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PERFORMANCE OF SHILLER LAG ESTIMATORS: SOME ADDITIONAL EVIDENCE

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In distributed lag models, collinearity among lagged independent variables often leads to imprecise estimates of the parameters when unrestricted ordinary least squares (OLS) is used. A widely used alternative-- the Almon (1965) procedure--has also come under attack (Maddala 1977) because it imposes strong restrictions on the parameters. The tendency for these restrictions to produce severely distorted shapes for the lag distribution has been verified in an extensive Monte Carlo study conducted by Cargill and Meyer (1974). Their study indicated the following (p. 1041): "(e)stimates obtained with a second degree polynomial, whether constrained or unconstrained, yielded very large biases which in many cases were over 50 percent of the true value of the coefficient. In addition, the mean coefficients were often unable to correctly describe the shape of the lagged relation. While increasing the polynomial to a fourth degree yielded a reduction in the size of the biases, they were still very large in magnitude compared to OLS." Misspecification of the lag length and the presence of serial correlation tended to increase these biases further.

A technique, which imposes less severe restrictions and includes the unrestricted OLS and Almon procedures as special cases, has been developed by Shiller (1973). Evidence regarding this procedure, while scanty, is encouraging. One of Shiller's applications involved estimating a lag distribution of known shape by the OLS, Almon and Shiller procedures. The results showed OLS producing a very jagged representation of the true shape. The

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Almon estimates, in general, did a poor job of representing the tails of the true distribution. The Shiller estimates, by contrast, produced a smooth shaped distribution that tracked the true distribution very well. More recently, Fomby (1979) applied the Shiller methodology to the Almon data and to data used by Griliches, et al. (1962). For the Almon data, when a polynomial of degree two is chosen, both the Almon and Shiller procedures produced estimates with a lower mean squared error (MSE)[1/] than the OLS estimates. However, the sum of the lag coefficients for the Shiller procedure had a smaller downward bias than the corresponding sum for the Almon procedure. Thus for the Almon data, the Shiller procedure provides plausible lag shapes with smaller bias in the estimated long-run effect. For the Griliches, et al. data Fomby reports only on results pertaining to a first-degree polynomial restriction. Here the Almon estimator leads to a rejection of the hypothesis of mean squared error superiority at the α = .05 level. The corresponding Shiller estimator produces results which one could claim has a smaller MSE than OLS.

A number of studies designed to determine the economic effectiveness of generic milk advertising has employed the Almon procedure in estimating the relationship between milk sales and advertising expenditures (Thompson, Eiler 1975; Thompson, Eiler, Forker 1976; Thompson 1978). The purpose of this paper is to explore what improvement, if any, can be expected from the use of the less restrictive Shiller procedure in the context of the data and model used in these studies. A test developed by Fomby will be used to determine whether the Almon or Shiller estimates can be considered mean squared superior to the unrestricted OLS estimates. The relatively small sample size (26 observations) used in the tests should yield some evidence regarding the small sample

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1/ The MSE of a parameter estimate is its variance plus bias squared.

properties of these estimators.[2/]

The Shiller methodology is reviewed, and then details regarding the evaluation procedure are presented. The subsequent section discusses the empirical results.

The Shiller Method

The idea underlying the Shiller method is that the researcher generally has some a priori notions about the likely appearance of the lag shape and that these notions should be incorporated explicitly into the analysis to increase the efficiency of the estimates. One such generally held belief is that the lag distribution should trace a smooth curve. The Shiller approach provides an extremely flexible means of incorporating this assumption. Specifically, suppose the model is

$$Y_t = \sum_{i=0}^{n} \beta_i X_{t-i} + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{1}$$

where Y_t and X_t are scaler time series at time t. The smoothness restriction can be imposed by requiring

$$\Delta^{d+1} \beta_i = w_i \qquad w_i \sim IN(0, \sigma_w^2) \tag{2}$$

where Δ is the difference operator (e.g., $\Delta \beta_i = \beta_i - \beta_{i-1}$). The d term

[2/] Some early work done by Swamy and Mehta (1969) suggests that the gains on efficiency that one would expect from using the Shiller method are considerable and not much affected by sample size. The question of bias in small samples still remains, however.

is the "degree of smoothness" imposed. Thus, for example, zero degree smoothness implies that first differences in the β_i are small, i.e., $\Delta \beta_i = \beta_i - \beta_{i-1}$ is approximately zero for all i. The smoothness restriction is made stochastic to allow specifying degrees of precision regarding our prior beliefs.

While Shiller used a Bayesian framework in the development of his procedure, Taylor (1974) has shown that equivalent results are achieved by using the more familiar Theil-Goldberger mixed estimation framework. Under this framework, equation (2) is rewritten in matrix notation as

$$0 = R_d\beta + w \tag{2'}$$

where R_d is a (n-d) x (n+1) matrix of restriction coefficients.[3/]

Combining (1) and (2'),

$$\begin{bmatrix} Y \\ 0 \end{bmatrix} = \begin{bmatrix} X \\ R_d \end{bmatrix} \beta + \begin{bmatrix} \varepsilon \\ w \end{bmatrix} \text{ where } E\begin{pmatrix} \varepsilon \\ w \end{pmatrix} \begin{pmatrix} \varepsilon' & w' \end{pmatrix} = \begin{bmatrix} \sigma_\varepsilon^2 I & 0 \\ 0 & \sigma_w^2 I \end{bmatrix} . \tag{3}$$

The best linear unbiased estimator of β for this model (if $k = \sigma_\varepsilon/\sigma_w$ is known a priori) is the Theil-Goldberger (1961) estimator

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3/ The Maddala (1977) discussion of the Shiller procedure contains an incorrect example of the R_2 matrix (p. 383)--the signs of the coefficients should be reversed. In addition, he incorrectly states the row dimension of the R_d matrix as (n-1); it should be (n-d).

$$\hat{\beta}_s = \left[X'X + k^2 R_d' R_d\right]^{-1} \left[X'Y\right] . \tag{4}$$

The best quadratic unbiased estimate of $\sigma_\varepsilon{}^2$ is

$$\hat{\sigma}_\varepsilon{}^2 = \left[\tilde{Y} - \tilde{Y}\hat{\beta}_s\right]' \left[\tilde{Y} - \tilde{X}\hat{\beta}_s\right] \Big/ \left(T - n - 1\right) \tag{5}$$

where $\tilde{Y}' = \left[Y' \quad 0\right]$

and $\tilde{X}' = \left[X' \quad k\, R_d'\right]$.

That the OLS estimator is a special case of the Shiller estimator is apparent from (4) by setting $k = 0$. It can also be shown (Shiller 1973) that

$$\lim_{k \to \infty} \hat{\beta}_s = \hat{\beta}_A . \tag{6}$$

That is, the Shiller and Almon estimates (based on a d-degree polynomial) are equivalent for sufficiently large k.

The Shiller methodology, in addition to subsuming the OLS and Almon estimators as special cases, has the further advantage of being less likely to fail to deal with the multicollinearity problem than the Almon procedure, since a degree of zero or one is probably adequate for the Shiller method, but not the Almon.

The Evaluation Procedure

The test criterion and statistic

Imposing restrictions on the parameters increases the efficiency (reduces the standard errors) of the parameter estimates but, unless correct, the restrictions produce bias. The potential for bias grows (and efficiency gains become larger) as the restrictions become more stringent. The efficiency gains from applying too stringent restrictions can outweigh the bias when multicollinearity is severe. This fact underlies the justification of the Almon procedure since the stringency of the restriction in the Shiller framework grows with k and Almon estimates are obtained by setting $k = \infty$.

The tradeoff between efficiency and bias, implied by the use of estimating procedures such as the Shiller method, can be effectively measured using the mean squared error criterion. Applying this MSE criterion to the Shiller estimator we can form the hypotheses

$$H_N: \quad E\left[(\beta_s - \beta)' \; (\beta_s - \beta)\right] \leq E\left[(\hat{\beta} - \beta)' \; (\hat{\beta} - \beta)\right] \qquad (7a)$$

$$H_A: \quad H_N \text{ not true,} \qquad (7b)$$

where $\hat{\beta}$ is the OLS estimator.[4/] Rejection of H_N implies that the Shiller estimator is <u>not</u> superior to the OLS estimator in a MSE sense. Since the Almon estimator is a special case of the Shiller estimator, the test is general.

[4/] The expression in H_N is the "weak" mean squared error criterion discussed in Wallace (1977) p. 434. A strong mean squared error criterion requires that for β_s to be better in MSE than $\hat{\beta}$, the MSE of every linear combination for β_s must be no larger than the same linear combinations for $\hat{\beta}$. Under H_N, β_s is better in MSE than $\hat{\beta}$ if β_s is closer, on the average, to β in squared Euclidian distance.

To test the null hypothesis (7a), Fomby (1979) developed the following test statistic

$$\gamma = \frac{\left[RSS(\beta_s) - RSS(\hat{\beta})\right]/(n-d)}{RSS(\hat{\beta}) \;/\; (T - 2n - 1)} \tag{8}$$

where $RSS(\beta_s)$ and $RSS(\hat{\beta})$ are, respectively, the residual sum of squares from the Shiller and OLS models. The γ statistic has a noncentral F-distribution with n-d and T-2n-1 degrees of freedom. A table of critical values for testing hypothesis (7a) is available in Wallace and Toro-Vizcarrondo (1969). By comparing the computed value of γ with the corresponding critical values, the appropriateness of the chosen degree of smoothness (the d parameter) as well as how stringently this degree smoothness should be allowed to modify the OLS estimates (the k parameter) can be objectively evaluated.

The model and data

The model used in this study is the milk sales response function developed by Thompson (1978):

$$\ln q_t = \alpha + \sum_{j=1}^{11} \phi_j z_{jt} + \theta \ln I_{t-1} + \delta \ln p^c_{t-1} + \lambda \ln p^m_{t-1} + \sum_{i=0}^{4} \beta_i \ln a_{t-i} + \varepsilon_t \tag{9}$$

where q_t = per capita daily milk sales,

z_j = monthly seasonality dummy variable with December as the base class,

I = real per capita personal income before taxes,

p^c = the real price of cola,

p^m = the real price of fluid milk, and

a = real per capita generic milk advertising expenditures.

Previous analyses by Thompson indicated that a lag length of four in advertising was appropriate for this data.

The data, which pertain to the New York City market, are presented in the appendix along with a more precise definition of the variables. The effective sample period covers May 1975 through June 1977 for 26 observations. For months with zero advertising expenditures an arbitrarily small value of .0001 was used to accomodate the double-log specification.

To implement the Shiller methodology, a value for the "tightness" parameter k must be selected. Shiller suggested that when d = 1, one rule-of-thumb procedure is to set $\hat{\sigma}_w = 8\ s/n$ where s is the sum of the lag coefficients (obtained from OLS regression) and then to compute $k = \hat{\sigma}_\varepsilon / \hat{\sigma}_w$ where $\hat{\sigma}_\varepsilon$ is the standard error of the OLS regression. Although this procedure seemed to have worked well for the experiments performed by Shiller (1973), it has the disadvantage that the k so computed is not invariant to changes in the units of measurement of the variables.

An alternative procedure used by Lindley and Smith (1972) and recommended by Maddala (p. 387) is to compute $\hat{\sigma}_\beta^2 = 1/n \sum_{i=0}^{n} (\hat{\beta}_i - \bar{\beta})^2$ where $\bar{\beta}$ is the mean of the OLS lagged coefficients $\hat{\beta}_i$ and then to set $\hat{k} = \hat{\sigma}_\varepsilon / \hat{\sigma}_\beta$. This procedure is used here. The sensitivity of the MSE test to the selected value of k is then analyzed by using four alternative values for k: $1/2\ \hat{k}$, $\hat{k}$, $2\ \hat{k}$, and $4\ \hat{k}$.

Shiller suggests that first-degree smoothness prior is probably adequate in most applications. To test this proposition and to provide results meaningful for the Almon procedure, zero-degree and second-degree smoothness assumptions are also examined. Thus, three sets of results are presented corresponding to $d = 0$, $d = 1$, and $d = 2$. Within each set, six alternative estimates are generated corresponding to $k = 0$ (the OLS estimates), $\hat{k} = \hat{\sigma}_\varepsilon / \hat{\sigma}_\beta$, $1/2\ \hat{k}$, $2\ \hat{k}$, $4\ \hat{k}$, and $k = 1{,}000{,}000$ (the Almon estimates for a d degree polynomial).

Empirical Results

The empirical results were obtained using the TROLL econometric software package. Initially, the experimental SHILLER LAG routine within TROLL was used. However, this program was found to be highly inefficient in terms of computer time and produced results that were inconsistent with corresponding OLS and Almon estimates. Therefore, the standard TROLL regression package was used on the appropriately augmented data matrices. To verify that this indirect procedure produced correct results, the estimates for the special cases, $k = 0$ and $k = \infty$, were compared to the standard TROLL-produced OLS and Almon estimates, respectively. This exercise not only verified the correctness of the indirect procedure, but also revealed that the standard errors produced by the TROLL Almon command implicitly assume that the imposed restrictions are correct, which is unlikely (see Cargill and Meyer 1974). These estimates should be multiplied by $\hat{\sigma}_\varepsilon / \hat{\sigma}_A$, the standard error of the OLS regression divided by the standard error of the Almon regression, if the Shiller estimates of the standard errors are desired.

The results for the various degrees of smoothness assumptions are presented in Tables 1 - 3. End-point constraints were not imposed in any of the tests. As Maddala notes (p. 386), when $d = 0$ and $k = \infty$, the Shiller procedure produces Lindley-Smith estimates. These are contained in the last column of table one.

Table 1. MILK SALES RESPONSE TO GENERIC ADVERTISING EXPENDITURES
Alternative Lag Estimates Based on Zero-Degree Smoothness Priors
New York City Data, May 1975 to June 1977

Coefficient of[a]	OLS Estimates	Shiller Estimates (d = 0)				Lindley-Smith Estimates
	(k = 0)	1/2 $\hat{k}$	$\hat{k}$[b]	2 $\hat{k}$	4 $\hat{k}$	(k = ∞)
a_t	.00322 (.00475)	.00326 (.00471)	.00340 (.00459)	.00395 (.00432)	.00507 (.00390)	.00643 (.00348)
a_{t-1}	-.00370 (.00549)	-.00307 (.00538)	-.00160 (.00513)	.00137 (.00457)	.00432 (.00394)	.00643 (.00348)
a_{t-2}	.00975 (.00620)	.00958 (.00606)	.00917 (.00569)	.00822 (.00492)	.00719 (.00407)	.00643 (.00348)
a_{t-3}	.02022 (.00560)	.01935 (.00549)	.01734 (.00521)	.01326 (.00462)	.00926 (.00396)	.00643 (.00348)
a_{t-4}	.01063 (.00471)	.01055 (.00468)	.01031 (.00459)	.00950 (.00435)	.00805 (.00393)	.00643 (.00348)
Sum	.04012	.03967	.03862	.03630	.03389	.02572
RSS[c]	.4557	.55100	.7720	1.2120	1.630	1.9130
γ		.889	2.950	7.053	10.952	13.591

F(1/2, 4, 17, .25) = 1.861 F(1/2, 4, 17, .10) = 2.868 F(1/2, 4, 17, .05) = 3.670

a/ Asymptotic standard errors are in parenthesis

b/ $\hat{k} = \hat{\sigma}_\varepsilon / \hat{\sigma}_\beta = .9757$

c/ Actual values multiplied by 1000

Table 2. MILK SALES RESPONSE TO GENERIC ADVERTISING EXPENDITURES
Alternative Lag Estimates Based on First-Degree Smoothness Priors
New York City Data, May 1975 to June 1977[a]

Coefficient of[a]	OLS Estimates (k = 0)	Shiller Estimates (d = 1)				Almon Estimate (k = ∞)
		1/2 $\hat{k}$	$\hat{k}$[b]	2 $\hat{k}$	4 $\hat{k}$	
a_t	.00322 (.00475)	.00306 (.00472)	.00279 (.00468)	.00241 (.00463)	.00218 (.00459)	.00206 (.00459)
a_{t-1}	-.00370 (.00549)	-.00246 (.00537)	-.00021 (.00516)	.00256 (.00469)	.00390 (.00417)	.00430 (.00376)
a_{t-2}	.00975 (.00620)	.00954 (.00599)	.00910 (.00565)	.00824 (.00500)	.00729 (.00419)	.00654 (.00346)
a_{t-3}	.02022 (.00560)	.01887 (.00546)	.01629 (.00520)	.01264 (.00471)	.01015 (.00420)	.00877 (.00382)
a_{t-4}	.01063 (.00471)	.01067 (.00471)	.01075 (.00472)	.01088 (.00471)	.01096 (.00468)	.01101 (.00468)
Sum	.04012	.03968	.03872	.03673	.03448	.03268
RSS[c]	.45570	.6262	.9435	1.370	1.628	1.890
γ	----	2.121	6.066	11.369	14.578	17.836

$F(1/2, 3, 17, .25) = 2.009$ $F(1/2, 3, 17, .10) = 3.220$ $F(1/2, 3, 17, .05) = 4.194$

a/ Asymptotic standard errors are in parenthesis

b/ $\hat{k} = \hat{\sigma}_\varepsilon/\hat{\sigma}_\beta = .9757$

c/ Actual values multiplied by 1000

Table 3. MILK SALES RESPONSE TO GENERIC ADVERTISING EXPENDITURES
Alternative Lag Estimates Based on Second-Degree Smoothness Priors
New York City Data, May 1975 to June 1977[a]

Coefficient of[a]	OLS Estimates (k = 0)	Shiller Estimates (d = 2)				Almon Estimate (k = ∞)
		$1/2\ \hat{k}$	$\hat{k}$[b]	$2\ \hat{k}$	$4\ \hat{k}$	
a_t	.00322 (.00475)	.00297 (.00468)	.00268 (.00461)	.00236 (.00459)	.00218 (.00459)	.00210 (.00460)
a_{t-1}	-.00370 (.00549)	-.00237 (.00542)	.00005 (.00526)	.00324 (.00509)	.00514 (.00499)	.00603 (.00494)
a_{t-2}	.00975 (.00620)	.00939 (.00584)	.00913 (.00559)	.00894 (.00548)	.00886 (.00546)	.00882 (.00546)
a_{t-3}	.02022 (.00560)	.01900 (.00548)	.01659 (.00529)	.01333 (.00509)	.01138 (.00498)	.01045 (.00492)
a_{t-4}	.01063 (.00471)	.01064 (.00470)	.01071 (.00468)	.01083 (.00468)	.01091 (.00469)	.01093 (.00469)
Sum	.04012	.03963	.03916	.03870	.03821	.03833
RSS[c]	.4557	.6270	.9428	1.3630	1.6120	1.730
γ	----	3.195	9.086	16.924	21.568	23.769

F(1/2, 2, 17, .25) = 2.286 F(1/2, 2, 17, .10) = 3.904 F(1/2, 2, 17, .05) = 5.221

a/ Asymptotic standard errors are in parenthesis

b/ $\hat{k} = \hat{\sigma}_\varepsilon / \hat{\sigma}_\beta = .9757$

c/ Actual values multiplied by 1000

The OLS results (which are invariant to the degree of smoothness) indicate that the lagged coefficients for the first two months are not statistically different from zero at the usual levels of significance. This, combined with the fact that the coefficient of a_{t-1} has the "wrong" sign, may lead the researcher to suspect that multicollinearity is preventing OLS from producing precise estimates of the lag parameters. If this is the case, then imposing the restrictions inherent in the Shiller procedure should improve the precision of these estimates by reducing their standard errors. However, the stronger the restriction (the lower the d or the higher the k parameter) the greater the probability of introducing bias as well.

The value of the γ-statistic (the bottom line in the tables) provides an objective means for determining whether the efficiency-bias tradeoff implicit in the use of restricted estimating procedures such as the Shiller method is sufficient to warrant their use. For instance, a γ-value greater than 2.868 for the zero-degree smoothness assumption means that the hypothesis that Shiller estimates are MSE superior to OLS estimates is rejected at the 10 percent level of significance.

In general, the MSE of the Shiller method is not significantly smaller than the MSE of the unrestricted OLS procedure. The only case in which the MSE superiority hypothesis is not rejected (at the $p \leq .05$ level) is under zero-degree and first degree smoothness with $k < \hat{k}$. In these cases OLS parameters are only slightly modified and gains in efficiency are modest. These results suggest that multicollinearity is <u>not</u> responsible for the large relative standard errors in the a_t, a_{t-1} coefficients. Hence, the advertising effect apparently did not begin to take hold until two months following the initial exposure--a not unreasonable finding.

The Almon procedure appears to be especially inappropriate for these data; the considerable gains in efficiency obtained by imposing the stringent restrictions of the Almon procedure are more than outweighed by the accompanied increase in bias. The bias, however, seems to be in the pattern of the lagged response. The long-run effect (the sum of the lag coefficients) is only slightly downward biased when the higher degree polynomial is chosen.[5/] Thus, the inappropriate application of the Almon procedure will not have too serious consequences if only the long-run effect is of interest, but if the pattern of the lag distribution is important, then the Almon procedure can produce highly misleading results. This finding corroborates evidence from both Monte Carlo studies (Cargill and Meyer 1974) and investigations involving actual data (Fomby 1979). This suggests that investigators using the Almon procedure would do well to use the less restrictive Shiller procedure, particularly if the estimated pattern of the lag response is of key importance.

The long-run advertising elasticity is a key parameter in the economic model developed by Thompson et al. to determine the optimal level of generic milk advertising in various markets. A study based upon the same data analyzed in this paper (Thompson 1978) used an estimated long-run advertising elasticity ($\eta_{s \cdot a}$ = .02931) that was 27 percent smaller than the OLS estimate obtained here ($\eta_{s \cdot a}$ = .0412).[6/] As a result, recommendations regarding the appropriate level of generic milk advertising in the New York City market for the period July 1976 to June 1977 may have been understated by as much as $660,452 (in 1976 dollars).

[5/] Even if a tail constraint is used the Almon estimate of the long-run effect was only 6.23 percent less than the corresponding OLS estimate (for a second-degree polynomial).

[6/] Not all of the difference in these estimates is attributable to the use of the Almon procedure. A computer software package with a less efficient regression algorithm than TROLL may be responsible for the remaining difference.

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A P P E N D I X

Appendix Table 1. Milk Sales, Generic Advertising Expenditures, and Other Data for New York City SMSA (January 1975 to June 1977)

		Adjusted Per Capita Sales[a/] (Ounces)	Per Capita Monthly Advertising (Dollars)[b/]	Per Capita Personal Income[c/] (Dollars)	Retail Milk Price Quart[d/] (Dollars)	Population SMSA (000)[e/]	Population MCA (000)[f/]	Consumer Price[g/] Index	Cost of Advertising Index[h/]	Retail[i/] Cola Price 72 oz. Carton (Dollars)
1975	Jan.	9.15	---	6,929.8	.433	12,082.8	18,595.5	153.2	140.0	1.985
	Feb.	9.30	.01742	6,930.5	.430	12,152.6	18,574.8	154.8	140.0	1.970
	Mar.	9.18	.00430	6,931.3	.431	12,222.4	18,554.1	155.3	140.0	1.995
	Apr.	9.03	.00404	6,932.0	.423	12,292.2	18,533.4	155.6	154.0	1.999
	May	8.86	.00610	6,932.8	.436	12,362.0	18,512.7	156.3	154.0	1.921
	Jun.	8.55	.01686	6,933.5	.423	12,432.8	18,492.0	156.4	154.0	1.894
	Jul.	7.95	.00898	6,934.3	.415	12,501.5	18,471.3	157.0	139.0	1.894
	Aug.	8.19	.00427	6,979.2	.424	12,502.5	18,472.0	158.5	139.0	1.821
	Sep.	8.73	.00925	7,024.0	.421	12,503.4	18,472.7	162.4	139.0	1.811
	Oct.	8.93	.00958	7,068.9	.415	12,504.4	18,473.4	163.1	159.0	1.805
	Nov.	8.85	.01288	7,113.7	.430	12,505.3	18,474.1	164.6	159.0	1.805
	Dec.	8.96	---	7,158.6	.436	12,506.2	18,474.8	165.2	159.0	1.804
1976	Jan.	8.82	.00128	7,203.5	.445	12,507.2	18,475.5	165.2	175.0	1.801
	Feb.	8.46	.00700	7,248.3	.443	12,508.1	18,476.2	166.5	175.0	1.811
	Mar.	8.94	.00702	7,293.1	.450	12,509.0	18,476.9	167.7	175.0	1.810
	Apr.	8.53	.01128	7,337.9	.449	12,510.0	18,477.6	168.1	191.0	1.810
	May	8.57	.01394	7,382.8	.446	12,510.9	18,478.3	168.7	191.0	1.813
	Jun.	8.31	.01059	7,427.5	.448	12,511.8	18,479.0	170.0	191.0	1.815
	Jul.	7.95	.00579	7,472.4	.446	12,512.8	18,479.6	170.3	175.0	1.815
	Aug.	7.93	.01451	7,536.4	.444	12,481.9	18,448.2	171.9	175.0	1.811
	Sep.	8.46	.00475	7,600.6	.450	12,451.0	18,416.8	173.6	175.0	1.812
	Oct.	8.73	.01592	7,665.3	.459	12,420.0	18,385.4	173.9	205.0	1.830
	Nov.	8.68	.01041	7,730.2	.450	12,389.1	18,354.0	174.4	205.0	1.851
	Dec.	8.85	---	7,795.5	.449	12,358.2	18,322.5	174.7	205.0	1.863
1977	Jan.	8.75	---	7,861.0	.453	12,327.3	18,291.1	174.7	202.0	1.872
	Feb.	8.68	.00391	7,926.9	.455	12,296.4	18,259.7	175.5	202.0	1.863
	Mar.	8.70	.00840	7,993.1	.455	12,265.5	18,228.3	176.4	202.0	1.864
	Apr.	8.06	.00085	8,059.6	.454	12,234.6	18,196.8	176.9	220.0	1.863
	May	8.40	.00779	8,126.5	.456	12,203.7	18,165.4	176.7	220.0	1.869
	Jun.	8.30	.01275	8,193.7	.459	12,172.8	18,134.0	178.4	220.0	1.883

FOOTNOTES FOR APPENDIX TABLE 1

a/ The net sales within the Standard Metropolitan Statistical Area (SMSA) were adjusted for the type of days in the month, i.e., number of Sundays, Mondays, etc. The sales were also placed on a per capita basis according to the population in the SMSA. Source for adjusting data for calendar composition: John P. Rourke, Adjusting In-Area Sales Data for Calendar Composition, USDA, Agr. Mktg. Ser. Fed. Milk Order Mktg. Stat., MOMS, No. 196, April 1976 and FMOMS No. 210, June 1977.

b/ Includes media advertising expenditures for television, radio and newspaper. Advertising expenditures were placed on a per capita basis according to the population in the media coverage area (MCA). Source: Advertising invoices of American Dairy Association and Dairy Council of Syracuse, New York.

c/ Personal income within SMSA before taxes. Personal income was placed on a per capita basis according to the population of the SMSA. Source: New York State Department of Commerce, Personal Income, New York State By County, 1974 and 1975, July 11, 1977. Historical growth rates were used to estimate 1976 and the first three months of 1977.

d/ Prevailing food store Metro Area fluid whole milk price in dollars per quart. Source: Survey of Prices Charged for Milk on Retail Routes, Food Stores and Dairy Stores 25 Upstate Markets, various monthly issues.

e/ SMSA counties for NYC Metro are: Nassau, New York City--five boroughs, Rockland, Suffolk, Westchester, and Bergen, New Jersey. Population source: New York State Statistical Yearbook, various issues.

f/ Media Coverage Area (MCA) population. Estimated population viewing television stations of a given market. Source: New York State Statistical Yearbook and Federal Population Series, P-26, various issues. Nonlinear population estimates were made for 1976 and 1977.

g/ Consumer Price Index (CPI) for all items less food in New York, 1967=100. Source: United States Department of Labor, The Consumer Price Index: U.S. City Average and Selected Areas, various monthly issues.

h/ Cost of Advertising Index (composite of all time periods) where first quarter 1971=100. This index reflects variations in the cost of prime-time spot television. Source: United Dairy Industry Association, correspondence, Barbara J. Deering, January 7, 1976. Estimates for 1976 and 1977 were made in consultation with personnel from D'Aray-MacManus & Masius, Inc.

1/ Retail price of cola drink (throwaway, 72 oz. carton) in the New York-Northeastern, New Jersey area, for the NYC market and retail price of cola in the Buffalo, New York area for both the Albany and Syracuse markets. Source: United States Department of Labor, Bureau of Labor Statistics, Estimated Retail Food Prices by City, various monthly issues.