_{j=1}^{n} a_{ij}\, x_j - \bar{b}_i}{\sigma_j}\right] \le \left[\frac{b_i - \bar{b}_i}{\sigma_i}\right]\right\} \ge \alpha.$$

If we let:

$$(95) \quad Z = \frac{b_i - \bar{b}_i}{\sigma_i},$$

and assume knowledge of the probability distribution of b_i one can find the value Z_α which is a critical value from the probability distribution such that values less than this occur α percent of the time and manipulate (94) to be:

$$(96) \quad P\left[\frac{\sum_{i=1}^{n} a_{ij}\, x_j - \bar{b}_i}{\sigma_j} \le Z_\alpha\right] \ge \alpha.$$

This can be rewritten to give a linear programming constraint:

$$(97) \quad \sum_{j=1}^{n} a_{ij}\, x_j \le \bar{b}_i - Z_\alpha\, \sigma_i,$$

which states that resource use ($\sum_j a_{ij}\, x_j$) must be less than or equal to average resource availability ($\bar{b}_i$) less the standard deviation of $b_i(\sigma_i)$ times a critical value (Z_α) associated with the probability level α.

Values of Z_α may be determined in two ways. The first is by making and testing hypotheses about the form of the probability distribution of b_i. Unfortunately, it is often difficult to identify these distributions and normality rarely can be assumed because resource supplies cannot go below zero. The second strategy is to rely on the conservative estimates generated by using Chebyshev's inequality, which states the probability of an estimate falling greater than M standard deviations away from the mean (two-tailed) is less than or equal to $1/M^2$. To use Chebyshev inequality in its "one-tailed" form one needs to solve for that value of M such that $2(1-\alpha)$ equals $1/M^2$. Thus, given a probability α, the Chebyshev value of Z_α is given by the equation $Z_\alpha = (2(1-\alpha))^{0.5}$.

This chance constrained formulation applies when only one element of the right-hand side vector is random. To generalize the procedure to jointly distributed RHS's one must replace the probability restriction on a single constraint with a single probability restriction across all random constraints. Following Wagner (1969, p. 668) the constraint becomes:

$$(98)\quad P\left[\sum_{j=1}^{n} a_{g+1,j}\ x_j \le b_{g+1}, \ldots, \sum_{j=1}^{n} a_{mj}\ x_j \le b_m\right] \ge \beta,$$

where $0 < \beta \le 1$.

Letting $F_i(b) = P[b_i < b]$ and assuming the b_i's are independent, then

the joint distribution function is $\prod_{i=g+1}^{m} F_i$. Finally, letting $G_i = 1-F_i$,

the deterministic constraints become:

$$(99)\quad -y_i + \sum_{j=1}^{n} a_{ij}\ x_j = 0 \quad (i=g+1,\ldots,m); \text{ and}$$

$$(100)\quad \prod_{i=g+1}^{m} G_i(y_i) \ge \beta,$$

where y_i is unconstrained in sign. The difficulty in applying this model is that constraint (100) is non-linear and will rarely be concave. The distributions of the b_i are Normal, Gamma or uniform distributions then:

$$(101)\quad \sum_{i=g+1}^{m} \ln G_i(y_i) \ge \ln \beta,$$

is concave. These problems are now solved routinely by MINOS.

Despite the fact that chance constrained programming is a well-known technique and has been applied to agriculture (e.g., Boisvert, 1976; Boisvert and Jensen, 1973; and Danok, McCarl and White, 1980) and water management (e.g. Eisel, 1972; Loukes, 1975; and Maji and Heady, 1978) its use has been limited and controversial.

The model's major advantage is its simplicity; it leads to an equivalent linear or non-linear programming problem of about the same size and the only additional data requirements are the fractiles on the unconditional or joint distributions on the RHS coefficients (Wagner 1969). However, as reported by Pfaffenberger and Walker (1976), its only decision theoretic underpinnings is Simon's principle of satisficing.

A more fundamental problem is seen by comparing chance constrained programming (CCP) to stochastic programming with recourse (SPR). Hogan *et al.*, (1981) "... emphasize that recourse problems characterize almost all real decision problems involving risk" (p. 699). They describe the problem in the following way:

> Consider the decision problem where a decision x is made before the random event r occurs. The observed outcome (in suitable units) is a function f(x,r) of both the decision and the random event. Once the random variable r is observed, a recourse action is taken which affects the outcome of the decision-event combination through f. An optimal decision solves the SPR problem: $\max\{E_r u[f(r,x)] | x \in K\}$, where r has values in R^m, $K \subset R^n$ denotes the set of feasible decisions, E_r denotes mathematical expectation with respect to r, and u is a utility function (p. 699).

If this recourse model is complete (that is, it actually specifies the implications for expected utility of recourse actions for all values of the random variable), then, at least in theory, it is possible to calculate the expected value of perfect information. However, the normal chance constrained problem ignores the recourse decisions; it does not indicate what to do if the recommended solution is not feasible. In this sense, it is a special case of an "incomplete" SPR model where the expected value of perfect information would not be bounded from below at zero as it should be. From this perspective, Hogan *et al.*, (1981), conclude that "... there is little evidence that CCP is used with the care that is necessary" (p. 698). Thus, for those considering a risk decision problem where important resource supplies are considered random, careful attention should be given to recourse actions and the potential for incorporating them into the decision framework.[20]

A Quadratic Programming Approach

Paris proposed a quadratic programming model which permits RHS and objective function uncertainty to be treated jointly or independently. Uncertainty in the objective function is treated in the same fashion as an E-V model. In contrast to chance constrained programming, the RHS part of the formulation was an attempt to include specifications of inter-dependencies between the RHS's. Paris developed the RHS part of the model through an application of non-linear duality theory. Paris' formulation is given by:

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[20]References in Hogan *et al.* (1981) provide a good review of the conceptual and computational difficulties involved in SPR.

(102) Maximize $\bar{C}X - \phi X' \sum_c X - \theta Y' \sum_b Y$

(103) $AX - \theta \sum_b Y \leq \bar{b}$

(104) $X, Y \geq 0,$

where $\bar{C}$ is the expected returns; X is the vector of activities; ϕ and θ are the risk aversion coefficients with respect to variance in returns and the RHS, respectively; $\sum_c$ and $\sum_b$ are variance-covariance matrices of returns and the RHS's, respectively; Y is the vector of dual variables, A is the matrix of technical coefficients, and $\bar{b}$ is the vector of expected values of the RHS's.

This particular model explicitly introduces the variance-covariance matrix of the RHS's, as well as the dual variables, into the primal formulation. The solution then contains both primal and dual variables. The problem may also be cast as a symmetric dual or self-dual complementary program problem (Cottle, 1963) as Paris explains. However, the solutions are not what one might expect (e.g., as risk aversion on the right-hand side terms increases, the objective function also increases in terms of expected income). The reason for this situation lies in the duality implications of the model. Risk aversion affects the dual problem by making its objective function worse. Since the dual problem always has an objective function value greater than that of the primal problem, risk aversion in the dual improves the primal objective function. The manifestation of this occurs in the way the risk terms enter the constraints. Notice above that, given a value of θ and $\sum_b$ positive, then the sum involving θ and Y on the left-hand side augments the availability of the resources. Thus, as the risk aversion parameter increases under certain selections of the dual variables, so can the implicit supplies of resources. This offsets the negative risk terms in the objective function in the example and can do so in other applications.

In a recent issue of the *AJAE*, Dubman *et al.* (1989) elaborate on these problems. Paris (1989), in his reply, argues that there is "... no theorem (under uncertain output prices and input supplies) that establishes the necessity for a risk-adverse entrepreneur to procure inputs in quantities smaller than their expected values" (p. 810). Therefore, he argues that the problem in his original formulation must be due to the fact that the objective function for the risk averse entrepreneur is higher than that for the risk neutral one. He goes on to argue that this is a direct result of not explicitly considering the covariance between output prices and input supplies.

Paris reformulates the problem by defining random profit as:

(105) $\Pi = p'X - b'Y$

with

(106) $E(\Pi) = E(p)'X - E(b)'Y$

and

(107) $\text{var}(\Pi) = X' \sum_p + Y' \sum_b - 2X'\sum_{pb}Y.$

He goes on to state that when p and b are assumed to be multivariate normal and there is negative exponential utility, the solution to the problem is found by solving the primal and dual constraints:

(108) $AX + \phi \sum_{bp}X - \phi \sum_b Y \leq E(b)$

(109) $A'Y - \phi \sum_{pb}Y + \phi \sum_p X \geq E(p).$

Thus, as in the original formulation, the terms involving the variance of b and p tend to enlarge the feasible regions of the primal and dual, respectively. In this revised formulation, these effects are potentially offset by the covariance terms, "... with the outcome determined by the empirical information" (Paris, 1989, p. 812).

This interchange in the *AJAE* is a productive one, and is required reading by anyone wishing to use this model or a variation on it. Both the original commentors, and Paris, in his reply, make important points, but until this new formulation is digested and applied, the jury remains out on the model's ultimate value as a programming tool. To these authors, the fact that an important special case (where $\sum_{bp}$ is zero) leads to contradictory results, remains troublesome.

Uncertain Technical Coefficients

The third type of uncertainty to be considered involves elements within the matrix of technical coefficients. The literature contains three approaches to this type of uncertainty. One approach is similar to the one used in E-V analysis (Merrill, 1965), one is similar to MOTAD (Wicks and Guise, 1978), while the third takes a sequential approach to decision problems and is covered in the next section.

Merrill (1965) formulated a non-linear programming problem including the mean and variance of the uncertain a_{ij}'s into the constraint matrix. Given an equation containing uncertain a_{ij}'s, one may write the mean of the uncertain part as $\sum_j \bar{a}_{ij} x_j$ and its variance as $\sum_k \sum_j x_i x_j \sigma_{ikj}$, where a_{ij} is the mean value of $\bar{a}_{ij}$ and σ_{ikj} are the covariance of a_{ij} coefficients for activities k and j (variance when k = j). Thus, a constraint containing uncertain coefficients is rewritten as:

(110) $\sum_j \bar{a}_{ij} x_j - \theta \sum_k \sum_j x_j x_k \sigma_{ikj} \leq b_i$

or, using standard deviations:

(111) $\sum_j \bar{a}_{ij} x_j - \theta (\sum_k \sum_j x_j x_k \sigma_{ikj})^{.5} \leq b_i.$

The parameter θ needs to be specified exogenously and could be done using distributional assumptions (such as normality) or Chebyshev's inequality as suggested in McCarl and Bessler (1989).

Merrill's approach has remained virtually unused since its development principally because of its incompatibility with available software. However, today algorithms such as MINOS (Murtaugh and Saunders, 1979) do provide capabilities for handling the non-linear constraints. Nevertheless, the simpler approach by Wicks and Guise (1979) discussed below is more likely to be used, particularly for larger problems.

Wicks and Guise (1978) provided a LP version of an uncertain a_{ij} formulation by measuring dispersion in terms of absolute deviation. Specifically, given that the i^{th} constraint contains uncertain a_{ij}'s, the following constraints are formulated:

$$(112) \quad \sum_{j=1}^{n} \bar{a}_{ij} x_j + \Psi D_i \leq b_i$$

$$(113) \quad \sum_{j=1}^{n} e_{kij} x_j - d_{ki}^{+} + d_{ki}^{-} = 0 \quad \text{for all } k$$

$$(114) \quad \sum_{k=1}^{K} (d_{ki}^{+} + d_{ki}^{-}) - D_i = 0$$

Equation (112) relates the mean value of the uncertain resource usage plus a term for risk (ΨD_i) to the right-hand side. Equation (113) takes the deviation ($e_{kij} = a_{ijk} - \bar{a}_{ij}$ where a_{ijk} is the k^{th} observation on a_{ij}) incurred from the k^{th} joint observation on all a_{ij}'s and accumulates the combined deviations into a pair of "deviation" variables (d_{ki}^{+}, d_{ki}^{-}). These variables are summed into a measure of total absolute deviation (D_i) in equation (114). The term ΨD_i provides the risk adjustment to the mean resource use in constraint i, where Ψ is a coefficient of risk aversion.

Following Hazell (1971), Wicks and Guise (1978) recognize that an equivalent formulation can be constructed dropping d_{ki}^{-}. They also convert the total absolute deviations into an estimate of standard deviation using the constant relating the two which has already been discussed in the MOTAD section. Using the constant discussed in the MOTAD section above, we add one more constraint:

$$(115) \quad \Delta RD - \sigma = 0 \qquad \text{where } \Delta = (2\Pi/(n(n-1)))^{.5} .$$

The complete problem becomes:

$$(116) \quad \text{Maximize} \sum_{j=1}^{n} C_j x_j$$

s.t.

$$(117) \quad \sum_{j=1}^{n} \bar{a}_{ij} x_j + \phi \sigma_i \leq b_i$$

$$(118) \quad \sum_{j=1}^{n} e_{kij}\, x_j - d_{ki}^{+} \leq 0 \quad \text{for all k and i}$$

$$(119) \quad 2 \sum_{k=1}^{k} d_{ki}^{+} - D_i = 0 \quad \text{for all i}$$

$$(120) \quad \Delta RD_i - \sigma_i = 0 \quad \text{for all i}$$

$$(121) \quad FX \leq g$$

$$(122) \quad x_j, \quad d_{ki}^{+}, \quad D_i, \quad \sigma_i \geq 0$$

where $\Delta = (2\Pi/(n(n-1)))^{.5}$.

In examining this model, the reader should note that these "deviation" variables do not work well unless the constraint including the risk adjustment is binding. However, if it is not binding, then the uncertainty does not matter. This formulation has not been widely used. Other than the initial application by Wicks and Guise the only other application we know of is by Tice (1979).

Several other efforts have been made regarding solely uncertain a_{ij}'s. The method used in Townsley (1968) and later by Chen (1973) involves bringing a single uncertain constraint into the objective function. The method used in Rahman and Bender (1971) involves developing an overestimate of variance. None of these models has been widely used.

Multiple Sources and Timing of Risk

Because of the biological nature of agricultural production, there is always a significant amount of time between initial production decisions and the realization of output. This means that in reality there is potentially incomplete knowledge about all of the parameters of any programming formulation of agricultural decisions. The models discussed above deal only with the single most important source of risk for a particular decision situation. These models also reflect active decision problems whereby decisions are made prior to the resolution of the uncertainty. The models also presuppose that production is instantaneous. Both these assumptions are difficult to sustain if one wishes to treat multiple sources of risk simultaneously. In this case, the resolution of various kinds of uncertainty takes place at different points during the production season. To accommodate this situation, one must specifically take account of the sequential nature of the problem.

To illustrate the decision problem, suppose we reconsider the example in the introduction whereby uncertainty about prices, yields and available field working time changes over time within the context of crop farming. Before planting, the decision maker is uncertain about planting and harvest time, yields and prices. After planting, uncertainty in planting time is resolved but harvesting time, prices and yields are still uncertain. Additional information is now available on prices based on futures markets, and USDA planting reports among other sources. Under these circumstances, decision makers can adjust their decisions and plans as more information becomes available. Therefore, to accommodate these various sources of risk this adaptive behavior must be captured in the model, along with fixity of earlier decisions (a decision maker can not always undo earlier decisions or commonly incurs costs attempting to do so).

The model discussed here for handling these sequential decisions was originally developed as the "two-stage" LP formulation, independently by Dantzig (1955) and Beale (1961). Later, Cocks (1968) devised a model with N stages, calling it discrete stochastic programming (DSP). Interpreted broadly, the models belong to the general class of stochastic programming models under recourse (SPR) mentioned briefly above. Apland and Kaiser (1984), and Hansotia (1980) review the literature in these areas. We will refer to one area as SPR following the broader operations research literature.

A formal probability tree framework is embedded in this SPR model. The nodes of the tree represent decision points, while the branches of the tree represent alternative possible states of nature.[21] Let's use the simple sequential problem described above to present the decision tree and illustrate the model.

Suppose we have two times at which risk arises - 1) planting season, and 2) harvesting season. Assume that a farmer needs to select the mix of crops grown. The items which are considered unknown at each time in this example are:

Pre-planting:	- time required to plant an acre
	- time available for planting
	- yield
	- time required to harvest an acre
Post-Planting:	- yield
	- time required to harvest an acre
Pre-Harvesting:	- time required to harvest an acre
Post-harvesting:	- all uncertainties resolved.

Now suppose a probability distribution can be estimated for various values of the uncertain parameters conditional on the events that have occurred. Assuming that there are two possible pre- to post-planting states of nature and two pre- to post-harvest states of nature, a decision tree may be constructed as in Figure 3.

This tree begins with an initial certain pre-planting position A. Then, the next section of the tree represents planting state B_1 (the end of the planting season) in which the amount of time required to plant or available for planting is known. This state occurs with probability P_{B1}.

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[21]As discussed by Rae (1971), this type of model can accommodate alternative information structures classified as: "At the beginning of stage t of the decision process, the outcomes of stages t-i, t-i-1,..., are known with certainty by the decision-maker but the outcomes of stages t-i+1, t-i+2,..., are known only in the form of probability distributions of outcomes conditional on the known outcomes of past time periods. Hence, if i=0, the decision-maker has complete knowledge of past and present; if i=1, he has complete knowledge of the past; and if i>1, he has incomplete knowledge of the past" (p. 449). The information structure can also accommodate forecast, but little has been done to use this framework to value information.

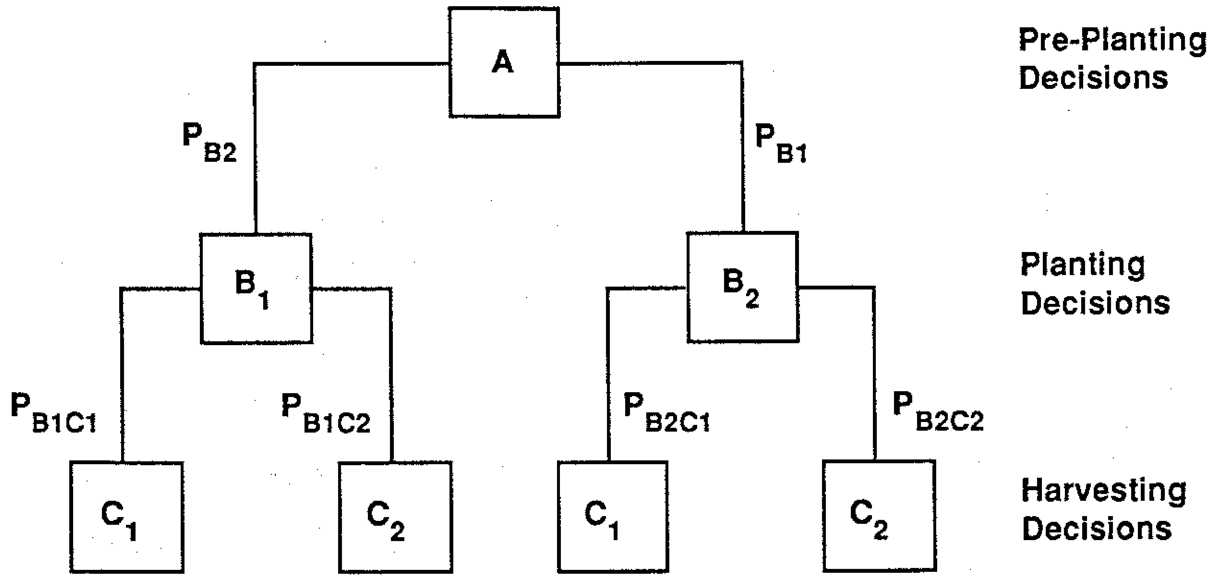

Figure 3. Decision tree for sequential programming example

Planting state B_2 occurs with probability P_{B2} and is associated with a different set of outcomes. At states B the harvest time outcome of yield and harvest weather are unknown, but information on their probabilities can be formulated. Once B_1 is reached, terminal events under B_2 cannot be reached and the probabilities of C_1 are conditional on having arrived at B_1. Given that B_1 is reached, then P_{B1C1} and P_{B1C2} represent the conditional probabilities of reaching states C_1 or C_2 when crop yield and harvesting time have been determined.

This decision tree represents the uncertainties and allows potential adaptive behavior. Decision makers can make decisions or possibly change previous decisions depending upon the way the uncertainty has been resolved; i.e., different post-planting decisions may be made under the B_1 and B_2 states of nature or a replanting decision can be made. An SPR model captures such behavior.

A general formulation of an SPR problem with three stages is as follows:

$$(123) \quad \text{Max} \sum_j c_j x_j + \sum_i P_i \left(\sum_k d_{ik} y_{ik} + \sum_\ell Q_{i\ell} \sum_m e_{i\ell m} z_{i\ell m}\right)$$

s.t.

$$(124) \quad \sum_j a_{rj} x_j \leq b_r \quad \text{for all } r$$

$$(125)\quad -\sum_j LKAB_{jip}\, x_j + \sum_k LKBA_{ipk}\, y_{ik} \le 0 \quad \text{for all } i \text{ and } p$$

$$(126)\quad \sum_k f_{iks}\, y_{ik} \le h_{is} \quad \text{for all } i \text{ and } s$$

$$(127)\quad -\sum_k LKBC_{i\ell kq}\, Y_{ik} + \sum_m LKCB_{i\ell mq}\, z_{i\ell m} \le 0 \quad \text{for all } i\ \ell \text{ and } q$$

$$(128)\quad \sum_m g_{i\ell mt}\, Z_{i\ell m} \le rhs_{i\ell m} \quad \text{for all } i\ \ell \text{ and } t$$

$$(129)\quad x_j,\ y_{ik},\ z_{i\ell m} \ge 0 \quad \text{for all } i\ j\ k\ \ell\ m$$

where x_j is the j^{th} activity at (the certain) decision point A;
C_j is the objective function coefficient of x_j;
P_i is the probability of reaching state B_i;
y_{ik} is the k^{th} activity at Stage B given that we are in state B_i;
d_{ik} is the objective function coefficient of Y_{ik} under state B_i;
$Q_{i\ell}$ is the probability of reaching state $C_{i\ell}$;
$z_{i\ell m}$ is the m^{th} activity at Stage C given that we are in state $C_{i\ell}$;
$e_{i\ell m}$ is the objective function coefficient of $Z_{i\ell m}$ under state $C_{i\ell}$;
a_{rj} is the per unit use of resource r by activity x_j;
f_{iks} is the per unit use of resource s by activity Y_{ik} under state B_i;
$g_{i\ell mt}$ is the per unit use of resource t by activity $Z_{i\ell m}$ under state $C_{i\ell}$;
b_r is the endowment of resource r;
h_{is} is the endowment of resource s under state B_i;
$rhs_{i\ell t}$ is the endowment of resource t under state $C_{i\ell}$;
$LKAB_{jip}$ are coefficients which link the j^{th} activity in Stage A with those which follow in B_i via the p^{th} link;
$LKBC_{i\ell kq}$ are coefficients linking the k^{th} activity in Stage B_i with those which follow in $C_{i\ell}$ via the q^{th} link;
$LKBA_{ipk}$ are coefficients linking the k^{th} activity in Stage B_i with those which precede A via the p^{th} link; and
$LKCB_{i\ell mq}$ are coefficients which link the m^{th} activity in Stage $C_{i\ell}$ with those which precede B_i via the q^{th} link.

Several points should be noted about this formulation. First, let us note what is risky. In each stage the coefficients expressing the resource endowment or activity's resource usage and objective function coefficient are dependent upon the state. Thus, under stage 1 all the coefficients dealing with the x's (b_r, a_{rj}, C_j) are certain. However, in stage 2 the coefficients (h_{is}, f_{iks}, d_{ik}, $LKAB_{jip}$, $LKBA_{ipk}$) all depend upon the state (i). In stage 3 the coefficients ($g_{i\ell mt}$, $rhs_{i\ell t}$, $e_{i\ell m}$, $LKBC_{i\ell kq}$, $LKCB_{i\ell mq}$) depend upon the state of both stage 3 $C(\ell)$ and stage 2 B(i). The important point is that all types of coefficients (b_i, c_j and a_{ij}) are potentially unknown and their values depend upon the path through the decision tree. Unlike the models above, such as the chance constrained model, this formulation reflects all possible states of nature and the decision variables reflect the optimal adaptive behavior (e.g. the optimal recourse) at each stage and state given the potential future states of nature and what has happened up to that point.

This formulation also highlights a potentially "risky" link between preceding and successive activities. (If the activities are not linked in this way, the problem is not a sequential decision problem.) If these links exist, they may involve the weighted sum of a number of variables in the various stages (i.e., acreage planted to corn via several methods is linked with acreage harvested via several methods.) Multiple links may also be present (i.e., there may be several crops), as reflected by the subscripts p and q.

A third comment relates to the nature of uncertainty resolution. The formulation reflects all uncertainty into the objective function, which becomes maximization of expected income. Rearranging the objective function yields the following:

$$(130) \quad \text{Maximize} \sum_i \sum_\ell P_i Q_{i\ell} \left(\sum_j C_j x_j + \sum_k d_{ik} y_{ik} + \sum_m e_{i\ell m} z_{i\ell m}\right).$$

Here the term in parentheses is income under the ith state of nature and the term outside the parentheses is the product of marginal probabilities which yields the probability of this state of nature.

An expected value may not be the appropriate optimization criterion and one might wish to include risk aversion. Many of the approaches to incorporating risk aversion discussed above can be built into this model. Specifically, Cocks developed an E-V model with the variance derived based on the theory of the multinomial distribution. Theoretically, the variance-covariance matrix between income states is:

$$(131) \quad S = \begin{bmatrix} P_1 (1 - P_1) & - P_1 P_2 & \cdots & -P_1 P_n \\ -P_2 P_1 & P_2 (1 - P_2) & \cdots & -P_2 P_n \\ \vdots & \vdots & & \vdots \\ -P_n P_1 & - P_n P_2 & \cdots & P_n (1 - P_n) \end{bmatrix}.$$

Formal incorporation of this into the above model is done by introducing new constraints to the model wherein:

$$(132) \quad y_{i\ell} = \sum_j C_j x_j + \sum_K d_{iK} y_{iK} + \sum_m e_{i\ell m} z_{i\ell m} \qquad \text{for all } i\ell.$$

Defining the relevant variance-covariance matrix elements as products of the probability:

$$(133) \quad S_{i\ell i\ell} = P_i Q_{i\ell} (1 - P_i Q_{i\ell}) \text{ and}$$

$$S_{i\ell KM} = -P_i Q_{i\ell} P_j Q_{jk},$$

the objective function becomes:

(134) maximize $\sum_i \sum_\ell P_i Q_{i\ell} Y_{i\ell} - \sum_i \sum_\ell \sum_K \sum_M S_{i\ell KM}\, Y_{i\ell}\, Y_{KM}$.

Similarly O'Brien (1981) presented a MOTAD formulation of the problem. Using the above notation this involves the new $Y_{i\ell}$ constraints and the equations:

(135) $\bar{Y} - \sum_i \sum_\ell P_i\, Q_{i\ell}\, Y_{i\ell} = 0$

$Y_{i\ell} + d^-_{i\ell} - d^+_{i\ell} = 0$

(136) $MAD - \sum_i \sum_\ell P_i\, Q_{i\ell}(d^+_{i\ell} + d^-_{i\ell}) = 0,$

where Y is mean income and d^+_i and $d^-_{i\ell}$ are deviation variables for state of nature $i\ell$. In turn, the objective function becomes:

(137) maximize $Y - \Psi\, MAD$.

This model is perhaps the most comprehensive and realistic of the risk models discussed because conceptually it incorporates all potential sources of uncertainty and treats decisions in a sequential fashion. Some applications of this model that we know of are those of Johnson, Tefertiller and Moore (1967); Yaron and Horowitz (1972); Klemme (1980); Tice (1979); Apland (1979); Apland, McCarl and Baker (1981); Lambert and McCarl (1985, 1989); O'Brien (1981); Leatham (1983); Leatham and Baker (1988); McCarl and Parandvash (1988); Kaiser and Apland (1989), and the early papers by Rae (1971a,b). The paper by Shumway and Gebremskal (1978) is closely related.

As the example in Appendix A illustrates, however, the model has two serious shortcomings. First, all uncertain parameters must be characterized in terms of discrete distributions. Second, the model suffers from the "curse of dimensionality". Each possible final state of nature leads to many activities within the model and large models can result from relatively simple problems. With only ten values of two right-hand sides, which were independently distributed for example, there would be 100 terminal states or sets of rows. Such models can be computationally intractable, although the sparsity and repeated structure tend to make such problems easier to solve than their size would imply (Kaiser and Apland, 1989). It is advisable to concentrate on the most important sources of uncertainty to be modeled; random variables not critical to the problem or that add little risk can be modeled deterministically or with few states of nature. Decision variables that are likely to be non-optimal can be eliminated *a priori*. Finally, since decisions at a particular stage are likely to be influenced less by prospects in future stages, it may be useful to sacrifice detail at later stages, derive approximate solutions for decisions at earlier stages, and restore the detail at later stages, thus using an adaptive solution process.

Other Methods

To this point, our discussion of risk programming models has been limited to those that have been used frequently in the agricultural economics literature. To a large extent, these models can be classified as active (here and now) decision models where decisions must be made prior to the resolution of the uncertainty. This is in contrast to the passive (wait and see) models where decision makers know the values of the random components prior to certain decisions. There have been few applications of the passive approach, although Tintner's original stochastic programming formulation is a passive model. The discrete stochastic programming model combines elements of both the passive and active approaches, but the major conceptual contributions in this general passive stochastic programming literature have been by people like Bereanu, 1980; Ewbank *et al.*, 1974; and Dempster, 1980). At a conceptual level, Boisvert, 1985 and Luckyn-Malone, 1984, have shown these models to be helpful in resolving some capital investment problems, but the computational difficulties for problems of realistic size have yet to be resolved.

Two of the more common methods for stochastic analysis have also been used to examine agricultural risks. First, some programming models are manipulated specifically to evaluate the consequences from specific scenarios and/or in some way describe the extremes of the probability distributions (i.e. see Adams, Hamilton, and McCarl, 1986). This approach simply entails solving the model with both optimistic and pessimistic estimates of the uncertain parameters. This approach, although not commonly used, has been extended whereby the models are run repeatedly under Monte Carlo conditions. Two important applications in this regard are the risk efficient Monte Carlo programming (REMP) model developed by Anderson (1975) and the generalized risk efficient Monte Carlo programming (GREMP) model by King and Oamek (1983). These types of strategies might well help resolve the computational problems in more general classes of stochastic programming.

The second approach involves Markovian based stochastic dynamic programming, as developed by Howard (1960) and used extensively by Burt and others. These models are beyond the scope of this review because they would involve a thorough discussion of dynamic programming. Stochastic control theory is also related to these types of models, and for discussions of these methods, the reader is referred to texts such as Howard, (1960), Nemhauser (1966), Kennedy (1986) and Neck (1984).

SUMMARY AND CONCLUSIONS

The purpose of this bulletin is to provide a survey of the literature covering the variety of mathematical programming techniques used to study agricultural problems involving risk and uncertainty. It begins with a characterization of the risky nature of agricultural decisions and the theoretical foundations of risk decision criteria. The major programming techniques for dealing with risk in prices, production and resource use and availability are described and evaluated. Their consistency with risk decision criteria is discussed. Appendix A contains example illustrations of many of the models and in Appendix B, the numerous applications of these methods are categorized, both by type of model and risk under study and by subject area.

Throughout the bulletin, several aspects of modeling agricultural risk are emphasized. The first relates to the question: should risk and uncertainty be considered explicitly in programming analysis, and if so, how should it be accomplished? Because agricultural production and marketing occurs in a risky environment, one might think the answer to the first question is obviously "yes". However, throughout the discussion, it has been emphasized that in order to consider risk and uncertainty explicitly, the models become more complex and the data requirements are more extensive. The empirical results from the models are more difficult to interpret and to explain to farmers and policy makers. Therefore, the extent to which risk should be incorporated into programming models depends both on the severity of the risk and the nature of decision makers' adjustments in response to the risky situation. If their responses are important, the results generated in empirical analysis will not resemble actual decisions unless the risk is explicitly reflected in the model. In any empirical application, the decisions about which sources of risk are most important and should be considered explicitly depend on the researchers' experience and subjective judgement.

Once the decision is made to include a particular type of risk in an analysis, a researcher must still select the particular model to be used and convert the decision problem into its deterministic equivalent. As illustrated above, there are a number of different models designed to incorporate various types of uncertainty. This is particularly true for incorporating price and yield risk into the objective function. In those situations where different models are likely to yield similar results, the choice of model might well be based on the model's simplicity and the ease with which the results of a particular model can be explained to decision or policy makers. While a number of decision makers may not relate directly to expected utility, they can certainly understand the notion of discounting a farm plan based on its variability in returns as is reflected in E-V and MOTAD models. Chance constrained models and safety-first models can be discussed in terms of the probability with which certain levels of income or resource supplies are available. The discrete stochastic programming model can be described as a decision tree representing the sequential nature of the decision process. Even if the decision problem is too complex to incorporate into a DSP model, communication with decision makers can be enhanced by viewing the problem within the context of a decision tree.

In addition, much work with what we have called risk efficiency analysis has been an attempt to provide useful information to decision makers without requiring explicit elicitation of individual risk preferences. This is a direct result of the difficulties encountered in measuring risk preferences and a desire to provide information useful to decision makers with a range of attitude toward risk. Thus, in some situations it may be sufficient to categorize efficient alternatives as being applicable to decision makers with low, moderate or high aversion to risk.

Regardless of the type of model used, researchers must formulate estimates of the joint probability distributions of the uncertain parameters. Although a discussion of how these distributions are to be estimated is beyond the scope of this paper, it is important to emphasize that the development of a satisfactory representation of these joint distributions is difficult to obtain. In the past, these empirical distribution functions have been developed through subjective elicitation and objective synthesis of time series data. In other cases, some underlying distribution is assumed and data are used to obtain maximum likelihood estimates of the parameters.

Since mathematical programming models are quite often "normative" or prescriptive in nature, some would argue that probability distributions of the important parameters should be elicited subjectively if the model is being developed for a particular decision maker. (The theory of subjective probability elicitation is summarized by Bessler (1984) and Perry (1986) using a model in which all the distributions were subjectively estimated.) While this strategy is appealing, it is not without substantial problems. In particular, if there are a number of uncertain parameters, it may be quite difficult to elicit the marginal distributions and next to impossible to elicit the full nature of the joint distributions among all the random variables. The problems would be very difficult in the DSP case where joint distribution of several prices, crop yields by time period and field time would have to be elicited.

As an alternative, most studies rely on the "objective" development of probability distributions based on either a panel or time series. In such cases, the joint distribution can be developed for a number of parameter values providing that a series of observations on the parameter values can be obtained. This method's major drawback is that it is necessary to assume that each of the historical or cross sectional observations are equally likely sample points from the true distribution. This may not be the case. Also, the practitioner may need to devote considerable effort to remove trend (as pointed out by Chen, 1971) and distill a pure set of random elements (e.g. see Lambert and McCarl, 1985 or the review in Young, 1980). Approaches to resolving this problem have involved the use of moving averages (Brink and McCarl, 1978), regression (Tice, 1979), and time series analysis (Lambert and McCarl, 1988) and variance components models (Adams, Menkhaus and Woolery, 1980). Regardless of the methods used, there is no guarantee that the distributions will be relevant for the current time period or for any reasonable time into the future.

The third alternative is to assume an underlying distribution and use the data to estimate the parameters. Studies have adopted such disparate distributions as the normal, log normal or gamma. Others have developed the parameters of the distributions using data from simulation models (e.g. Dillon, Mjelde and McCarl, 1989).

Regardless of the methods used to generate the probability distributions on parameters, sampling errors are likely to result and be propagated throughout the expected utility or risk efficiency analysis. Pope and Ziemer (1984), and more recently Collender (1989) and Gbur and Collins (1989), are among the only people to address the issues surrounding sampling errors in efficiency analysis. Their studies were not designed to look at programming studies per se. Pope and Ziemer's Monte Carlo results, however, suggest that empirical distributions performed better in risk efficiency analysis than did maximum likelihood methods which presume knowledge of underlying distributions. Gbur and Collins found that the relative performance of non-parametric and parametric specifications depends on sample size and level of risk aversion. Collender demonstrates that confidence regions around points on efficient sets are conditional on the allocation of resources but it may be statistically impossible to distinguish among many efficient combinations. These findings suggest that more research is needed to identify the level of "estimation risk" and the value of reducing it.

APPENDIX A

EXAMPLES OF RISK MODEL APPLICATIONS

The purpose of this appendix is to illustrate a number of the risk programming models discussed in the text. These examples are included primarily to increase the value of the report for use in a learning environment.

In the models reflecting risk in the objective function, a portfolio problem with only one constraint is used to illustrate each of the methods. While this is not an agricultural example, it does help to isolate the effects of the various risk decision criteria on the optimal solutions. In problems with more than one resource constraint, for example, much of the diversification in the optimal solution involves considerations surrounding resource usage. By abstracting from this issue, the diversification due to the response to risk is isolated.

Slightly more complex examples are needed to demonstrate the models in which there is uncertainty in the right-hand sides or in the technical coefficients. The discrete stochastic programming model is the most complex.

For each of the examples, the data are provided, as are the specific algebraic formulations of the models and the solutions. They are presented in the same order as they are discussed in the text.

The models that can be formulated as linear programming problems can be solved using any conventional linear programming software, whereas the others can only be solved using nonlinear programming methods. Jefferson and Boisvert (1989) illustrate how to prepare the data inputs efficiently and solve a variety of risk programming models with objective function uncertainty using GAMS-MINOS.

OBJECTIVE FUNCTION UNCERTAINTY

Mean-Variance Analysis

Data for an example mean-variance portfolio application are given in Table A-1. The first stage in model application is to compute mean returns and the variance-covariance matrix of total net returns. After calculating mean returns for the four stocks and the variance-covariance matrix (following equation (23)) from the data, the objective function is given by:

$$\text{Maximize } [13.38 \quad 9.18 \quad 13.13 \quad 16.00] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

$$- \phi \; [X_1 \; X_2 \; X_3 \; X_4] \begin{bmatrix} 7.788 & 5.561 & -2.996 & 11.960 \\ 5.561 & 5.102 & -2.513 & 15.660 \\ -2.996 & -2.513 & 12.542 & -48.890 \\ 11.960 & 15.660 & -48.890 & 381.000 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Table A-1. Annual Returns to Stocks for E-V model

Observation	Stock 1	Stock 2	Stock 3	Stock 4
1	$13.20	$ 9.40	$23.00	$-17.00
2	10.90	5.70	14.00	- 7.00
3	14.90	12.20	11.00	13.00
4	17.40	12.30	10.00	33.00
5	8.10	6.00	13.30	13.00
6	13.80	9.20	12.00	33.00
7	16.40	11.40	10.50	43.00
8	10.20	7.00	13.50	33.00
9	13.10	9.60	11.00	23.00
10	15.80	9.00	13.00	-7.00
Purchase Price	$100.00	$80.00	$95.00	$95.00

In scalar notation, the objective function is:

$$\begin{aligned}\text{Maximize}\quad & 13.38\ X_1 + 9.18\ X_2 + 13.13\ X_3 + 16\ X_4 - \phi(7.788\ X_1^2 + 5.561\ X_1X_2 \\ & - 2.996\ X_1X_3 + 11.960\ X_1X_4 + 5.561\ X_2X_1 + 5.102\ X_2^2 - 2.513\ X_2X_3 \\ & + 15.660\ X_2X_4 - 2.996\ X_3X_1 - 2.513\ X_3X_2 + 12.542\ X_3^2 - 48.890\ X_3X_4 \\ & + 11.960\ X_4X_1 + 15.660\ X_4X_2 - 48.890\ X_4X_3 + 381.000\ X_4^2)\end{aligned}$$

This objective function is maximized subject to a constraint on investable funds:

$$100\ X_1 + 80\ X_2 + 95\ X_3 + 95\ X_4 \le 1000$$

and non-negativity conditions on the variables:

$$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0.$$

This problem is solved for $\phi = 0$ to ∞. The solutions, at selected values of ϕ, are shown in Table A-2.

The model yields the profit maximizing solution ($X_1 = X_2 = X_3 = 0$, $X_4 = 10.53$) for small risk aversion parameters ($\phi<0.0003$), but as the risk aversion parameter increases, X_3 enters the solution. The diversification between X_3 and X_4, coupled with their negative covariance, reduces the variance of total returns. As the risk aversion parameter increases, more is invested in X_3 and less in X_4 until at $\phi = 0.004$ expected returns have fallen by $25 or 14.6%, while the standard deviation of total returns has fallen by $178 or 86.7%. Thus, a 14.6% reduction in expected returns leads to an 86.7% reduction in risk exposure. For values of the risk aversion parameter between 0.004 and 0.225, investment in X_1 increases.

Table A-2. Solutions to Quadratic Programming Formulation of the Investment Example

Risk Aversion Parameter	0.00025	0.0003	0.0005	0.0007	0.001	0.0015	0.002	0.003	0.004	0.0075	0.05	0.10	0.20	0.225	0.25	0.30	0.50
Objective Function Value	406.69	404.38	399.23	396.96	395.16	393.60	392.68	391.44	390.54	388.07	374.34	360.16	332.03	325.01	318.01	306.68	284.01
X_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	3.81	4.08	4.22	4.24	4.18	3.48	2.09
X_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X_3	0.00	0.45	3.95	5.45	6.58	7.46	7.90	8.33	8.55	8.19	5.76	5.46	5.44	5.43	5.33	4.45	2.67
X_4	10.53	10.08	6.57	5.07	3.94	3.07	2.63	2.19	1.97	1.54	0.75	0.68	0.65	0.64	0.63	0.52	0.31
Average Income	168.42	167.14	157.08	152.77	144.53	147.02	145.76	144.50	143.87	142.31	138.68	138.36	138.20	138.18	136.02	113.35	68.01
Variance in Income	42216	38259	14118	7467	3933	2047	1387	916	751	509	258	254	253	253	245	170	61
Standard Deviation in Income	205.47	195.60	118.82	86.41	62.70	45.24	37.24	30.26	27.40	22.55	16.07	15.94	15.90	15.90	15.65	13.03	7.82
Shadow Price	0.15	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.13	0.11	0.08	0.03	0.00	0.00	0.00	0.00
Idle Funds	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	15.58	179.65	507.90

Three other aspects of these results are worth noting. First, the shadow price on investable capital continually decreases as the risk aversion parameter (ϕ) increases. This reflects the increasing risk discount as risk aversion increases. Second, solutions are reported only for selected values of ϕ. However, any change in ϕ leads to a change in the solution and an infinite number of alternative ϕ's are possible; e.g., all solutions between $\phi = 0.0003$ and 0.004 are convex combinations of the $\phi = 0.0003$ and 0.004 solutions. Finally, when ϕ becomes sufficiently large, the model does not use all its resources. In this particular case, when ϕ exceeds 0.225, not all funds are invested.

A Linear Approximation - MOTAD

The MOTAD model is given by equations (55) through (58). This example of MOTAD uses the same data as in the E-V example above. The mean returns, used as a measure of expected value, equal 13.38, 9.18, 13.13, and 16. Deviations from the means ($C_{kj} - C_j$) for the stocks are shown in Table A-3.

The MOTAD formulation is:

$$\text{Maximize} \quad 13.38\,X_1 + 9.18\,X_2 + 13.13\,X_3 + 16\,X_4 - \alpha\,\sigma$$

$$\begin{aligned}
\text{s.t.} \quad & 100\,X_1 + 80\,X_2 + 95\,X_3 + 95\,X_4 \le 1{,}000 \\
& -0.18\,X_1 + 0.22\,X_2 + 9.87\,X_3 - 33\,X_4 + d_1^- \ge 0 \\
& -2.48\,X_1 - 3.48\,X_2 + .87\,X_3 - 23\,X_4 + d_2^- \ge 0 \\
& 1.52\,X_1 + 3.02\,X_2 - 2.13\,X_3 - 3\,X_4 + d_3^- \ge 0 \\
& 4.02\,X_1 + 3.12\,X_2 - 3.13\,X_3 + 17\,X_4 + d_4^- \ge 0 \\
& -5.28\,X_1 - 3.18\,X_2 + 0.17\,X_3 - 3\,X_4 + d_5^- \ge 0 \\
& 0.42\,X_1 + 0.02\,X_2 - 1.13\,X_3 + 17\,X_4 + d_6^- \ge 0 \\
& 3.02\,X_1 + 2.22\,X_2 - 2.63\,X_3 + 27\,X_4 + d_7^- \ge 0 \\
& -3.18\,X_1 - 2.18\,X_2 + 0.37\,X_3 + 17\,X_4 + d_8^- \ge 0 \\
& -0.28\,X_1 + 0.42\,X_2 - 2.13\,X_3 + 7\,X_4 + d_9^- \ge 0 \\
& 2.42\,X_1 - 0.18\,X_2 - 0.13\,X_3 - 23\,X_4 + d_{10}^- \ge 0 \\
& -3.7846\,\sigma + d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6^- \\
& \quad + d_7^- + d_8^- + d_9^- + d_{10}^- = 0 \\
& X_j,\ \sigma,\ d_k^-, \ge 0
\end{aligned}$$

Table A-3. Estimated Deviations From Mean Return for the Investment Example

Observation	Stock 1	Stock 2	Stock 3	Stock 4
1	$-0.18	$ 0.22	$ 9.87	$-33.00
2	-2.48	-3.48	0.87	-23.00
3	1.52	3.02	-2.13	- 3.00
4	4.02	3.12	-3.13	+17.00
5	-5.28	-3.18	0.17	- 3.00
6	0.42	0.02	-1.13	17.00
7	3.02	2.22	-2.63	27.00
8	-3.18	-2.18	0.37	17.00
9	-0.28	0.42	-2.13	7.00
10	2.42	-0.18	-0.13	-23.00

Solutions to this model are obtained for a range of values of α. The solutions which occur at the basis changes as α is varied are reported in Table A-4.

The above MOTAD solutions give the levels of investment in each X_j, the unused funds, the mean absolute deviation, and the approximation to the standard error. Also, the true variances and standard errors are calcualted based on the portfolio chosen. This problem's solutions are similar to that of the E-V model. Note that for risk aversion coefficients (α) less than 0.12, the profit maximizing solution is obtained. For α's between 0.12 and 0.13 investing all funds in X_3 and X_4 -- 8.07 units of X_3 and 2.46 units of X_4 -- is optimal. Solutions for the other values of the risk aversion parameters are similar to those in the above E-V analysis example, but these solutions are not convex combinations dependent on the risk aversion parameters as under the E-V model. Rather they are constant for the range of the risk aversion parameters specified. Perhaps the only difference in the numerical results worth noting is that these solutions behave in basically the same manner as those of the E-V formulation, although at high risk aversion, a small investment in X_2 is indicated. Subsequently, however, X_2 drops to zero.

The reader should also note the approximate nature of the standard error relationship. For example, the approximated standard deviation of the distribution at the first risk aversion range is 236.4, but the actual standard deviation of the portfolio is 205.67. In fact, the MOTAD approximation initially overstates the standard deviation from 6 and 15%, whereas later it is understated by 22%. The E-V and MOTAD frontiers correspond very closely, although this is not adequate proof that the solutions will always be close (see Thomson and Hazell (1972) for a comparison between the methods).

Table A-4. Solutions to MOTAD Problem for Ranges in Risk Aversion Coefficients

α	$\alpha \le 0.12$	$0.12 \le \alpha \le 0.13$	$0.13 \le \alpha \le 0.26$	$0.26 \le \alpha \le 0.39$	$0.39 \le \alpha \le 1.14$	$1.14 \le \alpha \le 6.03$	$6.03 \le \alpha \le 9.53$	$9.53 \le \alpha$
Income	168.421	145.2585	145.1659	142.9097	138.11	137.61	137.60	0
X_1	0	0	0	0	3.24	3.57	3.57	0
X_2	0	0	0	0	0	0	.003	0
X_3	0	8.07	8.10	8.89	6.66	6.43	6.43	0
X_4	10.53	2.46	2.42	1.64	0.46	0.34	0.34	0
Unused funds	0	0	0	0	0	0	0	1000
MAD	178.95	27.79	27.24	20.69	11.26	10.93	10.93	0
σ	236.41	36.71	35.99	27.33	14.88	14.44	14.44	0
Variance	42216.06	1176.69	1141.07	589.11	325.12	339.61	339.63	0
$(\text{Variance})^{.5}$	205.67	34.30	33.78	24.27	18.03	18.43	18.43	0

The Focus Loss Example

The E-V example data are again used in the focus loss example (equations (59) through (63)). Given that we set K equal to 3 and the activity F_j's to the standard deviation of the risky activities (corresponding to a t-value of one), the focus loss formulation is:

$$\text{Maximize} \quad 13.38\, X_1 + 9.18\, X_2 + 13.13\, X_3 + 16\, X_4$$

$$\begin{aligned}
\text{s.t.} \quad & 13.38\, X_1 + 9.18\, X_2 + 13.13\, X_3 + 16\, X_4 - L &= M \\
& 100\, X_1 + 80\, X_2 + 95\, X_3 + 95\, X_4 &\le 1000 \\
& 2.94\, X_1 - L/3 &\le 0 \\
& 2.38\, X_2 - L/3 &\le 0 \\
& 3.73\, X_3 - L/3 &\le 0 \\
& 20.56\, X_4 - L/3 &\ge 0 \\
& L;\ X_j &\ge 0
\end{aligned}$$

M has not been specified and is parameterized from $-\infty$ to ∞. These solutions are shown in Table A-5.

Any value of M less than -480 results in the undiversified LP solution. As the value of M becomes larger than -480, X_3 enters the solution. From $-480 < M \le 43$, a set of solutions is obtained (and their associated convex combinations) which include only X_3 and X_4. For M above 43.2 but below 91.2, X_1 is included in the convex combinations along with X_3 and X_4. Above 91.25, X_2 enters the solution; it increases in value until M = 99.33. For values of M exceeding 99.36 the problem is infeasible. For M = 99.33, the focus loss model includes all four X's at non-zero levels in the solution.

Table A-5. Solutions to the Focus Loss Model

Values of M	<-480.00	-480.00	43.00	43.20	91.20	91.25	99.33[a]
Average Income	168.42	168.38	142.86	142.84	138.05	138.05	130.04
X_1	0.00	0.00	0.00	0.006	5.30	5.31	3.48
X_2	0.00	0.00	0.00	0.00	0.00	0.01	4.30
X_3	0.00	0.01	8.91	8.90	4.19	4.18	2.74
X_4	10.53	10.51	1.62	1.62	0.76	0.76	0.50
L	1162.42	648.38	699.86	99.64	46.85	46.80	30.71

[a]For M > 99.36, the model is infeasible.

Target MOTAD

Using the data from the earlier examples and assuming each state of nature is equally probable ($P_k = 1/10$) the target MOTAD formulation (equations (64) through (68)) is:

$$\text{Maximize } 13.38\ X_1 + 9.18\ X_2 + 13.13\ X_3 + 16\ X_4$$

$$
\begin{array}{llllll}
\text{s.t.} & 100\ X_1 + & 80\ X_2 + & 95\ X_3 + 95\ X_4 & & \leq 1000 \\
 & 13.2\ X_1 + & 9.4\ X_2 + & 23.0\ X_3 - 17\ X_4 & + Y_1 & \geq T \\
 & 10.9\ X_1 + & 5.7\ X_2 + & 14.0\ X_3 - 7\ X_4 & + Y_2 & \geq T \\
 & 14.9\ X_1 + & 12.2\ X_2 + & 11.0\ X_3 + 13\ X_4 & + Y_3 & \geq T \\
 & 17.4\ X_1 + & 12.3\ X_2 + & 10.0\ X_3 + 33\ X_4 & + Y_4 & \geq T \\
 & 8.1\ X_1 + & 6.0\ X_2 + & 13.3\ X_3 + 13\ X_4 & + Y_5 & \geq T \\
 & 13.8\ X_1 + & 9.2\ X_2 + & 12.0\ X_3 + 33\ X_4 & + Y_6 & \geq T \\
 & 16.4\ X_1 + & 11.4\ X_2 + & 10.5\ X_3 + 43\ X_4 & + Y_7 & \geq T \\
 & 10.2\ X_1 + & 7.0\ X_2 + & 13.5\ X_3 + 33\ X_4 & + Y_8 & \geq T \\
 & 13.2\ X_1 + & 9.6\ X_2 + & 11.0\ X_3 + 23\ X_4 & + Y_9 & \geq T \\
 & 15.8\ X_1 + & 9.0\ X_2 + & 13.0\ X_3 - 7\ X_4 & + Y_{10} & \geq T
\end{array}
$$

$$0.1Y_1 + 0.1Y_2 + 0.1Y_3 + 0.1Y_4 + 0.1Y_5 + 0.1Y_6 + 0.1Y_7 + 0.1Y_8 + 0.1Y_9 + 0.1Y_{10} \leq \lambda$$

The solutions to this problem for selected values of T and λ are shown in Table A-6.

The Target MOTAD model solutions reflect much the same behavior as did the other models. For income targets exceeding 160 with a λ limit of greater than 85, we obtain the LP optimal solution. As the target values decrease, X_3 comes into the solution, along with X_4. This is a continuous process across different values of the target parameter down to the point where the target parameter equals 20.75. At that point X_1 enters the solution and increases in value from there until λ falls to 19.67. For λ below 19.67, the solution

Table A-6. Solutions to Target MOTAD Model

T	168.4211	160.00	160.00	160.00	160.00	160.00	160.00	160.00	160.00	160.00
λ	89.52631	85.00[a]	31.01[b]	31.00	22.24[b]	22.22	20.80[b]	20.75[b]	19.68	19.67[b]
X_1	0	0	0	0	0	0	0	0.51	2.04	2.16
X_2	0	0	0	0	0	0	0	0	0	0
X_3	0	0.03	6.84	6.84	8.14	8.12	8.47	8.43	6.89	6.80
X_4	10.53	10.49	3.68	3.68	2.38	2.38	2.06	2.04	1.49	8.46
Average Income	168.42	168.33	148.79	148.78	145.05	145.04	144.12	144.04	141.59	141.45

[a]Any value λ above 85.00 will yield the LP solution.

[b]Convex combinations of this solution and the one to the left are also feasible. The solution is infeasible for T = 160 and any value of λ less than 19.65.

becomes infeasible. Target MOTAD, as does E-V, leads to a continuous, not a discrete, set of plans. Any convex combination of the plans, for example, for λ between 85 and 31.01, leads to a convex combination of the solutions for X_3 and X_4.

Mean-Gini Efficiency Analysis

The investment problem is also solved using the Mean-Gini formulation given by equations (75) through (79). Because the formulation involves keeping track of all pairs of absolute differences in yearly returns, the model is rather large even with only 4 investment alternatives and 10 years of data. For this reason, summation notation is used to facilitate the presentation of the model. Despite its size, the code to formulate the model was easy to develop in GAMS (see Jefferson and Boisvert, 1989). The formulation for this example investment problem is:

$$\text{Minimize} \quad \sum_{j=2}^{10} (Y_{1j}^{+}+Y_{1j}^{-}) + \sum_{j=3}^{10} (Y_{2j}^{+}+Y_{2j}^{-}) + \sum_{j=4}^{10} (Y_{3j}^{+}+Y_{3j}^{-}) + \sum_{j=5}^{10} (Y_{4j}^{+}+Y_{4j}^{-})$$

$$+ \sum_{j=6}^{10} (Y_{5j}^{+}+Y_{5j}^{-}) + \sum_{j=7}^{10} (Y_{6j}^{+}+Y_{6j}^{-}) + \sum_{j=8}^{10} (Y_{7j}^{+}+Y_{7j}^{-}) + \sum_{j=9}^{10} (Y_{8j}^{+}+Y_{8j}^{-})$$

$$+ Y_{910}^{+}+Y_{910}^{-}$$

s.t.

$$\begin{array}{rrrrll}
100X_1 + & 8X_2 + & 95X_3 + & 95X4 & & \leq 1000 \\
13.38X_1 + & 9.18X_2 + & 13.13X_3 + & 16X_4 & & = \mu \\
2.3X_1 + & 3.7X_2 + & 9X_3 - & 10X_4 & + Y_{12}^{+}+Y_{12}^{-} & = 0 \\
-1.7X_1 - & 2.8X_2 + & 12X_3 - & 30X_4 & + Y_{13}^{+}+Y_{13}^{-} & = 0 \\
-4.2X_1 - & 2.9X_2 + & 13X_3 - & 50X_4 & + Y_{14}^{+}+Y_{14}^{-} & = 0 \\
5.1X_1 + & 3.4X_2 + & 9.7X_3 - & 30X_4 & + Y_{15}^{+}+Y_{15}^{-} & = 0 \\
-0.6X_1 + & 0.2X_2 + & 11X_3 - & 50X_4 & + Y_{16}^{+}+Y_{16}^{-} & = 0 \\
-3.2X_1 - & 2X_2 + & 12.5X_3 - & 60X_4 & + Y_{17+}+Y_{17}^{-} & = 0 \\
3X_1 + & 2.4X_2 + & 9.5X_3 - & 50X_4 & + Y_{18}^{+}+Y_{18}^{-} & = 0 \\
0.1X_1 - & 0.2X_2 + & 12X_3 - & 40X_4 & + Y_{19+}+Y_{19}^{-} & = 0 \\
-2.6X_1 + & 0.4X_2 + & 10X_3 - & 10X_4 & + Y_{110}^{+}+Y_{110}^{-} & = 0 \\
-4X_1 - & 6.5X_2 + & 3X_3 - & 20X_4 & + Y_{23}^{+}+Y_{23}^{-} & = 0 \\
-6.5X_1 - & 6.6X_2 + & 4X_3 - & 40X_4 & + Y_{24}^{+}+Y_{24}^{-} & = 0 \\
2.8X_1 - & 0.3X_2 + & 0.7X_3 - & 20X_4 & + Y_{25}^{+}+Y_{25}^{-} & = 0 \\
-2.9X_1 - & 3.5X_2 + & 2X_3 - & 40X_4 & + Y_{26}^{+}+Y_{26}^{-} & = 0 \\
-5.5X_1 - & 5.7X_2 + & 3.5X_3 - & 50X_4 & + Y_{27}^{+}+Y_{27}^{-} & = 0 \\
0.7X_1 - & 1.3X_2 + & 0.5X_3 - & 40X_4 & + Y_{28}^{+}+Y_{28}^{-} & = 0 \\
-2.2X_1 - & 3.9X_2 + & 3X_3 - & 30X_4 & + Y_{29}^{+}+Y_{29}^{-} & = 0 \\
-4.9X_1 - & 3.3X_2 + & X_3 & & + Y_{210}^{+}+Y_{210}^{-} & = 0 \\
-2.5X_1 - & 0.1X_2 + & X_3 - & 20X_4 & + Y_{34}^{+}+Y_{34}^{-} & = 0 \\
6.8X_1 + & 6.2X_2 - & 2.3X_3 & & + Y_{35}^{+}+Y_{35}^{-} & = 0 \\
1.1X_1 + & 3.0X_2 - & X_3 - & 20X_4 & + Y_{36}^{+}+Y_{36}^{-} & = 0 \\
-1.5X_1 + & 0.8X_2 + & 0.5X_3 - & 30X_4 & + Y_{37}^{+}+Y_{37}^{-} & = 0 \\
4.7X_1 + & 5.2X_2 - & 2.5X_3 - & 20X_4 & + Y_{38}^{+}+Y_{38}^{-} & = 0 \\
1.8X_1 + & 2.6X_2 & - & 10X_4 & + Y_{39}^{+}+Y_{39}^{-} & = 0 \\
-0.9X_1 + & 3.2X_2 - & 2X_3 + & 20X_4 & + Y_{310}^{+}+Y_{310}^{-} & = 0 \\
9.3X_1 + & 6.3X_2 - & 3.3X_3 + & 20X_4 & + Y_{45}^{+}+Y_{45}^{-} & = 0 \\
3.6X_1 + & 3.1X_2 - & 2X_3 & & + Y_{46}^{+}+Y_{46}^{-} & = 0 \\
1.0X_1 + & 0.9X_2 - & 0.5X_3 - & 10X_4 & + Y_{47}^{+}+Y_{47}^{-} & = 0
\end{array}$$

$$\begin{array}{rrrrrl}
7.2X_1 + & 5.3X_2 - & 3.5X_3 & & + Y_{48}^{+}+Y_{48}^{-} & = 0 \\
4.3X_1 + & 2.7X_2 - & X_3 + & 10X_4 & + Y_{49}^{+}+Y_{49}^{-} & = 0 \\
1.6X_1 + & 3.3X_2 - & 3X_3 + & 40X_4 & + Y_{410}^{+}+Y_{410}^{-} & = 0 \\
-5.7X_1 - & 3.2X_2 + & 1.3X_3 - & 20X_4 & + Y_{56}^{+}+Y_{56}^{-} & = 0 \\
-8.3X_1 - & 5.4X_2 + & 2.8X_3 - & 30X_4 & + Y_{57}^{+}+Y_{57}^{-} & = 0 \\
-2.1X_1 - & X_2 - & 0.2X_3 - & 20X_4 & + Y_{58}^{+}+Y_{58}^{-} & = 0 \\
-5X_1 - & 3.6X_2 + & 2.3X_3 - & 10X_4 & + Y_{59}^{+}+Y_{59}^{-} & = 0 \\
-7.7X_1 - & 3X_2 + & 0.3X_3 + & 20X_4 & + Y_{510}^{+}+Y_{510}^{-} & = 0 \\
-2.6X_1 - & 2.2X_2 + & 1.5X_3 - & 10X_4 & + Y_{67}^{+}+Y_{67}^{-} & = 0 \\
3.6X_1 + & 2.2X_2 - & 1.5X_3 & & + Y_{68}^{+}+Y_{68}^{-} & = 0 \\
0.7X_1 - & 0.4X_2 + & X_3 + & 10X_4 & + Y_{69}^{+}+Y_{69}^{-} & = 0 \\
-2X_1 + & 0.2X_2 - & X_3 + & 40X_4 & + Y_{610}^{+}+Y_{610}^{-} & = 0 \\
6.2X_1 + & 4.4X_2 - & 3X_3 + & 10X_4 & + Y_{78}^{+}+Y_{78}^{-} & = 0 \\
3.3X_1 + & 1.8X_2 - & 0.5X_3 + & 20X_4 & + Y_{79}^{+}+Y_{79}^{-} & = 0 \\
0.6X_1 + & 2.4X_2 - & 2.5X_3 + & 50X_4 & + Y_{710}^{+}+Y_{710}^{-} & = 0 \\
-2.9X_1 - & 2.6X_2 + & 2.5X_3 + & 10X_4 & + Y_{89}^{+}+Y_{89}^{-} & = 0 \\
-5.6X_1 - & 2X_2 + & 0.5X_3 + & 40X_4 & + Y_{810}^{+}+Y_{810}^{-} & = 0 \\
-2.7X_1 + & 0.6X_2 - & 2X_3 + & 30X_4 & + Y_{910}^{+}+Y_{910}^{-} & = 0
\end{array}$$

$$X_j \geq 0 \quad j=1,\dots,4$$

$$Y_{tk}^{+} \text{ and } Y_{tk}^{-} \geq 0 \quad t=1,\dots,9,\ k>t\ (k=2,\dots,10)$$

This problem is solved for seven different values of μ (expected income). These values are the same as the expected income levels implied by the solutions to the MOTAD formulation in Table A-4. The Mean-Gini solutions are in Table A-7.

The solutions to this Mean-Gini formulation are almost identical to the MOTAD solutions for the cases where expected income is above 143. For the other solutions, there is slightly higher investment in X_4 and less in both X_1 and X_3 then in the MOTAD solutions. Furthermore, all solutions are on the $M - \Gamma$ efficiency frontier, but the solution with the lowest expected income is dominated by another in an SSD sense.

Table A-7. Solutions to the Mean-Gini Example

Expected Income (μ)	168.42	145.26	145.17	142.91	138.11	137.61	137.60
X_1	0	0	0	0.44	3.03	3.16	3.16
X_2	0	0	0	0	0	0.14	0.14
X_3	*	8.07	8.10	8.36	6.91	6.72	6.72
X_4	10.53	2.46	2.42	1.70	0.43	0.37	0.37
Objective Function ($s^2\Gamma$)	11473.66	1937.61	1906.96	1368.38	829.89	819.24	819.15
Γ	114.74	19.38	19.07	13.68	8.30	8.19	8.19
$M - \Gamma$	53.68	125.88	126.10	129.23	129.81	129.41	129.40
Unused Funds	0	0	0	0	0	0	0

*less than 0.001.

A DEMP Example

Suppose an individual with the utility function for wealth $U = W^{.5}$ has an initial wealth (W_0) of 250. The portfolio example formulated as a DEMP model (equations (80) through (83)) becomes:

$$\text{Maximize} \quad 0.1(W_1)^{.5} + 0.1(W_2)^{.5} + 0.1(W_3)^{.5} + 0.1(W_4)^{.5} + 0.1(W_5)^{.5} + 0.1(W_6)^{.5} + 0.1(W_7)^{.5} + 0.1(W_8)^{.5} + 0.1(W_9)^{.5} + 0.1(W_{10})^{.5}$$

$$\begin{array}{llrlrlrlrl}
\text{s.t.} & & 100 & X_1 + 80 & X_2 + 95 & X_3 + 95 & X_4 & \le 1000 \\
 & W_1 - & 13.2 & X_1 - & 9.4 X_2 - & 20.0 X_3 + & 17 X_4 & = 250 \\
 & W_2 - & 10.9 & X_1 - & 5.7 X_2 - & 14.0 X_3 + & 7 X_4 & = 250 \\
 & W_3 - & 14.9 & X_1 - & 12.2 X_2 - & 11.0 X_3 - & 13 X_4 & = 250 \\
 & W_4 - & 17.4 & X_1 - & 12.3 X_2 - & 10.0 X_3 - & 33 X_4 & = 250 \\
 & W_5 - & 8.1 & X_1 - & 6.0 X_2 - & 13.3 X_3 - & 13 X_4 & = 250 \\
 & W_6 - & 13.8 & X_1 - & 9.2 X_2 - & 12.0 X_3 - & 33 X_4 & = 250 \\
 & W_7 - & 16.4 & X_1 - & 11.4 X_2 - & 10.5 X_3 - & 43 X_4 & = 250 \\
 & W_8 - & 10.2 & X_1 - & 7.0 X_2 - & 13.5 X_3 - & 33 X_4 & = 250 \\
 & W_9 - & 13.2 & X_1 - & 9.6 X_2 - & 11.0 X_3 - & 23 X_4 & = 250 \\
 & W_{10} - & 15.8 & X_1 - & 9.0 X_2 - & 13.0 X_3 + & 7 X_4 & = 250 \\
 & W_R; & X_j \ge 0 & & & & &
\end{array}$$

The solution to this problem is:

$W_1 = 266.43$	$W_5 = 308.31$	$W_8 = 502.12$	$X_1 = 0.00$
$W_2 = 278.89$	$W_6 = 494.78$	$W_9 = 433.49$	$X_2 = 0.00$
$W_3 = 377.07$	$W_7 = 543.89$	$W_{10} = 274.00$	$X_3 = 4.88$
$W_4 = 485.03$			$X_4 = 5.64$

The value of the objective function equals 19.95, while average wealth after the stock investment equals 404.40. The total funds shadow price equals 0.0360. Following Lambert and McCarl, this may be converted into an approximate value in dollar space by dividing by the marginal utility of average income; i.e., dividing the shadow prices by the factor $u^* \sim u/(\partial u(w)/\partial w)$ where

$$\frac{\partial(U(\overline{W}))}{\partial W} = 0.5(404.40)^{-.5} = 0.02486; \text{ and } u^* = 0.036/.02486 = 0.1447,$$

indicating that in this plan the marginal returns from more investable capital (the shadow price) is approximately \$0.1447.

RIGHT-HAND SIDE UNCERTAINTY

Chance Constrained Programming

Given the problem:

$$\text{Maximize } 4X_1 + 6X_2$$

$$\begin{array}{lrl}
\text{s.t. } 3X_1 + X_2 & \le & b \\
X_1 + 2X_2 & \le & 20 \\
X_1, \quad X_2 & \ge & 0
\end{array}$$

Suppose b is distributed normally with mean 30 and standard deviation 10. Then, in a chance constrained framework, we wish to guarantee that the constraint is feasible with probability greater than or equal to α:

$$P(3X_1 + X_2 \leq b) \geq \alpha$$

Since $3X_1 + X_2$ is nonstochastic we need only find a value of $b(b^*)$ such that by holding $3X_1 + X_2 \leq b^*$ the probability of the constraint holding for any b is at least α. Equivalently, b^* can be set as in equation (97) at $b^* = b - Z_\alpha \sigma = 30 - 10Z_\alpha$ and if $Z_\alpha = 2$ then b^* equals 10. The solutions for alternative values of Z_α developed from the standard normal table are shown in Table A-8.

Note that as one becomes more conservative in terms of insuring that the constraint is satisfied, then Z_α is increased, which leads to a RHS decrease. In turn this leads to an objective function decrease, as well as changes in the solution values.

TECHNICAL COEFFICIENT UNCERTAINTY

Wicks and Guise Approach

To illustrate the approach taken by Wicks and Guise to a decision problem in which there is uncertainty in the technical coefficients, suppose we have the following linear program:

$$\begin{array}{rl} \text{Maximize} & 4X^1 + 6X^2 \\ \text{s.t.} & a^1X^1 + a^2X^2 \leq 30 \\ & X^1 + 2X^2 \leq 18 \\ & X^1, X^2 \geq 0 \end{array}$$

where a^1 and a^2 are uncertain; but we have a set of observations on them in Table A-9. The formulation (following equations (116-122)) becomes:

Table A-8. Optimal Solutions to the Chanced Constrained Example

α	Z_α	b_1	Objective Function	X_1	X_2	U_1[a]	U_2[a]
0.5	0	30.0	68.00	8.00	6.00	0.4	2.8
0.9	1.28	17.20	62.88	2.88	8.56	0.4	2.8
0.95	1.65	13.46	61.38	1.38	9.31	0.4	2.8
0.99	2.33	6.70	4.2	0	6.7	6.0	0

[a]U_1 and U_2 are shadow prices.

Table A-9. Data to Illustrate Wicks and Guise Model

Observation Number	Observations a^1	a^2	Deviations from Means d^{k1}	d^{k2}
1	3	4	0.5	-1
2	2	1	-0.5	-2
3	1	3	-1.5	0
4	2.5	5	0	2
5	4	2	1.5	-1
Mean	2.5	3		

$$
\begin{array}{ll}
\text{Maximize} & 4X^1 + 6X^2 \\
\text{s.t.} & 2.5X^1 + 3X^2 + \Psi\sigma \leq 30 \\
& .5X^1 + 1X^2 + d^1 \geq 0 \\
& -.5X^1 - 2X^2 + d^2 \geq 0 \\
& -1.5X^1 + d^3 \geq 0 \\
& 2X^2 + d^4 \geq 0 \\
& 1.5X^1 - X^2 + d^5 \geq 0 \\
& 2d^1 + 2d^2 + 2d^3 + 2d^4 + 2d^5 - D = 0 \\
& + 0.5604\,D - \sigma = 0 \\
& X^1 + 2X^2 \leq 18 \\
& X^j, d^k, D, \sigma \geq 0 \quad \text{for all } j \text{ and } k
\end{array}
$$

The solutions to the problem for various values of Ψ are shown in Table A-10. Notice in these solutions the approximated standard error (σ) of the program with risky a^{ij}'s decreases from a value of 23.54 to a value of 10.67 as the risk aversion parameter Ψ is increased. Simultaneously, the values of the X variables change and the objective function decreases.

MULTIPLE SOURCES and TIMING OF RISK

Discrete Stochastic Programming

In setting up an example of discrete stochastic programming, suppose that, due to plowing practices, pesticides and inputs (seed, etc.), land use must be allocated to crops before the growing season and two crops can be grown on a total of 3100 acres. Stage A activities involve land allocation to two crops subject to the total land constraint. At this time several of the parameters involved with planting and harvesting are unknown. Crop prices are known to be \$2.50 for Crop 1 and \$7 for Crop 2. Further, assume that there are two possible planting periods for each crop and that the input requirements and time availabilities for planting vary with the state of nature. The two states are shown in Table A-11.

Once an acre has been planted, fall harvest resource use and crop sale depend on yields. Yields, in turn, depend on planting date and type of weather between planting and harvest. Harvest time uncertainties form the

Table A-10. Solutions to the Wicks and Guise Model

Ψ	Objective Function	X^1	X^2	d^1	d^2	d^3	d^4	d^5	D	σ	U^{1a}	U^{2a}
0	57.00	3.0	7.50	13.50	0	4.50	0	3	42.00	23.54	1	1.50
0.25	48.02	3.69	5.54	9.24	0	5.54	0	0	29.55	16.56	1.60	0
0.50	42.20	3.25	4.86	8.11	0	4.87	0	0	25.97	14.55	1.40	0
0.75	37.64	2.89	4.34	7.23	0	4.34	0	0	23.16	12.98	1.25	0
1.0	33.96	2.61	3.92	6.53	0	3.92	0	0	20.90	11.71	1.13	0
1.25	30.94	2.38	3.57	5.95	0	3.57	0	0	19.04	10.67	1.03	0

[a]U^1 and U^2 are shadow prices.

Stage C uncertainties. Suppose there are two states of nature for harvesting and their probabilities are independent of the planting outcome (Table A-11).

Harvesting may be performed in two periods in which time available is uncertain; the same weather patterns which influence yield are assumed to affect harvest losses and time availabilites. The harvest data are shown in Table A-11.

The LP formulation of this discrete stochastic problem is given in Table A-12. This formulation merits explanation. Initially in Stage A crop choice is made between Crop 1 (C^1) and Crop 2 (C^2). These choices then are linked to the planting stage (B) and require that the acreage planted to each crop equal the Stage A committed regardless of the Stage B outcome. However, planting date flexibility within each state of Stage B is governed by the availability of planting time (note the separate constraints for each state). Thus, variables P^{ij} are defined under each state giving the amount of crop i planted in period j. The B stage then is linked to the Stage C activities by yield available. In Stage C the acreage harvested will equal that planted in A and the exact yield harvested will be a function of harvest weather, planting weather and planting patterns. Thus, four sets of harvesting activities (m^{ij}) are present representing the harvest of crop i in period j. We also introduce sales activities (s^i) for each crop; and an overall income activity. The solution to this model is shown in Table A-13.

This solution represents the solution of four different problems. It gives best "first move" acreage allocation X^A's which is 115.15 acres to crop 1 and 484.85 acres to crop 2 then plans (X^B's, X^C's) contingent upon resolution of the uncertainties.

As an alternative to this formulation, one might argue that 4 individual LP's should be solved. For example, certainty that event C^{11} will occur results in the deterministic LP shown in Table A-14. Solving the LP's for

Table A-11. Data for Discrete Programming Example

CROP PLANTING DATA		
Spring State of Nature	B^1	B^2
Probability	0.6	0.4
Planting Period 1		
Hours Available	100	130
Planting Speed (acres/hr)		
Crop 1	0.33	0.33
Crop 2	0.4	0.42
Planting Period 2		
Hours Available	140	160
Planting Speed (acres/hr)		
Crop 1	0.5	0.25
Crop 2	0.4	0.33
Planting Cost ($/acre)		
Crop 1	72	81
Crop 2	46	45
CROP YIELD DATA		
Fall State of Nature	C^1	C^2
Probability	0.2	0.8
Yield for Planting Period 1		
Crop 1	200	130
Crop 2	42	62
Yield for Planting Period 2		
Crop 1	130	140
Crop 2	40	42
HARVEST DATA		
Fall State of Nature	C^1	C^2
Harvest Period 1		
Hours Available	300	400
Time Required (hrs/100 units)		
Crop 1	0.2	0.22
Crop 2	0.4	0.40
Harvest Loss (percent)		
Crop 1	0.0	3.0
Crop 2	2.0	2.0
Harvest Period 2		
Hours Available	285	450
Time Required (hrs/100 units)		
Crop 1	0.25	0.24
Crop 2	0.45	0.40
Harvest Loss (percent)		
Crop 1	5.0	4.0
Crop 2	4.0	2.0

Table A-12. Formulation of Farm Planning Problem with Joint Planting and Yield Risk

	Stage A		Stage B														
			B^1				B^2				B^1C^1						
	X^1	X^2	P^{11}	P^{12}	P^{21}	P^{22}	P^{11}	P^{12}	P^{21}	P^{22}	H^{11}	H^{12}	H^{21}	H^{22}	S^1	S^2	V^{11}
A																	
Obj. Function																	.12
Land	1	1															
A→B																	
Link Crop 1	-1		1	1													
Link Crop 2		-1			1	1											
Link Crop 1	-1						1	1									
Link Crop 2		-1							1	1							
B^1																	
Plant Per. 1			.33		.4												
Plant Per. 2				.5		.4											
B^2																	
Plant Per. 1							.33		.42								
Plant Per. 2								.25		.33							
$B^1 \rightarrow C^1$																	
Link Crop 1			-200	-130							100	100					
Link Crop 2					-42	-40							100	100			
$B^1 \rightarrow C^2$																	
Link Crop 1			-130	-140													
Link Crop 2					-42	-62											
$B^2 \rightarrow C^1$																	
Link Crop 1							-200	-130									
Link Crop 2									-42	-40							
$B^2 \rightarrow C^2$																	
Link Crop 1							-130	-140									
Link Crop 2									-42	-62							
$C^1 \rightarrow B^1$																	
Harvest Per. 1											.2		.4				
Harvest Per. 2												.25		.45			
Crop 1											-100	-95			1		
Crop 2													-98	-96		1	
Income			72	72	46	46									-2.5	-7	1
$C^1 \rightarrow B^2$																	
Harvest Per. 1																	
Harvest Per. 2																	
Crop 1																	
Crop 2																	
Income							81	81	45	45							
$C^2 \rightarrow B^1$																	
Harvest Per. 1																	
Harvest Per. 2																	
Crop 1																	
Crop 2																	
Income			72	72	46	46											
$C^2 \rightarrow B^2$																	
Harvest Per. 1																	
Harvest Per. 2																	
Crop 1																	
Crop 2																	
Income							81	81	45	45							

Table A-12. Formulation of Farm Planning Problem with Joint Planting and Yield Risk (cont.)

Stage C																					Rig Har Sid
B^1C^2							B^2C^1							B^2C^2							
H^{11}	H^{12}	H^{21}	H^{22}	S^1	S^2	V^{12}	H^{11}	H^{12}	H^{21}	H^{22}	S^1	S^2	V^{21}	H^{11}	H^{12}	H^{21}	H^{22}	S^1	S^2	V^{22}	
						.48							.08							.32	M
																					≤ 6
																					≤
														8							≤
																					≤
																					≤
																					≤ 1
																					≤ 1
																					≤ 1
																					≤ 1
																					≤
																					≤
100	100																				≤
		100	100																		≤
							100	100													≤
									100	100											≤
														100	100						≤
																100	100				≤
																					≤
																					≤
																					≤
																					≤
																					≤
																					≤
.2		.4																			≤
	.25		.45																		≤
-100	-95			1																	≤
		-98	-96		1																≤
				-2.5	-7	1															≤
							.22		.4												≤
								.24		.4											≤
							-97	-96			1										≤
									-98	-98		1									≤
											-2.5	-7	1								≤
														.22		.4					≤
															.24		.4				≤
														-97	-96			1			≤
																-98	-98		1		≤
																		-2.5	-7	1	≤

Table A-13. Optimal Solution to the Discrete Stochastic Farm Planning Model

Stage	State	Label	Solution Value
A		X^1	115.15
		X^2	484.85
B	B^1	P^{11}	115.15
		P^{12}	0.0
		P^{21}	134.85
		P^{22}	350.00
	B^2	P^{11}	115.15
		P^{12}	0.0
		P^{21}	0.0
		P^{22}	484.85
C	B^1C^1	H^{11}	230.30
		H^{12}	0.0
		H^{21}	196.64
		H^{22}	0.0
		S^1	23030.30
		S^2	19270.36
		V^{11}	161874.40
	B^1C^2	H^{11}	149.70
		H^{12}	0.0
		H^{21}	237.64
		H^{22}	0.0
		S^1	23339.40
		S^2	14520.61
		V^{12}	193993.30
	B^2C^1	H^{11}	230.30
		H^{12}	0.0
		H^{21}	0.0
		H^{22}	193.94
		S^1	19270.36
		S^2	26816.36
		V^{21}	158297.00
	B^2C^2	H^{11}	149.70
		H^{12}	0.0
		H^{21}	0.0
		H^{22}	300.61
		S^1	19006.06
		S^2	29459.39
		V^{22}	211371.80

Table A-14. LP Formulation for One State of Nature

		Allocate Land		Plant				Harvest				Sell		
		X^1	X^2	P^{11}	P^{12}	P^{21}	P^{22}	H^{11}	H^{12}	H^{21}	H^{22}	S^1	S^2	RHS
Maximize				-72	-72	-46	-46					2.5	7	
Land		1	1											≤ 600
C1		-1		1	1									≤ 0
C2			-1			1	1							≤ 0
Plant.	P^1			.33		.4								≤ 100
Time	P^2				.5		.4							≤ 140
Yield	C^1			-200	-130			100	100					≤ 0
	C^2					-42	-40			100	100			≤ 0
Sal-	C^1							-100	-95			1		≤ 0
able	C^2									-98	-96		1	≤ 0
Harv.	P^1							.2		.4				≤ 300
Time	P^2								.25		.45			≤ 285

each of the 4 states of nature leads to the solutions shown in Table A-15 in terms of overall acreage. Note that these plans are different individually and on average from the earlier plan, as is the average over the plans. This points toward the need for the adaptive formulation implicit in the descrete stochastic model above.

Applying the MOTAD formulation to the discrete stochastic example requires the objective function be changed to:

$$\text{Maximize} \quad V - \Psi \sigma$$

and requires the following additional constraints:

$$0.12\, V^{11} + 0.48\, V^{12} + 0.08\, V^{21} + 0.32\, V^{22} - \bar{V} = 0$$

$$V^{11} - \bar{V} + d^{1-} \geq 0$$

$$V^{12} - \bar{V} + d^{2-} \geq 0$$

$$V^{21} - \bar{V} + d^{3-} \geq 0$$

$$V^{22} - \bar{V} + d^{4-} \geq 0$$

$$0.5\, MAD - 0.12\, d^{1-} - 0.48\, d^{2-} - 0.82\, d^{3-} - 0.32\, d^{4-} = 0$$
$$1.2596\, MAD - \sigma = 0$$

where the coefficient for MAD in the last constraint is calculated assuming n=100.

The resultant solutions to this model for alternative values of Ψ are shown in Table A-16.

Note that, as before, the expected income ($\bar{V}$) and risk measure (σ) fall as risk aversion increases. In this model the plan is altered so as to reduce the variation among the income variables under the states of nature (V^{ij}). At the highest risk aversion ($\Psi = 2.5$) the model has adjusted the plan so all the income levels are equal, reducing the risk to zero.

Table A-15. Optimal Acreage Allocation Under Four States of Nature

Final State of Nature	Probability	X^1	X^2
C^{11}	0.12	515.15	84.85
C^{12}	0.48	250.00	350.00
C^{21}	0.08	495.65	105.35
C^{22}	0.32	0.00	600.00
Weighted Average		221.47	378.53

Table A-16. MOTAD Version of the Discrete Stochastic Programming Example

Ψ	X^1	X^2	V^{11}	V^{12}	V^{21}	V^{22}	OBJ	$\bar{V}$	σ
0.0	115.15	484.85	161874	193993	159297	211371	192844	192924	16326
0.5	250.00	350.00	18694	194112	183190	191674	190046	191597	3102
0.1	263.00	337.00	189753	190442	185617	189753	188920	189753	833
2.5	274.00	326.00	188188	188188	188188	188188	188188	188188	0

APPENDIX B

RECENT APPLICATIONS OF RISK PROGRAMMING MODELS IN AGRICULTURAL ECONOMICS

In this appendix, we provide a bibliography of recent applications of risk programming models in agricultural economics. Other less recent articles which have, in our judgment, been important contributions to the field are also included, as are some reviews of the literature. For the most part, the items in the bibliography are from the *American Journal of Agricultural Economics*, the various regional agricultural economics journals and the agricultural economics journals in England, Canada and Australia. However, a limited number of applications in other journals with which we are familiar are also included, as are selected research reports. We have not attempted to compile an exhaustive list of applications from experiment stations, the International Centers or those included in M.S. and Ph.D. theses from Agricultural Economics Departments. It would have been difficult to compile an exhaustive list of these kinds of publications and we believe that many of them are probably referenced in journal publications.

We also have made no attempt to provide a complete annotation for the papers listed. We have, however, placed them in a number of categories, depending on the type of risk being analyzed (e.g., whether the uncertainty is in the objective function, the technical coefficients, the right-hand side or some combination of the three). The citations are listed by technique, year and author with the earlier work appearing first. The particular subject matter area or subarea of the application is listed as well.

Table B-1. Classification Code References in Subsequent Tables

- AREA - GENERAL AREAS OF INQUIRY

PRODUCTION	RESOURCES
FINANCE	MIXTURE
MARKETING	INT TRADE
POLICY	

- SUBAREA - SUBJECT MATTER OF APPLICATION

CROPS -- CROPPING DECISIONS
LIVESTOCK -- LIVESTOCK DECISIONS
FARM -- WHOLE FARM DECISIONS
NON FARM -- NON FARM SETTING
POLLUTION -- POLLUTION CONCERNS
POLICY -- POLICY CONCERNS
FIN STRUC -- FINANCIAL STRUCTURE OF THE FIRM
IRRIG -- IRRIGATION SYSTEMS
LIT REV -- LITERATURE REVIEW
AG SECTOR -- PERTAINING TO SECTORAL OR GOVERNMENTAL DECISIONS
TENANCY -- LAND TENANACY DECISIONS
SOIL CONS -- SOIL CONSERVATION
FORESTRY -- FORESTRY DECISION MAKING
INPUT USE -- DEMAND FOR AGRICULTURAL INPUTS

- METHOD - TYPES OF RISK METHOD EMPLOYED

MOTAD -- MOTAD
EV -- EXPECTED VALUE-VARIANCE
FOC LOSS -- FOCUS LOSS
DEMP -- DIRECT EXPECTED UTILITY MAXIMIZATION
MN GINI -- MEAN GINI
TRGT INC -- FORM OF A TARGET INCOME MODEL
CHAN CON -- CHANCE CONSTRAINTS
JNT CNCON -- JOINT CHANCE CONSTRAINTS
PARIS SQP -- SYMMETRIC QUADRATIC PROGRAMMING
WICK GUI -- WICKS - GUISE AIJ UNCERTAINTY METHOD

- METHOD - TYPE OF RISK METHOD EMPLOYED (cont.)

MERRILL -- MERRILL AIJ UNCERTAINTY METHOD
STOCH DP -- MARKOVIAN BASED STOCHASTIC DYNAMIC PROGRAMMING
DSP -- DISCRETE STOCHASTIC PROGRAMMING, ALSO STOCHASTIC PROGRAMMING WITH RECOURSE
EUMGF -- EXPECTED UTILITY MOMENT GENERATING FUNCTION
SCENARIO -- COMPARATIVE ANALYSIS USING SCENARIOS MOTIVATED BY UNCERTAINTY
SNGL INDX -- SINGLE INDEX PORTFOLIO METHOD
EV, CHCON -- EV AND CHANCE CONSTRAINED SIMULTANEOUSLY
EV MULTI -- MULTI PERIOD EV MODELING
STOCH CON -- STOCHASTIC OPTIMAL CONTROL
OTHER -- MISCELLANEOUS OTHER METHODS
GAMES -- GAME THEORY
GREMP -- GENERALIZED RISK EFFICIENT MONTE CARLO PROGRAMMING
COMPAR -- COMPARATIVE ANALYSIS OF METHODS
TG MOTAD -- TARGET MOTAD
DSP WG -- DISCRETE STOCHASTIC AND WICKS AND GUISE
FL,MOTAD -- FOCUS LOSS AND MOTAD

- RISK TYPE - TYPE OF RISK PRESENT IN MODEL

OBJ -- OBJECTIVE FUNCTION RISK
AIJ -- TECHNICAL COEFFICIENT RISK
RHS -- RIGHT-HAND SIDE RISK
MIX -- MULTIPLE TYPES OF RISK

- OBJECT -- OBJECTIVE OF STUDY

ANAL -- MODEL USED PRIMARILY FOR ANALYTICAL PURPOSES
EST RAP -- MODEL USED PRIMARILY FOR RISK AVERSION COEFFICIENT ESTIMATION
SENSIT -- MODEL USED PRIMARILY FOR DECISION SENSITIVITY
NORM -- MODEL USED PRIMARILY FOR GENERATING SOLUTIONS

Table B-2. Reviews of Uncertainty Model Applications

Authors	Area	Subarea	Risk Type	Method	Object
Boussard, J-M.(1967)		LIT REV			
Anderson, J.R.(1979)		LIT REV			
Anderson, J.R., Dillon, J.L. and Hardaker, J.B.(1977)		LIT REV			
Barry, P.J., ed.(1984)		LIT REV			
McCarl, B.A.(1984)		LIT REV			
Boisvert, R.N.(1985)		LIT REV			
Kennedy, J.O.S.(1981)		LIT REV		STOCH DP	
Kennedy, J.O.S.(1986)		LIT REV		STOCH DP	
Hansotia, B.J.(1980)		LIT REV		DSP	
Apland, J.D. and Kaiser, H.(1984)		LIT REV		DSP	
Dillon, J.L.(1963)		LIT REV		GAMES	

Note: See Table B-1 for key to classification codes.

Table B-3. Applications of Objective Function Uncertainty

Authors	Area	Subarea	Risk Type	Method	Object
Brink, L. and McCarl, B.A.(1978)	PRODUCTION	CROPS	OBJ	MOTAD	EST RAP
Schurle, B.W., Erven, B. L.(1979)	PRODUCTION	CROPS	OBJ	MOTAD	SENSIT
Baker, T.G. and McCarl, B.A.(1982)	PRODUCTION	CROPS	OBJ	MOTAD	SENSIT
Brink, L. and McCarl, B.A.(1979)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Mapp, H.P., Hardin, M.L., Walker, O.L., Persand, T.(1979)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Schurle, B.W., Erven, B. L.(1979)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Apland, J.D., Barnes, R.N. and Justus, F.(1984)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
El-Nazer, T. and McCarl, B.A.(1986)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Teague, P.W. and Lee, J.G.(1988)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Weimar, M.R. and A. Hallam(1988)	PRODUCTION	CROPS	OBJ	MOTAD	NORM
Hanf, C.H. and Mueller, R.(1979)	PRODUCTION	LIVESTOCK	OBJ	MOTAD	SENSIT
Whitson, R.E., Barry, P.J., Lacewell, R.D.(1976)	PRODUCTION	LIVESTOCK	OBJ	MOTAD	NORM
Shumway, C.R., Gebremeskal, T.(1978)	PRODUCTION	LIVESTOCK	OBJ	MOTAD	NORM
Kaiser, E. and Boehlje, M.D.(1980)	PRODUCTION	LIVESTOCK	OBJ	MOTAD	NORM
Angirasa, A.K, Shumway, C.R., Nelson, T.C. and Cartwright, T.T.(1981)	PRODUCTION	LIVESTOCK	OBJ	MOTAD	NORM

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Norton, G.W., Easter, K.W., Roe, T.L.(1980)	PRODUCTION	FARM	OBJ	MOTAD	NORM
Held, L.J. and Zink, R.J.(1982)	PRODUCTION	FARM	OBJ	MOTAD	NORM
Apland, J.D., McCarl, B.A. and Miller, W.L.(1980)	PRODUCTION	IRRIG	OBJ	MOTAD	NORM
Hazell, P.B.R., Norton, R.D., Parthasarthy, M. and Pomereda, C.(1983)	PRODUCTION	AG SECTOR	OBJ	MOTAD	NORM
Brandao, E., McCarl, B.A. and Schuh, G.E.(1984)	PRODUCTION	TENANCY	OBJ	MOTAD	NORM
Mills, W.L.,Jr., Hoover, W.L.(1982)	PRODUCTION	FORESTRY	OBJ	MOTAD	NORM
Klinefelter, D.A.(1979)	MARKETING	CROPS	OBJ	MOTAD	NORM
Gembremeskel, T. and Shumway, C.R.(1979)	MARKETING	LIVESTOCK	OBJ	MOTAD	NORM
Nieuwoudt, W.L., Bullock, J.B., Mathia, G.A.(1976)	POLICY	CROPS	OBJ	MOTAD	NORM
Pomereda, C., Samayoa, O.(1979)	POLICY	AG SECTOR	OBJ	MOTAD	NORM
Hazell, P.B.R. and Pomereda, C.(1981)	POLICY	AG SECTOR	OBJ	MOTAD	NORM
Simmons, R.L., Pomereda, C.(1975)	INT TRADE	AG SECTOR	OBJ	MOTAD	NORM
Jabara, C.L. and Thompson, R.L.(1980)	INT TRADE	AG SECTOR	OBJ	MOTAD	NORM
Johnson, S.R.(1967)	PRODUCTION	CROPS	OBJ	EV	ANAL
Wolgin, J.M.(1975)	PRODUCTION	CROPS	OBJ	EV	ANAL
Lin, W., Carman, H.F., Moore, C.V., Dean, G.W.(1974)	PRODUCTION	CROPS	OBJ	EV	SENSIT

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Lin, W., Dean, G.W., Moore, C.V.(1974)	PRODUCTION	CROPS	OBJ	EV	SENSIT
Musser, W.N., McCarl, B.A., Smith, G.S.(1986)	PRODUCTION	CROPS	OBJ	EV	SENSIT
McSweeny, W.T., D.E. Kenyon, and R.A. Kramer(1987)	PRODUCTION	CROPS	OBJ	EV	SENSIT
Freund, R.J.(1956)	PRODUCTION	CROPS	OBJ	EV	NORM
Heady, E.O. and Candler, W.(1958)	PRODUCTION	CROPS	OBJ	EV	NORM
McFarquhar, A.M.M.(1961)	PRODUCTION	CROPS	OBJ	EV	NORM
Camm, B.M.(1962)	PRODUCTION	CROPS	OBJ	EV	NORM
Stovall, J.(1966)	PRODUCTION	CROPS	OBJ	EV	NORM
Thomas, W., Blakeslee, L., Rogers, L., Whittlesey, N.(1972)	PRODUCTION	CROPS	OBJ	EV	NORM
Weins, T.B.(1976)	PRODUCTION	CROPS	OBJ	EV	NORM
Musser, W.N., Stamoulis, K.G.(1981)	PRODUCTION	CROPS	OBJ	EV	NORM
Dillon, C.R., Mjelde, J.W., McCarl, B.A.(1989)	PRODUCTION	CROPS	OBJ	EV	NORM
Woolery, B.A. and R.M. Adams(1979)	PRODUCTION	LIVESTOCK	OBJ	EV	NORM
Musser, W.N., Shurley, W.D., Williams, F.W.(1980)	PRODUCTION	LIVESTOCK	OBJ	EV	NORM
Adams, R.M., Menkhaus, D.J. and Woolery, B.A.(1980)	PRODUCTION	FARM	OBJ	EV	SENSIT
Connor, J.R., Freund, R.J. and Godwin, M.R.(1972)	PRODUCTION	IRRIG	OBJ	EV	NORM

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Talpaz, H. and J.W. Mjelde(1988)	PRODUCTION	IRRIG	OBJ	EV	NORM
House, R.(1983)	PRODUCTION	AG SECTOR	OBJ	EV	EST RAP
Lee, J., Brown, D.J., Lovejoy, S.(1985)	PRODUCTION	SOIL CONS	OBJ	EV	NORM
McCamley, F.P., Kliebenstein, J.B.(1985)	PRODUCTION	INPUT USE	OBJ	EV	NORM
Robison, L.J., Brake, J.R.(1979)	FINANCE	FIN STRUC	OBJ	EV	ANAL
Featherstone, A.M., Moss, C.B., Baker, T.G. and Preckel, P.V.(1988)	FINANCE	FIN STRUC	OBJ	EV	ANAL
Robison, L.J., Barry, P.J.(1977)	FINANCE	FIN STRUC	OBJ	EV	NORM
Robison, L.J., Barry, P.J.(1980)	FINANCE	FIN STRUC	OBJ	EV	NORM
Barry, P.J., Baker, C.B. and Sanint, L.R.(1981)	FINANCE	FIN STRUC	OBJ	EV	NORM
Tauer, L.W., Boehlje, M.(1981)	FINANCE	FIN STRUC	OBJ	EV	NORM
Dixon, B.L. and Barry, P.J.(1983)	FINANCE	FIN STRUC	OBJ	EV	NORM
Young, R.P. and Barry, P.J.(1987)	FINANCE	FIN STRUC	OBJ	EV	NORM
Ward, R.W., Fletcher, L.B.(1971)	MARKETING	CROPS	OBJ	EV	ANAL
Peck, A.E.(1975)	MARKETING	CROPS	OBJ	EV	ANAL
Miller, S.(1986)	MARKETING	CROPS	OBJ	EV	ANAL
Heifner, R.G.(1966)	MARKETING	CROPS	OBJ	EV	NORM

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Eddleman, B.R. and Moya-Rodriguez, J.E.(1979)	MARKETING	CROPS	OBJ	EV	NORM
Berck, P.(1981)	MARKETING	CROPS	OBJ	EV	NORM
Alexander, V.J., Musser, W.N. and Mason, G.(1986)	MARKETING	CROPS	OBJ	EV	NORM
Johnson, D.A. and Boehlje, M.D.(1983)	MIXTURE	CROPS	OBJ	EV	NORM
Boussard, J-M. and Petit, M.(1967)	PRODUCTION	CROPS	OBJ	FOC LOSS	NORM
Boussard, J-M.(1971)	PRODUCTION	CROPS	OBJ	FOC LOSS	NORM
Musser, W.N., Ohannesian, J., Benson, F.J.(1981)	PRODUCTION	CROPS	OBJ	TRGT INC	NORM
Hauser, R.J. and Anderson, D.K.(1987)	MARKETING	CROPS	OBJ	TRGT INC	NORM
Hauser, R.J. and Eales, J.S.(1987)	MARKETING	CROPS	OBJ	TRGT INC	NORM
Atwood, J.A., Watts, M.J. and Helmers, G.A.(1988)	FINANCE	FIN STRUC	OBJ	CHAN CON	NORM
Collender, R.N. and Zilberman, D.(1985)	PRODUCTION	CROPS	OBJ	EUMGF	ANAL
Babcock, B.A., Chalfant, J.A., Collender, R.N.(1987)	PRODUCTION	CROPS	OBJ	EUMGF	ANAL
Moffitt, L.J., Burrows, T.M., Baritelle, J.L., Sevacherian, V.(1984)	PRODUCTION	CROPS	OBJ	EUMGF	NORM
Collender, R.N. and Chalfant, J.A.(1986)	PRODUCTION	CROPS	OBJ	EUMGF	NORM
Johnson, S.R., Tefertiller, K.R. and Moore, D.(1967)	PRODUCTION	CROPS	OBJ	SCENARIO	NORM

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Pope, A., Shumway, R.E.(1984)	PRODUCTION	LIVESTOCK	OBJ	SCENARIO	NORM
Chien, Y.I. and Bradford, G.L.(1976)	PRODUCTION	FARM	OBJ	SCENARIO	NORM
Barry, P.J. and Baker, C.B.(1971)	FINANCE	FIN STRUC	OBJ	SCENARIO	NORM
Adams, R.M., Hamilton, S.A. and McCarl, B.A.(1986)	RESOURCES	AG SECTOR	OBJ	SCENARIO	NORM
Collins, R.A. and Barry, P.J.(1986)	PRODUCTION	CROPS	OBJ	SNGL INDX	NORM
Perry, G.M.(1986)	PRODUCTION	CROPS	OBJ	EV MULTI	NORM
Barry, P.J. and Willmann, D.R.(1976)	FINANCE	FIN STRUC	OBJ	EV MULTI	NORM
Sanint, L. R., Barry, P.J.(1983)	FINANCE	FIN STRUC	OBJ	EV MULTI	NORM
Kawaguchi, T., Maruyama, Y.(1972)	PRODUCTION	CROPS	OBJ	GAMES	NORM
Low, A.R.C.(1974)	PRODUCTION	CROPS	OBJ	GAMES	NORM
Zering, K.D., McCorkle, C.O., Jr. and Moore, C.V.(1987)	PRODUCTION	CROPS	OBJ	GREMP	NORM
King, R.P., Oamek, G.E.(1979)	FINANCE	CROPS	OBJ	GREMP	NORM
King, R.P., Lybecker, D.W.(1983)	MARKETING	CROPS	OBJ	GREMP	NORM
Watts, M.J., Held, L.J., Helmers, G.A.(1984)	PRODUCTION	CROPS	OBJ	COMPAR	SENSIT
Atwood, J.A., Held, L.J., Helmers, G.A. and Watts, M.J.(1986)	PRODUCTION	CROPS	OBJ	COMPAR	SENSIT

Table B-3. Applications of Objective Function Uncertainty (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Reid, D.W. and B.V. Tew(1987)	PRODUCTION	CROPS	OBJ	COMPAR	SENSIT
Wicks, J.A.(1978)	PRODUCTION	CROPS	OBJ	COMPAR	NORM
Helmers, G.A., Held, L., Watts, J. and Atwood, J.,(1984)	PRODUCTION	CROPS	OBJ	TG MOTAD	NORM
Zimet, D.J. and Spreen, T.H.(1986)	PRODUCTION	FARM	OBJ	TG MOTAD	NORM
Curtis, C.E., Pfieffer, G.H., Lutgen, L.L. and Frank, S.D.(1987)	MARKETING	CROPS	OBJ	TG MOTAD	NORM
Frank, S.D., Irwin, S.H., Pfeiffer, G.H., Curtis, C.E.(1989)	MARKETING	CROPS	OBJ	TG MOTAD	NORM
Kennedy, J.O.S., Francisco, E.M.(1974)	PRODUCTION	FARM	OBJ	FL,MOTAD	SENSIT

Note: See Table B-1 for key to classification codes.

Table B-4. Applications of A_{ij} Uncertainty

Authors	Area	Subarea	Risk Type	Method	Object
Wicks, J.A., Guise, J.W.B.(1978)	PRODUCTION	CROPS	AIJ	WICK GUI	NORM
Tice, T.F.(1979)	PRODUCTION	CROPS	MIX	DSP WG	NORM
Rahman, S.A., Bender, F.E.(1971)	PRODUCTION	LIVESTOCK	AIJ	MERRILL	NORM
Chen, J.T.(1973)	PRODUCTION	LIVESTOCK	AIJ	MERRILL	NORM
Segarra, E., Kramer, R.A., Taylor, D.B.(1985)	PRODUCTION	SOIL CONS	AIJ	MERRILL	NORM
Babbar, M.M., Tintner, G. and Heady, E.O.(1955)	PRODUCTION	CROPS	AIJ	SCENARIO	NORM
Townsley, R.(1968)	PRODUCTION	LIVESTOCK	AIJ	OTHER	NORM

Note: See Table B-1 for key to classification codes.

Table B-5. Applications of RHS Uncertainty

Authors	Area	Subarea	Risk Type	Method	Object
Boisvert, R.N. and Jensen, H.R.(1973)	PRODUCTION	CROPS	RHS	CHAN CON	NORM
Boisvert, R.N.(1976)	PRODUCTION	CROPS	RHS	CHAN CON	NORM
Danok, A.B., McCarl, B.A. and White, T.K.(1980)	PRODUCTION	CROPS	RHS	CHAN CON	NORM
Loucks, D.(1975)	RESOURCES	IRRIG	RHS	CHAN CON	NORM
Maji, C., Heady, E.(1978)	RESOURCES	IRRIG	RHS	CHAN CON	NORM
Kieth, J.E., Martinez, G.A., Snyder, D.L., Glover, T.F.(1989)	RESOURCES	IRRIG	RHS	CHAN CON	NORM
Lambert, D.K.(1984)	MARKETING	CROPS	RHS	DSP	NORM
Candler, W.(1956)	PRODUCTION	CROPS	RHS	SCENARIO	NORM

Note: See Table B-1 for key to classification codes.

Table B-6. Applications of Multiple Uncertainty Models

Authors	Area	Subarea	Risk Type	Method	Object
Kramer, R.A., McSweeny, W.T., Stavros, R.W.(1983)	PRODUCTION	SOIL CONS	MIX	PARIS SQP	NORM
McSweeney, W.T. and R.A. Kramer(1986)	PRODUCTION	SOIL CONS	MIX	PARIS SQP	NORM
Taylor, C.R.(1983)	PRODUCTION	CROPS	MIX	STOCH DP	ANAL
Taylor, C.R.(1986)	PRODUCTION	CROPS	MIX	STOCH DP	ANAL
Burt, O.R. and Johnson, R.D.(1967)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Stauber, M.S., Burt, O.R., Linse, F.(1975)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Klemme, R.M.(1980)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
O'Brien, D.(1981)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Taylor, C.R., Burt, O.R.(1984)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Zacharias, T.P. and Grube, A.H.(1986)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Zacharias, T.P., Liebman, J.S. and Noel, G.R.(1986)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Mjelde, J.W., Sonka, S.T., Dixon, B.L., and Lamb, P.J.(1988)	PRODUCTION	CROPS	MIX	STOCH DP	NORM
Smith, B.J.(1973)	PRODUCTION	LIVESTOCK	MIX	STOCH DP	NORM
Henderson, R.A. and Toft, H.I.(1979)	PRODUCTION	LIVESTOCK	MIX	STOCH DP	NORM
Toft, H.I., O'Hanlon, P.W.(1979)	PRODUCTION	LIVESTOCK	MIX	STOCH DP	NORM
Rodriguez, A. and Taylor, R.G.(1988)	PRODUCTION	LIVESTOCK	MIX	STOCH DP	NORM
Burt, O.R. and Stauber, M.S.(1971)	PRODUCTION	IRRIG	MIX	STOCH DP	NORM

Table B-6. Applications of Multiple Uncertainty Models (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Biere, A.W. and Lee, I.M.(1972)	PRODUCTION	IRRIG	MIX	STOCH DP	NORM
McGuckin, J.T., C. Mapel, R.R. Lansford, and T.W. Sammis(1987)	PRODUCTION	IRRIG	MIX	STOCH DP	NORM
Burt, O.R.(1982)	PRODUCTION	LIT REV	MIX	STOCH DP	
Burt, O.R.(1965)	FINANCE	FIN STRUC	MIX	STOCH DP	NORM
Weersink, A. and S. Stauber(1988)	FINANCE	FIN STRUC	MIX	STOCH DP	NORM
Mjelde, J.W., C.R. Taylor, and G.L. Cramer(1985)	MARKETING	CROPS	MIX	STOCH DP	NORM
Yager, W.A., Greer, R.C., Burt, O.R.(1980)	MARKETING	LIVESTOCK	MIX	STOCH DP	NORM
Burt, O.R., Koo, W.W. and Dudley, N.J.(1980)	POLICY	AG SECTOR	MIX	STOCH DP	NORM
Koo, W.W., Burt,O.R.(1982)	POLICY	AG SECTOR	MIX	STOCH DP	NORM
Yaron, D. and Olian, A.(1973)	RESOURCES	IRRIG	MIX	STOCH DP	NORM
Burt, O.R.(1981)	RESOURCES	SOIL CONS	MIX	STOCH DP	NORM
Kennedy, J.O.S.(1979)	INT TRADE	AG SECTOR	MIX	STOCH DP	NORM
Rae, A.N.(1971)	PRODUCTION	CROPS	MIX	DSP	NORM
Burt, O.R. and Allison, J.R.(1979)	PRODUCTION	CROPS	MIX	DSP	NORM
Apland, J.D., McCarl, B.A. and Baker, T.(1981)	PRODUCTION	CROPS	MIX	DSP	NORM
Garoian, L., Conner, J.R. and Scifres, C.J.(1987)	PRODUCTION	LIVESTOCK	MIX	DSP	NORM
Lambert, D.K.(1989)	PRODUCTION	LIVESTOCK	MIX	DSP	NORM

Table B-6. Applications of Multiple Uncertainty Models (cont.)

Authors	Area	Subarea	Risk Type	Method	Object
Trebeck, D.B., Hardaker, J.B.(1972)	PRODUCTION	FARM	MIX	DSP	NORM
Kaiser, H.M., Apland, J.D.(1989)	PRODUCTION	FARM	MIX	DSP	NORM
Yaron, D. and Horowitz, U.(1972)	PRODUCTION	FIN STRUC	MIX	DSP	NORM
Leatham, D.J., and Baker, T.G.(1988)	PRODUCTION	FIN STRUC	MIX	DSP	NORM
Lambert, D.K., and McCarl, B.A.(1989)	MARKETING	CROPS	MIX	DSP	NORM
Brown, C. and Drynan, R.(1986)	MARKETING	NON FARM	MIX	DSP	NORM
McCarl, B.A. and G.H. Parandvash(1988)	POLICY	IRRIG	MIX	DSP	NORM
Falatoonzadeh, H., Conner, J.R. and Pope, R.D.(1985)	PRODUCTION	CROPS	MIX	EV, CHCON	NORM
Paris, Q., Easter, C.D.(1985)	PRODUCTION	AG SECTOR	MIX	EV, CHCON	NORM
Harris, T.R. and Mapp, H.P. Jr.(1980)	PRODUCTION	IRRIG	MIX	STOCH CON	NORM
Zavaleta, L.R., Lacewell, R.D. and Taylor, C.R.(1980)	PRODUCTION	IRRIG	MIX	STOCH CON	NORM
Dixon, B.L. and Howitt, R.E.(1980)	PRODUCTION	FORESTRY	MIX	STOCH CON	NORM
Tice, T.F.(1979)	PRODUCTION	CROPS	MIX	DSP WG	NORM

Note: See Table B-1 for key to classification codes.

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