December 1980 A.E. Res. 80-20

USING MARKOV MODELS TO PREDICT THE SIZE DISTRIBUTION OF DAIRY FARMS, NEW YORK STATE, 1968-1985

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Rapid changes in the number and size distribution of dairy farms in most areas of the United States have come rapidly during the years since World War II. For example, in the twenty-year period between 1958 and 1977, the number of farms delivering milk to plants in New York State decreased from over 45,000 to 16,500. Over the same time span, milk production in the State fluctuated between 9.8 and 11.0 billion pounds per year with a peak in 1965 and low points in 1959 and 1973.

Every indication suggests that further changes, both in the size distribution of dairy farms and in total milk production will occur in the future. Between 1975 and 1980 farm numbers continued to decline while milk production held steady and then increased modestly. Current methods for predicting such changes, particularly for periods of two or more years into the future, have proven to be deficient. This study was undertaken to improve forecasts of the numbers of farms in different size classes and to assess the impact such changes would have on milk production in future years.

During the past three decades, applied economists have utilized various formulations of Markov models to examine changes in size distributions and to project such changes to future time periods. One of the first applications of a Markov model to economics was by Solow (1951) in a study of wage and price distributions. This was followed by the work of Champernowne (1953) and Prais (1955). Hart and Prais (1956) were the first to apply Markov processes to the study of the size distribution of firms. Since that time, a variety of formulations have been applied by agricultural economists to various size distributions. A useful bibliography of applications to economic problems is provided by Lee, Judge and Zellner (1977).

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This publication summarizes work initially presented in Stavins' M.S. thesis, "Forecasting the Size Distribution of Farms: A Methodological Analysis of the Dairy Industry in New York State," Cornell University, 1979. K. L. Robinson's careful review of an earlier draft of this manuscript improved both the clarity of presentation and the content. T. D. Mount made helpful suggestions for the sections on the use of the multinomial logit model. The authors appreciate the consistent and careful work of Cheryl Morse in typing and organizing the manuscript materials. Errors of omission or commission rest with the authors.

Purpose and Data Sources

This publication reports some of the results of a study which considered nine alternative methods of estimating the size distribution of dairy farms from a sizable sample of time-series and cross-sectional farm production data (Stavins 1979). Emphasis in this report is given to the three Markov procedures which were examined in that study and which yielded some of the most useful results. The data for the research came from a study area consisting of twenty counties in New York State where virtually all of the milk produced was sold under the New York-New Jersey Milk Marketing Order. During the ten-year period from January 1968 through December 1977, there were 14,272 farms which sold milk at some time in the area. A systematic list sample of 1,012 producing units, stratified by counties and by entry and exit behavior, was compiled. Monthly milk sales data on each farm were obtained from the Market Administrator's office for the ten-year period. To eliminate the effects of seasonality, monthly data were aggregated into sequential, annual totals. Following this, annual monthly averages were calculated (Table 1).

Table 1. FARM NUMBERS AND AVERAGE MONTHLY PRODUCTION 20 New York Counties, 1968-1977

Year	Number of farms in sample	Average monthly production per farm
		pounds
1968	948	30,239
1969	875	33,146
1970	819	35,620
1971	792	38,124
1972	761	39,128
1973	730	38,962
1974	697	41,584
1975	670	43,994
1976	650	45,779
1977	650	47,312
Percent change 1968-77	-32%	+56%

Producing units were classified into size categories on the basis of pounds of milk sold per month per farm. The class interval used, 20,000 pounds, roughly corresponds to production from 20 cows, assuming average annual sales of 12,000 pounds per cow (1,000 pounds per cow per month). Farms were allocated to one of the ten categories formed. An entry/exit class with zero production and eight other classes, roughly equivalent to 1-19, 20-39, 40-59 cows, etc., were used. The last category was an open-ended class, 160,000 pounds of milk per month or more (Table 2).

Table 2. CHANGES IN THE SIZE DISTRIBUTION OF FARMS
Monthly Sales Per Farm, Central New York, 1968 and 1977

	Size-class in 1968	Number of farms in 1968	Exit farms	Milk sold per month 1977								
				(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Pounds milk sold per month			number of farms								
(1)	0	ENTRY FARMS →		19	20	14	4	2	1	1	1	2
(2)	1 - 19,999	349	213	58	54	11	10	2	0	1	0	0
(3)	20,000 - 39,999	360	117	35	116	63	21	5	2	0	1	0
(4)	40,000 - 59,999	154	27	1	22	50	38	9	4	1	0	2
(5)	60,000 - 79,999	59	11	2	0	9	16	9	5	4	1	2
(6)	80,000 - 99,999	13	3	0	0	1	2	3	3	0	1	0
(7)	100,000 - 119,999	8	0	0	0	0	1	1	1	1	2	2
(8)	120,000 - 139,999	1	0	0	0	0	0	0	0	0	0	1
(9)	140,000 - 159,999	3	1	0	0	0	0	0	0	0	1	1
(10)	160,000 or more	1	0	0	0	0	0	0	0	0	0	1
	Number of farms in 1977 →		372	115	212	148	92	31	16	8	7	11
	TOTAL FARMS (1968) = 948						TOTAL FARMS (1977) = 640					

At no time during the ten-year period were all of the 1,012 farms selling milk simultaneously. Approximately one-third of the farms terminated their sales during the period, while a relatively small number of others began or resumed milk production. As indicated in Table 1, net farm numbers declined steadily during the period, while monthly sales per farm increased. In this respect, the sample closely approximated aggregate numbers for the twenty-county area and for New York State as a whole.

One way of systematically examining changes over time in a size distribution is illustrated in Table 2. The original distribution in 1968 is compared with the distribution in 1977. Farms which moved into higher production categories or initiated production during the ten-year period are included in numbers located above and to the right of the diagonal, where no change occurred. Decreases in size and exits from active dairying are found below and to the left of the diagonal. The format used is that required for analysis with finite Markov processes and incorporates many of the features of a Markov transition matrix.

During the ten-year period represented in the matrix of Table 2, 64 new farms came into production while 372 discontinued milk sales. Of the 576 farms that produced milk continuously, 74 (13 percent) decreased production enough to drop one or more size classes. Another 246 farms (43 percent) remained stationary in the same size category (diagonal elements in the matrix) and 256 farms (44 percent) increased by one or more size groups.

Data Requirements of Markov Models: Micro-Data vs. Macro-Data

Time-series information on changes in the size of individual units within an industry has been referred to in the literature as "micro-data." This is in contrast to what has been described as "macro-data." This second classification refers to the situation in which only the size distributions of two or more time periods are specified. No information is provided regarding changes in individual firms, although it is such (micro) changes which in fact bring about any observed changes in the overall distributions. The clear advantage of a predictive model built upon micro-data is that it allows consideration of how the economic data were generated. By identifying the processes by which the distributions change, a sounder base is developed for predicting the future time path of the distribution in question.

Markov Models with Stationary Transition Probabilities

Projection techniques based upon Markov processes assume that the elements of individual matrices are constant over time, i.e., have stationary transition probabilities. This is a crucial assumption which can and often does lead to inaccurate predictions of future changes in size distributions. If detailed micro-data are available for several time periods, i.e., if the elements are m_{ijt} are available for t = 1, 2, ..., T time periods, then the maximum likelihood estimators (Anderson and Goodman) of the stationary transition probabilities over the entire sample period are:1/

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{T} m_{ijt}}{\sum_{j=1}^{n} \sum_{t=1}^{T} m_{ijt}} \tag{1}$$

A Stationary Micro-Data Markov Model

If micro-data are available and a researcher is willing to accept the validity of the assumption of stationary transition probabilities, it is a relatively simple matter to use Markov matrices to predict firm-size distributions. The first step is the construction of the relevant transition matrix in which the ij^{th} element indicates the number of firms which have moved from size-class i to size class j during the period in question. This matrix is transformed into the transition probability matrix by simply dividing each element by its respective row sum. The distribution, X_t, is obtained by matrix multiplication:

$$\hat{X}_t = X_0 P^t. \tag{2}$$

Given the constant probabilities, only one matrix and a starting-state vector of farm sizes are needed to predict any future distribution.

This approach has been used extensively in economics to project the size distribution of firms and has drawn heavily upon research by Hart and Prais (1956), by Adelman (1958) and Collins and Preston (1961). Applications of the method in agricultural economics include: Judge and Swanson (1961); Preston and Bell (1961); Padberg (1962); Williams and Alexander (1963); Farris and Padberg (1964); Kottke (1964); Reilly (1964); Alexander and Williams (1965); Stanton (1966); Conneman (1967); Stanton and Kettunen (1967); Colman (1967); Farris (1967); Johnson and Schneidau (1967); O'Dwyer (1968); Conneman and Harrington (1969); Colman and Leech (1970); Power and Harris (1971); Duncan (1972); Farris (1973); Ward and Smoleny (1973); Cleveland, Just and Salkin (1974); Ching, Davulis and Frick (1974); Igoe (1974); Cleveland and Salkin (1975); and Colman (1977).

1/ Colman (1977, p. 47) points out that when the data base is not continuous, this estimator is biased. In the empirical applications of this study, however, the sample is entirely continuous over the full sample period and so the Anderson and Goodman estimator is utilized.

Assumptions of the Model

Before developing a Markov model it is important to ask whether the real-world situation is consistent with the assumptions of the Markov process. The basic model implies four critical assumptions about the size distribution of dairy farms:

i. Dairy farms can be grouped into size-classes according to some criteria;

ii. The evolution of a dairy farm through size-classes can be regarded as a stochastic process;

iii. The probability of a dairy farm moving from one size class to another is a function only of the basic stochastic process; and

iv. The transition probabilities remain constant over time.

The second assumption is the most fundamental. The relevant question is whether growth patterns in the New York dairy industry can reasonably be regarded as a stochastic process. If structural change in the dairy industry is entirely the result of actions by individual dairy farms, then the probabilistic model is inappropriate. However, "if general environmental factors dictate a general type of structural development within an industry, a probabilistic model may approximate this development pattern" (Padberg 1962, p. 191). Williams and Alexander (1963), in a study of the size distribution of Louisiana dairy farms, supported the assumption of a stochastic process. They maintained that technical progress and uncertainty have the effect of stochastic elements in the growth patterns of farms.

Identifying a Pool of Potential Entrants

In developing the basic Markov model, a decision must be made regarding the number of firms to be assumed as potential entrants. The nature of this assumption directly affects the value in the first row and first column of the transition matrix. Adelman maintained that "this arbitrary selection does not affect the economically relevant portion of our results" (p. 899). Stanton and Kettunen, however, pointed out that although this is true in terms of projections of the proportion of firms in each size-class, it is not true in regard to projections of the actual number of firms in each class.

The value assumed as the a_{11} element is critical for two reasons:

i. This value determines the probabilities of the first row of the transition probability matrix, and these values affect every class of the projected distribution vector.

ii. The value directly affects the first element of the starting-state distribution vector and thus has an impact on the first element of subsequently predicted distribution vectors.

In these two ways, then, the assumed size of this pool of potential entrants may have a significant impact upon the model's predictions.

Fortunately, the choice of size of this pool need not be arbitrary. A number of approaches have been suggested for making reasonable assumptions about the level of potential entrants.2/ In this study, an attempt was made to identify the actual number of enterprises which had the capacity during the sample period to enter active dairying.

The total number of commercial non-dairy farms in the study area was identified as the pool of potential entrants to active dairying. The 1974 Census of Agriculture was used to determine the total number of farms with gross sales of $5,000 per year or more in the twenty-county area. From this was subtracted the average number of dairy farms over the ten-year time frame. An estimate of 1,687 commercial, non-dairy farms in the area was obtained which translated into a pool of 120 potential entrants in terms of the sample data set.

The Transition Probability Matrix

One useful, yet rarely used, method of evaluating alternative economic, predictive models is to utilize the alternative models in making actual predictions of the variable in question. This may be done by using only a portion of the available time-series data to estimate the parameters of the given models and then using each model in turn to make *ex post* "predictions" of the relevant variable for a time period for which its true value is known. Thus, data for the period 1968-1974 was used to predict the sample distribution as it existed in 1977. In this way it is possible to assess the predictive accuracy of the Markov model under consideration.

The first step in predicting the size distribution of farms for 1977 using the static micro-data Markov model involves the construction of a transition matrix for the period 1968-1974. Information on the changing size, entry and exit behavior of individual farms is presented in Table 3. From an initial sample size of 948 farms in 1968, there was a decline to 697 in 1974.

2/ Alternative approaches to the problem are suggested in Stanton and Kettunen; Colman and Leech; Colman 1967; Duncan and Lin; Ward and Smoleny; and Williams and Alexander.

During that period there were 296 exits from dairying and 45 entries. Farms that remained in the same size class are included within the boxed diagonal. Those above and to the right entered dairying or increased their output by one or more size classifications. Those to the left or below the diagonal either decreased in size or exited from dairying altogether.

Table 3. CHANGE IN THE SIZE DISTRIBUTION OF FARMS
Sample from 20 Counties, New York, 1968-74

Size class in 1968		Number of farms in 1968	Exit farms (1)	Milk sold per month in 1974 (2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Pounds sold per month				number of farms								
(1)	0	0	0	20	8	10	2	2	2	1	0	0
(2)	1 - 19,999	349	179	102	47	16	3	1	0	1	0	0
(3)	20,000 - 39,999	360	91	34	157	67	9	1	1	0	0	0
(4)	40,000 - 59,999	154	16	2	21	70	33	10	0	2	0	0
(5)	60,000 - 79,999	59	8	1	1	8	22	11	3	3	2	0
(6)	80,000 - 99,999	13	1	0	1	2	2	3	2	1	1	0
(7)	100,000 - 119,999	8	0	0	0	0	1	2	1	1	1	2
(8)	120,000 - 139,999	1	0	0	0	0	0	0	0	0	0	1
(9)	140,000 - 159,999	3	1	0	0	0	0	0	0	0	1	1
(10)	160,000 or more	1	0	0	0	0	0	0	0	0	0	1
Total farms		948	296	159	235	173	72	30	9	9	5	5

In the next step, a transition probability matrix is derived from this transition matrix (Table 4) using Anderson and Goodman's maximum likelihood estimator (Equation 1). First, the assumed number of potential entrants, 120 farms, is entered as the (1,1) element in the transition matrix. Then, each element in the matrix is divided by its respective row sum to yield the transition probability matrix (Table 4). Thus the first element in the second row, .513, is obtained by dividing 179 by 349. The elements of this new matrix express the probability that a farm in any size-class in 1968 will remain in that class or move to any other class by the year 1974.

Table 4. TRANSITION PROBABILITY MATRIX
Sample of Dairy Farms, New York, 1968-74

Size-class in 1968	Size-class in 1974 (1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
(1)	.728	.121	.048	.061	.012	.012	.012	.006	.000	.000
(2)	.513	.292	.135	.046	.009	.003	.000	.003	.000	.000
(3)	.253	.094	.436	.186	.025	.003	.003	.000	.000	.000
(4)	.104	.013	.136	.455	.214	.065	.000	.013	.000	.000
(5)	.136	.017	.017	.136	.373	.186	.051	.051	.034	.000
(6)	.177	.000	.077	.154	.154	.231	.154	.077	.077	.000
(7)	.000	.000	.000	.000	.125	.250	.125	.125	.125	.250
(8)	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000
(9)	.333	.000	.000	.000	.000	.000	.000	.000	.333	.333
(10)	.000	.000	.000	.000	.000	.000	.000	.000	.000	1.000

Interpreting the Markov Matrices

The information found in the transition probability matrix of Table 4 may be summarized by grouping the probabilities into four categories as shown in Table 5. For each of the nine size-classes, the table indicates the probabilities of leaving dairy farming altogether (exits), decreasing in size (decline), staying in the same size-class (stationary), and increasing in size (expansion).

Table 5. TYPES OF CHANGES IN SIZE OF DAIRY FARM
Sample Data, New York, 1968-74

Size-class in 1968	Number of farms in 1968	Type of Change in Size Between 1968 and 1974: Exit	Decline	Stationary	Expansion
Pounds sold per month			percent of farms		
1 - 19,999	349	51	0	29	20
20,000 - 39,999	360	25	9	44	22
40,000 - 59,999	154	10	15	46	29
60,000 - 79,999	59	14	17	37	32
80,000 - 99,999	13	8	38	23	31
100,000 - 119,999	8	0	38	13	49
120,000 - 139,999	1	0	0	0	100
140,000 - 159,999	3	33	0	33	34
160,000 and over	1	0	0	100	0
Total	948	31	8	38	23

It is helpful to examine those farms which remained in production over the six-year period (Table 6), and to observe the proportion of farms by size-class which declined, remained the same, or increased in size. The smallest and the largest, open-ended interval are not included because their definitions preclude the possibility of decline or growth, respectively. Relative stability is indicated by the percentage of farms of a given size-class which remained stationary over the sample time frame. The data support findings of previous Markov analyses of the dairy industry which suggested that smaller dairy farms are, for the most part, relatively more stable than larger ones (Willett and Saupe).

Table 6. CHANGES IN SIZE OF CONTINUOUS DAIRY FARMS
Sample Data, New York, 1968-74

Size-class in 1968	Type of Change in Size Between 1968 and 1974		
	Decline	Stationary	Expansion
Pounds sold per month		percent of farms	
20,000 - 39,999	12	59	29
40,000 - 59,999	17	51	32
60,000 - 79,999	20	43	37
80,000 - 99,999	41	25	34
100,000 - 119,999	38	13	49
120,000 - 139,999	0	0	100
140,000 - 159,999	0	50	50

Projections to 1977

By multiplying the 1968-1974 transition probability matrix (Table 4) by the actual size distribution vector for 1971, the distribution is projected to the year 1977:

$$\hat{X}_{77} = X_{71}P_{68:74} \qquad (3)$$

The predicted 1977 distribution (Table 7) approximates the actual distribution reasonably well (Figure 1). A quantitative indication of the accuracy of the prediction is provided by the square root of the sum of the squared deviations, 34.45.

Figure 1. ACTUAL AND PREDICTED 1977 FREQUENCY DISTRIBUTION OF SAMPLE FARMS - MICRO-DATA MARKOV MODEL (1968-1974 BASE PERIOD)

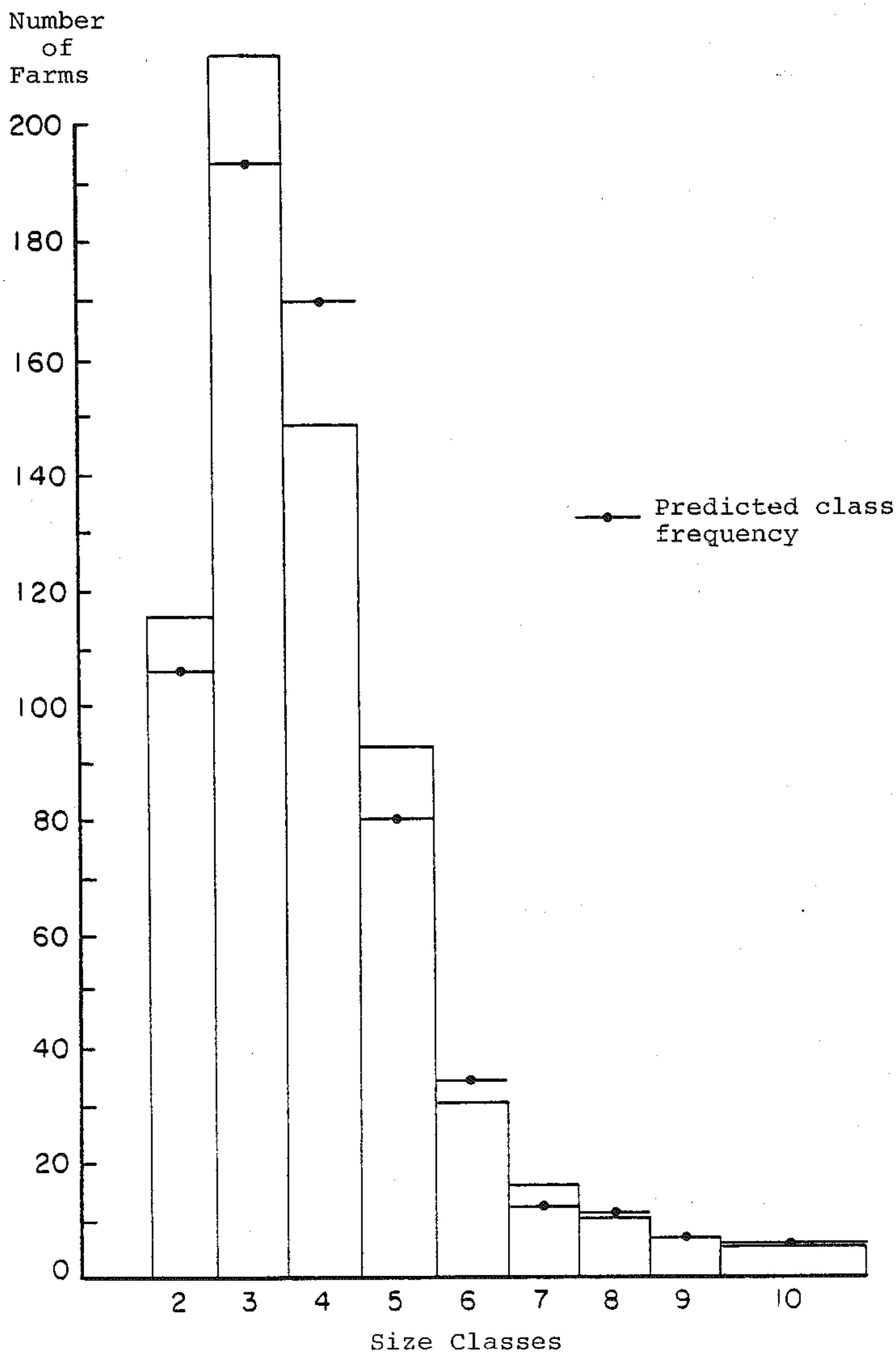

Table 7. PREDICTED AND ACTUAL SIZE DISTRIBUTIONS
Micro-Data, Dairy Farms, New York, 1977

Pounds of milk sold per month	Estimated 1977	Actual 1977	Deviations
1 - 19,999	105	115	-10
20,000 - 39,999	193	212	-19
40,000 - 59,999	170	148	+22
60,000 - 79,999	78	92	-14
80,000 - 99,999	35	31	+ 4
100,000 - 119,999	11	16	- 5
120,000 - 139,999	10	8	+ 2
140,000 - 159,999	7	7	0
160,000 and over	12	11	+ 1

As a test of the stationarity assumption, alternative estimates of the 1977 distribution were made using different base periods within the period 1968-1974. The transition probability matrices for 1968-1969 and 1968-1972 were used to project the sample size distribution through the year 1977. The results are summarized in Table 8. Not surprisingly, the predictive accuracy is greatest when the probabilities for the entire 1968-1974 period are used and least when only 1968-1969 data are utilized.

Table 8. ALTERNATIVE MARKOV ESTIMATES OF THE 1977 DISTRIBUTION
Probability Distributions, New York, 1968-74

Pounds of milk sold per month	Actual number 1977	Estimates of size distribution, 1977: 1968-69 base	1968-72 base	1968-74 base
			number of farms	
1 - 19,999	115	93	108	105
20,000 - 39,999	212	234	204	193
40,000 - 59,999	148	131	174	170
60,000 - 79,999	92	54	78	78
80,000 - 99,999	31	28	48	35
100,000 - 119,999	16	14	10	11
120,000 - 139,999	8	60	4	10
140,000 - 159,999	7	3	7	7
160,000 and over	11	8	19	12
Square root of sum of squared deviations		73.8	37.3	34.5

There are two factors which are responsible for different projections using alternative base years. First, the longer data-bases capture more accurately the long-term trends in the number of farms in each size-class. And second, the projection period is shortest with the 1968-1974 matrix and thus the error is multiplied less than with the longer projections. Since the model yields rather different predictions of the 1977 distribution depending on the particular data-period chosen, the assumption of stationary probabilities must be viewed with some caution. The static micro-data Markov model can be expected to be predictively accurate only to the extent that the factors which affect the changing size of firms (and entry and exit of firms) during the sample period are qualitatively and quantitatively the same during the projection period. The model does not explain the factors which underlie, cause or are associated with change. Rather, it serves only as a detailed description of such patterns of change.

Stationary Macro-Data Markov Models

In the methodology just presented, the Markov transition matrix was derived directly from available micro-data on the movements over time of individual producing units within the industry. Macro-data models, on the other hand, are designed for situations where only aggregate data of distributions are available. In these situations, where limited information is available, the problem is to estimate the transition matrix (or the transition probability matrix) as logically as possible. There have been two basic approaches to this problem -- rather naive rule-of-thumb methods and statistical estimation procedures.

Krenz (1964) developed a model to predict the future time-path of the size distribution (by acreage class) of census farms in North Dakota based only upon (macro) census data from the period 1935-1960. In order to estimate the elements of a Markov transition matrix, Krenz began with four assumptions:

i. Operators of any size farm will expand their acreage if possible.

ii. The farmers that are most likely to expand are those that are initially larger than average.

iii. Increases in farm size are likely to come about by gradual increases in acreage.

iv. Decreases in size of farms are not likely to occur (i.e., because of economies of size, farmers are not likely to decrease the size of their holdings voluntarily). Therefore, a farm is more likely to disappear altogether than to become smaller.

Based upon these assumptions, Krenz specified three rules which are utilized to formulate the transition matrix:

1. Farms in the largest size-class remain in that class;

2. Increases in the number of farms in any class come from the next smaller class; and

3. Any decrease in number of farms in any class, other than that resulting from rule 2, above, results in a movement out of farming (exit).

A fourth, implicit rule is that entry into farming is impossible.

By applying these rules to size distributions for different years, the (absolute) transition matrix can be estimated. From this, the transition probability matrix is calculated in the usual way. Lastly, the same procedure as outlined previously, $\hat{X}_t = X_0\hat{P}^t$, is used to predict future distributions.

The theoretical inadequacy of this approach is clear. Lee, Judge and Takayama (1965) pointed out the circular reasoning which is involved: "This approach has the uncomfortable aspect of postulating the behavior pattern for the units which it was the initial objective to investigate." In spite of this weakness, the method outlined above has been used in a number of farm-size studies (Daly 1967; Furniss and Gustafsson 1968; Ching et al 1974; Harrison 1975; and Keane 1976). Similar sets of rather arbitrary rules were adopted and utilized by Daly, Dempsey and Cobb (1972); Dean, Johnson and Carter (1963); and Lin, Coffman and Penn (1980).

Krenz's rules and procedures were used with the macro-data for 1968 and 1974, and a Markov transition probability matrix was estimated. The matrix was pre-multiplied by the 1971 distribution vector to yield a projection of the 1977 distribution (Table 9). The square root of the sum of the squared deviations is 56.46, an inferior fit compared to that of the classic, micro-data Markov model. However, the Krenz model, like the micro Markov model, captures the correct shape of the distribution (Figure 2), while requiring significantly less information.

Figure 2. ACTUAL AND PREDICTED 1977 FREQUENCY DISTRIBUTION OF SAMPLE FARMS - MACRO-DATA KRENZ-TYPE MODEL (1968-1974)

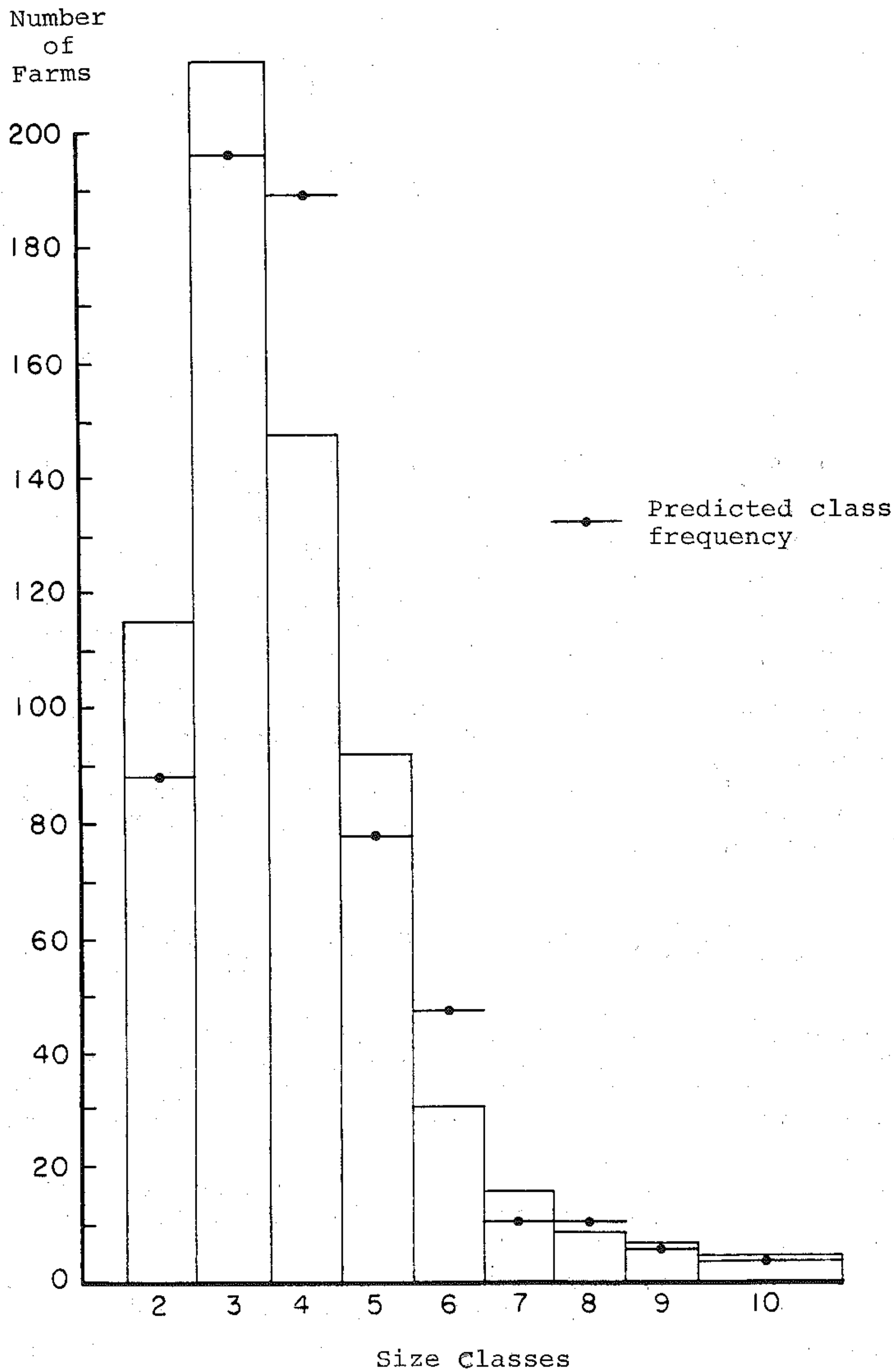

Table 9. PREDICTED AND ACTUAL SIZE DISTRIBUTIONS
Macro-Data, Krenz-Type Model:
Dairy Farms, New York, 1977

Pounds of milk sold per month	Estimated 1977	Actual 1977	Deviations
	number of farms		
1 - 19,999	89	115	-26
20,000 - 39,999	196	212	-16
40,000 - 59,999	190	148	+42
60,000 - 79,999	78	92	-14
80,000 - 99,999	47	31	+16
100,000 - 119,999	11	16	- 5
120,000 - 139,999	10	8	+ 2
140,000 - 159,999	6	7	- 1
160,000 and over	8	11	- 3

The theoretical limitations of the method are formidable, including the fact that statistical measures of reliability are not available (Krenz, p. 83). In the absence of detailed micro-data the Markov process loses much of its usefulness as a tool for prediction. The strength of the Krenz method comes from its ability to deal with a much broader range of actual situations (than the classic micro-data Markov model). Furthermore, micro-data usually are available only for small geographic areas and/or partial samples of the population. When using a macro-approach, census data can be utilized and sampling errors thus reduced considerably (Colman 1977). Because of the significant practical advantages of working with simple macro-data, there has been a substantial effort given to more sophisticated means of developing a Markov transition probability matrix based only upon information from aggregate distributions.

Statistical Estimation of the Transition Probability Matrix From Aggregate Time Series Data[3/]

Statistical estimation of transition probabilities from aggregate time series data has been attributed to an idea advanced by Miller (1952), refined and extended by Goodman (1953), Madansky (1959) and Telser (1963). A voluminous literature has developed on alternative estimators, including least squares[4/], Bayesian[5/],

3/ For a more complete survey of the literature and detailed analytical comparison of the various estimation procedures, see Lee, Judge and Zellner 1977.

4/ Miller 1952; Madansky 1969; Telser 1963; Lee et al 1965; Dent and Ballintine 1971; Colman 1977; Lee et al 1977.

5/ Martin 1967; Dent and Ballintine 1971; Colman 1977; Lee et al 1977.

maximum likelihood[6/], quadratic programming[7/], restricted minimum absolute deviations[8/], restricted least squares[9/], and minimum chi-square[10/].

Relative to the simple rule-of-thumb procedures developed by Krenz, these statistical estimation methods suffer from two distinct disadvantages. First, they are vastly more complicated and more expensive to apply. And second, whereas in the Krenz-type model, macro-data for only two consecutive time periods are required to estimate the probability matrix, with the statistical estimation procedures the number of time periods must be at least one greater than the number of states or size-classes. In fact, the number of time-series observations of the frequency distribution should probably exceed the number of classes in the distribution by at least three or four (Colman 1977, p. 67).

Markov Models with Variable Transition Probabilities

In the standard, static Markov model all economic factors which affect the growth pattern of individual firms are implicitly represented by a single variable, i.e., size. This is equivalent to maintaining that all other economic factors are correlated with size (Reilly, p. 14). Under close examination, it becomes clear that the theoretical base of the stationary Markov model is a rather weak one upon which to base predictions of future size-distributions.

There is no reason, in the vast majority of cases, to expect transition probabilities to be constant. On the contrary, there is good reason to expect them to vary. Numerous factors, such as technology, product prices, input costs, and legal requirements are likely to produce non-stationary transition probabilities. A number of firm-size studies have confirmed the existence of changing probabilities (Padberg 1962; Hart and Prais).

6/ Dent and Ballintine 1971; Hurtado 1977; Colman 1977; Lee *et al* 1977.

7/ Lee *et al* 1965; Theil and Ray 1966; Dent 1967; Dent and Ballintine 1971; Lee *et al* 1977.

8/ Fisher 1961; Lee et al 1965; Dent and Ballintine 1971; Lee *et al* 1977.

9/ Goodman 1953; Telser 1963; Lee *et al* 1965; Furniss and Gustafsson 1968; Dent and Ballintine 1971; Lee *et al* 1977.

10/ Lee *et al* 1977.

Accepting the notion of variable probabilities, some researchers have sought to develop models which might incorporate such a changing structure within the basic Markov framework. As with stationary models, the variable ones have been of two types -- those based upon aggregate macro-data and those utilizing detailed micro-data. Applications of the former type have been relatively few[11/] and none have involved distributions of agricultural firms. Furthermore, the assumptions and procedures involved in developing such models are exceptionally difficult. Consequently, this group of variable Markov models will not be considered.

Variable Micro-Data Markov Models

The probabilities in a Markov matrix have been observed to vary over time and have been hypothesized as being associated with exogenous factors. There are two possible approaches, then, to constructing a variable micro-data Markov model. First, the probabilities can be viewed simply as functions of time in a time-series regression framework (Salkin, Just and Cleveland). Second, a structural model may be developed, in which the transition probabilities are thought to be associated with changes in causal, exogenous variables (Hallberg 1969; MacMillan *et al* 1974).

Hallberg's Structural Regression Model

When a series of size distribution transition probability matrices are found to be changing over time, it is possible to modify the Markov model in such a way as to incorporate the variability. In some cases, *a priori* information about the particular industry may suggest a functional relationship between the changing probabilities and specific exogenous factors. Such was the case in Hallberg's investigation of the changing size distribution of frozen milk product manufacturing plants in Pennsylvania during the period 1944-1963 (Hallberg 1969).

Hallberg hypothesized that the changes in the probabilities were associated with changes in various factors which influence the demand for and costs of producing manufactured dairy products. Five exogenous variables were specified:

i. Index of hourly earnings of works engaged in food manufacturing industries in the U.S., deflated;

ii. Population in Pennsylvania;

11/ Goodman 1959; Telser 1963; Dent 1972; Dent and Ballintine 1971; Dent 1973a; Dent 1973b; Lee, Judge and Zellner 1977.

iii. Per capita income in Pennsylvania, deflated;

iv. Price per hundredweight received by Pennsylvania farmers for milk, deflated; and

v. Index of retail price of all dairy products in U.S., deflated.

In essence, Hallberg's approach comes down to fitting a least squares regression of the form,

$$\hat{p}_{ijt} = \hat{\alpha}_{ij} + \sum_{k=1}^{K} \hat{\beta}_{ijk} x_{kt}, \qquad (4)$$

to each of the n^2 cells of the transition probability matrix, where the set of exogenous variables is denoted by x_{kt} for $k = 1, 2, \ldots, K$.

In order to meet the two Markov requirements,

$$p_{ijt} \geq 0 \text{ for all } i, j, t \qquad (5)$$

$$\sum_{j=1}^{n} p_{ijt} = 1 \text{ for all } i \text{ and } t, \qquad (6)$$

Hallberg estimates the n + nk parameters of a given row in a single regression equation. If all parameters of the n equations in a row are to be estimated, it is appropriate to use ordinary least squares regression procedures. In most applications of Markov analysis to the size distribution of firms, however, many of the transition probability elements will be zero over the entire time frame being observed. This occurs because most firms remain the same size, some grow a little larger, a few decline in size, but virtually none grow or decline by large amounts in a single year. The result is often a matrix which looks something like this:

$$\begin{bmatrix} L & s & v & 0 & 0 \\ v & L & s & v & 0 \\ 0 & v & L & s & v \\ 0 & 0 & v & L & s \\ 0 & 0 & 0 & v & L \end{bmatrix}$$

where L indicates various large probabilities, s represents smaller ones and v stands for very small probabilities. Also, it is reasonable to expect that the exogenous factors which affect some of the probabilities in a given row would have no effect on certain other ones. For both reasons, then, it is often the case that some of the parameters will be assumed a priori to be zero. In this case, OLS estimation is unsatisfactory and some type of restricted least squares approach is advisable. Hallberg selected the procedures suggested by Goldberger (1964) and imposed the restriction that the non-zero intercept terms should sum to unity and the non-zero slope coefficients for a given exogenous variable should sum to zero in any row of the transition probability matrix.

When the estimation procedure is completed, one has, in essence, a system of n^2 equations (where there are n size-classes) relating the transition probabilities to a set of exogenous variables. The complete matrix for a time period t is then calculated by substituting the appropriate values of the exogenous variables into a set of equations of the form of equation 4, above. A simulation procedure is utilized in which a series of matrix-vector calculations of the form,

$$\hat{X}_t = \hat{X}_{t-1}\hat{P}_{t-1:t} \quad (7)$$ */

lead recursively to a conditional forecast of the future size distribution of firms.

One obvious problem with this type of forecasting model is that the prediction of the future is dependent upon a prediction of the exogenous variables for the model. In the final analysis, this structural model may rest on no more than simple and naive projection techniques. This limitation also applies to another variable Markov model considered later in this paper, and, for that matter, is a basic limitation of virtually all econometric models which are dependent on estimates of exogenous variables in future time periods.

Although Hallberg's restricted least squares approach ensures that the Markov condition that all rows sum to unity (Equation 6) not be violated, it does not deal directly with the constraint requiring that all probabilities be greater than or equal to zero (Equation 5). That is, within the restricted least squares estimation framework used, it is possible to predict p_{ijt}'s which are less than zero or greater than unity. Hallberg dealt with the problem in a rather ad hoc manner: If a negative probability turned up, it was assumed to be zero. If a probability greater than unity occurred, it was assumed to be equal to one. Hallberg states: "This rule is admittedly arbitrary, but is believed reasonable and, in the absence of a satisfactory alternative, was

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*/ The subscript of the matrix P indicates that this is the transition probability matrix which represents the change from period t-1 to period t.

used in the analysis..." (Hallberg 1969, p. 294). Other researchers, however, have felt that Hallberg's "rule" is not reasonable and have pointed out possible alternative approaches. Lee maintained that "the suggested procedure is simple and practical but deficient in theory " (Lee 1970, p. 613).

Lee went on to suggest the use of quadratic programming in conjunction with Aitken's generalized least squares so that estimated values of the probabilities would be restricted to the range of zero to one. Hallberg (1970, p. 615) replied to this suggestion by pointing out that although Lee's estimation procedure would guarantee that values of p_{ijt} during the sample period would fall between zero and one, there is no assurance with the method that the restriction would be satisfied during the forecast period. Thus, Hallberg's original model and Lee's suggested alternative both depend upon arbitrary judgments when used for predictive purposes.

A Note on Alternative Micro-Data Non-Stationary Markov Models

Only one alternative to Hallberg's structural model has been proposed and fully developed.12/ This is a time-series regression model developed by Salkin, Just and Cleveland (1976), in which the changing size-structure of Oklahoma cotton warehouses is described in terms of a series of Markov transition probability matrices. Two models are formulated. In the first, the probabilities are hypothesized as being linear functions of time. The problem, of course, is that the predicted path of future probabilities quickly goes below zero or above unity. Also, the fit of the data to the equations (in terms of R^2) is extremely poor, since the probabilities do not correspond to a smooth linear trend over time.

In a second model, the probabilities are posited as functions of a geometric transformation of time. That is, the magnitude of change in the transition probabilities from one matrix is described as changing at a constant rate. The result is that predicted probabilities will always fall between zero and unity. Although the second model performs well in terms of traditional goodness-of-fit criteria, it is not possible to assess the method in terms of its predictive capabilities since the authors chose not to forecast a distribution which could be contrasted with an actual one.

12/ Also, MacMillan et al (1974) developed a model in which transition probability matrices were formulated on the basis of simple, Krenz-type rules from macro-data. The probabilities were adjusted on the basis of information from a separate structural, econometric analysis based on cross-sectional data for individual farms.

Salkin et al maintains that since the exogenous variables in Hallberg's structural regression model must themselves be predicted (possibly by extrapolative methods) in order to forecast a future distribution, such a structural model is not preferable to their own time-series regression method (Salkin et al, p. 81). On the other hand, a time-dependent model cannot alter the rate and direction of structural change in response to exogenous factors over time.

Salkin, Just and Cleveland themselves state that better non-stationary models are possible and that one alternative would be the use of a "multivariate generalization of the logit or probit models" (Salkin et al, p. 81). Within a multinomial logit framework it is possible to retain the structural characteristics of Hallberg's approach (i.e., relating the probabilities to exogenous causal factors) while meeting requirements for logical transition probability predictions without adopting arbitrary rules and procedures.

A Micro-Data Variable Markov Multinomial Logit Model[13/]

The advantage of Hallberg's model is that it incorporates the concept of variable probabilities and does this in such a way that those probabilities are functionally related to logical, causal factors. In contrast to the time-series regression model of Salkin et al (1976), it is possible to develop conditional forecasts of the future size distribution of firms within a given industry if causal forces can be properly specified and estimated. A further refinement of Hallberg's basic structural approach is to specify the equations required in a multinomial logit framework.

Each of the N rows of a time-series of T transition probability matrices may be handled as a separate multinomial logit model (MNLM). For a given row, say the first row, we posit that the N transition probabilities are functions of exogenous, socio-economic factors. For the time being, f_{1jt} (j = 1, ..., N) will stand for such unspecified functions. The flexibility of the model is due to the range of possible specifications of the functions, f_{1j}. By using an exponential function it is a simple matter to ensure that all predicted values of the probabilities will be positive and sum to unity:

$$\hat{p}_{1jt} = e^{f_{1jt}} \Big/ \sum_{k=1}^{N} e^{f_{1kt}} \qquad (8)$$

13/ For a concise description of a basic multinomial logit model, see Theil (1971), pp. 632-633.

If f_{1jt} is linear in the unknown parameters, e.g., $f_{1jt} = \beta_{1j1} X_{1t} + \beta_{1j2} X_{2t} + \ldots\ldots u_{1jt}$, where the β's are unknown parameters, the X's are observed exogenous variables and u is an unobserved residual, the following transformation can be used if all $p_{1jt} > 0$:

$$\ln \frac{p_{1jt}}{p_{1dt}} = f_{1jt} = f_{1dt} \text{ for } j = 1,2,\ldots\ldots, N \text{ and } j \neq d. \quad (9)$$

This is the form of the multinomial logit model (for row #1) that can be estimated using linear regression techniques. Note that the choices of the first group (i=1) and the common denominator (j=d) are both arbitrary and do not affect the ultimate results.

Once the set of N-1 equations (Equation type 9) has been estimated, it is possible to derive a set of N - 1 predicted values of the ratios,

$$(\widehat{p_{1jt}/p_{1dt}}) \;,$$

given any set of values of the exogenous variables.

From this set of (N-1) predictions, simple arithmetic leads to the predicted value of the probability for the denominator.

$$\hat{p}_{1dt} = (1 + \sum_{j \neq d} \widehat{p_{1jt}/p_{1dt}})^{-1} \quad (10)$$

Then, the other N - 1 probabilities in the row can be calculated, using the predicted value of p_{1dt} from Equation 10:

$$\hat{p}_{1jt} = \hat{p}_{1dt} \, (\widehat{p_{1jt}/p_{1dt}}) \text{ for all } j \neq d \quad (11)$$

Thus the very structure of the model ensures that the probabilities in each row will sum to unity for every set of values of the exogenous variables. Both of the Markov constraints have been met without resorting to arbitrary or ad hoc procedures. Furthermore, the model is extremely flexible in that the specification of the function, f_{1jt}, may be different for every element of the transition probability matrix.

A time-series of predicted transition probability matrices is developed from the model, running from the present time up until the final prediction year, each year's matrix being calculated using a specified set of exogenous variable values for that particular year and the previous probability matrix. As with the

Hallberg model, a simulation procedure is used in which a series of matrix-vector calculations of the form,

$$\hat{X}_t = \hat{X}_{t-1}\hat{P}_{t-1:t}, \qquad (7)$$

lead recursively to a conditional forecast of the future size distribution of firms.

Testing the Assumption of Stationary Probabilities

Before estimating the parameters of the multinomial logit model for the sample of dairy farms, two other matters deserve discussion. One is the issue of variable vs. stationary probabilities, since the multinomial logit specification is warranted only if the Markov transition probability matrices are variable through time. The second issue is the identification of causal factors which may be associated with the changing size-structure of dairy farms.

Anderson and Goodman have developed a chi-square procedure for testing the null hypothesis that the true transition probabilities are indeed stationary. It simply provides an additional statistical procedure to assist a research worker in examining the micro-data available and his knowledge of the nature of change over time in the size distribution study. The calculated value of χ^2 helps one decide if the null hypothesis of stationary probabilities should be accepted or rejected.

In the present application, the test involves each of the six annual transitiion probability matrices of the period 1968-1974 and the overall transition probability matrix for the seven year period. The value of χ^2 for the sample period, 1968-1974, was found to be 2804.3. The degrees of freedom following the Anderson-Goodman procedure are:

$$(T-1)(n-1)n = (6-1)(10-1)10 = (5)(9)(10) = 450.$$

Table values of χ^2 for such large degrees of freedom are not available, but Mood and Graybill (p. 428) give an approximation of χ^2:

$$\chi^2_v = 1/2(x_a + \sqrt{2v-1})^2 \qquad (12)$$

where x_a is the a-point of the cumulative normal distribution. The approximated value of χ^2_{450} is 522.0 for the 99 percent confidence level. Thus, the null hypothesis of a stationary probability matrix is rejected.

The data from this study provide strong confirmation of the variable nature of the transition probabilities, however, other researchers, using the same chi-square test have concluded that the assumption of constant transition probabilities cannot be rejected. Hallberg (1969) has pointed out several examples in which matrices deemed stationary by the test, function very poorly as predictors precisely because of structural change, i.e., changing transitional probabilities. Colman (1977) has explained the anomaly as follows: According to the Anderson and Goodman test, in order for a matrix to be judged stationary the null hypothesis must be not rejected. This, of course, is quite the opposite from the usual situation in hypothesis testing, where inferences are typically drawn only when the null hypothesis is rejected in favor of some alternative hypothesis. Thus the chi-square test as used is "a very weak form of test compared to the usual form of significance testing" (Colman 1977, p. 46). Normally, we set a one percent or five percent significance level in order to minimize the probability of rejecting a true null hypothesis (Type I error), but this is always at the risk of not rejecting a false null hypothesis (Type II error). With the chi-square procedure as outlined by Anderson and Goodman, we are in fact minimizing the probability of rejecting true stationary probabilities, at the clear risk of possibly accepting a true, non-stationary probability matrix as being stationary.

Factors Affecting the Changing Size Distribution of Dairy Farms

From a theoretical point of view, Quandt (p. 418) maintains that four groups of factors should affect the transition probabilities of the sizes of firms in an industry:

i. The nature of the short-run cost function;

ii. The nature of the long-run cost function;

iii. The nature of oligopolistic arrangements in the industry; and

iv. The general configuration of competing products, changes in relative technology, and changes in relative demands.

Dairy farmers have limited opportunities in the short-run (say, three to six months) to adjust to changes in the price of milk or changes in the price of most inputs (Hallberg and Fallert, p. 23). Dairymen may decide to liquidate or at least cull their herds when slaughter-cattle prices are high relative to milk prices. Average output per cow also may be affected by changing price relationships. Farmers are likely to adjust the composition and quantity of rations when feed prices are high relative to that of milk. Furthermore, milk output per farm may be increased as a result of enlargement of the enterprise through acquisition of additional land made available by some farmers shifting to nonfarm employment or retiring.

Specification of the Multinomial Logit Model

One of the important attributes of the multinomial logit model (MNLM) is its flexibility. Each of the model's (N-1)N[14/] equations (see Equation 9) may be separately specified, both in terms of the exogenous variables chosen and the appropriate functional form. In the current study, this flexibility was severely constrained by practical considerations. The purpose behind developing the multinomial logit model in this study was simply to demonstrate the feasibility of the method and provide a first step toward the construction of a more reasonable and complete model. The MNLM used in this study consisted of 90 individual equations, and therefore the specification of the equations was kept as simple as possible. Thus, all equations were specified identically with a single exogenous variable.

Thirty variables which might affect the growth, decline, entry and/or exit of dairy farms were examined by graphic analysis: Time, consumer price index (U.S.), product price index (U.S.), average farm price of milk, price of dairy ration, milk-feed price ratio, May rainfall, price of hay, price of milk cows, number of plants accepting milk in cans, number of farmers delivering milk in cans, upstate business activity index, factory output index, upstate non-agricultural employment, unemployment rate, hourly earnings in manufacturing, average milk production per cow, average grain consumption per cow, average interest rate, slaughter cow price, index of prices paid by dairy farmers, and ratio of prices received to prices paid by dairy farmers.[15/]

The graphic analysis, supported by a review of the literature, indicated that if the model were to be restricted to a single exogenous variable, the most likely choice would be either the New York State milk-feed price ratio or the ratio of prices received to prices paid by New York dairy farmers. Preliminary regression analysis was carried out with the sample data, using various formulations of the two variables. As a result of this regression analysis and the review of literature, the two-year first difference of the New York State milk-feed price ratio was selected as the single exogenous variable for all ninety equations of the multinomial logit model.[16/]

14/ For each of the N rows of the Markov matrix, the MNLM has N - 1 defining equations.

15/ Unless otherwise noted, all variables were specific to New York State.

16/ The ratio was calculated as follows:

$$R = \frac{20M}{F}, \qquad (13)$$

where M is the annual average price received by New York State dairy farmers for all milk sold wholesale per hundredweight, and F is the average annual price in New York State of 16-percent protein mixed dairy feed, expressed in dollars per ton. The source for both statistics was New York State Department of Agriculture and Markets, New York Agricultural Statistics (1971, 1973, 1977).

Estimation of Parameters of the Multinomial Logit Model

Each row of the time-series of six transition probability matrices constitutes the data set for the dependent variable for a multinomial logit model. Thus, the overall model consists of ten such MNLM's. For each MNLM (i.e., for each row), the nine equations of the form of Equation 12 were independently estimated using ordinary least squares procedures.[17/] As expected, the simple regression specification did not account for a very large proportion of the variation in the transition probabilities over time (Table 10). One might expect further improvement if additional variables were added as appropriate for the different elements of the matrix.

Table 10. COEFFICIENTS OF DETERMINATION: MULTINOMIAL LOGIT MODEL 90 Equations, Dairy Farms, 1968-74

Rows	Columns 1	2	3	4	5	6	7	8	9	10
	Coefficients of Determination - R^2									
1	-	.59	.05	.11	.60	.07	.10	.36	.36	.36
2	.25	-	.08	.00	.06	.22	.22	.22	.22	.22
3	.70	.07	-	.03	.06	.61	.06	.61	.61	.61
4	.15	.54	.01	-	.24	.32	.00	.11	.11	.11
5	.18	.00	.14	.01	-	.04	.68	.06	.06	.06
6	.03	.07	.16	.10	.13	-	.12	.37	.07	.07
7	.19	.04	.04	.04	.20	.02	-	.01	.50	.04
8	.04	.05	.05	.05	.05	.05	.16	-	.07	.18
9	.10	.10	.10	.10	.01	.09	.09	.18	-	.18
10	.19	.19	.19	.19	.19	.19	.19	.19	.19	-

Derivation of the Variable Transition Probabilities

Having estimated 90 equations, the actual values of the exogenous variable over the period from 1974 to 1977 were used to derive a set of the ratios,

$$p_{ijt}/\hat{p}_{idt},$$

17/ It is reasonable to expect that the error terms of the N-1 equations are correlated. This suggests the use of Zellner's (1962) generalized least squares estimation technique for "seemingly unrelated regressions." However, Tyrrell and Mount (1978) point out that "if exactly the same set of regressors occurs in each of the N-1 equations, then the GLS and OLS procedures give identical estimates of the coefficients" (p. 11).

for the ten MNLM's. These estimates were then used to obtain the predicted value of the selected denominator (Equation 10) for each future time period. By Equation 11, the other N - 1 probabilities of each row were then calculated. The results of this procedure are three transition probability matrices, 1974:1975, 1975:1976 and 1976:1977. A simulation procedure was used in which a series of matrix-vector calculations of the form,

$$\hat{X}_t = \hat{X}_{t-1}\hat{P}_{t-1:t}, \qquad (7)$$

led recursively to a forecast of the 1977 size distribution of sample dairy farms (Table 11).

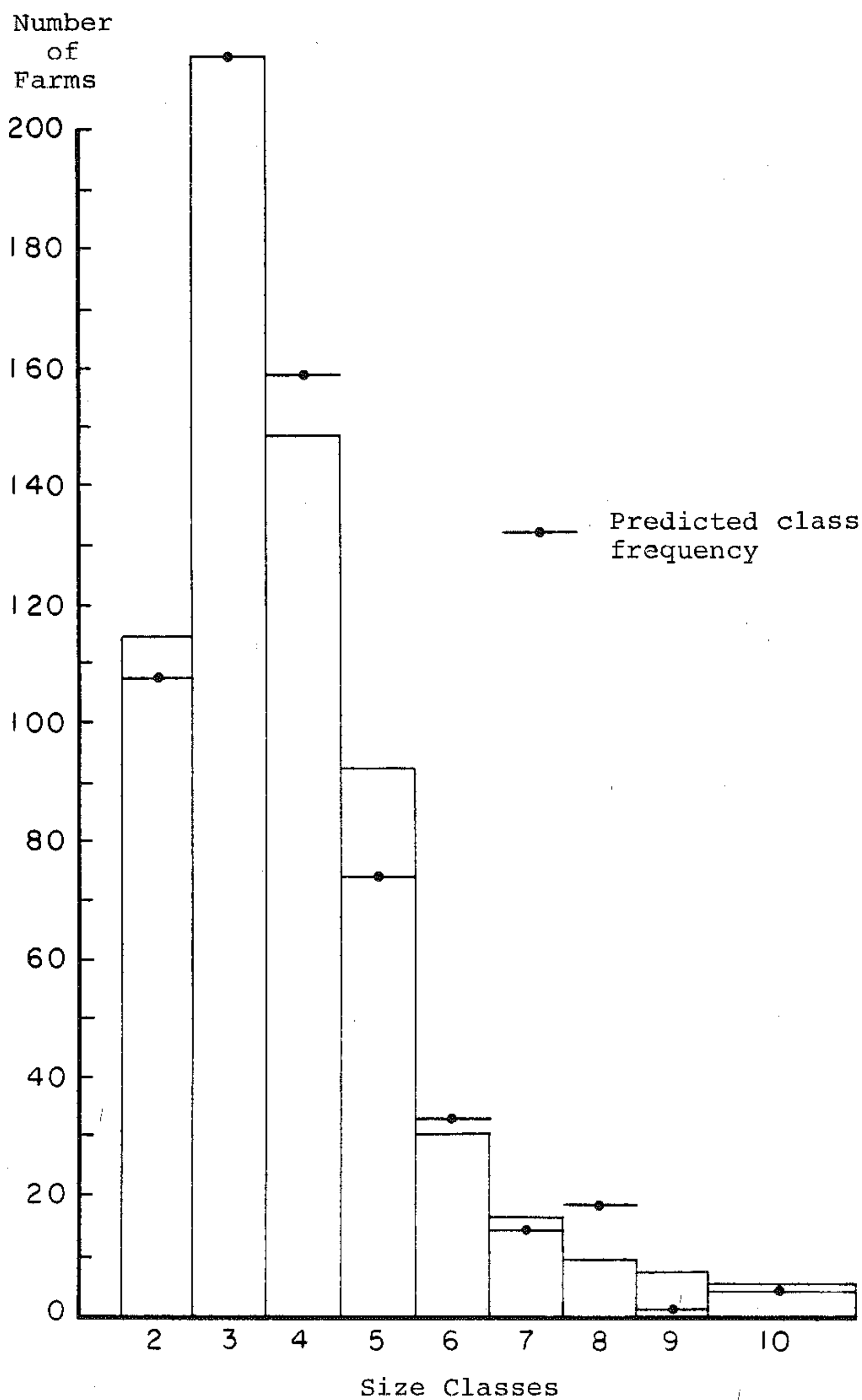
Table 11. PREDICTED AND ACTUAL SIZE DISTRIBUTIONS
Multinomial Logit Model, Variable Micro Data, New York, 1977

Pounds milk sold per month	Estimated 1977	Actual 1977	Deviation
1 - 19,999	108	115	- 7
20,000 - 39,999	213	212	+ 1
40,000 - 59,999	159	148	+11
60,000 - 79,999	74	92	-18
80,000 - 99,999	35	31	+ 4
100,000 - 119,999	14	16	- 2
120,000 - 139,999	21	8	+13
140,000 - 159,999	1	7	- 6
160,000 and over	8	11	- 3

The predictive accuracy of this model is the best of any considered in this study. The square root of the sum of the squared deviations is 27.00. The predicted distribution shows approximately the correct shape (Figure 3), although a secondary mode is mistakenly predicted for size-class (8).

The multinomial logit specification of the variable Markov matrix requires the same data as that of Hallberg's approach but offers an important theoretical advantage in that the transition probabilities will always lie between zero and one and sum to unity for each row. Thus, predicted matrices will always be of reasonable magnitudes for any levels of the exogenous variables. Furthermore, the model is more flexible than Hallberg's because it is possible to specify a different functional form with different variables for each of the transition probability elements of the variable matrix.

Figure 3. ACTUAL AND PREDICTED 1977 FREQUENCY DISTRIBUTION OF SAMPLE FARMS - MICRO-DATA VARIABLE MARKOV MULTINOMIAL LOGIT MODEL



When one considers predictive accuracy and the logic of the procedures used, the micro-data variable Markov multinomial logit model appears to be the best of the methods utilized to make *ex post* predictions of the sample farm size distributions for the year 1977. While the data and computational requirements of this approach are substantial, the model has many advantages, both practical and theoretical. Consequently, the parameters of the multinomial logit model were re-estimated using data from the entire sample time frame, 1968-1977, so that the model might be applied to predictions of the size distribution of farms for the year 1985.

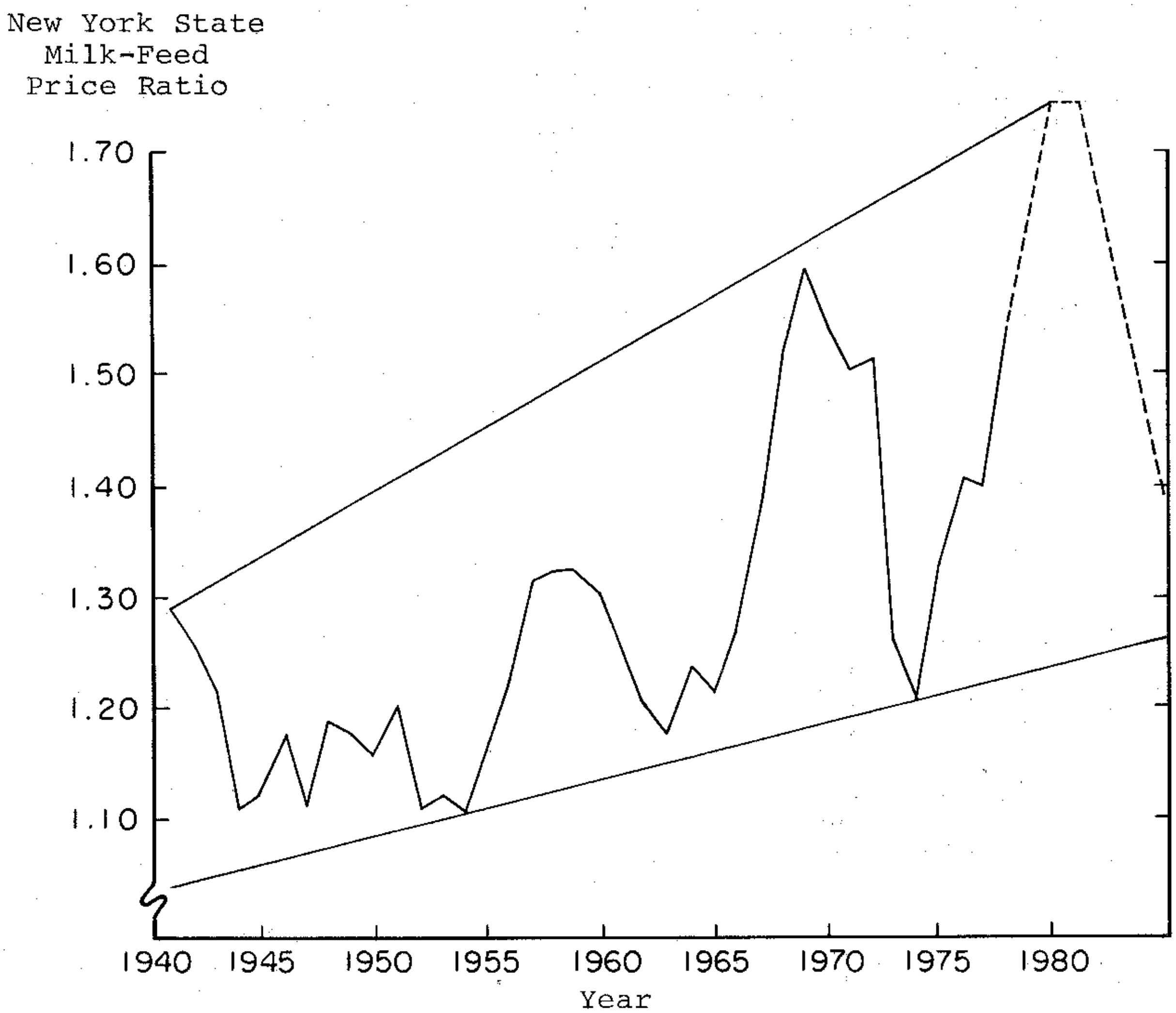
Re-Estimation of the Multinomial Logit Model Parameters Based Upon 1968-1977 Sample Data

For each row of the time-series of nine transition probability matrices, the nine equations (of the form of Equation 9) were independently estimated using ordinary least squares procedures. As was the case for 1968-1974, this simple regression specification does not account for a very large proportion of the variation in the transition probabilities over time.

The exogenous variable of the model is the two-year first difference of the New York State milk-feed price ratio. The estimated range for this variable during the forecast period was based upon an historical analysis of the price ratio from 1940 through 1978 (Figure 4). The analysis indicated a broad range through which the ratio is likely to vary. The analysis also indicated the maximum amount by which the ratio tends to change over any two-year period.

A sensitivity analysis program (MLAM) developed by Tyrrell (1978) was employed to derive a set of predicted probability matrices given the range of values of the exogenous variable (-.50 to +.48). Working with one row of the matrices at a time, the estimated parameters (Equation 9) and the specification of the equations constitute the input of the MLAM program. The output consists of the predicted probabilities for the given range of values of the exogenous variable. In this way, a set of 100 transition probability matrices was derived for the specified range of the first difference of the milk-feed price ratio, a different matrix being calculated for each .02 increment between the values of -.50 and +.48.

Figure 4. MILK FEED PRICE RATIO, NEW YORK STATE (1940-1978)



Conditional Forecasts of the Size Distribution of Sample Farms in the Year 1985

Predicting the 1978 Distribution

Milk-feed price ratio data were available for the first eight months of 1978 at the time of the study. Therefore, prediction of the 1978 distribution was treated differently from those for future years. The average milk-feed price ratio for 1978 was estimated to be 1.55 (the highest since 1972). This translates into a two-year first difference of +0.14. From the MLAM program, the appropriate transition probability matrix for 1977:1978 is derived for this given value of the exogenous variable. By vector-matrix multiplication,

$$\hat{X}_t = X_{t-1}\hat{P}_{t-1:t}, \qquad (7)$$

the 1978 distribution was then calculated (Table 12).

Table 12. ESTIMATED SIZE DISTRIBUTION OF FARMS MNLM, Micro-data, New York, 1978

Pounds of milk sold per month	Number of farms
(2) 1 - 19,999	105
(3) 20,000 - 39,999	204
(4) 40,000 - 59,999	145
(5) 60,000 - 79,999	90
(6) 80,000 - 99,999	34
(7) 100,000 - 119,999	16
(8) 120,000 - 139,999	12
(9) 140,000 - 159,999	5
(10) 160,000 or more	13
	624

Forecasting the 1985 Size Distribution of Sample Farms

Four scenarios were developed for the purpose of making alternative conditional forecasts of the size distribution of sample dairy farms in the year 1985. In Scenario #1, the milk-feed price ratio is assumed to be constant from 1978 through 1985. Thus the value of the exogenous variable, the two-year first difference of this ratio, is equal to zero for the seven transitions between 1978 and 1985 (Table 13).

Table 13. FOUR SCENARIOS FOR NEW YORK MILK-FEED PRICE RATIO Alternative Formulations, 1978-85

Matrix Years	Scenario #1 MFR[a/]	x[b/]	Scenario #2 MFR	x	Scenario #3 MFR	x	Scenario #4 MFR	x
1978:79	1.55	0	1.61	+0.12	1.49	-0.12	1.65	+0.24
1979:80	1.55	0	1.67	+0.12	1.43	-0.12	1.75	+0.20
1980:81	1.55	0	1.73	+0.12	1.37	-0.12	1.75	+0.10
1981:82	1.55	0	1.79	+0.12	1.31	-0.12	1.65	-0.10
1982:83	1.55	0	1.85	+0.12	1.25	-0.12	1.55	-0.20
1983:84	1.55	0	1.91	+0.12	1.19	-0.12	1.45	-0.20
1984:85	1.55	0	1.97	+0.12	1.13	-0.12	1.39	-0.16

a/ Milk-feed price ratio for New York State (annual averages).
b/ x = two-year first difference of milk-feed price ratio (annual averages) for New York State.

The predicted matrix for x = 0.0 permits a conditional forecast of the 1985 distribution:18/

$$\hat{X}_{1985} = \hat{X}_{1978}(\hat{P}_{x=0})^7. \qquad (14)$$

The forecast, and its time path from the year 1978, is presented in Table 14.

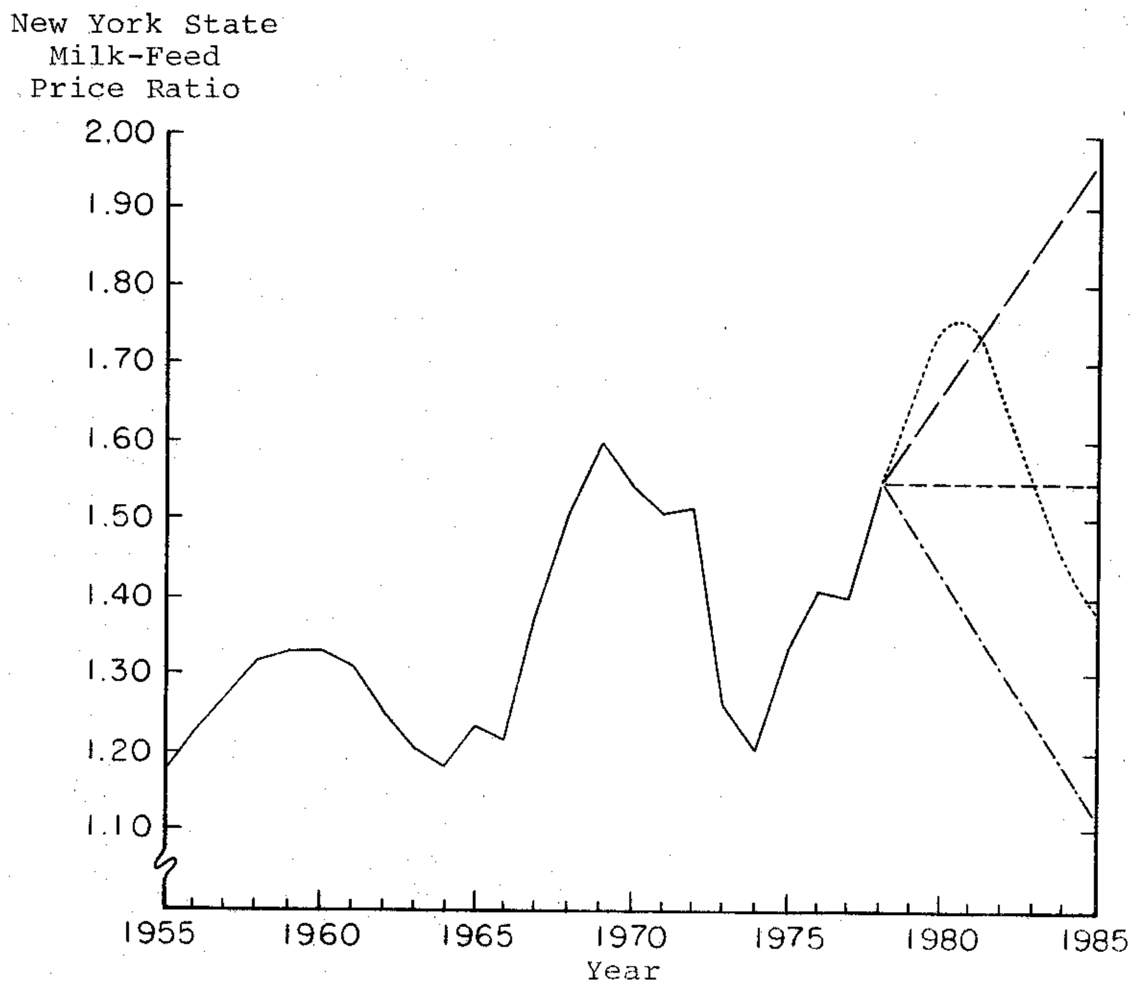
Table 14. PREDICTED SIZE DISTRIBUTIONS, CONSTANT MILK-FEED PRICE RATIO Multinomial Logit Model, New York, 1978-85

	Average monthly milk sales per farm								
	1-19,999	20,000-39,999	40,000-59,999	60,000-79,999	80,000-99,999	100,000-119,999	120,000-139,999	140,000-159,999	160,000 and over
	number of farms								
1978	105	204	145	90	34	16	12	5	13
1979	101	194	146	86	36	16	16	5	14
1980	98	185	146	83	36	16	20	5	14
1981	95	179	144	82	36	16	24	5	15
1982	93	173	142	80	36	17	27	5	15
1983	92	169	140	79	36	17	31	6	16
1984	90	165	138	78	36	17	34	6	17
1985	89	161	136	77	35	17	37	7	18

Scenario #2 is based upon a constantly increasing milk-feed price ratio, where the change is equal to +0.06 per year. Hence, the value of x is constant at +0.12, and the price ratio varies from 1.61 in 1979 up to 1.97 in 1985 (Figure 5). The 1985 distribution is estimated as follows:

18/ The small "x" stands for the exogenous variable of the MNLM, the second difference of the milk-feed price ratio, but large "X" represents a size distribution vector.

Figure 5. VALUES OF THE EXOGENOUS VARIABLE, MILK-FEED PRICE RATIO, FOR FOUR SCENARIOS, MULTINOMIAL LOGIT MODEL, 1978-1985



——— Historical path of actual MF ratio (including prediction of 1978

----- Scenario 1: Constant MF ratio

—— —— Scenario 2: Constantly increasing MF ratio

—·—·— Scenario 3: Constantly decreasing MF ratio

·········· Scenario 4: Historical fluctuation of MF ratio

Source: New York State Department of Agriculture and Markets. New York State Agricultural Prices and Cash Receipts from Farm Marketings, 1940-1963 and New York Agricultural Statistics 1971, 1973, 1977.

$$\hat{X}_{1985} = \hat{X}_{1978}(\hat{P}_{x=+0.12})^7. \qquad (15)$$

The predicted distributions through 1985 are found in Table 15.

Table 15. PREDICTED SIZE DISTRIBUTIONS, INCREASING MILK-FEED PRICE RATIO
Multinomial Logit Model, New York, 1978-85

	Average monthly milk sales per farm								
	1-19,999	20,000-39,999	40,000-59,999	60,000-79,999	80,000-99,999	100,000-119,999	120,000-139,999	140,000-159,999	160,000 and over
	number of farms								
1978	105	204	145	90	34	16	12	5	13
1979	98	196	142	88	36	17	16	4	15
1980	93	188	139	86	36	17	20	3	16
1981	89	182	136	85	37	18	24	3	17
1982	86	176	133	83	37	19	28	2	18
1983	83	171	130	82	37	19	32	2	19
1984	82	166	127	80	37	19	36	3	20
1985	80	162	124	79	37	19	39	3	21

In Scenario #3, the milk-feed price ratio decreases at the constant rate of -0.06 per year, such that x takes on the constant value of -0.12. The hypothesized value of the ratio decreases from 1.49 in 1979 to 1.13 in 1985. The 1985 distribution is estimated in the usual way:

$$\hat{X}_{1985} = \hat{X}_{1978}(\hat{P}_{x=0.12})^7. \qquad (16)$$

The sequence of predicted distributions resulting from Scenario #3 is presented in Table 16.

Table 16. PREDICTED SIZE DISTRIBUTIONS, DECREASING MILK-FEED PRICE RATIOS
Multinomial Logit Model, New York, 1978-85

	Average monthly milk sales per farm								
	1-19,999	20,000-39,999	40,000-59,999	60,000-79,999	80,000-99,999	100,000-119,999	120,000-139,999	140,000-159,999	160,000 and over
	number of farms								
1978	105	204	145	90	34	16	12	5	13
1979	105	191	150	84	35	15	16	5	13
1980	104	182	152	81	35	15	20	6	13
1981	103	176	152	78	35	15	23	7	14
1982	102	170	152	76	35	15	26	8	14
1983	101	166	151	75	34	15	29	9	14
1984	100	163	149	74	34	15	32	10	14
1985	100	160	147	73	33	15	34	12	15

Each of the first three scenarios represents a possibility of future trends in the milk-feed price ratio in New York State. The second and third scenarios are indicative of the expected limits of change for the period 1978-1985. Scenario #4 describes a specific pattern of fluctuations in the milk-feed price ratio, based upon a cycle similar to changes observed from 1940 to 1978 (Figure 5). In this scenario, the ratio starts out in 1979 at 1.65, and peaks between 1979 and 1980. The ratio then trends downward through 1985. Because the ratio changes at a variable rate, the value of the exogenous variable, the first difference of the ratio, is not constant; instead x takes on values between +0.24 and -0.20 (Table 13). A simulation procedure is used in which a series of vector-matrix calculations of the form,

$$\hat{X}_t = \hat{X}_{t-1}\hat{P}_{t-1:t}, \qquad (7)$$

lead recursively to the fourth set of conditional forecasts of the size distribution of sample farms in the year 1985 (Table 17).

Table 17. PREDICTED SIZE DISTRIBUTIONS, FLUCTUATING MILK-FEED PRICE RATIOS Multinomial Logit Model, New York, 1978-85

	Average monthly milk sales per farm								
	1-19,999	20,000-39,999	40,000-59,999	60,000-79,999	80,000-99,999	100,000-119,999	120,000-139,999	140,000-159,999	160,000 and over
	number of farms								
1978	105	204	145	90	34	16	12	5	13
1979	95	198	138	89	35	17	16	2	16
1980	88	190	133	88	36	18	20	1	17
1981	85	181	132	85	37	18	24	1	17
1982	87	170	136	80	37	17	28	2	17
1983	91	161	141	76	36	16	31	4	17
1984	93	154	143	73	35	15	34	6	17
1985	93	150	142	71	34	15	36	7	17

A secondary mode was predicted for size-class (8), 120,000 -139,999 pounds of milk per month, over this span of years. There is no logical reason for such a secondary mode to develop given the technology and economics of dairy production. This mode may be attributed to an error of prediction which results from an inconsistency in the estimation procedure. Logically one would expect a unimodal distribution with the open ended class at the tail of the distribution increasing over time.

The differences among the various predicted values for 1985 based upon the four scenarios of the milk-feed price ratio, were quite modest but of some interest (Table 18). Constantly increasing prices of milk relative to feed were associated with the movement of small-scale producers into the larger size categories. At the opposite extreme, the scenario of a continuously decreasing milk-feed price ratio yielded a structure which had a greater total number of farms in production but at substantially lower levels of monthly output.

Table 18. PREDICTED 1985 DISTRIBUTIONS OF NEW YORK DAIRY FARMS
Multinomial Logit Models, Four Milk-Feed Price Ratio Scenarios

Average monthly milk sales per farm	Assumptions about milk-feed price ratios			
	(1) Constant at 1.55	(2) Increasing	(3) Decreasing	(4) Fluctuating
pounds		number of farms		
1 - 19,999	89	80	100	93
20,000 - 39,999	161	162	160	150
40,000 - 59,999	136	124	147	142
60,000 - 79,999	77	79	73	71
80,000 - 99,999	35	37	33	34
100,000 - 119,999	17	19	15	15
120,000 - 139,999	37	39	34	36
140,000 - 159,999	7	3	12	17
160,000 and over	18	21	15	17
Total farms	577	564	589	566
Estimated New York supply (bill. lbs.)	10.96	11.01	10.86	10.66

Estimating the Total Milk Supply in New York For the Year 1985

The alternative predictions of the 1985 size distribution of sample New York State dairy farms may be used to make estimates of total milk supply in that year. A number of assumptions are required. First, it is assumed that the stratified random and systematic sampling method which was employed produced a sample which was representative (in terms of size distribution) of all New York-New Jersey market order farms in the twenty-county sample area. The sample consisted of 1,012 out of 14,272 market order farms in the area. Thus, the conversion factor between sample farms and all market order farms in the twenty counties is $\frac{14,272}{1,012}$.

Second, in order to extrapolate from the market order farms of the sample area to all dairy farms across the State, it is necessary to assume that the dairy farms in the sample area are representative of all dairy farms across the State (in terms of size distribution). While this assumption is subject to question, a chi-square test supported the hypothesis of no significant difference between the distributions. In December of 1977, there were 8,444 market order farms in the sample area and in the same month it is estimated that there were approximately 16,900 dairy farms selling milk in the entire State. Thus, to convert the sample statistics to State-wide estimates, the following conversion factor was calculated:

$$\frac{14,272}{1,012} \cdot \frac{16,900}{8,444} = 28.2256 \qquad (17)$$

The next step in estimating total milk supply in 1985 is to estimate the supply of milk produced on the sample farms in that year. It was assumed that the midpoint of each size-class was the mean for that group and that 200,000 pounds per month was the mean for the largest, open-ended size-class. The assumed mean of each class was multiplied by the frequency of farms in the respective class. These nine figures were summed and multiplied by the conversion ratio derived above (Equation 17) to yield an estimate of the total average monthly supply of milk. Multiplying this number by twelve months gives the estimate of annual milk supply:

$$\hat{S}_{Milk} = 12(28.2256)\sum_{\text{All Classes}}\{(\text{Mean Farm Size})(\text{Class Frequency})\} \qquad (18)$$

This procedure was checked by testing it with the actual sample distribution for the year 1977. Utilizing Equation 18, the 1977 State supply was estimated to be 10,215 million pounds of milk. The actual supply for New York State that year was 10,228 million pounds, giving an error of prediction of less than 0.13 percent. Following these same procedures, estimates of total State supply in 1985 were made for each of the four scenarios and are presented in Table 18.

The greatest supply is predicted for Scenario #2, in which the milk-feed price ratio is constantly increasing over the period 1978 through 1985. When the ratio is constantly decreasing, the supply is less than for the constant or increasing milk-feed price ratios. For the fluctuating milk-feed price ratio (Scenario #4), the supply estimate is lower than for any of the other scenarios, including the constantly decreasing ratio. This supports the hypothesis that it is variability in the milk-feed and other price ratios which curtails milk supply as much as it is the absolute level of the prices and price ratios in question (Willett and Saupe; Mathis 1970; Manchester 1978).

Concluding Observations

Three alternative formulations of Markov models were used to forecast the future size distribution of New York State dairy farms. In terms of predictive accuracy, the multinomial logit model incorporating variable transition probabilities and making use of estimates of the milk-feed price ratio as an explanatory variable, gave the best results. The stationary micro-data Markov model yielded the second best predictions. The macro-data model, which did not utilize information about specific changes in the size of individual farms, was the least accurate approach.

The variable Markov multinomial logit model, while theoretically sound and yielding predictions with the smallest deviations from actual distributions, has the most substantial data requirements and the greatest computational costs of the three models considered in this study.19/ Once the model has been estimated for a given data-set, however, it is relatively easy to update the model with more recently available information. In situations where regular forecasts of size distributions are needed, this type of analysis may be conducted without great difficulty and at relatively modest cost.

Reducing the complex real-world of economic behavior to manageable, but simplified models inevitably requires substantial assumptions. There is always the danger that a model may be constructed in such a way that it implicitly or explicitly postulates a certain pattern of future change, even though the identification of a pattern of future change is in fact the initial objective of the modelling exercise. This type of problem was illustrated in the present study by the assumptions built into the macro-data, Krenz-type Markov model. In general, the more complex the economic model, the more likely the danger of circular reasoning, in which a researcher proceeds from preconceived notions through a maze of mathematical manipulations to a set of foregone conclusions.

The multinomial logit model combined with individual farm data and variable transition probabilities related to exogenous economic variables deserves further exploration in estimating changes in size distributions. The data requirements of the model are less onerous than they may seem at first. Whenever cross-sectional data on changes in production are available over time, there is the possibility of learning more about why size distributions change as they do. The interplay of prices, technology and market structure includes only a few of the forces at work. The multinomial logit model allows consideration of some of these variables in making size distribution forecasts.

19/ In the broader research upon which this study is based (Stavins 1979) the micro-data variable Markov multinomial logit model produced the most accurate predictions of the 1977 sample size distribution of all the nine methods considered.

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